Exponents

Exercise 5A

$a \times a \times a \times a \dots \times a$
 $\leftarrow$ n times $\rightarrow$
 $-5^4 = -5 \times -5 \times -5 \times -5 = 625$

a^n $\rightarrow$ Power
 $\swarrow$ Base

$a^0 = 1$
 $a^1 = a$

where 'a' is a non zero rational number

Standard Form

$\underbrace{a} \times \underbrace{10^b}$
 where integer
 $1 \leq a < 10$ power of 10

$79,345 = 7.9345 \times 10^4$

Negative Exponents

a^{-n} is the reciprocal of a^n

$a^{-n} = \frac{1}{a^n}$
 $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Rules of Exponents or Laws of Exponents	
Multiplication Rule	$a^x \times a^y = a^{x+y}$
Division Rule	$a^x \div a^y = a^{x-y}$
Power of a Power Rule	$(a^x)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Exponent Code	Multiplication
2^3	$2 \cdot 2 \cdot 2$ (= 8)
3^4	$3 \cdot 3 \cdot 3 \cdot 3$ (= 81)
5^3	$5 \cdot 5 \cdot 5$ (= 125)
10^3	$10 \cdot 10 \cdot 10$ (= 1,000)
x^3	$x \cdot x \cdot x$ or (xxx)
x^4	$x \cdot x \cdot x \cdot x$ or (xxxx)

Q1

Answer :

$$(i) \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} = \left(\frac{5}{7}\right)^4$$

$$(ii) \left(\frac{-4}{3}\right) \times \left(\frac{-4}{3}\right) \times \left(\frac{-4}{3}\right) \times \left(\frac{-4}{3}\right) \times \left(\frac{-4}{3}\right) = \left(\frac{-4}{3}\right)^5$$

$$(iii) \left(\frac{-1}{6}\right) \times \left(\frac{-1}{6}\right) \times \left(\frac{-1}{6}\right) = \left(\frac{-1}{6}\right)^3$$

$$(iv) (-8) \times (-8) \times (-8) \times (-8) \times (-8) = (-8)^5$$

Q2

Answer :

$$(i) \frac{25}{36} = \frac{5^2}{6^2} \quad [\text{since } 25 = 5^2 \text{ and } 36 = 6^2]$$

$$= \left(\frac{5}{6}\right)^2$$

$$(ii) \frac{-27}{64} = \frac{(-3)^3}{4^3} \quad [\text{since } -27 = (-3)^3 \text{ and } 64 = 4^3]$$

$$= \left(\frac{-3}{4}\right)^3$$

$$(iii) \frac{-32}{243} = \frac{(-2)^5}{3^5} \quad [\text{since } -32 = (-2)^5 \text{ and } 243 = 3^5]$$

$$= \left(\frac{-2}{3}\right)^5$$

$$(iv) \frac{-1}{128} = \frac{(-1)^7}{2^7} \quad [\text{since } (-1)^7 = -1 \text{ and } 128 = 2^7]$$

$$= \left(\frac{-1}{2}\right)^7$$

Q3

Answer :

$$(i) \left(\frac{2}{3}\right)^5 = \frac{(2)^5}{(3)^5} = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = \frac{32}{243}$$

$$(ii) \left(\frac{-8}{5}\right)^3 = \frac{(-8)^3}{(5)^3} = \frac{(-8) \times (-8) \times (-8)}{5 \times 5 \times 5} = \frac{-512}{125}$$

$$(iii) \left(\frac{-13}{11}\right)^2 = \frac{(-13)^2}{(11)^2} = \frac{(-13) \times (-13)}{11 \times 11} = \frac{169}{121}$$

$$(iv) \left(\frac{1}{6}\right)^3 = \frac{(1)^3}{(6)^3} = \frac{1 \times 1 \times 1}{6 \times 6 \times 6} = \frac{1}{216}$$

$$(v) \left(\frac{-1}{2}\right)^5 = \frac{(-1)^5}{(2)^5} = \frac{(-1) \times (-1) \times (-1) \times (-1) \times (-1)}{2 \times 2 \times 2 \times 2 \times 2} = \frac{-1}{32}$$

$$(vi) \left(\frac{-3}{2}\right)^4 = \frac{(-3)^4}{(2)^4} = \frac{(-3) \times (-3) \times (-3) \times (-3)}{2 \times 2 \times 2 \times 2} = \frac{81}{16}$$

$$(vii) \left(\frac{-4}{7}\right)^3 = \frac{(-4)^3}{(7)^3} = \frac{(-4) \times (-4) \times (-4)}{7 \times 7 \times 7} = \frac{-64}{343}$$

$$(viii) (-1)^9 = -1 \quad [\text{Since } (-1) \text{ an odd natural number} = -1]$$

Q4

Answer :

$$(i) (4)^{-1} = \left(\frac{4}{1}\right)^{-1} = \left(\frac{1}{4}\right)^1 = \frac{1}{4} \quad \left[\text{since } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$(ii) (-6)^{-1} = \left(\frac{-6}{1}\right)^{-1} = \left(\frac{1}{-6}\right)^1 = \frac{-1}{6} \quad \left[\text{since } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$(iii) \left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3 \quad \left[\text{since } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$(iv) \left(\frac{-2}{3}\right)^{-1} = \left(\frac{3}{-2}\right)^1 = \frac{-3}{2} \quad \left[\text{since } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

Q5

Answer :

We know that the reciprocal of $\left(\frac{a}{b}\right)^m$ is $\left(\frac{b}{a}\right)^m$.

$$(i) \text{ Reciprocal of } \left(\frac{3}{8}\right)^4 = \left(\frac{8}{3}\right)^4$$

$$(ii) \text{ Reciprocal of } \left(\frac{-5}{6}\right)^{11} = \left(\frac{-6}{5}\right)^{11}$$

$$(iii) \text{ Reciprocal of } 6^7 = \text{Reciprocal of } \left(\frac{6}{1}\right)^7 = \left(\frac{1}{6}\right)^7$$

$$(iv) \text{ Reciprocal of } (-4)^3 = \text{Reciprocal of } \left(\frac{-4}{1}\right)^3 = \left(\frac{-1}{4}\right)^3$$

Q6

Answer :

$$(i) 8^0 = 1$$

$$(ii) (-3)^0 = 1$$

$$(iii) 4^0 + 5^0 = 1 + 1 = 2$$

$$(iv) 6^0 \times 7^0 = 1 \times 1 = 1$$

Note: $a^0 = 1$

Q7

Answer :

$$(i) \left(\frac{3}{2}\right)^4 \times \left(\frac{1}{5}\right)^2 = \frac{3^4}{2^4} \times \frac{1^2}{5^2} = \frac{81 \times 1}{16 \times 25} = \frac{81}{400}$$

$$(ii) \left(\frac{-2}{3}\right)^5 \times \left(\frac{-3}{7}\right)^3 = \frac{(-2)^5}{(3)^5} \times \frac{(-3)^3}{(7)^3} \\ = \frac{(-2)^5}{(7)^3} \times \frac{(-1)(3)^3}{(3)^5} \quad \left[\text{since } 3^{-2} = \frac{1}{9}\right] \\ = \frac{-32 \times -1 \times 3^{3-5}}{343} \\ = \frac{-32 \times -1 \times 3^{-2}}{343} \\ = \frac{-32 \times -1 \times 1}{343 \times 9} \\ = \frac{32}{3087}$$

$$(iii) \left(\frac{-1}{2}\right)^5 \times 2^3 \times \left(\frac{3}{4}\right)^2 = \frac{(-1)^5}{2^5} \times 2^3 \times \frac{3^2}{4^2} \\ = \frac{(-1)^5}{2^5} \times 2^3 \times \frac{3^2}{(2^2)^2} \\ = \frac{-1 \times 2^3 \times 3^2}{2^5 \times 2^4} \\ = \frac{-1 \times 2^2 \times 3^2}{2^9} = -1 \times 2^{3-9} \times 3^2 = -9 \times 2^{-6} = \frac{-9}{2^6} = \frac{-9}{64}$$

$$\left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right]$$

$$(iv) \left(\frac{2}{3}\right)^2 \times \left(\frac{-3}{5}\right)^3 \times \left(\frac{7}{2}\right)^2 = \frac{2^2}{3^2} \times \frac{(-3)^3}{5^3} \times \frac{7^2}{2^2} \\ = \frac{-1 \times 3^{3-2} \times 7^2}{5^3} = \frac{-1 \times 3^1 \times 7^2}{125} = \frac{-147}{125}$$

$$\begin{aligned}
 \text{(v)} \left\{ \left(\frac{-3}{4} \right)^3 - \left(\frac{-5}{2} \right)^3 \right\} \times 4^2 &= \left\{ \left(\frac{-3^3}{4^3} \right) - \left(\frac{-5^3}{2^3} \right) \right\} \times 4^2 \\
 &= \left\{ \left(\frac{-27}{64} \right) - \left(\frac{-125}{8} \right) \right\} \times 16 \\
 &= \left\{ \frac{-27}{64} + \frac{125}{8} \right\} \times 16 \\
 &= \left(\frac{-27+1000}{64} \right) \times 16 \\
 &= \left(\frac{973}{64} \times 16 \right) = \frac{973}{4}
 \end{aligned}$$

Q8

Answer :

$$\begin{aligned}
 \text{(i)} \left(\frac{4}{9} \right)^6 \times \left(\frac{4}{9} \right)^{-4} &= \left(\frac{4}{9} \right)^{6+(-4)} && [\text{since } a^n \times a^m = a^{n+m}] \\
 &= \left(\frac{4}{9} \right)^2 = \frac{(4)^2}{(9)^2} = \frac{4 \times 4}{9 \times 9} = \frac{16}{81}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \left(\frac{-7}{8} \right)^{-3} \times \left(\frac{-7}{8} \right)^2 &= \left(\frac{-7}{8} \right)^{(-3)+2} && [\text{since } a^n \times a^m = a^{n+m}] \\
 &= \left(\frac{-7}{8} \right)^{-1} \\
 &= \left(\frac{8}{-7} \right)^1 && \left[\text{since } \left(\frac{a}{b} \right)^{-1} = \left(\frac{b}{a} \right)^1 \right] \\
 &= \left(\frac{8 \times -1}{-7 \times -1} \right) = \frac{-8}{7}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \left(\frac{4}{3} \right)^{-3} \times \left(\frac{4}{3} \right)^{-2} &= \left(\frac{4}{3} \right)^{(-3)+(-2)} && [\text{since } a^n \times a^m = a^{n+m}] \\
 &= \left(\frac{4}{3} \right)^{-5} \\
 &= \left(\frac{3}{4} \right)^5 && \left[\text{since } \left(\frac{a}{b} \right)^{-1} = \left(\frac{b}{a} \right)^1 \right] \\
 &= \frac{(3)^5}{(4)^5} = \frac{3 \times 3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4 \times 4} = \frac{243}{1024}
 \end{aligned}$$

Q9

$$\text{(i)} 5^{-3} = \left(\frac{5}{1} \right)^{-3} = \left(\frac{1}{5} \right)^3 = \frac{(1)^3}{(5)^3} = \frac{1}{125}$$

$$\text{(ii)} (-2)^{-5} = \left(\frac{-2}{1} \right)^{-5} = \left(\frac{1}{-2} \right)^5 = \frac{(1)^5}{(-2)^5} = \frac{1 \times -1}{-32 \times -1} = \frac{-1}{32}$$

$$\text{(iii)} \left(\frac{1}{4} \right)^{-4} = \left(\frac{4}{1} \right)^4 = \frac{(4)^4}{(1)^4} = \frac{256}{1} = 256$$

$$\text{(iv)} \left(\frac{-3}{4} \right)^{-3} = \left(\frac{4}{-3} \right)^3 = \frac{(4)^3}{(-3)^3} = \frac{64}{-27} = \frac{64 \times -1}{-27 \times -1} = \frac{-64}{27}$$

$$\text{(v)} (-3)^{-1} \times \left(\frac{1}{3} \right)^{-1} = \left(\frac{1}{-3} \right)^1 \times \left(\frac{3}{1} \right)^1 = \left(\frac{1 \times 3}{-3 \times 1} \right)^1 = \left(\frac{3}{-3} \right)^1 = \frac{1}{-1} = \frac{1 \times -1}{-1 \times -1} = \frac{-1}{1} = -1$$

$$\text{(vi)} \left(\frac{5}{7} \right)^{-1} \times \left(\frac{7}{4} \right)^{-1} = \left(\frac{7}{5} \right)^1 \times \left(\frac{4}{7} \right)^1 = \left(\frac{7 \times 4}{5 \times 7} \right)^1 = \frac{4}{5}$$

$$\begin{aligned} \text{(vii)} \quad (5^{-1} - 7^{-1})^{-1} &= \left(\frac{1}{5} - \frac{1}{7}\right)^{-1} = \left(\frac{7-5}{35}\right)^{-1} \\ &= \left(\frac{2}{35}\right)^{-1} = \left(\frac{35}{2}\right)^1 = \frac{35}{2} \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad \left\{ \left(\frac{4}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} \right\}^{-1} &= \left\{ \left(\frac{3}{4}\right)^1 - \left(\frac{4}{1}\right)^1 \right\}^{-1} = \left(\frac{3}{4} - \frac{4}{1}\right)^{-1} \\ &= \left(\frac{3-16}{4}\right)^{-1} = \left(\frac{-13}{4}\right)^{-1} \\ &= \left(\frac{4}{-13}\right)^1 = \left(\frac{4 \times -1}{-13 \times -1}\right) \\ &= \frac{-4}{13} \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad \left\{ \left(\frac{3}{2}\right)^{-1} \div \left(\frac{-2}{5}\right)^{-1} \right\} &= \left\{ \left(\frac{2}{3}\right)^1 \div \left(\frac{5}{-2}\right)^1 \right\} \\ &= \left(\frac{2}{3} \times \frac{-2}{5}\right) \\ &= \frac{-4}{15} \end{aligned}$$

$$\text{(x)} \quad \left(\frac{23}{25}\right)^0 = 1 \quad [\text{since } a^0 = 1 \text{ for every integer } a]$$

Q10

Answer :

(i)

$$\begin{aligned} \left[\left\{ \left(-\frac{1}{4}\right)^2 \right\}^{-2} \right]^{-1} &= \left[\left(-\frac{1}{4}\right)^{2 \times -2} \right]^{-1} \quad \left[\text{since } \left\{ \left(\frac{a}{b}\right)^m \right\}^n = \left(\frac{a}{b}\right)^{mn} \right] \\ &= \left[\left(-\frac{1}{4}\right)^{-4} \right]^{-1} \\ &= \left(-\frac{1}{4}\right)^{(-4) \times (-1)} \\ &= \left(-\frac{1}{4}\right)^4 = \frac{(-1)^4}{(4)^4} \\ &= \frac{1}{256} \end{aligned}$$

(ii)

$$\begin{aligned} \left\{ \left(\frac{-2}{3}\right)^2 \right\}^3 &= \left(\frac{-2}{3}\right)^{2 \times 3} \quad \left[\text{since } \left\{ \left(\frac{a}{b}\right)^m \right\}^n = \left(\frac{a}{b}\right)^{mn} \right] \\ &= \left(\frac{-2}{3}\right)^6 \\ &= \frac{(-2)^6}{(3)^6} = \frac{64}{729} \quad [\text{since } (-2)^6 = 64 \text{ and } (3)^6 = 729] \end{aligned}$$

(iii)

$$\begin{aligned} \left(\frac{-3}{2}\right)^3 \div \left(\frac{-3}{2}\right)^6 &= \left(\frac{-3}{2}\right)^{3-6} \quad [\text{since } a^m \div a^n = a^{m-n}] \\ &= \left(\frac{-3}{2}\right)^{-3} \\ &= \left(\frac{2}{-3}\right)^3 \quad \left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1 \right] \\ &= \left(\frac{2 \times -1}{-3 \times -1}\right)^3 = \left(\frac{-2}{3}\right)^3 \\ &= \frac{(-2)^3}{(3)^3} = \frac{-8}{27} \end{aligned}$$

(iv)

$$\begin{aligned} \left(\frac{-2}{3}\right)^7 \div \left(\frac{-2}{3}\right)^4 &= \left(\frac{-2}{3}\right)^{7-4} \quad [\text{since } a^m \div a^n = a^{m-n}] \\ &= \left(\frac{-2}{3}\right)^3 \\ &= \frac{(-2)^3}{(3)^3} = \frac{-8}{27} \end{aligned}$$

Q11

Answer :

Let the required number be x .

$$(-5)^{-1} \times x = (8)^{-1}$$

$$\Rightarrow \frac{1}{-5} \times x = \frac{1}{8}$$

$$\therefore x = \frac{1}{8} \times (-5) = \frac{-5}{8}$$

Hence, the required number is $\frac{-5}{8}$.

Q12

Answer :

Let the required number be x .

$$(3)^{-3} \times x = 4$$

$$\Rightarrow \frac{1}{3^3} \times x = 4$$

$$\Rightarrow \frac{1}{27} \times x = 4$$

$$\therefore x = 4 \times 27 = 108$$

Hence, the required number is 108.

Q13

Answer :

Let the required number be x .

$$(-30)^{-1} \div x = 6^{-1}$$

$$\Rightarrow \frac{1}{(-30)} \times \frac{1}{x} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{(-30x)} = \frac{1}{6}$$

$$\therefore x = \frac{6}{(-30)} = \frac{1}{-5} \\ = \frac{-1}{5}$$

Hence, the required number is $\frac{-1}{5}$.

Q14

Answer :

$$\left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{3+(-6)} = \left(\frac{3}{5}\right)^{2x-1} \quad [\text{since } a^m \times a^n = a^{m+n}]$$

$$\Rightarrow \left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{2x-1}$$

On equating the exponents:

$$-3 = 2x - 1$$

$$\Rightarrow 2x = -3 + 1$$

$$\Rightarrow 2x = -2$$

$$\therefore x = \left(\frac{-2}{2}\right) = -1$$

Q15

Answer :

$$\frac{3^5 \times 10^8 \times 25}{5^7 \times 6^8} = \frac{3^5 \times (2 \times 5)^8 \times 5^2}{5^7 \times (2 \times 3)^8} \\ = \frac{3^5 \times 2^8 \times 5^5 \times 5^2}{5^7 \times 2^8 \times 3^8} \\ = \frac{3^5 \times 2^8 \times 5^7}{3^8 \times 2^8 \times 5^7} \\ = 3^{5-8} \times 2^{8-8} \times 5^{7-7} \\ = 3^0 \times 2^0 \times 5^0 \\ = 1 \times 1 \times 1 = 1$$

Q16

Answer :

$$\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+1}} \\ = \frac{2^4 \times 2^{n+1} - 2^2 \times 2^n}{2^4 \times 2^{n+2} - 2 \times 2^{n+1}} \\ = \frac{2^2 \times (2^{n+4} - 2^{n+1})}{2^2 \times (2^{n+4} - 2^{n+1})} \\ = \frac{2^n \times 2^3 - 2^n}{2^n \times 2^4 - 2^n \times 2} \\ = \frac{2^n (2^3 - 1)}{2^n (2^4 - 2)} = \frac{8-1}{16-2} = \frac{7}{14} = \frac{1}{2}$$

Q17

Answer :

$$(i) 5^{2n} \times 5^3 = 5^9$$
$$5^{2n+3} = 5^9 \quad [\text{since } a^n \times a^m = a^{m+n}]$$

On equating the coefficients:

$$2n + 3 = 9$$
$$\Rightarrow 2n = 9 - 3$$
$$\Rightarrow 2n = 6$$
$$\therefore n = \frac{6}{2} = 3$$

$$(ii) 8 \times 2^{n+2} = 32$$
$$\Rightarrow (2)^3 \times 2^{n+2} = (2)^5 \quad [\text{since } 2^3 = 8 \text{ and } 2^5 = 32]$$
$$\Rightarrow (2)^{3+(n+2)} = (2)^5$$

On equating the coefficients:

$$3 + n + 2 = 5$$
$$\Rightarrow n + 5 = 5$$
$$\Rightarrow n = 5 - 5$$
$$\therefore n = 0$$

$$(iii) 6^{2n+1} \div 36 = 6^3$$
$$\Rightarrow 6^{2n+1} \div 6^2 = 6^3 \quad [\text{since } 36 = 6^2]$$
$$\Rightarrow \frac{6^{2n+1}}{6^2} = 6^3$$
$$\Rightarrow 6^{2n+1-2} = 6^3 \quad [\text{since } \frac{a^m}{a^n} = a^{m-n}]$$
$$\Rightarrow 6^{2n-1} = 6^3$$

On equating the coefficients:

$$2n - 1 = 3$$
$$\Rightarrow 2n = 3 + 1$$
$$\Rightarrow 2n = 4$$
$$\therefore n = \frac{4}{2} = 2$$

Q18

Answer :

$$2^{n-7} \times 5^{n-4} = 1250$$
$$\Rightarrow \frac{2^n}{2^7} \times \frac{5^n}{5^4} = 2 \times 5^4 \quad [\text{since } 1250 = 2 \times 5^4]$$
$$\Rightarrow \frac{2^n \times 5^n}{2^7 \times 5^4} = 2 \times 5^4$$
$$\Rightarrow 2^n \times 5^n = 2 \times 5^4 \times 2^7 \times 5^4 \quad [\text{using cross multiplication}]$$
$$\Rightarrow 2^n \times 5^n = 2^{1+7} \times 5^{4+4} \quad [\text{since } a^m \times a^n = a^{m+n}]$$
$$\Rightarrow 2^n \times 5^n = 2^8 \times 5^8$$
$$\Rightarrow (2 \times 5)^n = (2 \times 5)^8 \quad [\text{since } a^n \times b^n = (a \times b)^n]$$
$$\Rightarrow 10^n = 10^8$$
$$\Rightarrow n = 8$$

Exponents

Exercise 5B

Q1

Answer :

- (i) $538 = 5.38 \times 10^2$ [since the decimal point is moved 2 places to the left]
- (ii) $6428000 = 6.428 \times 10^6$ [since the decimal point is moved 6 places to the left]
- (iii) $82934000000 = 8.2934 \times 10^{10}$ [since the decimal point is moved 10 places to the left]
- (iv) $940000000000 = 9.4 \times 10^{11}$ [since the decimal point is moved 11 places to the left]
- (v) $23000000 = 2.3 \times 10^7$ [since the decimal point is moved 7 places to the left]

Q2

Answer :

- (i) Diameter of the Earth = 1.2756×10^7 m
[since the decimal point is moved 7 places to the left]
- (ii) Distance between the Earth and the Moon = 3.84×10^8 m
[since the decimal point is moved 8 places to the left]
- (iii) Population of India in March 2001 = 1.027×10^9
[since the decimal point is moved 9 places to the left]
- (iv) Number of stars in a galaxy = 1.0×10^{11}
[since the decimal point is moved 11 places to the left]
- (v) Present age of the universe = 1.2×10^{10} years
[since the decimal point is moved 10 places to the left]

Q3

Answer :

- (i) $684502 = 6 \times 10^5 + 8 \times 10^4 + 4 \times 10^3 + 5 \times 10^2 + 0 \times 10^1 + 2 \times 10^0$
- (ii) $4007185 = 4 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 7 \times 10^3 + 1 \times 10^2 + 8 \times 10^1 + 5 \times 10^0$
- (iii) $5807294 = 5 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 7 \times 10^3 + 2 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$
- (iv) $50074 = 5 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$

Note: $a^0 = 1$

Q4

Answer :

- (i) $6 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 7 \times 10^1 + 8 \times 10^0$
 $= 6 \times 10000 + 3 \times 1000 + 0 \times 100 + 7 \times 10 + 8 \times 1 = 63078$
- (ii) $9 \times 10^6 + 7 \times 10^5 + 0 \times 10^4 + 3 \times 10^3 + 4 \times 10^2 + 6 \times 10^1 + 2 \times 10^0$
 $= 9 \times 1000000 + 7 \times 100000 + 0 \times 10000 + 3 \times 1000 + 4 \times 100 + 6 \times 10 + 2 \times 1 = 9703462$
- (iii) $8 \times 10^5 + 6 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 9 \times 10^1 + 6 \times 10^0$
 $= 8 \times 100000 + 6 \times 10000 + 4 \times 1000 + 2 \times 100 + 9 \times 10 + 6 \times 1 = 864296$

Exponents

Exercise 5C

Q1

Answer :

(d) 24

$$\begin{aligned}(6^{-1} - 8^{-1})^{-1} &= \left(\frac{1}{6} - \frac{1}{8}\right)^{-1} \\ &= \left(\frac{4-3}{24}\right)^{-1} \quad [\text{since L.C.M. of 6 and 8 is 24}] \\ &= \left(\frac{1}{24}\right)^{-1} \\ &= \left(\frac{24}{1}\right)^1 = 24 \quad \left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right]\end{aligned}$$

Q2

Answer :

(c) 15

We have:

$$\begin{aligned}(5^{-1} \times 3^{-1})^{-1} &= \left(\frac{1}{5} \times \frac{1}{3}\right)^{-1} \\ &= \left(\frac{1}{15}\right)^{-1} \\ &= \left(\frac{15}{1}\right)^1 = 15 \quad \left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right]\end{aligned}$$

Q3

Answer :

(c) $\frac{1}{16}$

We have:

$$\begin{aligned}(2^{-1} - 4^{-1})^2 &= \left(\frac{1}{2} - \frac{1}{4}\right)^2 \\ &= \left(\frac{2-1}{4}\right)^2 \quad [\text{since L.C.M. of 2 and 4 is 4}] \\ &= \left(\frac{1}{4}\right)^2 \\ &= \left(\frac{1}{4} \times \frac{1}{4}\right) = \frac{1}{16}\end{aligned}$$

Q4

Answer :

(b) 29

We have:

$$\begin{aligned}\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} &= \left(\frac{2}{1}\right)^2 + \left(\frac{3}{1}\right)^2 + \left(\frac{4}{1}\right)^2 && \left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right] \\ &= (2^2 + 3^2 + 4^2) \\ &= (4 + 9 + 16) \\ &= 29\end{aligned}$$

Q5

Answer :

(c) $\frac{6}{5}$

We have:

$$\begin{aligned}\left\{6^{-1} + \left(\frac{3}{2}\right)^{-1}\right\}^{-1} &= \left(\frac{1}{6} + \frac{2}{3}\right)^{-1} \\ &= \left(\frac{1+4}{6}\right)^{-1} && [\text{since L.C.M. of 3 and 6 is 6}] \\ &= \left(\frac{5}{6}\right)^{-1} \\ &= \left(\frac{6}{5}\right)^1 = \left(\frac{6}{5}\right) && \left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right]\end{aligned}$$

Q6

Answer :

(b) 64

We have:

$$\begin{aligned}\left(\frac{-1}{2}\right)^{-6} &= \left(\frac{2}{-1}\right)^6 && \left[\text{since } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right] \\ &= (-2)^6 \\ &= (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \\ &= 64\end{aligned}$$

Q7

Answer :

(b) $\frac{-3}{8}$

$$\begin{aligned}\left\{\left(\frac{3}{4}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} &= \left(\frac{4}{3} - \frac{4}{1}\right)^{-1} \\ &= \left(\frac{4-12}{3}\right)^{-1} && [\text{since L.C.M. of 1 and 3 is 3}] \\ &= \left(\frac{-8}{3}\right)^{-1} \\ &= \left(\frac{3}{-8}\right)^1 && \left[\text{since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right] \\ &= \left(\frac{3 \times -1}{-8 \times -1}\right) = \frac{-3}{8}\end{aligned}$$

Q8

Answer :

(a) $\frac{1}{16}$

$$\begin{aligned} \left[\left\{ \left(-\frac{1}{2} \right)^2 \right\}^{-2} \right]^{-1} &= \left[\left(-\frac{1}{2} \right)^{2 \times -2} \right]^{-1} && \left[\text{since } \left\{ \left(\frac{a}{b} \right)^m \right\}^n = \left(\frac{a}{b} \right)^{mn} \right] \\ &= \left[\left(-\frac{1}{2} \right)^{-4} \right]^{-1} \\ &= \left(-\frac{1}{2} \right)^{(-4) \times (-1)} \\ &= \left(-\frac{1}{2} \right)^4 = \frac{(-1)^4}{(2)^4} \\ &= \frac{1}{16} \end{aligned}$$

Q9

Answer :

(c) 1

(a) $0 = 1$

$\therefore \left(\frac{5}{6} \right)^0 = 1$

Q10

Answer :

(b) $\frac{243}{32}$

$$\begin{aligned} \left(\frac{2}{3} \right)^{-5} &= \left(\frac{3}{2} \right)^5 && \left[\text{since } \left(\frac{a}{b} \right)^{-n} = \left(\frac{b}{a} \right)^n \right] \\ &= \frac{3^5}{2^5} = \frac{3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 2} = \frac{243}{32} \end{aligned}$$

Q11

Answer :

(b) $\left(\frac{1}{3} \right)^8$

$$\left\{ \left(\frac{1}{3} \right)^2 \right\}^4 = \left(\frac{1}{3} \right)^{2 \times 4} = \left(\frac{1}{3} \right)^8 \quad \left[\text{since } \left\{ \left(\frac{a}{b} \right)^m \right\}^n = \left(\frac{a}{b} \right)^{mn} \right]$$

Q12

Answer :

(b) $\frac{-2}{3}$

We have:

$$\begin{aligned} \left(\frac{-3}{2} \right)^{-1} &= \left(\frac{2}{-3} \right)^1 && \left[\text{since } \left(\frac{a}{b} \right)^{-n} = \left(\frac{b}{a} \right)^n \right] \\ &= \frac{-2}{3} \end{aligned}$$

Q13

Answer :

(d) $\frac{135}{8}$

$$\begin{aligned} (3^2 - 2^2) \times \left(\frac{2}{3} \right)^{-3} &= (9 - 4) \times \left(\frac{3}{2} \right)^3 && \left[\text{since } \left(\frac{a}{b} \right)^{-1} = \left(\frac{b}{a} \right)^1 \right] \\ &= 5 \times \frac{3^3}{2^3} = 5 \times \frac{27}{8} = \frac{135}{8} \end{aligned}$$

Q14

Answer :

(a) $\frac{19}{64}$

We have:

$$\begin{aligned} \left\{ \left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3} \right\} \div \left(\frac{1}{4}\right)^{-3} &= \left\{ \left(\frac{3}{1}\right)^3 - \left(\frac{2}{1}\right)^3 \right\} \div \left(\frac{4}{1}\right)^3 \\ \left[\text{since } \left(\frac{a}{b}\right)^{-1} &= \left(\frac{b}{a}\right)^1 \right] \\ &= \left\{ (3^3) - (2^3) \right\} \div (4)^3 \\ &= (27 - 8) \div 64 \\ &= 19 \div 64 \\ &= 19 \times \frac{1}{64} = \frac{19}{64} \end{aligned}$$

Q15

Answer :

(c) $(-5)^5$

We have:

$$\begin{aligned} \left(\frac{-1}{5}\right)^3 \div \left(\frac{-1}{5}\right)^8 &= \left(\frac{-1}{5}\right)^{3-8} \quad [\text{since } a^m \div a^n = a^{m-n}] \\ &= \left(\frac{-1}{5}\right)^{-5} \\ &= \left(\frac{5}{-1}\right)^5 \quad \left[\text{Since } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1 \right] \\ &= \left(\frac{5 \times -1}{-1 \times -1}\right)^5 = \left(\frac{-5}{1}\right)^5 = (-5)^5 \end{aligned}$$

Q16

Answer :

(a) $\frac{4}{25}$

$$\begin{aligned} \left(\frac{-2}{5}\right)^7 \div \left(\frac{-2}{5}\right)^5 &= \left(\frac{-2}{5}\right)^{7-5} \quad [\text{since } a^m \div a^n = a^{m-n}] \\ &= \left(\frac{-2}{5}\right)^2 \\ &= \frac{(-2)^2}{(5)^2} = \frac{4}{25} \end{aligned}$$

Q17

Answer :

(c) $\frac{4}{9}$

$$\left(\frac{-2}{3}\right)^2 = \frac{-2}{3} \times \frac{-2}{3} = \frac{4}{9}$$

Q18

Answer :

(b) $\frac{-1}{8}$

We have:

$$\left(\frac{-1}{2}\right)^3 = \frac{-1}{2} \times \frac{-1}{2} \times \frac{-1}{2} = \frac{-1}{8}$$

Q19

Answer :

(c) $\frac{3}{4}$

$$\begin{aligned} \left(\frac{5}{3}\right)^{-5} \times \left(\frac{5}{3}\right)^{11} &= \left(\frac{5}{3}\right)^{8x} \\ \Rightarrow \left(\frac{5}{3}\right)^{-5+11} &= \left(\frac{5}{3}\right)^{8x} \quad [\text{since } a^m \times a^n = a^{m+n}] \\ \Rightarrow \left(\frac{5}{3}\right)^6 &= \left(\frac{5}{3}\right)^{8x} \end{aligned}$$

On equating the coefficients:

$$6 = 8x$$

$$\therefore x = \frac{6}{8} = \frac{3}{4}$$

Q20

Answer :

(c) $\frac{-4}{5}$

Let the required number be x .

$$(-8)^{-1} \times x = (10)^{-1}$$

$$\Rightarrow \frac{1}{-8} \times x = \frac{1}{10}$$

$$\therefore x = \frac{1}{10} \times (-8) = \frac{-4}{5}$$

Hence, the required number is $\frac{-4}{5}$.

Q21

Answer :

(c) 2.156×10^6

A given number is said to be in standard form if it can be expressed as $k \times 10^n$, where k is a real number such that $1 \leq k < 10$ and n is a positive integer.

For example: 2.156×10^6

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