ICSE Board
Class X Mathematics
Sample Paper 2

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1

(a) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by x + 3. [3]

(b)

Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$. [3]

Find the matrix X such that $A + 2X = 2B + C$.

(c) Divide 96 into four parts which are in A.P and the ratio between product of their means to product of their extremes is 15 :7 [4]

Q. 2.

(a) A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is:

I. a green ball
II. a white or a red ball.
III. Neither a green ball nor a white ball [3]

(b)

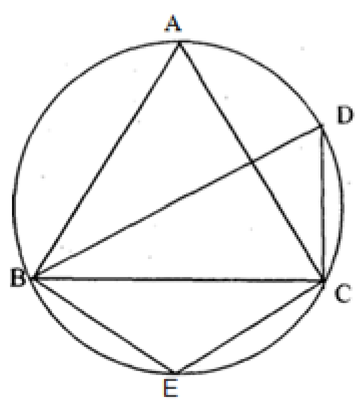
Solve the equation $9x^2 + \frac{3x}{4} + 2 = 0$, if possible, for real values of x. [3]

(c) In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate :

(i) ∠BDC

(ii) ∠BEC

(iii) ∠BAC



[4]

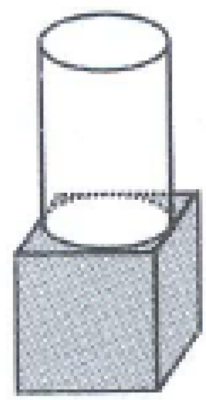
Q. 3.

(a) Peter has a recurring deposit account in Punjab National Bank at Sadar Bazar, Delhi for 4 years at 10% p.a. He will get ` 6,370 as interest on maturity. Find :

(i) monthly installment,

(ii) the maturity value of the account. [3]

(b) The given figure shows a solid formed of a solid cube of side 40cm and a solid cylinder of radius 20 cm and height 50 cm attached to the cube as shown.



Find the volume and the total surface area of the whole solid (Take $\pi = 3.14$) [3]

(c) Two vertices of a triangle are (-1, 4) and (5, 2). If the centroid is (0, -3), find the third vertex. [4]

Q. 4.

(a)

Find the values of x, which satify the inequation

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in W,.$ [3]

Graph the solution set on the number line.

(b) $\frac{\sec A - 1}{\sec A + 1} = \left(\frac{\sin A}{1 + \cos A}\right)^2$ [3]

(c) (Use a graph paper for this question). The daily pocket expenses of 200 students in a school are given below :

Pocket expenses (in Rs)	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students (frequency)	10	14	28	42	50	30	14	12

[4]

SECTION – B (40 Marks)

*(Answer **any four questions** from this Section)*

Q. 5.

(a) A shopkeeper sells an article at the listed price of ` 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ` 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler? [3]

(b) The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate :

(i) the radius of the sphere.

(ii) the number of cones recast. $\left(\text{Take } \pi = \frac{22}{7}\right)$ [3]

(c) Point A(1,-5) is mapped as A' on reflection in the line y=1. The point B(-5,1) is mapped as B' on reflection in the line y=4. Write the co-ordinates of A' and B'. Calculate AB'. [4]

Q. 6.

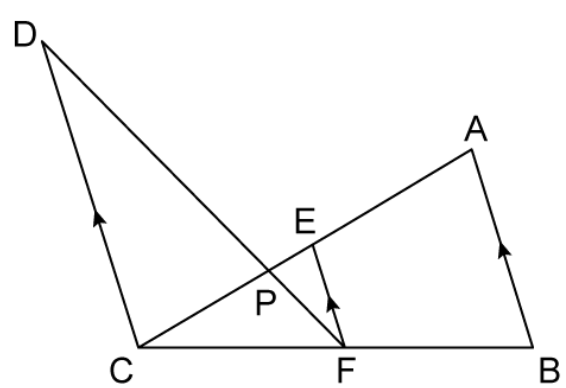
(a) Find the point on the x-axis equidistant from the points (5, 4) and (-2, 3). [3]

(b)

Given that 2 is a root of the equation $3x^2 - p(x + 1) = 0$ and that the equation $px^2 - qx + 9 = 0$ has equal roots, find the values of p and q.

[3]

(c) In the figure given below, AB || EF || CD. If AB = 22.5 cm, EP = 7.5 cm, PC = 15 cm and DC = 27 cm. Calculate : AC



[4]

Q. 7.

(a) A school has 630 students. The ratio of the number of boys to the number of girls is 3 : 2. This ratio changes to 7 : 5 after the admission of 90 new students. Find the number of newly admitted boys. [3]

(b) Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$. [3]

Find the matrix X such that $A + 2X = 2B + C$.

(c) In the following table, $\sum f = 200$ and mean = 73. Find the missing frequencies f_1, and f_2.

x	0	50	100	150	200	250
f	46	f_1	f_2	25	10	5

[4]

Q. 8.

(a) The marks obtained by 100 students in a mathematics test are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.

Use a scale of 2 cm = 10 units on both the axes.

Use the ogive to estimate :

(i) Median
(ii) Lower quartile
(iii) Number of students who obtained more than 85% marks in the test.
(iv) Number of students failed, if the pass percentage was 35. [6]

(b) In Δ ABC, D and E are the points on sides AB and AC respectively.

Find whether DE || BC, if [4]

Q. 9.

(a) Rohit invested ` 9,600 on ` 100 shares at ` 20 premium paying 8% dividend. Rohit sold the shares when the price rose to ` 160. He invested the proceeds (excluding dividend) in 10% ` 50 shares at ` 40. Find the :

a. Original number of shares.
b. Sale proceeds.
c. New number of shares.
d. Change in the two dividends. [3]

(b) Construct a ΔABC with BC = 6.5 cm, AB = 5.5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle. [3]

(c) Prove that $\sin(90^\circ - A).\cos(90^\circ - A) = \dfrac{\tan A}{1 + \tan^2 A}$ [4]

Q. 10.

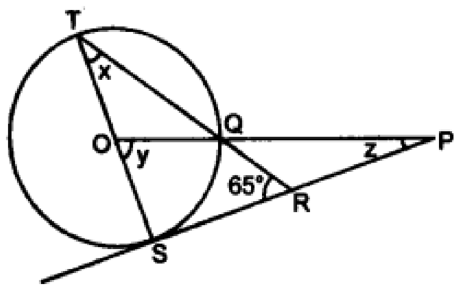
(a) A sum of 700 is to be paid to give seven cash prizes to the students of a school for their overall academic performance. If the cost of each prize is 20 less than its preceding prize; find the value of each of the prizes. [3]

(b) The second term of a G.P. is 9 and sum of its infinite terms is 48. Find its first three terms. [3]

(c) An aeroplane, at an altitude of 250 m, observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number. [4]

Q. 11.

(a) In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the values of x, y and z.



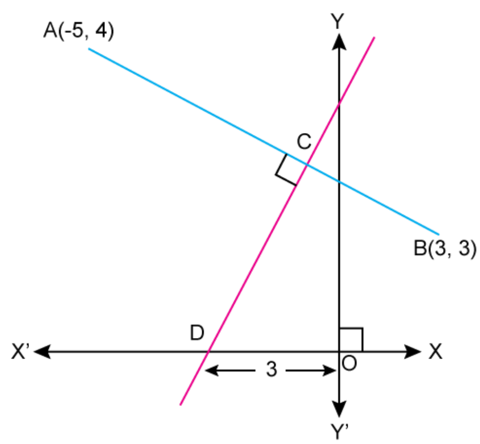
[3]

(b) Solve for x using the quadratic formula. Write your answer correct to two significant figures. $(x - 1)^2 - 3x + 4 = 0$ [3]

(c) Find the required equation

i. equation of AB

ii. equation of CD



[4]

ICSE Board
Class X Mathematics
Sample Paper 5 – Solution

SECTION – A (40 Marks)

Q. 1.

(a)

$x + 3 = 0 \Rightarrow x = -3$

Since, the given polynomials leave the same remainder when divided by $(x - 3)$,

Value of polynomial $ax^3 + 3x^2 - 9$ at $x = -3$ is same as value of polynomial $2x^3 + 4x + a$ at $x = -3$.

$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$

$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$

$\Rightarrow -27a + 18 = -66 + a$

$\Rightarrow 28a = 84$

$\Rightarrow a = \frac{84}{28}$

$\Rightarrow a = 3$

(b)

Given: $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Now, $A + 2X = 2B + C$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$

$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$

(c)

Let the four parts be $(a-3d),(a-d),(a+d)$ and $(a+3d)$.

Then, $(a-3d)+(a-d)+(a+d)+(a+3d)=96$

$\Rightarrow 4a=96$

$\Rightarrow a=24$

It is given that

$$\frac{(a-d)(a+d)}{(a-3d)(a+3d)}=\frac{15}{7}$$

$$\Rightarrow \frac{a^2-d^2}{a^2-9d^2}=\frac{15}{7}$$

$$\Rightarrow \frac{576-d^2}{576-9d^2}=\frac{15}{7}$$

$\Rightarrow 4032-7d^2=8640-135d^2$

$\Rightarrow 128d^2=4608$

$\Rightarrow d^2=36$

$\Rightarrow d=\pm 6$

When $a=24$, $d=6$

$a-3d=24-3(6)=6$

$a-d=24-6=18$

$a+d=24+6=30$

$a+3d=24+3(6)=42$

When $a=24$, $d=-6$

$a-3d=24-3(-6)=42$

$a-d=24-(-6)=30$

$a+d=24+(-6)=18$

$a+3d=24+3(-6)=6$

Thus, the four parts are (6, 18, 30, 42) or (42, 30, 18, 6).

Q. 2.

(a)

Number of white balls = 5

Number of red balls = 6

Number of green balls = 9

$\therefore$ Total number of balls = 5 + 6 + 9 = 20

(i) $P(\text{Green ball}) = \dfrac{\text{Number of Green balls}}{\text{Total number of balls}} = \dfrac{9}{20}$

(ii)
$$\begin{aligned}P(\text{White ball or Red ball}) &= P(\text{White ball}) + P(\text{Red ball})\\ &= \frac{\text{Number of White balls}}{\text{Total number of balls}} + \frac{\text{Number of Red balls}}{\text{Total number of balls}}\\ &= \frac{5}{20} + \frac{6}{20}\\ &= \frac{11}{20}\end{aligned}$$

(iii)
$$\begin{aligned}P(\text{Neither Green ball nor White ball}) &= P(\text{Red ball})\\ &= \frac{\text{Number of Red balls}}{\text{Total number of balls}}\\ &= \frac{6}{20} = \frac{3}{10}\end{aligned}$$

(b)

$$9x^2 + \frac{3x}{4} + 2 = 0$$

$$\Rightarrow \frac{36x^2 + 3x + 8}{4} = 0$$

$$\Rightarrow 36x^2 + 3x + 8 = 0$$

Here, a = 36, b = 3 and c = 8

$$\begin{aligned}\therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-3 \pm \sqrt{(3)^2 - 4 \times 36 \times 8}}{2 \times 36}\\ &= \frac{-3 \pm \sqrt{9 - 1152}}{72}\\ &= \frac{-3 \pm \sqrt{-1143}}{72}\end{aligned}$$

Since $\sqrt{-1143}$ is not possible, we cannot solve the given equation for x.

(c)

(i) Given that BD is a diameter of the circle.

The angle in a semicircle is a right angle.

$\therefore \angle BCD = 90^\circ$

Also given that $\angle DBC = 58^\circ$

In ΔBDC,

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$\Rightarrow 58^\circ + 90^\circ + \angle BDC = 180^\circ$

$\Rightarrow 148^\circ + \angle BDC = 180^\circ$

$\Rightarrow \angle BDC = 180^\circ - 148^\circ$

$\Rightarrow \angle BDC = 32^\circ$

(ii) We know that the opposite angles of a cyclic quadrilateral are supplementary.

Thus, in cyclic quadrilateral BECD,

$\angle BEC + \angle BDC = 180^\circ$

$\Rightarrow \angle BEC + 32^\circ = 180^\circ$

$\Rightarrow \angle BEC = 180^\circ - 32^\circ$

$\Rightarrow \angle BEC = 148^\circ$

(iii) In cyclic quadrilateral ABEC,

$\angle BAC + \angle BEC = 180^\circ$

$\Rightarrow \angle BAC + 148^\circ = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 148^\circ$

$\Rightarrow \angle BAC = 32^\circ$

Q. 3.

(a)

(i) Let the monthly instalment be Rs. P.

$n = 4$ years $= 4 \times 12$ months $= 48$ months

Rate of interest, $r = 10\%$

Interest $=$ Rs. 6370

Now, Interest $= P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$\Rightarrow 6370 = P \times \frac{48 \times 49}{24} \times \frac{10}{100}$

$\Rightarrow 6370 = P \times \frac{49}{5}$

$\Rightarrow P = \frac{6370 \times 5}{49} =$ Rs. 650

Thus, the monthly instalment is Rs. 650.

(ii) Total money deposited in the bank $= 48 \times$ Rs. 650 $=$ Rs. 31200

$\therefore$ Maturity value $=$ Total money deposited $+$ Interest

$=$ Rs. $(31200 + 6370)$

$=$ Rs. 37570

(b)

Edge of a cube $= l = 40$ cm

$\therefore$ Volume of a cube $= l^3 = (40)^3 = 64000\ cm^3$

Radius of a solid cylinder $= r = 20$ cm

Height of a solid cylinder $= h = 50$ cm

$\therefore$ Volume of cylinder $= \pi r^2 h = 3.14 \times 20 \times 20 \times 50 = 62800\ cm^3$

$\therefore$ Volume of whole solid $=$ Volume of cube $+$ Volume of cylinder

$= (64000 + 62800)\ cm^3$

$= 126800\ cm^3$

Total surface area of the whole solid

$=$ Total surface area of a cube $+$ Curved surface area of a cylinder

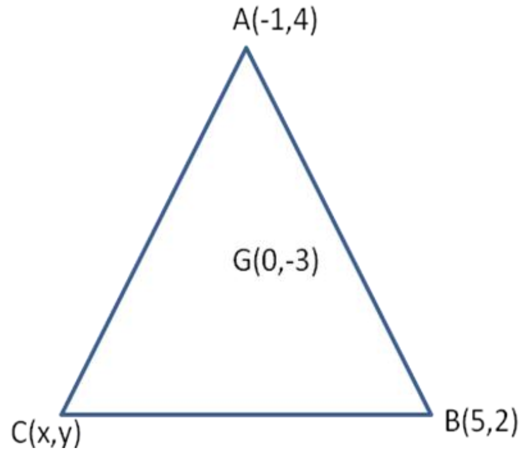
$= 6l^2 + 2\pi rh$

$= 6 \times (40)^2 + 2 \times 3.14 \times 20 \times 50$

$= 9600 + 6280$

$= 15880\ cm^2$

(c)



Let G be the centroid of ΔABC whose coordinates are (0,-3) and let C(x,y) be the coordianates of third vertex

Coordinates of G are,

$$G(0,-3) = G\left(\frac{-1+5+x}{3}, \frac{4+2+y}{3}\right)$$

$$O = \frac{4+x}{3}, -3 = \frac{6+4}{3}$$

x = -4, y = -15

Coordinates of third vertex are (-4,-15)

Q. 4.

(a)

We need to find the values of x, such that

x satisfies the inequation $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in W$

Consider the given inequation:

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$

$\Rightarrow \frac{-17}{6} < \frac{3-4x}{6} \leq \frac{12}{6}$

$\Rightarrow \frac{17}{6} > \frac{4x-3}{6} \geq \frac{-12}{6}$

$\Rightarrow 17 > 4x - 3 \geq -12$

$\Rightarrow -12 \leq 4x - 3 < 17$

$\Rightarrow -12 + 3 \leq 4x - 3 + 3 < 17 + 3$

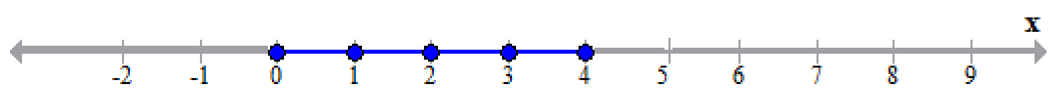
$\Rightarrow -9 \leq 4x < 20$

$\Rightarrow -\frac{9}{4} \leq \frac{4x}{4} < \frac{20}{4}$

$\Rightarrow -\frac{9}{4} \leq x < 5$

Since $x \in W$, the values of x are 0, 1, 2, 3, 4.

And the required line is



(b)

$\text{LHS} = \frac{\sec A - 1}{\sec A + 1}$

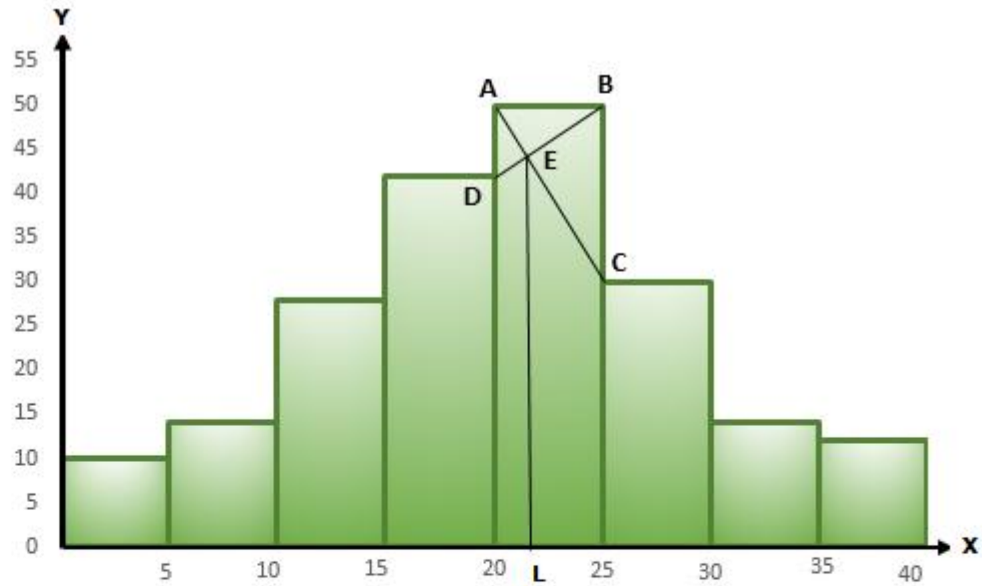
$= \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1} = \frac{1 - \cos A}{1 + \cos A}$

multiplying by $(1 + \cos A)$ both numerator and denominator,

$\text{LHS} = \frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}$

$= \frac{1 - \cos^2 A}{(1 + \cos A)^2} = \frac{\sin^2 A}{(1 + \cos A)^2} = \left(\frac{\sin A}{1 + \cos A}\right)^2 = \text{RHS}$

(c) Histogram is as follows:



In the highest rectangle which represents modal class draw two lines AC and BD intersecting at E.

From E, draw a perpendicular to x-axis meeting at L.

Value of L is the mode. Hence, mode = 21.5

SECTION – B (40 Marks)

Q. 5.

(a)

Suppose the shopkeeper buys the article from wholesaler at Rs. x.

Tax paid by shopkeeper $= \frac{12}{100} \times x$

Since, the shopkeeper sells the article for Rs. 1500
and charges sales-tax at the rate of 12%.

Tax charged by the shopkeeper $= 12\%$ of $1500 = \frac{12}{100} \times 1500$

Shopkeeper pays VAT of Rs. 36 to the Government.

Now, VAT = Tax charged by shopkeeper − Tax paid by shopkeeper

$\Rightarrow 36 = \frac{12}{100} \times 1500 - \frac{12}{100} \times x$

$\Rightarrow 36 = \frac{12}{100}(1500 - x)$

$\Rightarrow 1500 - x = \frac{36 \times 100}{12}$

$\Rightarrow 1500 - x = 300$

$\Rightarrow x =$ Rs. 1200

$\Rightarrow$ Tax paid by shopkeeper $= \frac{12}{100} \times 1200 =$ Rs. 144

Thus, total amount paid by shopkeeper inclusive of tax

= Rs. (1200 + 144)

= Rs. 1344

(b)

(i) Let R be the radius of a solid metallic sphere.

Surface area of a solid metallic sphere $= 2464 \text{ cm}^2$

$\Rightarrow 4\pi R^2 = 2464$

$\Rightarrow 4 \times \frac{22}{7} \times R^2 = 2464$

$\Rightarrow R^2 = \frac{2464 \times 7}{4 \times 22} = 196$

$\Rightarrow R = 14 \text{ cm}$

(ii) Volume of sphere melted $= \frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi \times 14 \times 14 \times 14$

Radius of each cone recasted $= r = 3.5$ cm

Height of each cone recasted $= h = 7$ cm

$\therefore$ Volume of each cone recasted $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 7$

$$\therefore \text{Number of cones recasted} = \frac{\text{Volume of sphere melted}}{\text{Volume of each cone formed}}$$

$$= \frac{\frac{4}{3} \times \pi \times 14 \times 14 \times 14}{\frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 7}$$

$$= 128$$

(c) A(1,-5), the co-ordinates of A' = (1, 2x1-(-5)) = (1,7)

B(-5,1), the co-ordinates of B' = (-5, 2x4-(1)) = (-5,7)

The distance AB' =

$$= \sqrt{(-5-1)^2 + (7-(-5))^2}$$

$$= \sqrt{(-6)^2 + 12^2}$$

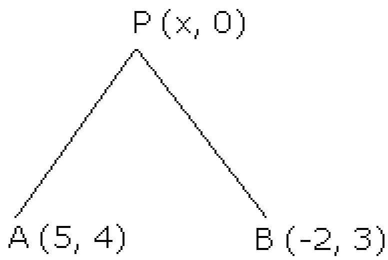
$$= \sqrt{36 + 144}$$

$$= \sqrt{180}$$

$$= 13.41 \text{units}$$

Q. 6.

(a) Let the point on x-axis be P (x, 0).



Given,

$PA = PB$

$PA^2 = PB^2$

$(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$

$x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$

$\Rightarrow -14x + 28 = 0$

$\Rightarrow 14x = 28$

$\Rightarrow x = 2$

$\therefore$ The point on x – axis is $(2, 0)$

(b)

Since 2 is a root of the equation $3x^2 - p(x+1) = 0$

$\Rightarrow 3(2)^2 - p(2+1) = 0$

$\Rightarrow 3 \times 4 - 3p = 0$

$\Rightarrow 12 - 3p = 0$

$\Rightarrow 3p = 12$

$\Rightarrow p = 4$

Now, the other equation becomes $4x^2 - qx + 9 = 0$

Here, $a = 4$, $b = -q$ and $c = 9$

Since the roots are equal, we have

$b^2 - 4ac = 0$

$\Rightarrow (-q)^2 - 4 \times 4 \times 9 = 0$

$\Rightarrow q^2 - 144 = 0$

$\Rightarrow q^2 = 144$

$\Rightarrow q = 12$

Hence, $p = 4$ and $q = 12$.

(c)

In ΔPCD and ΔPEF,

$\angle CPD = \angle EPF$ ....(vertically opposite angles)

$\angle DCE = \angle FEP$ ....(Since DC||EF.)

$\Delta PCD \sim \Delta PEF$ ....(AA criterion for Similarity)

$\Rightarrow \frac{27}{EF} = \frac{15}{7.5}$

$\Rightarrow EF = 13.5$ cm

Since EF||AB, $\Delta CEF \sim \Delta CAB$.

$\Rightarrow \frac{EC}{AC} = \frac{EF}{AB}$

$\Rightarrow \frac{22.5}{AC} = \frac{13.5}{22.5}$

$\Rightarrow AC = 37.5$ cm

Q.7.

(a)

Let the number of boys be 3x.

Then, number of girls = 2x

$\therefore 3x + 2x = 630$

$\Rightarrow 5x = 630$

$\Rightarrow x = 126$

$\Rightarrow$ Number of boys $= 3x = 3 \times 126 = 378$

And, Number of girls $= 2x = 2 \times 126 = 252$

After admission of 90 new students, we have total number of students $= 630 + 90 = 720$

Now, let the number of boys be 7x.

Then, number of girls = 5x

$\therefore 7x + 5x = 720$

$\Rightarrow 12x = 720$

$\Rightarrow x = 60$

$\Rightarrow$ Number of boys $= 7x = 7 \times 60 = 420$

And, Number of girls $= 5x = 5 \times 60 = 300$

$\therefore$ Number of newly admitted boys $= 420 - 378 = 42$

(b)

Given : $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Now, $A + 2X = 2B + C$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$

$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$

(c)

We have,

x	f	fx
0	46	0
50	f_1	$50f_1$
100	f_2	$100f_2$
150	25	3750
200	10	2000
250	5	1250
	$\sum f = 86 + f_1 + f_2$	$\sum fx = 7000 + 50f_1 + 100f_2$

Given, $\sum f = 200$

$\Rightarrow 86 + f_1 + f_2 = 200$

$\Rightarrow f_1 + f_2 = 114 \quad ....(i)$

$\text{Mean} = \frac{\sum fx}{\sum f}$

$\Rightarrow 73 = \frac{7000 + 50f_1 + 100f_2}{200}$

$\Rightarrow 7000 + 50f_1 + 100f_2 = 14600$

$\Rightarrow 50f_1 + 100f_2 = 7600$

$\Rightarrow f_1 + 2f_2 = 152 \quad ....(ii)$

Subtracting (ii) from (i), we get

$f_2 = 38$

$\Rightarrow f_1 = 114 - 38 = 76$

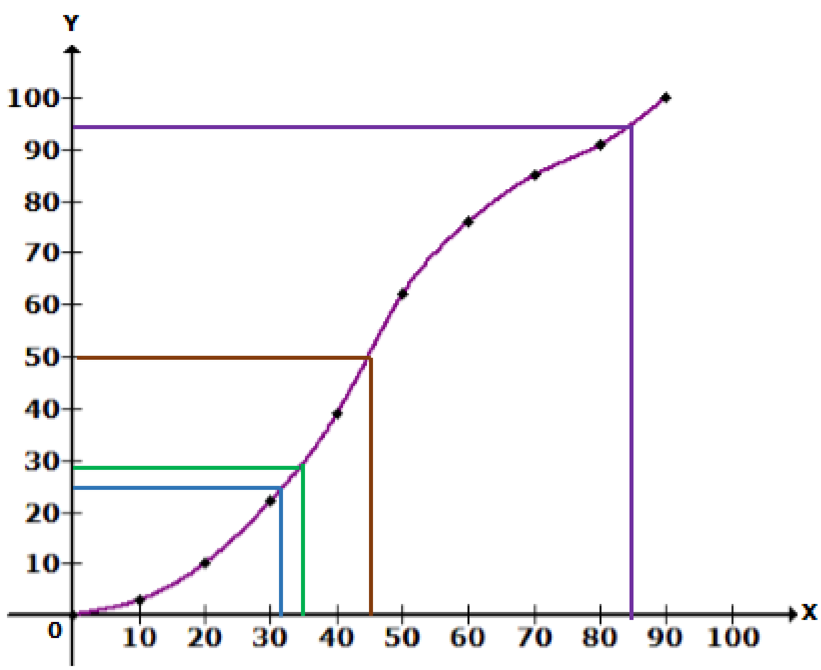
Hence, $f_1 = 76$ and $f_2 = 38$

Q.8.

(a)

Marks	Number of students (Frequency)	Cumulative Frequency
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

The ogive is as follows:



(i) Median $= \left(\frac{N}{2}\right)^{th}$ term $= \left(\frac{100}{2}\right)^{th}$ term $= 50^{th}$ term $= 45$

(ii) Lower quartile $= \left(\frac{N}{4}\right)^{th}$ term $= \left(\frac{100}{4}\right)^{th}$ term $= 25^{th}$ term $= 32$

(iii) Number of students who obtained more than 85% marks

$= 100 - 94$

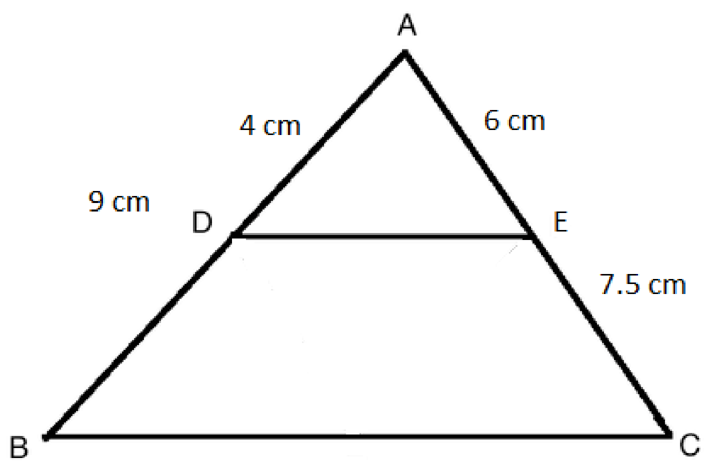
$= 6$

(iv) Number of students who failed

$= 29$

$= 6$

(b)



In $\triangle ADE$ and $\triangle ABC$,

$\frac{AE}{EC} = \frac{6}{7.5} = \frac{4}{5}$

$\frac{AD}{BD} = \frac{4}{5}$ ....(Since AB = 9 cm and AD = 4 cm)

So, $\frac{AE}{EC} = \frac{AD}{BD}$.

$\therefore DE \,||\, BC$ ....(By the Converse of Mid-point theorem)

Q. 9.

(a)

(i) 100 shares at Rs. 20 premium means

Nominal value of the share is Rs. 100

and its market value $= 100 + 20 =$ Rs. 120

Money required to buy 1 share $=$ Rs. 120

$\therefore$ Number of shares $= \dfrac{\text{Money Invested}}{\text{Market Value of 1 Share}} = \dfrac{9600}{120} = 80$

(ii) Each share is sold at Rs. 160

$\therefore$ Sale Proceeds $= 80 \times$ Rs. 160 $=$ Rs. 12,800

(iii) Now, investment $=$ Rs. 12800

Dividend $= 10\%$

Net Value $= 50$

Market Value $=$ Rs. 40

$\therefore$ Number of shares $= \dfrac{\text{Investment}}{\text{Market Value}} = \dfrac{12800}{40} = 320$

(iv) Now, dividend on 1 share $= 10\%$ of N.V. $= 10\%$ of $50 = 5$

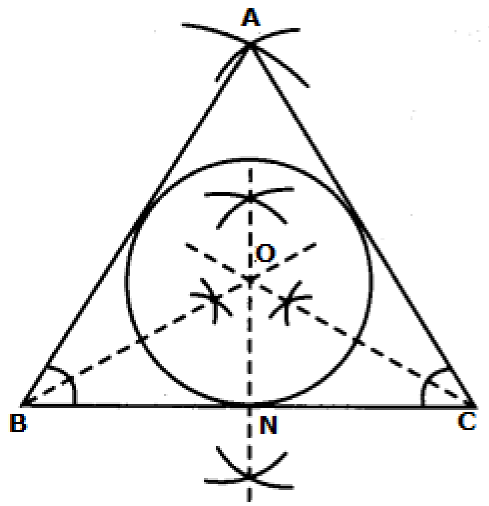
$\Rightarrow$ Dividend on 320 shares $= 320 \times 5 = 1600$

Thus, change in two dividends $= 1600 - 640 = 960$

(b)

Steps of construction:

1) Draw BC = 6.5 cm.
2) With B as centre, draw an arc of radius 5.5 cm.
3) With C as centre, draw an arc of radius 5 cm.
   Let this arc meets the previous arc at A.
4) Join AB and AC to get ΔABC.
5) Draw the bisectors of $\angle ABC$ and $\angle ACB$.
   Let these bisectors meet each other at O.
6) Draw $ON \perp BC$.
7) With O as centre and radius ON, draw a incircle
   that touches all the sides of ΔABC.
8) By measurement, radius ON = 1.5 cm



(c)

$$\text{LHS} = \sin(90^\circ - A).\cos(90^\circ - A)$$

$$\Rightarrow \cos A.\sin A$$

$$\text{RHS} = \frac{\tan A}{1 + \tan^2 A} = \frac{\tan A}{\sec^2 A} = \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^2 A}}$$

$$\Rightarrow \text{RHS} = \frac{\sin A}{\cos A}.\cos^2 A = \cos A.\sin A$$

$$\text{Thus, LHS} = \text{RHS}$$

$$\Rightarrow \sin(90^\circ - A).\cos(90^\circ - A) = \frac{\tan A}{1 + \tan^2 A}$$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)}$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = 1 + \frac{1}{\sin A \cos A}$$

$$= 1 + \operatorname{cosec} A \sec A$$

$$= \text{R.H.S}$$

Hence Proved

Q.10

(a)

Total amount of prize $= S_n = \text{Rs. } 700$

Let the value of the first prize be Rs. a.

Number of prizes $= n = 7$

Let the value of first prize be Rs. a.

Depreciation in next prize $= -\text{Rs. } 20$

We have,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 700 = \frac{7}{2}\left[2a + 6(-20)\right]$$

$$\Rightarrow 700 = \frac{7}{2}\left[2a - 120\right]$$

$$\Rightarrow 1400 = 14a - 840$$

$$\Rightarrow 14a = 2240$$

$$\Rightarrow a = 160$$

$\Rightarrow$ Value of 1^{st} prize $= \text{Rs. } 160$

Value of 2^{nd} prize $= \text{Rs. } (160 - 20) = \text{Rs. } 140$

Value of 3^{rd} prize $= \text{Rs. } (140 - 20) = \text{Rs. } 120$

Value of 4^{th} prize $= \text{Rs. } (120 - 20) = \text{Rs. } 100$

Value of 5^{th} prize $= \text{Rs. } (100 - 20) = \text{Rs. } 80$

Value of 6^{th} prize $= \text{Rs. } (80 - 20) = \text{Rs. } 60$

Value of 7^{th} prize $= \text{Rs. } (60 - 20) = \text{Rs. } 40$

(b)

Let a be the first term and r be the common ratio of a G.P.

2^{nd} term, $t_2 = ar = 9$

$\Rightarrow r = \frac{9}{a}$

Sum of its infinite terms, $S = 48$

$\Rightarrow \frac{a}{1-r} = 48$

$\Rightarrow \frac{a}{1-\frac{9}{a}} = 48$

$\Rightarrow \frac{a^2}{a-9} = 48$

$\Rightarrow a^2 = 48a - 432$

$\Rightarrow a^2 - 48a + 432 = 0$

$\Rightarrow a^2 - 36a - 12a + 432 = 0$

$\Rightarrow a(a-36) - 12(a-36) = 0$

$\Rightarrow (a-36)(a-12) = 0$

$\Rightarrow a = 36$ or $a = 12$

When $a = 36$, $r = \frac{9}{36} = \frac{1}{4}$

$\Rightarrow 1^{st}$ term $= 36$,

2^{nd} term $= ar = 36 \times \frac{1}{4} = 9$

3^{rd} term $= ar^2 = 36 \times \frac{1}{16} = \frac{9}{4}$

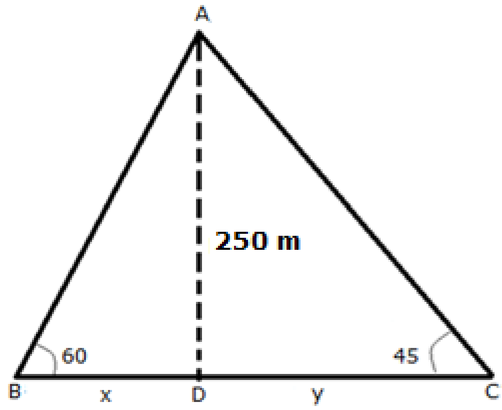
When $a = 12$, $r = \frac{9}{12} = \frac{3}{4}$

$\Rightarrow 1^{st}$ term $= 12$,

2^{nd} term $= ar = 12 \times \frac{3}{4} = 9$

3^{rd} term $= ar^2 = 12 \times \frac{9}{16} = \frac{27}{4}$

(c)



Let A be the position of the airplane and let BC be the river. Let D be the point in BC just below the airplane.

B and C be two boats on the opposite banks of the river with angles of depression 60° and 45° from A.

In ΔADC,

$\tan 45^\circ = \frac{AD}{DC}$

$\Rightarrow 1 = \frac{250}{y}$

$\Rightarrow y = 250 \text{ m} = DC$

In ΔADB,

$\tan 60^\circ = \frac{AD}{BD}$

$\Rightarrow \sqrt{3} = \frac{250}{x}$

$\Rightarrow x = \frac{250}{\sqrt{3}} = \frac{250\sqrt{3}}{3} = \frac{250 \times 1.732}{3} = 144.3 \text{ m} = BD$

$\therefore BC = BD + DC = 144.3 + 250 = 394.3 \approx 394 \text{ m}$

Thus, the width of the river is 394 m.

Q.11.

(a)

TS $\perp$ SP,

$\Rightarrow \angle TSR = 90^\circ$

In ΔTSR,

$\angle TSR + \angle TRS + \angle RTS = 180^\circ$

$\Rightarrow 90^\circ + 65^\circ + x = 180^\circ$

$\Rightarrow x = 180^\circ - 90^\circ - 65^\circ$

$\Rightarrow x = 25^\circ$

Now, $y = 2x$ ....(Angle subtended at the centre is double that of the angle subtended by the arc at the same centre)

$\Rightarrow y = 2 \times 25^\circ$

$\Rightarrow y = 50^\circ$

In ΔOSP,

$\angle OSP + \angle SPO + \angle POS = 180^\circ$

$\Rightarrow 90^\circ + z + 50^\circ = 180^\circ$

$\Rightarrow z = 180^\circ - 140^\circ$

$\Rightarrow z = 40^\circ$

Hence, $x = 25^\circ$, $y = 50^\circ$ and $z = 40^\circ$

(b)

$(x-1)^2 - 3x + 4 = 0$

$\Rightarrow x^2 - 2x + 1 - 3x + 4 = 0$

$\Rightarrow x^2 - 5x + 5 = 0$

Here, $a = 1$, $b = -5$ and $c = 5$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

$$= \frac{5 \pm 2.24}{2}$$

$$\therefore x = \frac{5 + 2.24}{2} \text{ or } x = \frac{5 - 2.24}{2}$$

$$\Rightarrow x = \frac{7.24}{2} \text{ or } x = \frac{2.76}{2}$$

$\Rightarrow x = 3.6$ or $x = 1.4$

(c)

(i) Slope of AB $= \frac{3-4}{3-(-5)} = \frac{-1}{8}$

$\therefore$ Equation of AB is given by

$y - 4 = -\frac{1}{8}(x - (-5))$

$8y - 32 = -(x + 5)$

$8y - 32 = -x - 5$

$x + 8y = 27$

(ii) AB and CD are perpendicular to each other.

Thus, product of their slopes $= -1$

Slope of AB $\times$ Slope of CD $= -1$

$\Rightarrow \frac{-1}{8} \times$ Slope of CD $= -1$

$\Rightarrow$ Slope of CD $= 8$

Now, from graph we have coordinates of D $= (-3, 0)$

$\therefore$ Equation of line CD is given by

$y - 0 = 8(x + 3)$

$y = 8x + 24$