6. Operations on Algebraic Expressions

Exercise 6A

1. Question

Add:

8ab, -5ab, 3ab, -ab

Answer

To add the expressions, we have to arrange the given expression in the form of rows and then add the expression column wise, so we have;

$$\begin{array}{r} 8ab \\ -5ab \\ 3ab \\ -ab \\ \hline 5ab \end{array}$$

2. Question

Add:

7x, - 3x, 5x, – x, -2x

Answer

To add the expressions, we have to arrange the given expression in the form of rows and then add the expression column wise, so we have;

$$\begin{array}{r} 7x \\ -3x \\ 5x \\ -x \\ -2x \\ \hline 6x \end{array}$$

3. Question

Add:

3a – 4b + 4c, 2a + 3b – 8c, a – 6b + c

Answer

To add the expressions, we have to arrange the given expression in the form of rows and then add the expression column wise, so we have;

$$\begin{array}{r} 3a - 4b + 4c \\ 2a + 3b - 8c \\ a - 6b + \ \ c \\ \hline 6a - 7b - 3c \end{array}$$

4. Question

Add:

5x – 8y + 2z, 3z – 4y – 2x, 6y – z – x and 3x – 2x – 3y

Answer

To add the expressions, we have to arrange the given expression in the form of rows and then add the expression column wise, so we have;

$$\begin{array}{r} 5x - 8y + 2z \\ -2x - 4y + 3z \\ -\ x + \ 6y - z \\ 3x - 3y - 2z \\ \hline 5x - 9y + 2z \end{array}$$

5. Question

Add:

6ax – 2by + 3cz, 6by -11ax – cz and 10 cz -2ax – 3by

Answer

To add the expressions, we have to arrange the given expression in the form of rows and then add the expression column wise, so we have;

$$\begin{array}{r} 6ax - 2by + 3cz \\ -11ax + 6by - cz \\ -2ax - 3by + 10cz \\ \hline -7ax + by + 12cz \end{array}$$

6. Question

Add:

$2x^3 - 9x^2 + 8$, $3x^2 - 6x - 5$, $7x^3 - 10x + 1$ and $3 + 2x - 5x^2 - 4x^3$

Answer

Let’s arrange the data in a table in the form of descending power of x,

We will get rows and columns; add the data column wise;

	$2x^3$	$-9x^2$	0	+8
	0	$3x^2$	−6x	−5
+	$7x^3$	0	−10x	+1
	$-4x^3$	$-5x^2$	+2x	+3
Total →	$5x^3$	$-11x^2$	-14x	+7

So, the answer after adding all the expressions will be;

$5x^3 - 11x^2 - 14x + 7$

7. Question

Add:

$6p + 4q - r + 3$, $2r - 5p - 6$, $11q - 7p + 2r - 1$ and $2q - 3r + 4$

Answer

To add the expressions, we have to arrange the given expression in the form of rows and then add the expression column wise, so we have;

$$\begin{array}{l} 6p + 4q - r + 3 \\ -5p + 0 + 2r - 6 \\ -7p + 11q + 2r - 1 \\ 0 + 2q - 3r + 4 \\ \hline -6p + 17q + 0 + 0 \end{array}$$

So, the answer is;

$-6p + 17q$

8. Question

Add:

$4x^2 - 7xy + 4y^2 - 3$, $5 + 6y^2 - 8xy + x^2$ and $6 - 2xy + 2x^2 - 5y^2$

Answer

By arrange the given expression in descending powers of x it will be easier To add the expressions,s,

So, we have;

$$\begin{array}{l} 4x^2 + 4y^2 - 7xy - 3 \\ x^2 + 6y^2 - 8xy + 5 \\ 2x^2 - 5y^2 - 2xy + 6 \\ \hline 7x^2 + 5y^2 - 17xy + 8 \end{array}$$

9. Question

Subtract:

$2a^2b$ from $-5a^2b$

Answer

We have to subtract $3a^2b$ from $-5a^2b$.

According to the rule when both the expressions have negative sign so we add both the expression and put negative sign only.

So, by arranging the data in rows and columns form we have;

$$\begin{array}{r} -5a^2b \\ 3a^2b \\ \hline -8a^2b \end{array}$$

10. Question

Subtract:

–8pq from 6pq

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

So, To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise,

Therefore, we have;

$$\begin{array}{r} 6pq \\ -8pq \\ \hline 14pq \end{array}$$

11. Question

Subtract:

–2abc from –8abc

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression, we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{r} -8abc \\ -2abc \\ \hline -6abc \end{array}$$

12. Question

Subtract:

–16p from –11 p

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{r} -11p \\ -16p \\ +\_\_\_\_\_ \\ 5p \end{array}$$

13. Question

Subtract:

2a − 5b + 2c − 9 from 3a − 4b − c + 6

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{l} 3a-4b-c+6 \\ 2a-5b+2c-9 \\ -\quad +\quad -\quad + \\ \hline a+b-3c+15 \end{array}$$

14. Question

Subtract:

−6p + q + 3r + 8 from p − 2q − 5r − 8

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{l} P-2q-5r-8 \\ -6p+q+3r+8 \\ +\quad -\quad -\quad + \\ \hline 7p-3q-8r-16 \end{array}$$

15. Question

Subtract:

$x^3 + 3x^2 - 5x + 4$ from $3x^3 - x^2 + 2x - 4$

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{r} 3x^3 - x^2 + 2x - 4 \\ x^3 + 3x^2 - 5x + 4 \\ - \quad - \quad + \quad - \\ \hline 2x^3 - 4x^2 + 7x - 8 \end{array}$$

16. Question

Subtract:

$5y^4 - 3y^3 + 2y^2 + y - 1$ from $4y^4 - 2y^3 - 6y^2 - y + 5$

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{r} 4y^4 - 2y^3 - 6y^2 - y + 5 \\ 5y^4 - 3y^3 + 2y^2 + y - 1 \\ - \quad + \quad - \quad - \quad + \\ \hline -4y^4 + y^3 - 8y^2 - 2y + 6 \end{array}$$

17. Question

Subtract:

$4p^2 + 5q^2 - 6r^2 + 7$ from $3p^2 - 4q^2 - 5r^2 - 6$

Answer

According to the rule of subtraction two negative becomes positive and the result will have negative sign.

To subtract the expression,we have to arrange the given expression in the form of rows and then subtract the expression column wise, so we have;

$$\begin{array}{r} 3p^2 - 4q^2 - 5r^2 - 6 \\ 4p^2 + 5q^2 - 6r^2 + 7 \\ - \quad - \quad + \quad - \\ \hline -p^2 - 9q^2 + r^3 - 13 \end{array}$$

18. Question

What must be subtracted from $3a^2 - 6ab - 3b^2 - 1$ to get $4a^2 - 7ab - 4ab^2 + 1$?

Answer

Let's suppose the required number be x,

So we have;

$(3a^2 - 6ab - 3b^2 - 1) - x = 4a^2 - 7a - 4b^2 + 1$

$(3a^2 - 6ab - 3b^2 - 1) - (4a^2 - 7a - 4b^2 + 1) = x$

So,

To get the required number we have to subtract $4a^2 - 7a - 4b^2 + 1$ from $3a^2 - 6ab - 3b^2 - 1$

$$\begin{array}{l} 3a^2 - 6ab - 3b^2 - 1 \\ 4a^2 - 7a - 4b^2 + 1 \\ \text{-} \quad + \quad + \quad \text{-} \\ \hline - a^2 + ab + b^2 - 2 \end{array}$$

So, the required number is $- a^2 + ab + b^2 - 2$

19. Question

The two adjacent sides of a rectangle are $5x^2 - 3y^2$ and $x^2 + 2xy$. Find the perimeter.

Answer

We know that;

Two adjacent sides of a rectangle are l and b;

$l = 5x^2 - 3y^2$

$b = x^2 + 2xy$

Perimeter of rectangle = (2l+2b)

Which is;

$2 (5x^2 - 3y^2) + 2(x^2 + 2xy)$

$= (10x^2 - 6y^2) + (2x^2 + 4xy)$

$$\begin{array}{l} 10x^2 - 6y^2 \\ 2x^2 + 0 + 4xy \\ \hline 12x^2 - 6y^2 + 4xy \end{array}$$

20. Question

The perimeter of triangle is $6p^2 - 4p + 9$ and two of its sides are $p^2 - 2p + 1$ and $3p^2 - 5p + 3$. Find the third side of the triangle.

Answer

Perimeter of the triangle = $6p^2 - 4p + 9$

Two sides are;

Side one = $p^2 - 2p + 1$ and

Side two = $3p^2 - 5p + 3$

Let's take third side be = x

As we know perimeter of a triangle = sum of all the sides

So, we have

$6p^2 - 4p + 9 = \{(p^2 - 2p + 1) + (3p^2 - 5p + 3) + (x)\}$

$6p^2 - 4p + 9 = p^2 - 2p + 1 + 3p^2 - 5p + 3 + x$

$6p^2 - 4p + 9 - p^2 + 2p - 1 - 3p^2 + 5p - 3 = x$

Let's make the pairs;

$(6p^2 - p^2 - 3p^2) + (- 4p + 2p + 5p) + (9 - 1 - 3) = x$

$2p^2 + 3p + 5 = x$

The required side is $2p^2 + 3p + 5$.

Exercise 6B

1. Question

Find each of the following products:

$(5x + 7) \times (3x + 4)$

Answer

To find the product of the given expression we have to Horizontal method;

Horizontal method is the method where each term of one expression is multiplied with each term of other expression.

So, by using horizontal method,

We have;

$= (5x + 7) \times (3x + 4)$

$= 5x (3x + 4) + 7 (3x + 4)$

$= 15x^2 + 20x + 21x + 28$

$= 15x^2\ 41x + 28$

2. Question

Find each of the following products:

$(4x + 9) \times (x - 6)$

Answer

By using horizontal method,

We have;

$= (4x + 9) \times (x - 6)$

$= 4x(x - 6) + 9(x - 6)$

$= 4x^2 - 24x + 9x - 54$

$= 4x^2 - 15x - 54$

3. Question

Find each of the following products:

$(2x + 5) \times (4x - 3)$

Answer

By using horizontal method,

We have;

$= (2x + 5) \times (4x - 3)$

$= 2x (4x - 3) + 5 (4x - 3)$

$= 8x^2 - 6x + 20x - 15$

$= 8x^2 + 14x - 15$

4. Question

Find each of the following products:

$(3y - 8) \times (5y - 1)$

Answer

By using horizontal method,

We have;

$= (3y - 8) \times (5y - 1)$

$= 3y(5y - 1) - 8(5y - 1)$

$= 15y^2 - 3y - 40y + 8$

$= 15y^2 - 43y + 8$

5. Question

Find each of the following products:

$(7x + 2y) \times (x + 4y)$

Answer

By using horizontal method,

We have;

$= (7x + 2y) \times (x + 4y)$

$= 7x(x + 4y) + 2y(x + 4y)$

$= 7x^2 + 28xy + 2xy + 8y^2$

$= 7x^2 + 30xy + 8y^2$

6. Question

Find each of the following products:

$(9x + 5y) \times (4x + 3y)$

Answer

By using horizontal method,

We have;

$= (9x + 5y) \times (4x + 3y)$

$= 9x(4x + 3y) + 5y(4x + 3y)$

$= 36x^2 + 27xy + 20xy + 15y^2$

$= 36x^2 + 47xy + 15y^2$

7. Question

Find each of the following products:

$(3m - 4n) \times (2m - 3n)$

Answer

By using horizontal method,

We have;

$= (3m - 4n) \times (2m - 3n)$

$= 3m(2m - 3n) - 4n(2m - 3n)$

$= 6m^2 - 9mn - 8mn + 12n^2$

$= 6m^2 - 17mn + 12n^2$

8. Question

Find each of the following products:

$(x^2 - a^2) \times (x - a)$

Answer

By using horizontal method,

We have;

$= (x^2 - a^2) \times (x - a)$

$= x^2(x - a) - a^2(x - a)$

$= x^3 - ax^2 - a^2x + a^3$

9. Question

Find each of the following products:

$(x^2 - y^2) \times (x + 2y)$

Answer

By using horizontal method,

We have;

$= (x^2 - y^2) \times (x + 2y)$

$= x^2 (x + 2y) - y^2(x + 2y)$

$= x^3 + 2x^2y - xy^2 - 2y^3$

10. Question

Find each of the following products:

$(3p^2 + q^2) \times (2p^2 - 3q^2)$

Answer

By using horizontal method,

We have;

$= (3p^2 + q^2) \times (2p^2 - 3q^2)$

$= 3p^2(2p^2 - 3q^2) + q^2(2p^2 - 3q^2)$

$= 6p^4 - 9p^2q^2 + 2p^2q^2 - 3q^4$

$= 6p^4 - 7p^2q^2 - 3q^4$

11. Question

Find each of the following products:

$(2x^2 - 5y^2) \times (x^2 + 3y^2)$

Answer

By using horizontal method,

We have;

$= (2x^2 - 5y^2) \times (x^2 + 3y^2)$

$= 2x^2(x^2 + 3y^2) - 5y^2(x^2 + 3y^2)$

$= 2x^4 + 6x^2y^2 - 5x^2y^2 - 15y^4$

$= 2x^4 + x^2y^2 - 15y^4$

12. Question

Find each of the following products:

$(x^3 - y^3) \times (x^2 + y^2)$

Answer

By using horizontal method,

We have;

$= (x^3 - y^3) \times (x^2 + y^2)$

$= x^3(x^2 + y^2) - y^3(x^2 + y^2)$

$= x^5 + x^3y^2 - x^2y^3 - y^5$

13. Question

Find each of the following products:

$(x^4 + y^4) \times (x^2 - y^2)$

Answer

By using horizontal method,

We have;

$= (x^4 + y^4) \times (x^2 - y^2)$

$= x^4(x^2 - y^2) + y^4(x^2 - y^2)$

$= x^6 - x^4y^2 + x^2y^4 - y^6$

14. Question

Find each of the following products:

$$\left(x^4+\frac{1}{x^4}\right)\times\left(x+\frac{1}{x}\right)$$

Answer

By using horizontal method,

We have;

$$=\left(x^4+\frac{1}{x^4}\right)\times\left(x+\frac{1}{x}\right)$$

$$=x^4\left(x+\frac{1}{x}\right)+\frac{1}{x^4}\left(x+\frac{1}{x}\right)$$

$$=x^5+x^3+\frac{1}{x^3}+\frac{1}{x^5}$$

15. Question

Find each of the following products:

$(x^2 - 3x + 7) \times (2x + 3)$

Answer

By using horizontal method,

We have;

$= (x^2 - 3x + 7) \times (2x + 3)$

$= 2x(x^2 - 3x + 7) + 3(x^2 - 3x + 7)$

$= 2x^3 - 6x^2 + 14x + 3x^2 - 9x + 21$

By arranging the expression in the form of descending powers of x,

We get;

$= 2x^3 - 6x^2 + 3x^2 + 14x - 9x + 21$

$= 2x^3 - 3x^2 + 5x + 21$

16. Question

Find each of the following products:

$(3x^2 + 5x - 9) \times (3x - 5)$

Answer

By using horizontal method,

We have;

$= (3x^2 + 5x - 9) \times (3x - 5)$

$= 3x(3x^2 + 5x – 9) – 5(3x^2 + 5x – 9)$

$= 9x^3 + 15x^2 – 27x – 15x^2 - 25x + 45$

By arranging the expression in the form of descending powers of x,

We get;

$= 9x^3 + 15x^2 – 15x^2 – 27x – 25x + 45$

$= 9x^3 – 52x + 45$

17. Question

Find each of the following products:

$(x^2 – xy + y^2) \times (x + y)$

Answer

By using horizontal method,

We have;

$= (x^2 – xy + y^2) \times (x + y)$

$= x(x^2 – xy + y^2) + y(x^2 – xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y – xy^2 + y^3$

By arranging the expression in the form of descending powers of x,

We get;

$= (x^3 + y^3)$

18. Question

Find each of the following products:

$(x^2 + xy + y^2) \times (x – y)$

Answer

By using horizontal method,

We have;

$= (x^2 + xy + y^2) \times (x - y)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y – xy^2 - y^3$

By arranging the expression in the form of descending powers of x,

We get;

$= (x^3 - y^3)$

19. Question

Find each of the following products:

$(x^3 – 2x^2 + 5) \times (4x - 1)$

Answer

By using horizontal method,

We have;

$= (x^3 – 2x^2 + 5) \times (4x – 1)$

$= 4x(x^3 – 2x^2 + 5) – 1(x^3 – 2x^2 + 5)$

$= 4x^4 – 8x^3 + 20x – 1x^3 + 2x^2 – 5$

By arranging the expression in the form of descending powers of x,

We get;

$= 4x^4 – 8x^3 – x^3 + 2x^2 + 20x – 5$

$= 4x^4 – 9x^3 + 2x^2 + 20x – 5$

20. Question

Find each of the following products:

$(9x^2 – x + 15) \times (x^2 – 3)$

Answer

By using horizontal method,

We have;

$= (9x^2 – x + 15) \times (x^2 – 3)$

$= x^2(9x^2 – x +15) – 3(9x^2 – x + 15)$

$= 9x^4 – x^3 + 15x^2 – 27x^2 + 3x – 45$

$= 9x^4 - x^3 – 12x^2 + 3x – 45$

21. Question

Find each of the following products:

$(x^2 – 5x + 8) \times (x^2 + 2)$

Answer

By using horizontal method,

We have;

$= (x^2 - 5x + 8) \times (x^2 + 2)$

$= x^2(x^2 - 5x + 8) + 2(x^2 - 5x + 8)$

$= x^4 - 5x^3 + 8x^2 + 2x^2 - 10x + 16$

$= x^4 - 5x^3 + 10x^2 - 10x + 16$

22. Question

Find each of the following products:

$(x^3 - 5x^2 + 3x + 1) \times (x^2 - 3)$

Answer

By using horizontal method,

We have;

$= (x^3 - 5x^2 + 3x + 1) \times (x^2 - 3)$

$= x^2(x^3 - 5x^2 + 3x + 1) - 3(x^3 - 5x^2 + 3x + 1)$

$= x^5 - 5x^4 + 3x^3 + x^2 - 3x^3 + 15x^2 - 9x - 3$

By arranging the expression in the form of descending powers of x,

We get;

$= x^5 - 5x^4 + 3x^3 - 3x^3 + x^2 + 15x^2 - 9x - 3$

$= x^5 - 5x^4 + 16x^2 - 9x - 3$

23. Question

Find each of the following products:

$(3x + 2y - 4) \times (x - y + 2)$

Answer

By using horizontal method,

We have;

$= (3x + 2y - 4) \times (x - y + 2)$

$= x(3x + 2y - 4) - y(3x + 2y - 4) + 2(3x + 2y - 4)$

$= 3x^2 + 2xy - 4x - 3xy - 2y^2 + 4y + 6x + 4y - 8$

By arranging the expression in the form of descending powers of x,

We get;

$= 3x^2 - 4x + 6x + 2xy - 3xy - 2y^2 + 4y + 4y - 8$

$= 3x^2 + 2x - xy - 2y^2 + 8y - 8$

24. Question

Find each of the following products:

$(x^2 - 5x + 8) \times (x^2 + 2x - 3)$

Answer

By using horizontal method,

We have;

$= (x^2 - 5x + 8) \times (x^2 + 2x - 3)$

$= x^2(x^2 - 5x + 8) + 2x(x^2 - 5x + 8) - 3(x^2 - 5x + 8)$

$= x^4 - 5x^3 + 8x^2 + 2x^3 - 10x^2 + 16x - 3x^2 + 15x - 24$

By arranging the expression in the form of descending powers of x,

We get;

$= x^4 - 3x^3 - 5x^2 + 31x - 24$

25. Question

Find each of the following products:

$(2x^2 + 3x - 7) \times (3x^2 - 5x + 4)$

Answer

By using horizontal method,

We have;

$(2x^2 + 3x - 7) \times (3x^2 - 5x + 4)$

$= 2x^2(3x^2 - 5x + 4) + 3x(3x^2 - 5x + 4) - 7(3x^2 - 5x + 4)$

$= 6x^4 - 10x^3 + 8x^2 + 9x^3 - 15x^2 + 12x - 21x^2 + 35x - 28$

Now, putting equal power terms together, we get,

$= 6x^4 - 10x^3 + 9x^3 + 8x^2 - 15x^2 - 21x^2 + 35x + 12x - 28$

$= 6x^4 - x^3 - 28x^2 + 47x - 28$

26. Question

Find each of the following products:

$(9x^2 - x + 15) \times (x^2 - x - 1)$

Answer

By using horizontal method,

We have;

$(9x^2 - x + 15) \times (x^2 - x - 1)$

$= 9x^2(x^2 - x - 1) - x(x^2 - x - 1) + 15(x^2 - x - 1)$

$= 9x^4 - 9x^3 - 9x^2 - x^3 + x^2 + x + 15x^2 - 15x - 15$

Putting equal power terms together, we get,

$= 9x^4 - 9x^3 - x^3 - 9x^2 + x^2 + 15x^2 - 15x + x - 15$

$= 9x^4 - 10x^3 + 7x^2 - 14x - 15$

Exercise 6C

1. Question

Divide:

(i) $24x^2y^3$ by $3xy$

(ii) $36xyz^2$ by $-9xz$

(iii) $-72x^2y^2z$ by $-12xyz$

(iv) $-56mnp^2$ by $7mnp$

Answer

(i) By dividing $24x^2y^3$ by $3xy$

We get;

$$= \frac{24x^2y^3}{3xy}$$

$= 8xy^2$

(ii) By dividing $36xyz^2$ by by $-9xz$

We get;

$$= \frac{36xyz^2}{-9xz}$$

$= -4yz$

(iii) By dividing $-72x^2y^2z$ by $-12xyz$

We get;

$$= \frac{-72x^2y^2z}{-12xyz}$$

= 6xy

(iv) By dividing – $56mnp^2$ by 7mnp

We get;

$$= \frac{-56mnp^2}{7mnp}$$

= - 8p

2. Question

Divide:

(i) $5m^3 - 30m^2 + 45m$ by 5m

(ii) $8x^2y^2 - 6xy^2 + 10x^2y^3$ by 2xy

(iii) $9x^2y - 6xy + 12xy^2$ by – 3xy

(iv) $12x^4 + 8x^3 - 6x^2$ by $- 2x^2$

Answer

(i) By dividing $5m^3 - 30m^2 + 45m$ by 5m

We get;

$$= \frac{5m^3 - 30m^2 + 45m}{5m}$$

$$= \frac{5m^3}{5m} - \frac{30m^2}{5m} + \frac{45m}{5m}$$

$= m^2 - 6m + 9$

(ii) By dividing $8x^2y^2 - 6xy^2 + 10x^2y^3$ by 2xy

We get;

$$= \frac{8x^2y^2 - 6xy^2 + 10x^2y^3}{2xy}$$

$$= \frac{8x^2y^2}{2xy} - \frac{6xy^2}{2xy} + \frac{10x^2y^3}{2xy}$$

$= 4xy - 3y + 5xy^2$

(iii) If we divide $9x^2y - 6xy + 12xy^2$ by – 3xy

We get;

$$= \frac{9x^2y - 6xy + 12xy^2}{-3xy}$$

$$= \frac{9x^2y}{-3xy} - \frac{6xy}{-3xy} + \frac{12xy^2}{-3xy}$$

= - 3x + 2 – 4y

(iv) If we divide $12x^4 + 8x^3 – 6x^2$ by $– 2x^2$

We get;

$$= \frac{12x^4 + 8x^3 - 6x^2}{-2x^2}$$

$$= \frac{12x^4}{-2x^2} + \frac{8x^3}{-2x^2} - \frac{6x^2}{-2x^2}$$

= $- 6x^2 – 4x + 3$

3. Question

Write the quotient and remainder when we divide:

$(x^2 – 4x + 4)$ by $(x – 2)$

Answer

If we divide $x^2 – 4x + 4$ by x -2;

$$\begin{array}{r} x - 2 \\ x - 2 \overline{)\, x^2 - 4x + 4} \\ x^2 - 2x \\ - \quad + \\ \hline - 2x + 4 \\ - 2x + 4 \\ + \quad - \\ \hline \times \\ \hline \end{array}$$

So, we get;

Quotient = x – 2 and remainder = 0

4. Question

Write the quotient and remainder when we divide:

$(x^2 – 4)$ by $(x + 2)$

Answer

If we divide $(x^2 - 4)$ by $(x + 2)$;

$$
\begin{array}{r|l}
x + 2 & x^2 - 4 \\
 & x^2 + 2x \\
 & - \quad - \\
 & \hline
 & -2x - 4 \\
 & -2x - 4 \\
 & + \quad + \\
 & \hline
 & \times \\
 & \hline
\end{array}
$$

So, we get;

Quotient = x – 2 and remainder = 0

5. Question

Write the quotient and remainder when we divide:

$(x^2 + 12x + 35)$ by $(x + 7)$

Answer

If we divide $(x^2 + 12x + 35)$ by $(x + 7)$

$$
\begin{array}{r|l}
x + 7 & x^2 + 12x + 35 \\
 & x^2 + 7x \\
 & - \quad - \\
 & \hline
 & 5x + 35 \\
 & 5x + 35 \\
 & - \quad - \\
 & \hline
 & \times \\
 & \hline
\end{array}
$$

So, we get;

Quotient = (x + 5) and remainder = 0

6. Question

Write the quotient and remainder when we divide:

$(15x^2 + x - 6)$ by $(3x + 2)$

Answer

If we divide $(15x^2 + x - 6)$ by $(3x + 2)$

$$
\begin{array}{r|l}
3x+2 & 5x-3 \\
\cline{2-2}
 & 15x^2+x-6 \\
 & 15x^2+10x \\
 & - \quad - \\
\cline{2-2}
 & -9x-6 \\
 & -9x-6 \\
 & + \quad + \\
\cline{2-2}
 & \times
\end{array}
$$

We get;

Quotient = (5x – 3) and remainder = 0

7. Question

Write the quotient and remainder when we divide:

$(14x^2 – 53x + 45)$ by $(7x – 9)$

Answer

If we divide $(14x^2 – 53x + 45)$ by $(7x – 9)$

So we get;

$$
\begin{array}{r|l}
7x-9 & 2x-5 \\
\cline{2-2}
 & 14x^2-53x+45 \\
 & 14x^2-18x \\
 & - \quad + \\
\cline{2-2}
 & -35x+45 \\
 & -35x+45 \\
 & + \quad - \\
\cline{2-2}
 & \times
\end{array}
$$

Quotient = 2x - 5 and remainder = 0

8. Question

Write the quotient and remainder when we divide:

$(6x^2 – 31x + 47)$ by $(2x – 5)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r}
3x - 8 \\
2x - 5 \overline{)\,6x^2 - 31x + 47} \\
6x^2 - 15x \\
- \quad + \\
\hline
-16x + 47 \\
-16x + 40 \\
+ \quad - \\
\hline
7 \\
\hline
\end{array}
$$

Quotient = (3x – 8) and remainder = 7

9. Question

Write the quotient and remainder when we divide:

$(2x^3 + x^2 – 5x – 2)$ by $(2x + 3)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r}
x^2 - x - 1 \\
2x + 3 \overline{)\,2x^3 + x^2 - 5x - 2} \\
2x^3 + 3x^2 \\
- \quad - \\
\hline
-2x^2 - 5x \\
-2x^2 - 3x \\
+ \quad + \\
\hline
-2x - 2 \\
-2x - 3 \\
+ \quad + \\
\hline
+1 \\
\hline
\end{array}
$$

Quotient = $(x^2 – x – 1)$ and remainder = 1

10. Question

Write the quotient and remainder when we divide:

$(x^3 + 1)$ by $(x + 1)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x + 1 & x^3 + 1 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -x^2 + 1 \\
 & -x^2 - x \\
 & + \quad + \\
\hline
 & +x + 1 \\
 & x + 1 \\
 & - \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

Quotient = $(x^2 - x + 1)$ and remainder = 0

11. Question

Write the quotient and remainder when we divide:

$(x^4 - 2x^3 + 2x^2 + x + 4)$ by $(x^2 + x + 1)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r|l}
 & x^2 - 3x + 4 \\
\hline
x^2 + x + 1 & x^4 - 2x^3 + 2x^2 + x + 4 \\
 & x^4 + x^3 + x^2 \\
 & - \quad - \quad - \\
\hline
 & -3x^3 + x^2 + x \\
 & -3x^3 - 3x^2 - 3x \\
 & + \quad + \quad + \\
\hline
 & 4x^2 + 4x + 4 \\
 & 4x^2 + 4x + 4 \\
 & - \quad - \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

Quotient = $x^2 - 3x + 4$ and remainder = 0

12. Question

Write the quotient and remainder when we divide:

$(x^3 - 6x^2 + 11x - 6)$ by $(x^2 - 5x + 6)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r|l}
x^2 - 5x + 6 & x - 1 \\
\cline{2-2}
 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 - 5x^2 + 6x \\
 & - \quad + \quad - \\
\cline{2-2}
 & -x^2 + 5x - 6 \\
 & -x^2 + 5x - 6 \\
 & + \quad - \quad + \\
\cline{2-2}
 & \times
\end{array}
$$

Quotient = (x – 1) and remainder = 0

13. Question

Write the quotient and remainder when we divide:

$(5x^3 - 12x^2 + 12x + 13)$ by $(x^2 - 3x + 4)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r|l}
x^2 - 3x + 4 & 5x + 3 \\
\cline{2-2}
 & 5x^3 - 12x^2 + 12x + 13 \\
 & 5x^3 - 15x^2 + 20x \\
 & - \quad + \quad - \\
\cline{2-2}
 & 3x^2 - 8x + 13 \\
 & 3x^2 - 9x + 12 \\
 & - \quad + \quad - \\
\cline{2-2}
 & x + 1
\end{array}
$$

Quotient = (5x + 3) and remainder = (x + 1)

14. Question

Write the quotient and remainder when we divide:

$(2x^3 - 5x^2 + 8x - 5)$ by $(2x^2 - 3x + 5)$

Answer

By dividing the given expressions we get;

$$
\begin{array}{r}
x-1 \\
2x^2-3x+5\overline{)\,2x^3-5x^2+8x-5} \\
2x^3-3x^2+5x \\
-\quad +\quad - \\
\hline
-2x^2+3x-5 \\
-2x^2+3x-5 \\
+\quad -\quad + \\
\hline
\times \\
\hline
\end{array}
$$

Quotient = (x – 1) and remainder = 0

15. Question

Write the quotient and remainder when we divide:

$(8x^4 + 10x^3 – 5x^2 – 4x + 1)$ by $(2x^2 – 3x + 5)$

Answer

If we divide $(8x^4 + 10x^3 – 5x^2 – 4x + 1)$ by $(2x^2 – 3x + 5)$

We get,

$$
\begin{array}{r}
4x^2+11x+4 \\
2x^2-3x+5\overline{)\,8x^4+10x^3-5x^2-4x+1} \\
8x^4-12x^3+20x^2 \\
-\quad +\quad - \\
\hline
22x^2-25x^2-4x \\
22x^3-33x^2+55x \\
-\quad +\quad - \\
\hline
8x^2-59x+1 \\
8x^2-12x+20 \\
-\quad +\quad - \\
\hline
-47x-19 \\
\hline
\end{array}
$$

So,

We get the quotient $4x^2 + 11x + 4$

And the remainder – 47x – 19

Exercise 6D

1. Question

Find each of the following products:

(i) (x + 6)(x+6)

(ii) $(4x + 5y)(4x + 5y)$

(iii) $(7a + 9b)(7a + 9b)$

(iv) $\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right)$

(v) $(x^2 + 7)(x^2 + 7)$

(vi) $\left(\frac{5}{6}a^2 + 2\right)\left(\frac{5}{6}a^2 + 2\right)$

Answer

(i) As we have $(x + 6)(x+6)$

$(x + 6)(x + 6) = (x + 6)^2$

By using the formula;

$[(a + b)^2 = a^2 + b^2 + 2ab]$

We get,

$(x + 6)^2 = x^2 + (6)^2 + 2\times (x) \times (6)$

$= x^2 + 36 + 12x$

By arranging the expression in the form of descending powers of x we get;

$= x^2 + 12x + 36$

(ii) Given;

$(4x + 5y)(4x + 5y)$

By using the formula;

$[(a + b)^2 = a^2 + b^2 + 2ab]$

We get,

$(4x + 5y)(4x + 5y) = (4x + 5y)^2$

$(4x + 5y)^2 = (4x)^2 + (5y)^2 + 2 \times (4x) \times(5y)$

$= 16x^2 + 25y^2 + 40xy$

(iii) Given,

$(7a + 9b)(7a + 9b)$

By using the formula;

$[(a + b)^2 = a^2 + b^2 + 2ab]$

We get,

$(7a + 9b)(7a + 9b) = (7a + 9b)^2$

$(7a + 9b)^2 = (7a)^2 + (9b)^2 + 2 \times (7a) \times (9b)$

$= 49a^2 + 81b^2 + 126ab$

(iv) $\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right)$

By using the formula $(a + b)^2$

We get;

$$= \left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right) = \left(\frac{2}{3}x + \frac{4}{5}y\right)^2$$

$$= \left(\frac{2}{3}x\right)^2 + \left(\frac{4}{5}y\right)^2 + 2 \times \left(\frac{2}{3}x\right) \times \left(\frac{4}{5}y\right)$$

$$= \frac{4}{9}x^2 + \frac{16}{25}y^2 + \frac{16}{15}xy$$

(v) $(x^2 + 7)(x^2 + 7)$

By using the formula $(a + b)^2$

We get;

$(x^2 + 7)(x^2 + 7) = (x^2 + 7)^2$

$= (x^2)^2 + (7)^2 + 2 \times (x^2) \times (7)$

$= x^4 + 49 + 14x^2$

(vi) $\left(\frac{5}{2}a^2 + 2\right)\left(\frac{5}{2}a^2 + 2\right)$

By using the formula $(a + b)^2$

We get;

$$\left(\frac{5}{2}a^2 + 2\right)\left(\frac{5}{2}a^2 + 2\right)$$

$$= \left(\frac{5}{2}a^2 + 2\right)^2$$

$$= \frac{5}{2}a^2 + 2^2 + 2 \times \frac{5}{2}a^2 \times 2$$

$$= \frac{25}{36}a^4 + 4 + \frac{10}{3}a^2$$

2. Question

Find each of the following products:

(i) (x – 4)(x – 4)

(ii) (2x – 3y)(2x – 3y)

(iii) $\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$

(iv) $\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)$

(v) $\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)$

(vi) $\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$

Answer

(i) Given,

(x – 4)(x – 4)

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$= (x - 4)^2$

$= (x)^2 - 2 \times (x) \times 4 + (4)^2$

$= x^2 - 8x + 16$

(ii) Given,

(2x – 3y)(2x – 3y)

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$= (2x - 3y)^2$

$= (2x)^2 - 2 \times (2x) \times (3y) + (3y)^2$

$= 4x^2 - 12xy + 9y^2$

(iii) $\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$

$=\left(\frac{3}{4}x-\frac{5}{6}y\right)^2$

$=\left(\frac{3}{4}x\right)^2-2\times\frac{3}{4}x\times\frac{5}{6}y+\left(\frac{5}{6}y\right)^2$

$=\frac{9}{16}x^2-\frac{15}{12}xy+\frac{25}{36}y^2$

(iv) $\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)$

By using the formula $(a-b)^2=a^2-2ab+b^2$

We get;

$\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)$

$=\left(x-\frac{3}{x}\right)^2$

$=(x)^2-2\times x\times\frac{3}{x}+\left(\frac{3}{x}\right)^2$

$=x^2-6+\frac{9}{x^2}$

(v) $\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)$

By using the formula $(a-b)^2=a^2-2ab+b^2$

We get;

$\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)=\left(\frac{1}{3}x^2-9\right)^2$

$=\left(\frac{1}{3}x^2\right)^2-2\times\frac{1}{3}x^2\times9+(9)^2$

$=\frac{1}{9}x^4-6x^2+81$

(vi) $\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$

By using the formula $(a-b)^2=a^2-2ab+b^2$

We get;

$$\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$$

$$=\left(\frac{1}{2}y^2-\frac{1}{3}y\right)^2$$

$$=\left(\frac{1}{2}y^2\right)^2-2\times\frac{1}{2}y^2\times\frac{1}{3}y+\left(\frac{1}{3}y\right)^2$$

$$=\frac{1}{4}y^4-\frac{1}{3}y^3+\frac{1}{9}y^2$$

3. Question

Expand:

(i) $(8a + 3b)^2$ (ii) $(7x + 2y)^2$

(iii) $(5x + 11)^2$ (iv) $\left(\frac{a}{2}+\frac{2}{a}\right)^2$

(v) $\left(\frac{3x}{4}+\frac{2y}{9}\right)^2$ (vi) $(9x - 10)^2$

(vii) $\left(x^2y-yz^2\right)^2$ (viii) $\left(\frac{x}{y}-\frac{y}{x}\right)^2$

(ix) $\left(3m-\frac{4}{5}n\right)^2$

Answer

(i) Given,

$(8a + 3b)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get;

$= (8a)^2 + (3b)^2 + 2 \times 8a \times 3b$

$= 64a^2 + 9b^2 + 48ab$

(ii) $(7x + 2y)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get;

$= (7x)^2 + (2y)^2 + 2 \times (7x) \times (2y)$

= $49x^2 + 4y^2 + 28xy$

(iii) $(5x + 11)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get;

= $(5x)^2 + (11)^2 + 2\times(5x) \times 11$

= $25x^2 + 121 + 110x$

(iv) $\left(\frac{a}{2}+\frac{2}{a}\right)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get;

$$\left(\frac{a}{2}+\frac{2}{a}\right)^2$$

$$=\left(\frac{a}{2}\right)^2+\left(\frac{2}{a}\right)^2+2\times\frac{a}{2}\times\frac{2}{a}$$

$$=\frac{a^2}{4}+\frac{4}{a^2}+2$$

(v) $\left(\frac{3x}{4}+\frac{2y}{9}\right)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get;

$$\left(\frac{3x}{4}+\frac{2y}{9}\right)^2=\left(\frac{3x}{4}\right)^2+\left(\frac{2y}{9}\right)^2+2\times\frac{3x}{4}\times\frac{2y}{9}$$

$$=\frac{9x^2}{16}+\frac{4y^2}{81}+\frac{1}{3}xy$$

(vi) $(9x - 10)^2$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$(9x - 10)^2$

= $(9x)^2 - 2 \times (9x) \times 10 + (10)^2$

= $81x^2 - 180x + 100$

(vii) $(x^2y - yz^2)^2$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$= (x^2y - yz^2)^2$

$= (x^2y)^2 - 2 \times (x^2y) \times yz^2 + (yz^2)^2$

$= x^4y^2 - 2x^2y^2z^2 + y^2z^4$

(viii) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$$\left(\frac{x}{y} - \frac{y}{x}\right)^2 = \left(\frac{x}{y}\right)^2 - 2 \times \frac{x}{y} \times \frac{y}{x} + \left(\frac{y}{x}\right)^2 = \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$$

(ix) $\left(3m - \frac{4}{5}n\right)^2$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get;

$$= (3m)^2 - 2 \times 3m \times \frac{4}{5}n + \left(\frac{4}{5}n\right)^2$$

$$= 9m - \frac{24}{5}mn + \frac{16}{25}n^2$$

4. Question

Find each of the following products:

(i) $(x + 3)(x - 3)$

(ii) $(2x + 5)(2x - 5)$

(iii) $(8 + x)(8 - x)$

(iv) $(7x + 11y)(7x - 11y)$

(v) $\left(5x^2 + \frac{3}{4}y^2\right)\left(5x^2 - \frac{3}{4}y^2\right)$

(vi) $\left(\frac{4x}{5} - \frac{5y}{3}\right)\left(\frac{4x}{5} + \frac{5y}{3}\right)$

(vii) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(viii) $\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)$

(ix) $\left(2a+\frac{3}{b}\right)\left(2a-\frac{3}{b}\right)$

Answer

(i) Given,

$(x + 3)(x - 3)$

By using the formula $(a + b)(a - b) = a^2 - b^2$

We get;

$= x(x + 3) - 3(x + 3)$

$= x^2 + 3x - 3x - 9$

$= x^2 - 9$

(ii) Given,

$(2x + 5)(2x - 5)$

By using the formula $(a + b)(a - b) = a^2 - b^2$

We get;

$= 2x(2x + 5) - 5(2x + 5)$

$= 4x^2 + 10x - 10x - 25$

$= 4x^2 - 25$

(iii) Given,

$(8 + x)(8 - x)$

By using the formula $(a + b)(a - b) = a^2 - b^2$

We get;

$= 8(8 + x) - x(8 + x)$

$= 64 + 8x - 8x - x^2$

$= 64 - x^2$

(iv) Given,

$(7x + 11y)(7x - 11y)$

By using the formula $(a + b)(a - b) = a^2 - b^2$

We get;

= 7x(7x + 11y) – 11y(7x + 11y)

= $49x^2 + 77xy – 77xy – 121y^2$

= $49x^2 – 121y^2$

(v) Given,

$$\left(5x^2+\frac{3}{4}y^2\right)\left(5x^2-\frac{3}{4}y^2\right)$$

By using the formula (a + b) (a – b) = $a^2 – b^2$

We get;

$$\left(5x^2+\frac{3}{4}y^2\right)\left(5x^2-\frac{3}{4}y^2\right)$$

$$=25x^4-\frac{9}{16}y^4$$

(vi) $\left(\frac{4x}{5}-\frac{5y}{3}\right)\left(\frac{4x}{5}+\frac{5y}{3}\right)$

By using the formula (a + b) (a – b) = $a^2 – b^2$

We get;

$$=\frac{16x^2}{25}-\frac{25y^2}{9}$$

(vii) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

By using the formula (a + b) (a – b) = $a^2 – b^2$

We get;

$$=x^2-\frac{1}{x^2}$$

(viii) $\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)$

By using the formula (a + b) (a – b) = $a^2 – b^2$

We get;

$$=\frac{1}{x^2}-\frac{1}{y^2}$$

(ix) $\left(2a+\frac{3}{b}\right)\left(2a-\frac{3}{b}\right)$

By using the formula (a + b) (a – b) = $a^2 – b^2$

We get;

$= 4a^2 - \frac{9}{b^2}$

5. Question

Using the formula for squaring a binomial, evaluate the following:

(i) $(54)^2$ (ii) $(82)^2$

(iii) $(103)^2$ (iv) $(704)^2$

Answer

(i) Given,

$(54)^2$

If we break the given number we get;

$(50 + 4)^2$

Now we can use the $(a + b)^2 = a^2 + b^2 + 2ab$

So,

$= (50 + 4)^2 = (50)^2 + (4)^2 + 2 \times 50 \times 4$

$= 2500 + 16 + 400$

$= 2916$

(ii) $(82)^2$

We can also write it as;

$(80 + 2)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get,

$= (80 + 2)^2 = (80)^2 + (2)^2 + 2 \times 80 \times 2$

$= 6400 + 4 + 320$

$= 6724$

(iii) $(103)^2$

We can also write it as;

$(100 + 3)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get,

$(100 + 3)^2 = (100)^2 + (3)^2 + 2 \times 100 \times 3$

= 10000 + 9 + 600

= 10609

(iv) $(704)^2$

We can also write it as;

$(700 + 4)^2$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

We get,

$= (700 + 4)^2 = (700)^2 + (4)^2 + 2 \times 700 \times 4$

= 490000 + 16 + 5600

= 495616

6. Question

Using the formula for squaring a binomial, evaluate the following:

(i) $(69)^2$ (ii) $(78)^2$

(iii) $(197)^2$ (iv) $(999)^2$

Answer

(i) Given,

$(69)^2$

We can also write it as;

$(70 - 1)^2$

Now,

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get,

$= (70 - 1)^2 = (70)^2 - 2 \times 70 \times 1 + (1)^2$

= 4900 – 140 + 1

= 4761

(ii) Given = $(78)^2$

We can also write it as;

$(80 - 2)^2$

Now,

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get,

$(80 - 2)^2 = (80)^2 - 2 \times 80 \times 2 + (2)^2$

$= 6400 - 320 + 4$

$= 6084$

(iii) $(197)^2$

We can also write it as;

$(200 - 3)^2$

Now,

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get,

$(200 - 3)^2 = (200)^2 - 2 \times 200 \times 3 + (3)^2$

$= 40000 - 1200 + 9$

$= 38809$

(iv) $(999)^2$

We can also write it as;

$(1000 - 1)^2$

Now,

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$

We get,

$(1000 - 1)^2 = (1000)^2 - 2 \times 1000 \times 1 + (1)^2$

$= 1000000 - 2000 + 1$

$= 998001$

7. Question

Find the value of:

(i) $(82)^2 - (18)^2$

(ii) $(128)^2 - (72)^2$

(iii) 197×203

(iv) $\frac{198 \times 198 - 102 \times 102}{96}$

(v) (14.7 × 15.3)

(vi) $(8.63)^2 - (1.37)^2$

Answer

(i) Given,

$(82)^2 - (18)^2$

By using $(a - b)(a + b) = a^2 - b^2$

$= (82 - 18)(82 + 18)$

$= (64)(100)$

$= 6400$

(ii) $(128)^2 - (72)^2$

By using $(a - b)(a + b) = a^2 - b^2$

$= (128 - 72)(128 + 72)$

$= (56)(200)$

$=11200$

(iii) 197 × 203

By converting the given number into the form of formula we get,

$= (200 - 3)(200 + 3)$

$= (200)^2 - (3)^2$

$= 40000 - 9$

$= 39991$

(iv) Given,

$$\frac{198 \times 198 - 102 \times 102}{96}$$

By using the formula $(a - b)(a + b) = a^2 - b^2$

We get;

$$= \frac{(198)^2 - (102)^2}{96}$$

$$= \frac{(198 - 102)(198 + 102)}{96}$$

$$= \frac{(96)(300)}{96} = 300$$

(v) (14.7 × 15.3)

By using $(a - b)(a + b) = a^2 - b^2$

We get;

$= (15 - 0.3)(15 + 0.3)$

$= (15)^2 - (0.3)^2$

$= 225 - 0.09$

$= 224.91$

(vi) $(8.63)^2 - (1.37)^2$

By using $(a - b)(a + b) = a^2 - b^2$

We get;

$= (8.63 - 1.37)(8.63 + 1.37)$

$= (7.26)(10)$

$= 72.6$

8. Question

Find the value of the expression $(9x^2 + 24x + 16)$, when x = 12.

Answer

Given,

$(9x^2 + 24x + 16)$

x = 12

So, we can also write it as;

$= (3x)^2 + 2(3x)(4) + (4)^2$

→ By the formula $(a + b)^2$ we get,

$= (3x + 4)^2$

$= [3\ (12) + 4]^2$

$= [36 + 4]^2$

$= [40]^2 = 1600$

Hence the value of the expression is 1600 when x =12.

9. Question

Find the value of the expression $(64x^2 + 81y^2 + 144xy)$, when x = 11 and $y = \frac{4}{3}$.

Answer

Given,

$(64x^2 + 81y^2 + 144xy)$

X = 11

Y = $\frac{4}{3}$

By using the formula $(a + b)^2$ we get;

$= (8x)^2 + (9y)^2 + 2(8x)(9y)$

$= (8x + 9y)^2$

$= [8(11) + 9\left(\frac{4}{3}\right)]^2$

$= (88 + 12)^2$

$= (100)^2 = 10000$

Hence the value of the expression is 10000.

10. Question

Find the value of the expression $(36x^2 + 25y^2 - 60xy)$ when $x = \frac{2}{3}$ and $y = \frac{1}{5}$.

Answer

Given,

$(36x^2 + 25y^2 - 60xy)$

X = $\frac{2}{3}$

Y = $\frac{1}{5}$

With the help of the formula $(a - b)^2$ we get;

$= (6x)^2 + (5y)^2 - 2(6x)(5y)$

$= (6x - 5y)^2$

$= \left(6\left(\frac{2}{3}\right) - 5\left(\frac{1}{5}\right)\right)^2$

$= (4 - 1)^2$

$= (3)^2 = 9$

11. Question

If $\left(x+\frac{1}{x}\right)=4$, find the values of

(i) $\left(x^2+\frac{1}{x^2}\right)$ (ii) $\left(x^4+\frac{1}{x^4}\right)$

Answer

(i) $\left(x^2+\frac{1}{x^2}\right)$

We know that,

From formula $(a+b)^2 = a^2 + b^2 + 2ab$

$= \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x} = x^2+\frac{1}{x^2}+2$

$= x+\frac{1}{x} = 4$ given

So, by putting the values , we get,

$= 4^2 = x^2+\frac{1}{x^2}+2$

$= x^2+\frac{1}{x^2} = 16-2 = 14$

(ii) $\left(x^4+\frac{1}{x^4}\right)$

We know that,

From formula $(a+b)^2 = a^2 + b^2 + 2ab$

$= \left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2} = x^4+\frac{1}{x^4}+2$

$= x^2+\frac{1}{x^2} = 14$ (previously calculated)

So, by putting the values , we get,

$= 14^2 = x^4+\frac{1}{x^4}+2$

$= x^2+\frac{1}{x^2} = 196-2 = 194$

12. Question

If $\left(x-\frac{1}{x}\right)=5$, find the value of

(i) $\left(x^2+\frac{1}{x^2}\right)$ (ii) $\left(x^4+\frac{1}{x^4}\right)$

Answer

(i) $\left(x^2+\frac{1}{x^2}\right)$

We know that,

From formula $(a-b)^2 = a^2 + b^2 - 2ab$

$= \left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x} = x^2+\frac{1}{x^2}-2$

$= x-\frac{1}{x} = 5 \text{ given}$

So, by putting the values, we get,

$= 5^2 = x^2+\frac{1}{x^2}-2$

$= x^2+\frac{1}{x^2} = 25+2 = 27$

(ii) $\left(x^4+\frac{1}{x^4}\right)$

We know that,

From formula $(a+b)^2 = a^2 + b^2 + 2ab$

$= \left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2} = x^4+\frac{1}{x^4}+2$

$= x^2+\frac{1}{x^2} = 27 \text{ (previously calculated)}$

So, by putting the values, we get,

$= 27^2 = x^4+\frac{1}{x^4}+2$

$= x^2+\frac{1}{x^2} = 729-2 = 727$

13. Question

Find the continued product:

(i) $(x+1)(x-1)(x^2+1)$

(ii) $(x-3)(x+3)(x^2+9)$

(iii) $(3x-2y)(3x+2y)(9x^2+4y^2)$

(iv) $(2p+3)(2p-3)(4p^2+9)$

Answer

(i) $(x+1)(x-1)(x^2+1)$

We know that, from formula,

$(a + b)(a - b) = a^2 - b^2$

$(x + 1)(x - 1)(x^2 + 1) = (x^2 - 1)(x^2 + 1)$

$= (x^2)^2 - 1 = x^4 - 1$

(ii) $(x- 3)(x + 3)(x^2 + 9)$

We know that, from formula,

$(a + b)(a - b) = a^2 - b^2$

$(x - 3)(x + 3)(x^2 + 9)$

$= (x^2 - 9)(x^2 + 9)$

$= (x^2)^2 - 9^2 = x^4 - 81$

(iii) $(3x - 2y)(3x + 2y)(9x^2 + 4y^2)$

We know that, from formula,

$(a + b)(a - b) = a^2 - b^2$

$(3x - 2y)(3x + 2y)(9x^2 + 4y^2)$

$= (9x^2 - 4y^2)(9x^2 + 4y^2)$

$= 81x^4 - 16y^4$

(iv) $(2p + 3)(2p - 3)(4p^2 + 9)$

We know that, from formula,

$(a + b)(a - b) = a^2 - b^2$

$(2p + 3)(2p - 3)(4p^2 + 9)$

$= (4p^2 - 9)(4p^2+ 9)$

$= (4p^2)^2 - 9^2 = 16p^4 - 81$

14. Question

If $x + y = 12$ and $xy = 14$, find the value of $(x^2 + y^2)$.

Answer

Given,

$x + y = 12$

Let's square the both sides,

We get;

$= (x + y)^2 = (12)^2$

$= x^2 + y^2 + 2xy = 144$

$= x^2 + y^2 = 144 - 2xy$

Also given,

$xy = 14$

$= x^2 + y^2 = 144 - 2(14)$

$= x^2 + y^2 = 144 - 28$

$= x^2 + y^2 = 116$

So, the value of $(x^2 + y^2)$ is 116.

15. Question

If $x - y = 7$ and $xy = 9$, find the value of $(x^2 + y^2)$.

Answer

$x - y = 7$ (given)

By squaring both the sides we get;

$= (x - y)^2 = (7)^2$

$= x^2 + y^2 - 2xy = 49$

$= x^2 + y^2 = 49 + 2xy$

Also given,

$xy = 9$

$= x^2 + y^2 = 49 + 2(9)$

$= x^2 + y^2 = 49 + 18$

$= x^2 + y^2 = 67$

So, the value of $x^2 + y^2$ is 67.

Exercise 6E

1. Question

The sum of (6a + 4b – c + 3), (2b – 3c + 4), (11b – 7a + 2c - 1) and (2c – 5a - 6) is

A. $(4a - 6b + 2)$

B. $(-3a + 14b - 3c + 2)$

C. $(-6a + 17b)$

D. $(-6a + 6b + c - 4)$

Answer

6a + 4b – c + 3
+ 2b – 3c + 4
-7a+ 11b +2c – 1
-5a + 2c – 6
- 6a + 17 b

2. Question

(3q + $7p^2$ – $2r^3$ + 4) – (4p^2 – 2q + $7r^3$ – 3) = ?

A. $(p^2 + 2q + 5r^2 + 1)$

B. $(11p^2 + q + 5r^3 + 1)$

C. $(-3p^2 - 5q + 9r^3 - 7)$

D. $(3p^2 + 5q - 9r^3 + 7)$

Answer

$(3q + 7p^2 - 2r^3 + 4) - (4p^2 - 2q + 7r^3 - 3) = ?$

After solving the bracket,

we get,

= 3q + $7p^2$ – $2r^3$ + 4 – 4p^2 + 2q – $7r^3$ + 3 = $7p^2$ – 4p^2 + 3q + 2q – $2r^3$ – $7r^3$ + 3 + 4

= 3p^2 + 5q – $9r^3$ + 7

3. Question

(x + 5) (x - 3) = ?

A. $x^2 + 5x - 15$ B. $x^2 - 3x - 15$

C. $x^2 + 2x + 15$ D. $x^2 + 2x - 15$

Answer

After solving the equation,

we get,

$(x + 5)(x - 3) = x(x - 3) + 5(x - 3)$

$= x^2 - 3x + 5x - 15$

$= x^2 + 2x - 15$

4. Question

$(2x + 3)(3x - 1) = ?$

A. $(6x^2 + 8x - 3)$ B. $(6x^2 + 7x - 3)$

C. $6x^2 - 7x - 3$ D. $(6x^2 - 7x + 3)$

Answer

After solving the equations,

we get,

$(2x + 3)(3x - 1) = 2x(3x - 1) + 3(3x - 1)$

$= 6x^2 - 2x + 9x - 3$

$= 6x^2 + 7x - 3$

5. Question

$(x + 4)(x + 4) = ?$

A. $(x^2 + 16)$ B. $(x^2 + 4x + 16)$

C. $(x^2 + 8x + 16)$ D. $(x^2 + 16x)$

Answer

We know that,

$(x + 4)(x + 4) = (x + 4)^2$

From formula, $(a + b)^2 = a^2 + b^2 + 2ab$

$(x + 4)^2 = x^2 + 4^2 + 2 \times x \times 4$

$= x^2 + 8x + 16$

6. Question

$(x - 6)(x - 6) = ?$

A. $(x^2 - 36)$ B. $(x^2 + 36)$

C. $(x^2 - 6x + 36)$ D. $(x^2 - 12x + 36)$

Answer

(x - 6)(x - 6)By component wise multiplication= x(x - 6) - 6(x - 6) (**from above we can see that,** $x.x = x^2$, $x.(-6) = -6x$, $-6.x = -6x$, and $-6.-6 = +36$)

$= x^2 - 6x - 6x + 36$

$= x^2 - 12x + 36$

Note: Multiplication of signs is given by-

(+) × (+) = +

(+) × (-) = -

(-)×(+)= -

(-) × (-) = +

7. Question

(2x + 5)(2x - 5) = ?

A. $(4x^2 + 25)$ B. $(4x^2 - 25)$

C. $(4x^2 - 10x + 25)$ D. $(4x^2 + 10x - 25)$

Answer

We know that,

From formula, $(a + b)(a - b) = a^2 - b^2$

$(2x + 5)(2x - 5) = (2x)^2 - (5)^2$

$= 4x^2 - 25$

8. Question

$8a^2b^3 \div (-2ab)$ = ?

A. $4ab^2$ B. $4a^2b$

C. $-4ab^2$ D. $-4a^2b$

Answer

If we divide $8a^2b^3$ by $(-2ab)$we get;

$= \left(\frac{8}{-2}\right)(a^{2-1})(b^{3-1})$

$= -4ab^2$

9. Question

$(2x^2 + 3x + 1) \div (x + 1) = ?$

A. $(x+1)$ B. $(2x+1)$

C. $(x+3)$ D. $(2x+3)$

Answer

By dividing $(2x^2 + 3x + 1)$ by $(x + 1)$

We get;

$$
\begin{array}{r}
2x+1 \\
x+1\overline{)2x^2+3x+1} \\
2x^2+2x \\
- \quad - \\
\hline
+1x+1 \\
+1x+1 \\
- \quad - \\
\hline
\times \\
\hline
\end{array}
$$

10. Question

$(x^2 - 4x + 4) \div (x - 2) = ?$

A. $(x-2)$ B. $(x+2)$

C. $(2-x)$ D. $(2+x+x^2)$

Answer

By dividing $(x^2 - 4x + 4)$ by $(x - 2)$

We get;

$$
\begin{array}{r}
x-2 \\
x-2\overline{)x^2-4x+4} \\
x^2-2x \\
- \quad + \\
\hline
-2x+4 \\
-2x+4 \\
+ \quad - \\
\hline
\times \\
\hline
\end{array}
$$

11. Question

$(a + 1)(a - 1)(a^2 + 1) = ?$

A. $\left(a^4-2a^2-1\right)$ B. $\left(a^4-a^2-1\right)$

C. $\left(a^4-1\right)$ D. $\left(a^4+1\right)$

Answer

We know that,

From formula, $(a + b)(a – b) = a^2 – b^2$

$(a + 1)(a – 1)(a^2 + 1) = (a^2 – 1)(a^2+ 1)$

Again applying the formula,

$(a^2 – 1)(a^2+ 1) = (a^2)^2 – (1^2)^2 = a^4 – 1$

12. Question

$\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)=?$

A. $\left(\frac{1}{x^2}-\frac{1}{y^2}\right)$ B. $\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$

C. $\left(\frac{1}{x^2}+\frac{1}{y^2}-\frac{1}{xy}\right)$ D. $\left(\frac{1}{x^2}-\frac{1}{y^2}+\frac{1}{xy}\right)$

Answer

We know that,

From formula, $(a + b)(a – b) = a^2 – b^2$

$$\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)=\left(\frac{1}{x}\right)^2-\left(\frac{1}{y}\right)^2=\frac{1}{x^2}-\frac{1}{y^2}$$

13. Question

If $\left(x+\frac{1}{x}\right)=5$, then $\left(x^2+\frac{1}{x^2}\right)=?$

A. 25 B. 27

C. 23 B. $25\frac{1}{25}$

Answer

We know that,

From formula, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x} = x^2+\frac{1}{x^2}+2$$

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 \text{ .....(i)}$$

And $x+\frac{1}{x}=5$, given,

Putting value of $x+\frac{1}{x}$ in equation (i), we get,

$(5)^2 = x^2+\frac{1}{x^2}+2$

$x^2+\frac{1}{x^2}$ = 25 – 2 = 23.

14. Question

If $\left(x-\frac{1}{x}\right)=6$, then $\left(x^2+\frac{1}{x^2}\right)$ = ?

A. 36 B. 38

C. 32 D. $36\frac{1}{36}$

Answer

We know that,

From formula, $(a-b)^2 = a^2+b^2-2ab$

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x} = x^2+\frac{1}{x^2}-2$$

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2 \text{ .....(i)}$$

And $x-\frac{1}{x}=6$, given,

Putting value of $x-\frac{1}{x}$ in equation (i), we get,

$(6)^2 = x^2+\frac{1}{x^2}-2$

$x^2+\frac{1}{x^2}$ = 36+ 2= 38.

15. Question

$(82)^2 - (18)^2$ = ?

A. 8218 B. 6418

C. 6400 D. 7204

Answer

$(82)^2 - (18)^2$

By using $(a - b)(a + b) = a^2 - b^2$

$= (82 - 18)(82 + 18)$

$= (64)(100)$

$= 6400$

16. Question

$(197 \times 203) = ?$

A. 39991 B. 39999

C. 40009 D. 40001

Answer

We can write following problem such as,

$(197 \times 203) = (200 - 3)(200 + 3)$

From the formula, $(a + b)(a - b) = a^2 - b^2$

We get,

$(200 - 3)(200 + 3) = 200^2 - 3^2 = 40000 - 9 = 39991.$

17. Question

If $(a + b) = 12$ and $ab = 14$, then $(a^2 + b^2) = ?$

A.172

B. 116

C. 162

D. 126

Answer

From the formula,

$(a + b)^2 = a^2 + b^2 + 2ab$

We get,

$= a + b = 12$ and $ab = 14$

By putting values, we get,

$12^2 = a^2 + b^2 + 2 \times 14$

$= a^2 + b^2 = 144 - 28 = 116.$

18. Question

If (a - b) = 7 and ab = 9, then ($a^2 + b^2$) = ?

A. 67 B. 31

C. 40 D. 58

Answer

From the formula,

$(a - b)^2 = a^2 + b^2 - 2ab$

We get,

= a – b = 7 and ab = 9

By putting values, we get,

$7^2 = a^2 + b^2 - 2\times 9$

$= a^2 + b^2 = 49 + 18 = 67$

19. Question

If x = 10, then find the value of ($4x^2 + 20x + 25$).

A. 256 B. 425

C. 625 D. 575

Answer

($4x^2 + 20x + 25$)

By using $(a + b)^2 = a^2 + b^2 + 2ab$,

We get;

$= (2x)^2 + (5)^2 + 2(2x)(5)$

$= (2x + 5)^2$

$= (2(10) + 5)^2$

$= (20 + 5)^2$

$= (25)^2$

= 625