2. Polynomials

Exercise 2A

1. Question

Which of the following expressions are polynomials?

(i) $x^5 - 2x^3 + x + 7$

(ii) $y^3 - \sqrt{3}y$

(iii) $t^2 - \frac{2}{5}t + \sqrt{2}$

(iv) $5\sqrt{z} - 6$

(v) $x - \frac{1}{x}$

(vi) $x^{108} - 1$

(vii) $\sqrt[3]{x} - 27$

(viii) $\frac{1}{\sqrt{2}}x^2 - \sqrt{2x} + 2$

(ix) $x^{-2} + 2x^{-1} + 3$

(x) 1

(xi) $-\frac{3}{5}$

(xii) $\sqrt[3]{2}y^2 - 8$

In case of a polynomial, write its degree.

Answer

(i) $x^5 - 2x^3 + x + 7$

Yes,

The given expression is a polynomial

This is because all the variables have integer exponents that are positive.

Since, the highest power of the variable is 5.

Hence, the degree of the polynomial is 5.

(ii) $y^3 - \sqrt{3}y$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 3

Hence, the degree of the polynomial is 3

(iii) $t^2 - \frac{2}{5}t + \sqrt{2}$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 2

Hence, the degree of the polynomial is 2

(iv) $5\sqrt{z} - 6$

No,

The given expression is not a polynomial

Since, the term has a fractional exponent.

(v) $x - \frac{1}{x}$

No,

The given expression is not a polynomial

Since, the term has a negative exponent.

(vi) $x^{108} - 1$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 108

Hence, the degree of the polynomial is 108

(vii) $\sqrt[3]{x} - 27$

No,

The given expression is not a polynomial

Since, the term has a fractional exponent.

(viii) $\frac{1}{\sqrt{2}}x^2-\sqrt{2x}+2$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 2

Hence, the degree of the polynomial is 2

(ix) $x^{-2}+2x^{-1}+3$

No,

The given expression is not a polynomial

Since, the term has a negative exponent.

(x) 1

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 0

Hence, the degree of the polynomial is 0

(xi) $-\frac{3}{5}$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 0

Hence, the degree of the polynomial is 0

(xii) $\sqrt[3]{2}y^2-8$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 2

Hence, the degree of the polynomial is 2

2. Question

Write the degree of each of the following polynomials:

(i) $2x-\sqrt{5}$

(ii) $3-x+x^2-6x^3$

(iii) 9

(iv) $8x^4-36x+5x^7$

(v) $x^9-x^5+3x^{10}+8$

(vi) $2-3x^2$

Answer

(i) $2x-\sqrt{5}$

Since,

In the given polynomial, the highest power of the variable is 1

Hence,

The degree of the polynomial is 1

(ii) $3-x+x^2-6x^3$

Since,

In the given polynomial, the highest power of the variable is 3

Hence,

The degree of the polynomial is 3

(iii) 9

Since,

In the given polynomial, the highest power of the variable is 0

Hence,

The degree of the polynomial is 0

(iv) $8x^4-36x+5x^7$

Since,

In the given polynomial, the highest power of the variable is 7

Hence,

The degree of the polynomial is 7

(v) $x^9 - x^5 + 3x^{10} + 8$

Since,

In the given polynomial, the highest power of the variable is 10

Hence,

The degree of the polynomial is 10

(vi) $2 - 3x^2$

Since,

In the given polynomial, the highest power of the variable is 2

Hence,

The degree of the polynomial is 2

3. Question

Write:

(i) Coefficient of x^3 in $2x + x^2 - 5x^3 + x^4$

(ii) Coefficient of x in $\sqrt{3} - 2\sqrt{2x} + 4x^2$

(iii) Coefficient of x^2 in $\frac{\pi}{3}x^2 + 7x - 3$

(iv) Coefficient of x^2 in $3x - 5$

Answer

(i) $2x + x^2 - 5x^3 + x^4$

Hence,

The Coefficient of x^3 in the given polynomial is -5

(ii) $\sqrt{3} - 2\sqrt{2x} + 4x^2$

Hence,

The Coefficient of x in the given polynomial is -22

(iii) $\frac{\pi}{3}x^2 + 7x - 3$

Hence,

The Coefficient of x^2 in the given polynomial is $\frac{\pi}{3}$

(iv) 3x – 5

Since,

There isn't any variable with exponent as 2

Hence,

The Coefficient of x^2 in the given polynomial, is 0

4 A. Question

Give an example of a binomial of degree 27.

Answer

An example of a binomial of degree 27 is a two-term polynomial with highest degree 27.

Hence,

The suitable example for the question can be $y^{27} - 29$.

4 B. Question

Give an example of a monomial of degree 16.

Answer

An example of a monomial of degree 16 is a single term polynomial with highest degree 16.

Hence,

The suitable example for the question can be y^{16}

4 C. Question

Give an example of a trinomial of degree 3.

Answer

An example of a trinomial of degree is a three-term polynomial with highest degree 3.

Hence,

The suitable example for the question can be $y^3 - y^2 + 29$

5. Question

Classify the following as linear, quadratic and cubic polynomials:

(i) $2x^2 + 4x$

(ii) $x - x^3$

(iii) $2 - y - y^2$

(iv) -7+z

(v) $5t$ (vi) p^3

Answer

(i) $2x^2+4x$

Since,

The degree of the given polynomial is 2

Hence,

The polynomial is a quadratic polynomial.

(ii) $x-x^3$

Since,

The degree of the given polynomial is 3

Hence,

The polynomial is a cubic polynomial.

(iii) $2-y-y^2$

Since,

The degree of the given polynomial is 2

Hence,

The polynomial is a quadratic polynomial.

(iv) -7+z

Since,

The degree of the given polynomial is 1

Hence,

The polynomial is a linear polynomial.

(v) $5t$

Since,

The degree of the given polynomial is 1

Hence,

The polynomial is a linear polynomial.

(vi) p^3

Since,

The degree of the given polynomial is 3

Hence,

The polynomial is a cubic polynomial.

Exercise 2B

1. Question

If $p(x) = 5 - 4x + 2x^2$, find

(i) p (0)

(ii)p(3)

(iii) p(-2)

Answer

(i) We have,

$p(x) = 5 - 4x + 2x^2$,

Now,

Put x = 0

$p(0) = 5 - 4(0) + 2(0)^2$

= 5 - 4(0) + 2(0)

= 5 − 0 + 0

= 5

Hence,

P(0) = 5

(ii) We have,

$p(x) = 5 - 4x + 2x^2$,

Now,

Put x = 3

$p(3) = 5 - 4(3) + 2(3)^2$

= 5 - 4(3) + 2(9)

= 5 − 12 + 18

= 11

Hence,

P (3) = 11

(iii) We have,

$p(x) = 5 - 4x + 2x^2$,

Now,

Put x = -2

$p(-2) = 5 - 4(-2) + 2(-2)^2$

= 5 - 4(-2) + 2(4)

= 5 + 8 + 8

= 21

Hence,

P (-2) = 21

2. Question

If $p(y) = 4 + 3y - y^2 + 5y^3$, find

(i) P(0)

(ii) p(2)

(iii) p(-1)

Answer

(i) We have,

$p(y) = 4 + 3y - y^2 + 5y^3$

Now,

Put y = 0

$p(0) = 4 + 3(0) - (0)^2 + 5(0)^3$

= 4 - 3(0) - (0) + 5(0)

= 4 – 0 – 0 + 0

= 4

Hence,

P (0) = 4

(ii) We have,

$p(y) = 4 + 3y - y^2 + 5y^3$

Now,

Put y = 2

$p(0) = 4 + 3(2) - (2)^2 + 5(2)^3$

$= 4 + 3(2) - (4) + 5(8)$

$= 4 + 6 - 4 + 40$

$= 46$

Hence,

P (2) = 46

(iii) We have,

$p(y) = 4 + 3y - y^2 + 5y^3$

Now,

Put y = -1

$p(-1) = 4 + 3(-1) - (-1)^2 + 5(-1)^3$

$= 4 + 3(-1) - (1) + 5(-1)$

$= 4 - 3 - 1 - 5$

$= -5$

Hence,

p(-1) = -5

3. Question

If $f(t) = 4t^2 - 3t + 6$, find

(i) f (0)

(ii) f (4)

(iii) f (-5)

Answer

(i) We have,

$f(t) = 4t^2 - 3t + 6,$

Now,

Put t = 0

$f(0) = 4(0)^2 - 3(0) + 6$

$= 4(0) - 3(0) + 6$

= 0 – 0 + 6

= 6

Hence,

f(0) = 6

(ii) We have,

$f(t) = 4t^2 - 3t + 6,$

Now,

Put t = 4

$f(4) = 4(4)^2 - 3(4) + 6$

= 4(16) - 3(4) + 6

= 64 – 12 + 6

= 58

Hence,

f(4) = 58

(iii) We have,

$f(t) = 4t^2 - 3t + 6,$

Now,

Put t = -5

$f(-5)=4(-5)^2 - 3(-5) + 6$

= 4(25) - 3(-5) + 6

= 100 + 15 + 6

= 121

Hence,

f(0) = 6

4. Question

Find the zero of the polynomial:

(i) P(x)=x-5

(ii) q(x)=x+4

(iii) p(t)=2t-3

(iv) f(x)=3x+1

(v) g(x)=5-4x

(vi) h(x)=6x-1

(vii) p(x)=ax + b,a≠0

(viii) q(x) = 4x

(ix) p(x)=ax,a≠0

Answer

(i) At first,

In order to find the zero of the polynomial we will,

Put p(x) = 0

Now,

We have,

P(x)=x-5

0 = x − 5

x = 5

Hence, 5 is the zero of the given polynomial.

(ii) At first,

In order to find the zero of the polynomial we will,

Put q(x) = 0

Now,

We have,

q(x)=x+4

0 = x + 4

x = - 4

Hence, -4 is the zero of the given polynomial.

(iii) At first,

In order to find the zero of the polynomial we will,

Put p(t) = 0

Now,

We have,

P(t)= 2t - 3

0 = 2t − 3

2t = 3

t = 3/2

Hence, 3/2 is the zero of the given polynomial.

(iv) At first,

In order to find the zero of the polynomial we will,

Put f(x) = 0

Now,

We have,

f(x)= 3x + 1

0 = 3x + 1

3x = -1

x = $\frac{-1}{3}$

Hence, -1/3 is the zero of the given polynomial.

(v) At first,

In order to find the zero of the polynomial we will,

Put g(x) = 0

Now,

We have,

g(x)= 5 – 4x

0 = 5 – 4x

4x = 5

x = 5/4

Hence, 5/4 is the zero of the given polynomial.

(vi) At first,

In order to find the zero of the polynomial we will,

Put h(x) = 6x - 1

Now,

We have,

h(x)= 6x- 1

0 = 6x – 1

6x = 1

x = 1/6

Hence, 1/6 is the zero of the given polynomial.

(vii) At first,

In order to find the zero of the polynomial we will,

Put p(x) = 0

Now,

We have,

p(x)= ax + b

0 = ax + b

ax = -b

x = (-b)/a

Hence, (-b)/a is the zero of the given polynomial.

(viii) At first,

In order to find the zero of the polynomial we will,

Put q(x) = 0

Now,

We have,

q(x)= 4x

0 = 4x

x = 0

Hence, 0 is the zero of the given polynomial.

(ix) At first,

In order to find the zero of the polynomial we will,

Put p(x) = 0

Now,

We have,

p(x)= ax

0 = ax

x = 0

Hence, 0 is the zero of the given polynomial.

5. Question

Verify that:

(i) 4 is a zero of the polynomial p(x)=x-4.

(ii) -3 is a zero of the polynomial p(x) = x + 3.

(iii) -1/2 is a zero of the polynomial p(y)=2y+1.

(iv) 2/5 is a zero of the polynomial p(x) =2-5x.

(v) 1 and 2 are the zeros of the polynomial p(x)=(x-1)(x-2)

(vi) 0 and 3 are the zeros of the polynomial p(x) =x^2 - 3x

(vii) 2 and-3 are the zeros of the polynomial p(x) =x^2 + x - 6

Answer

(i) We have, p(x) = x − 4

In order to verify the zero of the polynomial,Put p(x) = 4put x = 4 in the expression , we get, p(4) = 4 − 4p(4) = 0Since p(4) = 0

Hence, 4 is a zero of the polynomial p(x).

(ii) We have, p(x) = x + 3

In order to verify the zero of the polynomial,Put p(x) = -3p(-3) = - 3 + 3p(-3) = 0Since p(-3) = 0

Hence, -3 is a zero of the polynomial p(x).

(iii) We have, p(y) = 2y + 1

In order to verify the zero of the polynomial,Put p(y) = 1/2

p(1/2) = 2(1/2) + 1

p(1/2) = 1 + 1p(1/2) = 2

Since p(1/2) ≠ 0

Hence, 1/2 is not a zero of the polynomial p(y)

(iv) We have, p(x) = 2 - 5x

In order to verify the zero of the polynomial,Put p(x) = 2/5p(2/5) = 2 - 5 (2/5)p(2/5)= 2 - 2p(2/5)= 0

Since p(2/5) = 0

Hence, 2/5 is a zero of the polynomial p(x)

(v) We have, p(x) = (x-1)(x-2)

In order to verify the zero of the polynomial,

Case 1: Put x = 1, we get,

p(1) = (1-1)(1-2)p(1) = 0(-1)p(1) = 0Hence, 1 is a zero of the polynomial p(x)

Case 2: Put x = 2, we get,

p(3) = (2-1)(2-2)p(3) = (1)0p(3) = 0since p(3) = 0Hence, 2 is a zero of the polynomial p(x)

(vi) We have, p(x) = x^2 -3xIn order to verify the zero of the polynomial,

Case 1: Put x = 0

p(0) = $(0)^2$ – 3(0)p(0) = 0 - 0p(0) = 0

Since p(0) = 0Hence, 0 is a zero of the polynomial p(x)

Case 2: Put x = 3p(3) = $(3)^2$ – 3(3)p(3) = 9 - 9p(3) = 0

Since p(3) = 0Hence, 3 is a zero of the polynomial p(x)

(vii) We have, p(x) =x^2 + x - 6

In order to verify the zero of the polynomial,

Case 1: Put x = 2p(2) = $(2)^2$ + 2 - 6p(2) = 4 + 2 - 6p(2) = 0

Since p(2) = 0

Hence, 2 is a zero of the polynomial p(x)

Case 2: Put x = -3

p(3) = $(-3)^2$ + (−3) − 6p(3) = 9 - 3 - 6p(3) = 0

Since p(-3) = 0

Hence, -3 is a zero of the polynomial p(x)

Exercise 2C

1. Question

$\left(x^3 - 6x^2 + 9x + 3\right)$is divided by (x-1)

Answer

Let, f(x) = x^3 – 6x^2 + 9x + 3

Now,

As per the question,

x − 1 = 0

x = 1

Using Remainder theorem,

We know that when f(x)is divided by (x − 1), the remainder so obtained will be f(1).

Hence,

$f(1) = (1)^3 - 6(1)^2 + 9(1) + 3$

$= 1 - 6 + 9 + 3$

$= 13 - 6$

$= 7$

Therefore,

The required remainder is 7

2. Question

$\left(2x^3 5x^2 + 9x - 8\right)$ is divided by (x-3)

Answer

Let, $f(x) = 2x^3 - 5x^2 + 9x - 8$

Now,

As per the question,

$x - 3 = 0$

$x = 3$

Using Remainder theorem,

We know that when f(x)is divided by (x – 3), the remainder so obtained will be f(3).

Hence,

$f(3) = 2(3)^3 - 5(3)^2 + 9(3) - 8$

$= 2(27) - 5(9) + 27 - 8$

$= 54 - 45 + 19$

$= 28$

Therefore,

The required remainder is 28

3. Question

$\left(3x^4 - 6x^2 - 8x + 2\right)$ is divided by (x-2)

Answer

Let, $f(x) = 3x^4 - 6x^2 - 8x + 2$

Now,

As per the question,

x – 2 = 0

x = 2

Using Remainder theorem,

We know that when f(x)is divided by (x – 2), the remainder so obtained will be f(2).

Hence,

$f(2) = 3(2)^4 – 6(2)^2 - 8(2) + 2$

= 3(16) – 6(4) -16 + 2

= 48 -24 – 14

= 10

Therefore,

The required remainder is 10.

4. Question

$\left(x^3 - 7x^2 + 6x + 4\right)$ is divided by (x-6)

Answer

Let, $f(x) = x^3 – 7x^2 + 6x + 4$

Now,

As per the question,

x – 6 = 0

x = 6

Using Remainder theorem,

We know that when f(x) is divided by (x – 6), the remainder so obtained will be f(6).

Hence,

$f(6) = (6)^3 – 7(6)^2 + 6(6) + 4$

= (216) – 7(36) + 36 + 4

= 256 -252

= 4

Therefore,

The required remainder is 4.

5. Question

$\left(x^3 - 6x^2 + 13x + 60\right)$ is divided by (x+2)

Answer

Let, f(x) = $x^3 - 6x^2 + 13x + 60$

Now,

As per the question,

x + 2 = 0

x = -2

Using Remainder theorem,

We know that when f(x)is divided by (x + 2), the remainder so obtained will be f(-2).

Hence,

$f(-2) = (-2)^3 - 6(-2)^2 + 13(-2) + 60$

= - 8 – 6(4) - 26 + 60

= 60 - 58

= 2

Therefore,

The required remainder is 2.

6. Question

$\left(2x^4 + 6x^3 + 2x^2 + x - 8\right)$ is divided by (x+3)

Answer

Let, f(x) = $2x^4 + 6x^3 + 2x^2 + x - 8$

Now,

As per the question,

x - 3 = 0

x = 3

Using Remainder theorem,

We know that when f(x)is divided by (x – 3), the remainder so obtained will be f(3).

Hence,

$f(3) = 2(3)^4 + 6(3)^3 + 2(3)^2 + 3 - 8$

= 2(81) + 6(27) +18 – 5

= 18 - 11

= 7

Therefore,

The required remainder is 7.

7. Question

$\left(4x^3 - 12x^2 + 11x - 5\right)$ is divided by (2x-1)

Answer

Let, f(x) = $4x^3 - 12x^2 + 11x - 5$

Now,

As per the question,

2x − 1 = 0

2x = 1

x = $\frac{1}{2}$

Using Remainder theorem,

We know that when f(x)is divided by (2x − 1), the remainder so obtained will be $f\left(\frac{1}{2}\right)$.

Hence,

$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) - 5$

$= \frac{1}{2} - 3 + \frac{11}{2} - 5$

= -4/2

= -2

Therefore,

The required remainder is -2.

8. Question

$\left(81x^4 + 54x^3 - 9x^2 - 3x + 2\right)$ is divided by (3x+2)

Answer

Let, f(x) = $81x^4 + 54x^3 - 9x^2 - 3x + 2$

Now,

As per the question,

3x + 2 = 0

3x = -2

$x = \frac{-2}{3}$

Using Remainder theorem,

We know that when f(x)is divided by (2x − 1), the remainder so obtained will be $f\left(\frac{-2}{3}\right)$.

Hence,

$$f\left(\frac{-2}{3}\right) = 81\left(\frac{-2}{3}\right)^4 + 54\left(\frac{-2}{3}\right)^3 - 9\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) + 2$$

$$= 81\left(\frac{16}{81}\right) + 54\left(\frac{-8}{27}\right) - 9\left(\frac{4}{9}\right) + 2 + 2$$

= 16 − 16 + 4 - 4

= 0

Therefore,

The required remainder is 0.

9. Question

$x^3 - ax^2 + 2x - a$ is divided by (x-a)

Answer

Let, f(x) = x^3 - ax^2 + 2x - a

Now,

As per the question,

x - a = 0

x = a

Using Remainder theorem,

We know that when f(x)is divided by (x − a), the remainder so obtained will be f(a).

Hence,

f(a) = a^3 − $a(a)^2$ + 2(a) − a

= a^3 − a^3 + 2a − a

= 2a - a

= a

Therefore,

The required remainder is a.

10. Question

The polynomials $\left(ax^3+3x^2-3\right)$ and $\left(2x^3-5x+a\right)$ when divided by (x-4) leave the same remainder. Find the value of a.

Answer

Let f(x) = $ax^3 + 3x^2 - 3$

And,

g(x) = $2x^3 - 5x + a$

Now,

f(4) = $a(4)^3 + 3(4)^2 - 3$

= 64a + 48 - 3

= 64a + 45

And,

g(4) = $2(4)^3 - 5(4) + a$

= 128 – 20 + a

= 108 + a

According to the question,

f(4) = g(4)

64a + 45 = 108 + a

64a – a = 108 – 45

63a = 63

a = 1

Hence, the value of ‘a’ is 1.

11. Question

The Polynomial f(x) = $x^4 - 2x^3 + 3x^2 - ax + b$ when divided by (x-1) and (x+1) leaves the remainders 5 and 19 respectively. Find the values of a and b. Hence, find the remainder when f(x) is divided by (x-2).

Answer

Let f(x) = $x^4 - 2x^3 + 3x^2 - ax + b$

Now,

f(1) = $1^4 - 2(1)^3 + 3(1)^2 - a(1) + b$

5 = 1 – 2 + 3 – a + b

3 = - a + b (i)

And,

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b$

$19 = 1 + 2 + 3 + a + b$

$13 = a + b$ (ii)

Now,

Adding (i) and (ii),

$8 + 2b = 24$

$2b = 16$

$b = 8$

Now,

Using the value of b in (i)

$3 = - a + 8$

$a = 5$

Hence,

$a = 5$ and $b = 8$

Hence,

$f(x) = x^4 - 2(x)^3 + 3(x)^2 - a(x) + b$

$= x^4 - 2x^3 + 3x^2 - 5x + 8$

$f(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8$

$= 16 - 16 + 12 - 10 + 8$

$= 20 - 10$

$= 10$

Therefore, remainder is 10

Exercise 2D

1. Question

(x-2) is a factor of $\left(x^3 - 8\right)$

Answer

From the factor theorem, we have

$(x - 2)$ is the factor of f(x) if $f(2) = 0$

Here, we have

$f(2) = (2)^3 - 8$

$= 8 - 8$

$= 0$

Therefore,

$(x - 2)$ is a factor of $(x^3 - 8)$

2. Question

(x-3) is a factor of $\left(2x^3 + 7x^2 - 24x - 45\right)$

Answer

From the factor theorem, we have

$(x - 3)$ is the factor of $f(x)$ if $f(3) = 0$

Here, we have

$f(3) = 2 \times 3^3 + 7 \times 3^2 - 24 \times 3 - 45$

$= 54 + 63 - 72 - 45$

$= 117 - 117$

$= 0$

Therefore,

$(x - 3)$ is a factor of $(2x^3 + 7x^2 - 24x - 45)$

3. Question

(x-1) is a factor of $\left(2x^4 + 9x^3 + 6x^2 - 11x - 6\right)$

Answer

From the factor theorem, we have

$(x - 1)$ is the factor of $f(x)$ if $f(1) = 0$

Here, we have

$f(1) = 2 \times 1^4 + 9 \times 1^3 + 6 \times 1^2 - 11 \times 1 - 6$

$= 2 + 9 + 6 - 11 - 6$

$= 17 - 17$

$= 0$

Therefore,

$(x - 1)$ is the factor of $(2x^4 + 9x^3 + 6x^2 - 11x - 6)$

4. Question

(x+2) is a factor of $\left(x^4 - x^2 - 12\right)$

Answer

From the factor theorem, we have

(x + 2) is the factor of f(x) if (f-2) = 0

Here, we have

$f(-2) = (-2)^4 - (-2)^2 - 12$

= 16 – 4 – 12

= 16 – 16

= 0

Therefore,

(x + 2) is a factor of $(x^4 - x^2 - 12)$

5. Question

(x+5) is a factor of $\left(2x^3 + 9x^2 - 11x - 30\right)$

Answer

From the factor theorem, we have

(x + 5) is the factor of f(x) if f(-5) = 0

Here, we have

$f(-5) = 2(-5)^3 + 9(-5)^2 - 11(-5) - 30$

= -250 + 225 + 55 – 30

= -280 + 280

= 0

Therefore,

(x + 5) is the factor of $(2x^3 + 9x^2 - 11x - 30)$

6. Question

(2x-3) is a factor of $\left(2x^4 + x^3 - 8x^2 - x + 6\right)$

Answer

From the factor theorem, we have

(x – a) is the factor of f(x) if f(a) = 0

Here, we have

2x − 3 = 0

x = $\frac{3}{2}$

$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) + 6$

$= 2 \times \frac{81}{16} + \frac{27}{8} - 8 \times \frac{9}{4} - \frac{3}{2} + 6$

$= \frac{81}{8} + \frac{27}{8} - 18 - \frac{3}{2} + 6$

$= \frac{81+27-144-12+48}{8} = \frac{156-156}{8}$

= 0

Therefore,

(2x − 3) is a factor of ($2x^4 + x^3 - 8x^2 - x + 6$)

7. Question

$\left(x-\sqrt{2}\right)$ is a factor of $\left(7x^2 - 4\sqrt{2}x - 6\right)$

Answer

From the factor theorem, we have

(x − a) is the factor of f(x) if f(a) = 0

Here, we have

$f(\sqrt{2}) = 7\left(\sqrt{2}\right)^2 - 4\sqrt{2} \times \sqrt{2} - 6$

= 14 − 8 − 6

= 14 − 14

= 0

Therefore,

(x - $\sqrt{2}$) is a factor of ($7x^2 - 4\sqrt{2}x - 6$)

8. Question

$\left(x+\sqrt{2}\right)$ is a factor of $\left(2\sqrt{2}x^2 + 5x + \sqrt{2}\right)$

Answer

From the factor theorem, we have

(x − a) is the factor of f(x) if f(a) = 0

Here, we have

$f(-\sqrt{2}) = 2\sqrt{2}\,(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$

$= 2\sqrt{2}\times 2 - 5\sqrt{2} + \sqrt{2}$

$= 5\sqrt{2} - 5\sqrt{2}$

= 0

Therefore,

$(x + \sqrt{2})$ is the factor of $(2\sqrt{2}x^2 + 5x + \sqrt{2})$

9. Question

Find the value of k for which (x-1) is a factor of $(2x^3 + 9x^2 + x + k)$.

Answer

Let, $f(x) = 2x^3 + 9x^2 + x + k$

Now, we have

x − 1 = 0

x = 1

Hence,

$f(1) = 2 \times 1^3 + 9 \times 1^2 + 1 + k$

= 2 + 9 + 1 + k

= 12 + k

As per the question,

(x − 1) is the factor of f(x)

Now, by using factor theorem we get

(x − a) will be a factor of f(x) only if f(a) = 0 and hence f(1) = 0

So,

f(1) = 0

0 = 12 + k

k = -12

10. Question

Find the value of a for which (x-4) is a factor of $(2x^3 - 3x^2 - 18x + a)$.

Answer

Let, $f(x) = 2x^3 - 3x^2 - 18x + a$

Now, we have

$x - 4 = 0$

$x = 4$

Hence,

$f(4) = 2(4)^3 - 3(4)^2 - 18 \times 4 + a$

$= 128 - 48 - 72 + a$

$= 128 - 120 + 8$

$= 8 + a$

As per question,

$(x - 4)$ is the factor of $f(x)$

Now, by using factor theorem we get

$(x - a)$ will be the factor of $f(x)$ if $f(a) = 0$ and hence $f(4) = 0$

So,

$f(4) = 8 + a = 0$

$a = -8$

11. Question

Find the value of a for which the polynomial $\left(x^4 - x^3 - 11x^2 - x + a\right)$ is divisible by $(x+3)$.

Answer

Let, $f(x) = x^4 - x^3 - 11x^2 - x + a$

Now, we have

$x + 3 = 0$

$x = -3$

Hence,

$f(-3) = (-3)^4 - (-3)^3 - 11(-3)^2 - (-3) + a$

$= 81 + 27 - 11 \times 9 + 3 + a$

$= 81 + 27 - 99 + 3 + a$

$= 111 - 99 + a$

$= 12 + a$

As per question,

(x + 3) is the factor of f(x)

Now, by using factor theorem we get

(x – a) will be the factor of f(x) if f(a) = 0 and hence f(-3) = 0

So,

f(-3) = 12 + a = 0

a = -12

12. Question

For what value of a is the polynomial $\left(2x^3 + ax^2 + 11x + a + 3\right)$ exactly divisible by (2x-1)?

Answer

Let, f(x) = $2x^3 + ax^2 + 11x + a + 3$

Now, we have

2x – 1 = 0

x = 1/2

As per the question,

f(x) is exactly divisible by 2x – 1 which means that 2x – 1 is a factor of f(x)

Hence,

Using factor theorem,

(x – a) will be a factor of f(x) if f(a) = 0 and hence,

f($\frac{1}{2}$) $\neq$ 0

Hence,

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11 \times \frac{1}{2} + a + 3 = 0$$

$$= 2 \times \frac{1}{8} + a \times \frac{1}{4} + \frac{11}{2} + a + 3 = 0$$

$$= \frac{1}{4} + \frac{1}{4}a + \frac{11}{2} + a + 3 = 0$$

$$= \frac{1+a+22+4a+12}{4} = 0$$

$$= \frac{5a+35}{4} = 0$$

= 5a = -35

a = -7

Thus, the value of a is -7

13. Question

Find the values of a and b so that the polynomial $\left(x^3 - 10x^2 + ax + b\right)$ is exactly divisible by (x-1) as well as (x-2).

Answer

Let f(x) = $x^3 - 10x^2 + ax + b$

Now,

By using factor theorem,

(x – 1) and (x - 2) will be the factors of f(x) if f(1) = 0 and f(2) = 0

Hence,

f(1) = $1^3 - 10(1)^2 + a \times 1 + b0 = 1 - 10 + a + b$

a + b = 9 (i)

And,

f(2) = $2^3 - 10 \times 2^2 + a \times 1 + b$

0 = 8 – 40 + 2a + b

2a + b = 32 (ii)

Now, subtracting (i) from (ii)

a = 23

Using the value of a in (i), we get

23 + b = 9

b = 9 – 23

b = -14

Hence,

a = 23 and b = -14

14. Question

Find the values of a and b so that the polynomial $\left(x^4 + ax^3 - 7x^2 - 8x + b\right)$ is

exactly divisible by (x+2) as well as (x+3).

Answer

Let f(x) = $x^4 + ax^3 - 7x^2 - 8x + b$

Now,

$(x + 2) = 0$

$x = -2$

And,

$(x + 3) = 0$

$x = -3$

Now,

By using factor theorem,

$(x + 2)$ and $(x + 3)$ will be the factors of $f(x)$ if $f(-2) = 0$ and $f(-3) = 0$

Hence,

$f(-2) = (-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b0 = 16 - 8a - 28 + 16 + b$

$8a - b = 4$ (i)

And,

$f(-3) = (-3)^4 + a(-3)^3 - 10(-3)^2 - 8(-3) + b$

$0 = 81 - 27a - 63 + 24 + b$

$27a - b = 42$ (ii)

Now, subtracting (i) from (ii)

$19a = 38$

$a = 2$

Using the value of a in (i), we get

$8(2) - b = 4$

$16 - b = 4$

$b = 12$

Therefore,

$a = 2$ and $b = 12$

15. Question

Without actual division, show that $\left(x^3 - 3x^2 - 13x + 15\right)$ is exactly divisible by $\left(x^2 + 2x - 3\right)$.

Answer

Let $f(x) = x^3 - 3x^2 - 13x + 15$

Now, we have

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$(x + 3)(x - 1)$

Hence, f(x) will be exactly divisible by $x^2 + 2x - 3 = (x + 3)(x - 1)$

Now,

Using factor theorem,

If (x + 3) and (x − 1) are both the factors of f(x), then f(-3) = 0 and f(1) = 0

Now,

$f(-3) = (-3)^3 - 3(-3)^2 - 13(-3) + 15$

$= - 27 - 27 + 39 + 15$

$= - 54 + 54$

$= 0$

And,

$f(1) = (1)^3 - 3(1)^2 - 13(1) + 15$

$= 1 - 3 - 13 + 15$

$= 16 - 16$

$= 0$

Now,

Since, f(-3) = 0 and f(1) = 0

Therefore, $x^2 + 2x - 3$ divides f(x) completely.

16. Question

If $\left(x^3 + ax^2bx + 6\right)$ has (x-2) as a factor and leaves a remainder 3 when divided by (x-3), find the values of a and b.

Answer

Let $f(x) = (x^3 + ax^2 + bx + 6)$

By using remainder theorem,

If we divide f(x) by (x − 3) then it will leave a remainder as f(3)

So,

$f(3) = 3^2 + a \times 3^2 + b \times 3 + 6 = 3$

$27 + 9a + 3b + 6 = 3$

$9a + 3b + 33 = 3$

$9a + 3b = 3 - 33$

$9a + 3b = -30$

$3a + b = -10$ (i)

It is also given that,

(x − 2) is a factor of f(x)

Therefore,

By using factor theorem, we get

(x − a) is the factor of f(x) if f(a) = 0and also f(2) = 0

Now,

$f(2) = 2^3 + a \times 2^2 + b \times 2 + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4a + 2b = -14$

$2a + b = -7$ (ii)

Now by subtracting (ii) from (i), we get

$a = -3$

Putting the value of a in (i), we get

$3(-3) + b = -10$

$-9 + b = -10$

$b = -10 + 9$

$b = -1$

Therefore,

b = -1 and a = -3

Exercise 2E

1. Question

Factorize:

$9x^2 + 12xy$

Answer

We have,

$9x^2 + 12xy$

At first, we'll take common from the expression

$3x(3x+4y)$

Hence,

The given expression can be factorized as:

3x(3x + 4y)

2. Question

Factorize:

$18x^2y - 24xyz$

Answer

We have,

$18x^2y - 24xyz$

At first we'll take common from the expression

$6xy(3x-4z)$

Hence,

The given expression can be factorized as:

$6xy(3x-4z)$

3. Question

Factorize:

$27a^3b^3 - 45a^4b^2$

Answer

We have,

$27a^3b^3 - 45a^4b^2$

At first we'll take common from the expression

$9a^3b^2(3b-5a)$

Hence,

The given expression can be factorized as:

$9a^3b^2(3b-5a)$

4. Question

Factorize:

2a(x + y) -3b(x + y)

Answer

We have,

$2a(x+y)-3b(x+y)$

At first, we'll take common from the expression

$(x+y)(2a-3b)$

Hence,

The given expression can be factorized as:

$(x+y)(2a-3b)$

5. Question

Factorize:

$2x(p^2 + q^2) +4y (p^2 + q^2)$

Answer

We have,

$2x(p^2+q^2)+4y(p^2+q^2)$

At first, we'll take common from the expression

$=2[x (p^2 + q^2) + 2y (p^2 + q^2)]$

$=2(p^2+q^2)(x+2y)$

Hence,

The given expression can be factorized as:

$2(p^2+q^2)(x+2y)$

6. Question

Factorize:

x(a - 5) + y (5 - a)

Answer

We have,

$x(a-5)+y(5-a)$

At first we'll take common from the expression

= (a – 5) (x – y)

Hence,

The given expression can be factorized as:

$(a-5)(x-y)$

7. Question

Factorize:

4(a + b) – 6 (a + b)^2

Answer

We have,

$4(a+b)-6(a+b)^2$

At first, we'll take (a+b) common from the expression.

= (a + b) [4 – 6 (a + b)]

Take 2 common out of [4 - 6(a+b)]

= 2(a + b) (2 – 3a – 3b)

Hence,

The given expression can be factorized as:

$2(a+b)(2-3a-3b)$

8. Question

Factorize:

8(3a – 2b)^2 – 10 (3a – 2b)

Answer

We have,

$8(3a-2b)^2-10(3a-2b)$

At first we'll take common from the expression

= (3a – 2b) [8 (3a – 2b) – 10]

= (3a – 2b) 2[4 (3a – 2b) – 5]

= 2 (3a - 2b) (12a – 8b – 5)

Hence,

The given expression can be factorized as:

$2(3a-2b)(12a-8b-5)$

9. Question

Factorize:

$x(x + y)^3 - 3x^2y (x + y)$

Answer

We have,

$$x(x+y)^3 - 3x^2y(x+y)$$

At first we'll take common from the expression

$= x (x + y) [(x + y)^2 - 3xy]$

$= x (x + y) (x^2 + y^2 + 2xy - 3xy)$

$= x (x + y) (x^2 + y^2 - xy)$

Hence,

The given expression can be factorized as:

$$x(x+y)(x^2+y^2-xy)$$

10. Question

Factorize:

$x^3 + 2x^2 + 5x + 10$

Answer

We have,

$$x^3+2x^2+5x+10$$

At first we'll take common from the expression

$= x^2 (x + 2) + 5 (x + 2)$

$= (x^2 + 5) (x + 2)$

Hence,

The given expression can be factorized as:

$$(x+2)(x^2+5)$$

11. Question

Factorize:

$x^2 + xy - 2xz - 2yz$

Answer

We have,

$$x^2 + xy - 2xz - 2yz$$

At first we'll take common from the expression

= x (x + y) – 2z (x + y)

= (x + y) (x – 2z)

Hence,

The given expression can be factorized as:

$$(x + y)(x - 2z)$$

12. Question

Factorize:

$a^3b - a^2b + 5ab - 5b$

Answer

We have,

$$a^3b - a^2b + 5ab - 5b$$

At first we'll take common from the expression

= a^2b (a – 1) + 5b (a – 1)

= (a – 1) (a^2b + 5b)

= (a – 1) b (a^2 + 5)

= b (a – 1) (a^2 + 5)

Hence,

The given expression can be factorized as:

$$b(a - 1)(x^2 + 5)$$

13. Question

Factorize:

$8 - 4a - 2a^3 + a^4$

Answer

We have,

$$8 - 4a - 2a^3 + a^4$$

At first we'll take common from the expression

$= 4\ (2 - a) - a^3\ (2 - a)$

$= (2 - a)\ (4 - a^3)$

Hence,

The given expression can be factorized as:

$$(2-a)(4-a^3)$$

14. Question

Factorize:

$x^3 - 2x^2y + 3xy^2 - 6y^3$

Answer

We Have,

$$x^3 - 2x^2y + 3xy^2 - 6y^3$$

At first we'll take common from the expression

$= x^2\ (x - 2y) + 3y^2\ (x - 2y)$

$= (x - 2y)\ (x^2 + 3y^2)$

Hence,

The given expression can be factorized as:

$$(x-2y)(x^2+3y^2)$$

15. Question

Factorize:

$px - 5q + pq - 5x$

Answer

We have,

$$px - 5q + pq - 5x$$

At first we'll take common from the expression

$= p\ (x + q) - 5\ (q + x)$

$= (x + q)\ (p - 5)$

Hence,

The given expression can be factorized as:

$$(x+q)(p-5)$$

16. Question

Factorize:

$x^2 + y - xy - x$

Answer

We have,

$x^2 + y - xy - x$

At first we'll take common from the expression

$= x (x - y) - 1 (x - y)$

$= (x - y) (x - 1)$

Hence,

The given expression can be factorized as:

$(x - y)(x - 1)$

17. Question

Factorize:

$(3a - 1)^2 - 6a + 2$

Answer

We have,

$(3a - 1)^2 - 6a + 2$

At first, we'll take 2 common from - 6a + 2 in the expression.

$= (3a - 1)^2 - 2 (3a - 1)$

Now take (3a-1) common from above to get,

$= (3a - 1) [(3a - 1) - 2]$

$= (3a - 1) (3a - 3)$

$= 3(3a - 1) (a - 1)$

Hence,

The given expression can be factorized as:

$3(3a - 1)(a - 1)$

18. Question

Factorize:

$(2x - 3)^2 - 8x + 12$

Answer

We have,

$$(2x-3)^2 - 8x + 12$$

At first we'll take common from the expression

$= (2x - 3)^2 - 4(2x - 3)$

$= (2x - 3)(2x - 3 - 4)$

$= (2x - 3)(2x - 7)$

Hence,

The given expression can be factorized as:

$$(2x-3)(2x-7)$$

19. Question

Factorize:

$a^2 + a - 3a^2 - 3$

Answer

We have,

$$a^2 + a - 3a^2 - 3$$

At first we'll take common from the expression

$= a(a^2 + 1) - 3(a^2 + 1)$

$= (a - 3)(a^2 + 1)$

Hence,

The given expression can be factorized as:

$$(a-3)(a^2+1)$$

20. Question

Factorize:

$3ax - 6ay - 8by + 4bx$

Answer

We have,

$$3ax - 6ay - 8by + 4bx$$

At first we'll take common from the expression

$= 3a (x - 2y) + 4b (x - 2y)$

$= (x - 2y) (3a + 4b)$

Hence,

The given expression can be factorized as:

$(3a+4b)(x-2y)$

21. Question

Factorize:

$abx^2 + a^2x + b^2x + ab$

Answer

We have,

$abx^2 + a^2x + b^2x + ab$

At first we'll take common from the expression

$= ax (bx + a) + b (bx + a)$

$= (bx + a) (ax + b)$

Hence,

The given expression can be factorized as:

$(bx+a)(ax+b)$

22. Question

Factorize:

$x^3 - x^2 + ax + x - a - 1$

Answer

We have,

$x^3 - x^2 + ax + x - a - 1$

At first we'll take common from the expression

$= x^3 - x^2 + ax - a + x - 1$

$= x^2 (x - 1) + a (x - 1) + 1 (x - 1)$

$= (x - 1) (x^2 + a + 1)$

Hence,

The given expression can be factorized as:

$(x-1)(x^2+a+1)$

23. Question

Factorize:

2x + 4y – 8xy – 1

Answer

We have,

$2x+4y-8xy-1$

At first we'll take common from the expression

= 2x – 1 – 8xy + 4y

= (2x – 1) – 4y (2x – 1)

= (2x – 1) (1 – 4y)

Hence,

The given expression can be factorized as:

$(1-4y)(2x-1)$

24. Question

Factorize:

$ab(x^2 + y^2) - xy(a^2 + b^2)$

Answer

We have,

$ab(x^2+y^2)-xy(a^2+b^2)$

At first we'll take common from the expression

$= abx^2 + aby^2 - a^2xy - b^2xy$

$= abx^2 - a^2xy + aby^2 - b^2xy$

= ax (bx – ay) + by (ay – bx)

= (bx – ay) (ax – by)

Hence,

The given expression can be factorized as:

$(bx-ay)(ax-by)$

25. Question

Factorize:

$a^2 + ab(b + 1) + b^3$

Answer

We have,

$a^2 + ab(b+1) + b^3$

At first we'll take common from the expression

$= a^2 + ab^2 + ab + b^3$

$= a^2 + ab + ab^2 + b^3$

$= a\ (a + b) + b^2\ (a + b)$

$= (a + b)\ (a + b^2)$

Hence,

The given expression can be factorized as:

$(a+b)(a+b^2)$

26. Question

Factorize:

$a^3 + ab(1 - 2a) - 2b^2$

Answer

We have,

$a^3 + ab(1-2a) - 2b^2$

At first we'll take common from the expression

$= a^3 + ab - 2a^2b - 2b^2$

$= a\ (a^2 + b) - 2b\ (a^2 + b)$

$= (a^2 + b)\ (a - 2b)$

Hence,

The given expression can be factorized as:

$(a-2b)(a^2+b)$

27. Question

Factorize:

$2a^2 + bc - 2ab - ac$

Answer

We have,

$$2a^2 + bc - 2ab - ac$$

At first we'll take common from the expression

$= 2a^2 - 2ab - ac + bc$

$= 2a(a - b) - c(a - b)$

$= (a - b)(2a - c)$

Hence,

The given expression can be factorized as:

$$(2a - c)(a - b)$$

28. Question

Factorize:

$(ax + by)^2 + (bx - ay)^2$

Answer

We have,

$$(ax + by)^2 + (bx - ay)^2$$

At first we'll take common from the expression

$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$

$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

$= a^2x^2 + b^2x^2 + b^2y^2 + a^2y^2$

$= x^2(a^2 + b^2) + y^2(a^2 + b^2)$

$= (a^2 + b^2)(x^2 + y^2)$

Hence,

The given expression can be factorized as:

$$(a^2 + b^2)(x^2 + y^2)$$

29. Question

Factorize:

a(a + b – c) – bc

Answer

We have,

$a(a+b-c)-bc$

At first we'll take common from the expression

$= a^2 + ab - ac - bc$

$= a (a + b) - c (a + b)$

$= (a - c) (a + b)$

Hence,

The given expression can be factorized as:

$(a-c)(a+b)$

30. Question

Factorize:

a(a – 2b – c) + 2bc

Answer

We have,

$a(a-2b-c)+2bc$

At first we'll take common from the expression

$= a^2 - 2ab - ac + 2bc$

$= a (a - 2b) - c (a - 2b)$

$= (a - 2b) (a - c)$

Hence,

The given expression can be factorized as:

$(a-c)(a-2b)$

31. Question

Factorize:

$a^2x^2 + (ax^2 + 1)x + a$

Answer

We have,

$a^2x^2 + (ax^2 + 1)x + a$

At first, we'll take common from the expression

$= a^2x^2 + ax^3 + x + a$

$= ax^2 (a + x) + 1 (x + a)$

$= (ax^2 + 1) (a + x)$

Hence,

The given expression can be factorized as:

$(a + x)(ax^2 + 1)$

32. Question

Factorize:

$ab(x^2 + 1) + x(a^2 + b^2)$

Answer

We have,

$ab(x^2 + 1) + x(a^2 + b^2)$

At first, we'll take common from the expression

$= abx^2 + ab + a^2x + b^2x$

$= abx^2 + a^2x + ab + b^2x$

$= ax (bx + a) + b (bx + a)$

$= (bx + a) (ax + b)$

Hence,

The given expression can be factorized as:

$(ax + b)(bx + a)$

33. Question

Factorize:

$x^2 - (a + b)x + ab$

Answer

We have,

$x^2 - (a + b)x + ab$

At first we'll take common from the expression

$= x^2 - ax - bx + ab$

$= x(x - a) - b(x - a)$

$= (x - a)(x - b)$

Hence,

The given expression can be factorized as:

$(x-a)(x-b)$

34. Question

Factorize:

$x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$

Answer

We have,

$x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$

At first, we'll take common from the expression

$= (x - 1/x)^2 - 3(x - 1/x)$

$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$

Hence,

The given expression can be factorized as:

$\left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$

Exercise 2F

1. Question

Factorize:

$25x^2 - 64y^2$

Answer

We have,

$25x^2 - 64y^2$

We can also write the expression as:

$(5x)^2 - (8y)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(5x-8y)(5x+8y)$

Hence,

The given expression can be factorized as: $(5x-8y)(5x+8y)$

2. Question

Factorize:

$100 - 9x^2$

Answer

We have,

$100-9x^2$

We can also write the expression as:

$(10)^2 - (3x)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(10-3x)(10+3x)$

Hence,

The given expression can be factorized as: $(10-3x)(10+3x)$

3. Question

Factorize:

$5x^2 - 7y^2$

Answer

We have,

$100-9x^2$

We can also write the expression as:

$(\sqrt{5}x)^2 - (\sqrt{7}y)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$$\left(\sqrt{5}x-\sqrt{7}y\right)\left(\sqrt{5}x+\sqrt{7}y\right)$$

Hence,

The given expression can be factorized as: $\left(\sqrt{5}x-\sqrt{7}y\right)\left(\sqrt{5}x+\sqrt{7}y\right)$

4. Question

Factorize:

$(3x + 5y)^2 - 4z^2$

Answer

We have,

$$(3x+5y)^2-4z^2$$

We can also write the expression as:

$(3x + 5y)^2 - (2z)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$$(3x+5y-2z)(3x+5y+2z)$$

Hence,

The given expression can be factorized as: $(3x+5y-2z)(3x+5y+2z)$

5. Question

Factorize:

$150 - 6x^2$

Answer

We have,

$$150-6x^2$$

$= 6(25 - x^2)$

We can also write the expression as:

$6[(5\ )^2 - (x)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$6(5 - x)(5 + x)$

Hence,

The given expression can be factorized as: $6(5 - x)(5 + x)$

6. Question

Factorize:

$20x^2 - 45$

Answer

We have,

$20x^2 - 45$

$= 5(x^2 - 9)$

We can also write the expression as:

$= 5(x - 9)(x + 9)$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$5(2x-3)(2x+3)$

Hence,

The given expression can be factorized as: $5(2x-3)(2x+3)$

7. Question

Factorize:

$3x^3 - 48$

Answer

We have,

$3x^3 - 48$

$= 3x(x^2 - 16)$

We can also write the expression as:

$3x[(x)^2 - (4)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$3x(x-4)(x+4)$

Hence,

The given expression can be factorized as: $3x(x-4)(x+4)$

8. Question

Factorize:

$2 - 50x^2$

Answer

We have,

$2-50x^2$

$= 2(1 - 25x^2)$

We can also write the expression as:

$= 2[(1)^2 - (25x)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$2(1-5x)(1+5x)$

Hence,

The given expression can be factorized as: $2(1-5x)(1+5x)$

9. Question

Factorize:

$27a^2 - 48b^2$

Answer

We have,

$27a^2 - 48b^2$

$= 3(9a^2 - 16b^2)$

We can also write the expression as:

$= 3[(3a)^2 - (4b)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$3(3a-4b)(3a+4b)$

Hence,

The given expression can be factorized as: $3(3a-4b)(3a+4b)$

10. Question

Factorize:

$x - 64x^3$

Answer

We have,

$x-64x^3$

$= x(1 - 64x^2)$

We can also write the expression as:

$= x[(1)^2 - (8x)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$x(1-8x)(1+8x)$

Hence,

The given expression can be factorized as: $x(1-8x)(1+8x)$

11. Question

Factorize:

$8ab^2 - 18a^3$

Answer

We have,

$8ab^2-18a^3$

$= 2a(4b^2 - 9a^2)$

We can also write the expression as:

$= 2a[(2b)^2 - (3a)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$2a(2b-3a)(2b+3a)$

Hence,

The given expression can be factorized as: $2a(2b-3a)(2b+3a)$

12. Question

Factorize:

$3a^3b - 243ab^3$

Answer

We have,

$3a^3b-243ab^3$

$= 3ab(a^2 - 81b^2)$

We can also write the expression as:

$= 3ab[(a)^2 - (9b)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$3ab(a-9b)(a+9b)$

Hence,

The given expression can be factorized as: $3ab(a-9b)(a+9b)$

13. Question

Factorize:

$(a + b)^3 - a - b$

Answer

We have,

$(a+b)^3-a-b$

$= (a + b)^3 - (a + b)$

$= (a + b) [(a + b)^2 - 1]$

We can also write the expression as:

$= (a + b)[(a + b)^2 - (1)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$$(a+b)(a+b-1)(a+b+1)$$

Hence,

The given expression can be factorized as: $(a+b)(a+b-1)(a+b+1)$

14. Question

Factorize:

$108a^2 - 3(b - c)^2$

Answer

We have,

$$108a^2 - 3(b-c)^2$$

$= 3[36a^2 - (b - c)^2]$

We can also write the expression as:

$= 3[(6a)^2 - (b - c)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$$3(6a-b+c)(6a+b-c)$$

Hence,

The given expression can be factorized as: $3(6a-b+c)(6a+b-c)$

15. Question

Factorize:

$x^3 - 5x^2 - x + 5$

Answer

We have,

$$x^3 - 5x^2 - x + 5$$

$= x^2(x - 5) - 1(x - 5)$

$= (x - 5)[x^2 - (1)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(x-5)(x-1)(x+1)$

Hence,

The given expression can be factorized as: $(x-5)(x-1)(x+1)$

16. Question

Factorize:

$a^2 + 2ab + b^2 - 9c^2$

Answer

We have,

$a^2+2ab+b^2-9c^2$

$= (a + b)^2 - 9c^2$

We can also write the expression as:

$= [(a + b)^2 - (3c)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(a+b-3c)(a+b+3c)$

Hence,

The given expression can be factorized as: $(a+b-3c)(a+b+3c)$

17. Question

Factorize:

$9 - a^2 + 2ab - b^2$

Answer

We have,

$9-a^2+2ab-b^2$

$= [9 - (a^2 - 2ab + b)^2]$

We can also write the expression as:

$= [(3)^2 - (a - b)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(3+a-b)(3-a+b)$

Hence,

The given expression can be factorized as: $(3+a-b)(3-a+b)$

18. Question

Factorize:

$a^2 - b^2 - 4ac + 4c^2$

Answer

We have,

$a^2 - b^2 - 4ac + 4c^2$

$= a^2 - 2(a)(2c) + c^2 - b^2$

$= (a - c)^2 - (b)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(a-2c+b)(a-2c-b)$

Hence,

The given expression can be factorized as: $(a-2c+b)(a-2c-b)$

19. Question

Factorize:

$9a^2 + 3a - 8b - 64b^4$

Answer

We have,

$9a^2 + 3a - 8b - 64b^4$

$= 9a^2 - 64b^2 + 3a - 8b$

We can also write this as:

$= (3a)^2 - (8b)^2 + (3a - 8b)$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(3a-8b)(3a+8b+1)$

20. Question

Factorize:

$x^2 - y^2 + 6y - 9$

Answer

We have,

$x^2 - y^2 + 6y - 9$

$= [x^2 - (y^2 - 2(y)(3) + 3^2]$

We can also write the expression as:

$= [(x)^2 - (y - 3)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(x + y - 3)(x - y + 3)$

Hence,

The given expression can be factorized as: $(x + y - 3)(x - y + 3)$

21. Question

Factorize:

$4x^2 - 9y^2 - 2x - 3y$

Answer

We have,

$4x^2 - 9y^2 - 2x - 3y$

We can also write this as:

$= (4x)^2 - (9y)^2 - (2x + 3y)$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(2x + 3y)(2x - 3y - 1)$

22. Question

Factorize:

$x^4 - 1$

Answer

We have:

$x^4 - 1$

We can also write this as:

$=(x^2)^2 - 1^2$

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$=(x^2 + 1)(x^2 - 1)$

Again,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$=(x^2 + 1)(x + 1)(x - 1)$

23. Question

Factorize:

$a - b - a^2 + b^2$

Answer

We have:

$a - b - a^2 + b^2$

$= (a - b) - (a^2 - b^2)$

$= (a - b) - (a - b)(a + b)$

Hence,

The factorization of the given expression is,

$(a-b)(1-a-b)$

24. Question

Factorize:

$x^4 - 625$

Answer

We have:

$x^4 - 625$

We can also write this as:

$=(x^2)^2 - (25)^2$

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$=(x^2 + 25)(x^2 - 25)$

Again,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(x-5)(x+5)(x^2+25)$

Exercise 2G

1. Question

Factorize:

$x^2 + 11x + 30$

Answer

We have,

$x^2+11x+30$

Now by using middle-term splitting, we get

$= x^2 + 6x + 5x + 30$

$= x (x + 6) + 5 (x + 6)$

$= (x + 6) (x + 5)$

Hence,

The given expression can be factorized as:

$(x+6)(x+5)$

2. Question

Factorize:

$x^2 + 18x + 32$

Answer

We have,

$x^2+18x+32$

Now by using middle-term splitting, we get

$= x^2 + 16x + 2x + 32$

$= x (x + 16) + 2 (x + 16)$

$= (x + 16) (x + 2)$

Hence,

The given expression can be factorized as:

$(x+16)(x+2)$

3. Question

Factorize:

$x^2 + 7x - 18$

Answer

We have,

$x^2+7x-18$

Now by using middle-term splitting, we get

$= x^2 + 9x - 2x - 18$

= x (x + 9) - 2 (x + 9)

= (x + 9) (x - 2)

Hence,

The given expression can be factorized as:

$(x+9)(x-2)$

4. Question

Factorize:

$x^2 + 5x - 6$

Answer

We have,

x^2+5x-6

Now by using middle-term splitting, we get

$= x^2 + 6x - x - 6$

= x (x + 6) - 1 (x + 6)

= (x + 6) (x - 1)

Hence,

The given expression can be factorized as:

$(x+6)(x-1)$

5. Question

Factorize:

$y^2 - 4y + 3$

Answer

We have,

$y^2 - 4y + 3$

Now by using middle-term splitting, we get

$= y^2 - 3y - y + 3$

$= y (y - 3) - 1 (y - 3)$

$= (y - 3) (y - 1)$

Hence,

The given expression can be factorized as:

$(y-3)(y-1)$

6. Question

Factorize:

$x^2 - 21x + 108$

Answer

We have,

$x^2 - 21x + 108$

Now by using middle-term splitting, we get

$= x^2 - 12x - 9x + 108$

$= x (x - 12) - 9 (x - 12)$

$= (x - 12) (x - 9)$

Hence,

The given expression can be factorized as:

$(x-12)(x-9)$

7. Question

Factorize:

$x^2 - 11x - 80$

Answer

We have,

$x^2 - 11x - 80$

Now by using middle-term splitting, we get

= x^2 - 16x + 5x - 80

= x (x - 16) + 5 (x - 16)

= (x - 16) (x + 5)

Hence,

The given expression can be factorized as:

$(x-16)(x+5)$

8. Question

Factorize:

$x^2 - x - 156$

Answer

We have,

$x^2 - x - 156$

Now by using middle-term splitting, we get

= x^2 - 13x + 12x - 156

= x (x - 13) + 12 (x - 13)

= (x - 13) (x + 12)

Hence,

The given expression can be factorized as:

$(x-13)(x+12)$

9. Question

Factorize:

$z^2 - 32z - 105$

Answer

We have,

$z^2 - 32z - 105$

Now by using middle-term splitting, we get

$= z^2 - 35z + 3z - 105$

$= z (z - 35) + 3 (z - 35)$

$= (z - 35) (z + 3)$

Hence,

The given expression can be factorized as:

$(z-35)(z+3)$

10. Question

Factorize:

$40 + 3x - x^2$

Answer

We have,

$40+3x-x^2$

Now by using middle-term splitting, we get

$= 40 + 8x - 5x - x^2$

$= 8 (5 + x) - x (5 + x)$

$= (5 + x) (8 - x)$

Hence,

The given expression can be factorized as:

$(5+x)(8-x)$

11. Question

Factorize:

$6x - x - x^2$

Answer

We have,

$6-x-x^2$

Now by using middle-term splitting, we get

$= 6 + 2x - 3x - x^2$

$= 2 (3 + x) - x (3 + x)$

$= (3 + x) (2 - x)$

Hence,

The given expression can be factorized as:

$(2-x)(3+x)$

12. Question

Factorize:

$7x^2 + 49x + 84$

Answer

We have,

$7x^2+49x+84$

Now by using middle-term splitting, we get

$= 7 (x^2 +7x + 12)$

$= 7 [x^2 + 4x + 3x + 12]$

$= 7 [x (x + 4) + 3 (x + 4)]$

$= 7 (x + 4) (x + 3)$

Hence,

The given expression can be factorized as:

$7(x+4)(x+3)$

13. Question

Factorize:

$m^2 + 17mn – 84$

Answer

We have,

$m^2+17mn-84$

Now by using middle-term splitting, we get

$= m^2 + 21mn– 4mn – 84n^2$

$= m (m + 21n) – 4n (m + 21n)$

$= (m + 21n) (m – 4n)$

Hence,

The given expression can be factorized as:

$(m+21n)(m-4n)$

14. Question

Factorize:

$5x^2 + 16x + 3$

Answer

We have,

$5x^2+16x+3$

Now by using middle-term splitting, we get

$= 5x^2 + 15x + x + 3$

$= 5x (x + 3) + 1 (x + 3)$

$= (5x + 1) (x + 3)$

Hence,

The given expression can be factorized as:

$(x+3)(5x+1)$

15. Question

Factorize:

$6x^2 + 17x + 12$

Answer

We have,

$6x^2+17x+12$

Now by using middle-term splitting, we get

$= 6x^2 + 9x + 8x + 12$

$= 3x (2x + 3) + 4 (2x + 3)$

$= (2x + 3) (3x + 4)$

Hence,

The given expression can be factorized as:

$(3x+4)(2x+3)$

16. Question

Factorize:

$9x^2 + 18x + 8$

Answer

We have,

$9x^2 + 18x + 8$

Now by using middle-term splitting, we get

$= 9x^2 + 12x + 6x + 8$

$= 3x(3x + 4) + 2(3x + 4)$

$= (3x + 4)(3x + 2)$

Hence,

The given expression can be factorized as:

$(3x+4)(3x+2)$

17. Question

Factorize:

$14x^2 + 9x + 1$

Answer

We have,

$14x^2 + 9x + 1$

Now by using middle-term splitting, we get

$= 14x^2 + 7x + 2x + 1$

$= 7x(2x + 1) + 1(2x + 1)$

$= (7x + 1)(2x + 1)$

Hence,

The given expression can be factorized as:

$(2x+1)(7x+1)$

18. Question

Factorize:

$2x^2 + 3x - 90$

Answer

We have,

$2x^2+3x-90$

Now by using middle-term splitting, we get

= $2x^2$ - 12x + 15x - 90

= 2x (x - 6) + 15 (x - 6)

= (x - 6) (2x + 15)

Hence,

The given expression can be factorized as:

$(2x+15)(x-6)$

19. Question

Factorize:

$2x^2$ + 11x – 21

Answer

We have,

$2x^2+11x-21$

Now by using middle-term splitting, we get

= $2x^2$ + 14x - 3x - 21

= 2x (x + 7) - 3 (x + 7)

= (x + 7) (2x - 3)

Hence,

The given expression can be factorized as:

$(x+7)(2x-3)$

20. Question

Factorize:

$3x^2$ – 14x + 8

Answer

We have,

$3x^2-14x+8$

Now by using middle-term splitting, we get

= $3x^2$ - 12x - 2x + 8

= 3x (x - 4) - 2 (x - 4)

= (x - 4) (3x - 2)

Hence,

The given expression can be factorized as:

$(x-4)(3x-2)$

21. Question

Factorize:

$18x^2 + 3x - 10$

Answer

We have,

$18x^2+3x-10$

Now by using middle-term splitting, we get

= $18x^2$ - 12x + 15x - 10

= 6x (3x - 2) + 5 (3x - 2)

= (6x + 5) (3x - 2)

Hence,

The given expression can be factorized as:

$(6x+5)(3x-2)$

22. Question

Factorize:

$15x^2 + 2x - 8$

Answer

We have,

$15x^2+2x-8$

Now by using middle-term splitting, we get

= $15x^2$ - 10x + 12x - 8

= 5x (3x - 2) + 4 (3x - 2)

= (5x + 4) (3x - 2)

Hence,

The given expression can be factorized as:

$(5x+4)(3x-2)$

23. Question

Factorize:

$6x^2 + 11x - 10$

Answer

We have,

$6x^2+11x-10$

Now by using middle-term splitting, we get

$= 6x^2 + 15x - 4x - 10$

$= 3x (2x + 5) - 2 (2x + 5)$

$= (2x + 5) (3x - 2)$

Hence,

The given expression can be factorized as:

$(2x+5)(3x-2)$

24. Question

Factorize:

$30x^2 + 7x - 15$

Answer

We have,

$30x^2+7x-15$

Now by using middle-term splitting, we get

$= 30x^2 - 18x + 25x - 15$

$= 6x (5x - 3) + 5 (5x - 3)$

$= (5x - 3) (6x + 5)$

Hence,

The given expression can be factorized as:

$(6x+5)(5x-3)$

25. Question

Factorize:

$24x^2 - 41x + 12$

Answer

We have,

$24x^2 - 41x + 12$

Now by using middle-term splitting, we get

$= 24x^2 - 32x - 9x + 12$

$= 8x\ (3x - 4) - 3\ (3x - 4)$

$= (3x - 4)\ (8x - 3)$

Hence,

The given expression can be factorized as:

$(3x - 4)(8x - 3)$

26. Question

Factorize:

$2x^2 - 7x - 15$

Answer

We have,

$2x^2 - 7x - 15$

Now by using middle-term splitting, we get

$= 2x^2 - 10x + 3x - 15$

$= 2x\ (x - 5) + 3\ (x - 5)$

$= (x - 5)\ (2x + 3)$

Hence,

The given expression can be factorized as:

$(x - 5)(2x + 3)$

27. Question

Factorize:

$6x^2 - 5x - 21$

Answer

We have,

$6x^2 - 5x - 21$

Now by using middle-term splitting, we get

= $6x^2 + 9x - 14x - 21$

= $3x (2x + 3) - 7 (2x + 5)$

= $(3x - 7) (2x + 3)$

Hence,

The given expression can be factorized as:

$(3x-7)(2x+3)$

28. Question

Factorize:

$10x^2 – 9x – 7$

Answer

We have,

$10x^2 - 9x - 7$

Now by using middle-term splitting, we get

= $10x^2 + 5x - 14x - 7$

= $5x (2x + 1) - 7 (2x + 1)$

= $(2x + 1) (5x - 7)$

Hence,

The given expression can be factorized as:

$(5x-7)(2x+1)$

29. Question

Factorize:

$5x^2 – 16x – 21$

Answer

We have,

$5x^2 - 16x - 21$

Now by using middle-term splitting, we get

$= 5x^2 + 5x - 21x - 21$

$= 5x (x + 1) - 21 (x + 1)$

$= (x + 1) (5x - 21)$

Hence,

The given expression can be factorized as:

$(5x-21)(x+1)$

30. Question

Factorize:

$2x^2 - x - 21$

Answer

We have,

$2x^2 - x - 21$

Now by using middle-term splitting, we get

$= 2x^2 + 6x - 7x - 21$

$= 2x (x + 3) - 7 (x + 3)$

$= (x + 3) (2x - 7)$

Hence,

The given expression can be factorized as:

$(2x-7)(x+3)$

31. Question

Factorize:

$15x^2 - x - 28$

Answer

We have,

$15x^2 - x - 28$

Now by using middle-term splitting, we get

$= 15x^2 + 20x - 21x - 28$

$= 5x (3x + 4) - 7 (3x + 4)$

$= (3x + 4) (5x - 7)$

Hence,

The given expression can be factorized as:

$(5x-7)(3x+4)$

32. Question

Factorize:

$8a^2 - 27ab + 9b^2$

Answer

We have,

$8a^2 - 27ab + 9b^2$

Now by using middle-term splitting, we get

$= 8a^2 - 24ab - 3ab + 9b^2$

$= 8a(a - 3b) - 3b(a - 3b)$

$= (a - 3b)(8a - 3b)$

Hence,

The given expression can be factorized as:

$(a-3b)(8a-3b)$

33. Question

Factorize:

$5x^2 + 33xy - 14y^2$

Answer

We have,

$5x^2 + 33xy - 14y^2$

Now by using middle-term splitting, we get

$= 5x^2 + 35xy - 2xy - 14y^2$

$= 5x(x + 7y) - 2y(x + 7y)$

$= (x + 7y)(5x - 2y)$

Hence,

The given expression can be factorized as:

$(x+7y)(5x-2y)$

34. Question

Factorize:

$3x^3 - x^2 - 10x$

Answer

We have,

$3x^3 - x^2 - 10x$

Now by using middle-term splitting, we get

$= x\ (3x^2 - x - 10)$

$= x\ [3x^2 - 6x + 5x - 10]$

$= x\ [3x\ (x - 2) + 5\ (x - 2)]$

$= x\ (x - 2)\ (3x + 5)$

Hence,

The given expression can be factorized as:

$x(x-2)(3x+5)$

35. Question

Factorize:

$\frac{1}{3}x^2 - 2x - 9$

Answer

We have,

$\frac{1}{3}x^2 - 2x - 9$

Now by using middle-term splitting, we get

$= 1/3\ x^2 - 3x + x - 9$

$= x\ (x/3 - 3) + (x - 9)$

$= x/3\ (x - 9) + 1\ (x - 9)$

$= (x - 9)\ (x/3 + 1)$

$= (x - 9) \times (x - 3)/3$

$= 1/3\ (x - 9)\ (x + 3)$

Hence,

The given expression can be factorized as:

$\frac{1}{3}(x-9)(x+3)$

36. Question

Factorize:

$x^2-2x+\frac{7}{16}$

Answer

We have,

$x^2-2x+\frac{7}{16}$

Now by using middle-term splitting, we get

= 1/16 ($16x^2$ – 32x + 7)

= 1/16 ($16x^2$ – 4x – 28x + 7)

= 1/16 [4x (4x – 1) – 7 (4x – 1)]

= 1/16 (4x – 1) (4x – 7)

Hence,

The given expression can be factorized as:

$\frac{1}{16}(4x-7)(4x-1)$

37. Question

Factorize:

$\sqrt{2}x^2+3x+\sqrt{2}$

Answer

We have,

$\sqrt{2}x^2+3x+\sqrt{2}$

Now by using middle-term splitting, we get

$=\sqrt{2}x^2 + x + 2x + \sqrt{2}$

= x (√2 x+1) + √2 (√2 x+1)

= $\left(x+\sqrt{2}\right)\left(\sqrt{2}x+1\right)$

Hence,

The given expression can be factorized as:

$\left(x+\sqrt{2}\right)\left(\sqrt{2}x+1\right)$

38. Question

Factorize:

$\sqrt{5}x^2+2x-3\sqrt{5}$

Answer

We have,

$\sqrt{5}x^2+2x-3\sqrt{5}$

Now by using middle-term splitting, we get

= √5 × x × x + 5x − 3x - 3√5

= √5 x (x + √5) − 3 (x + √5)

= (√5 x − 3) (x + √5)

Hence,

The given expression can be factorized as:

$\left(x+\sqrt{5}\right)\left(\sqrt{5}x-3\right)$

39. Question

Factorize:

$2x^2+3\sqrt{3}x+3$

Answer

We have,

$2x^2+3\sqrt{3}x+3$

Now by using middle-term splitting, we get

= 2 × x × x + 2√3x + √3x + 3

= 2x (x+ √3) + √3 (x + √3)

= (x + √3) (2x + √3)

Hence,

The given expression can be factorized as:

$(x+\sqrt{3})(2x+\sqrt{3})$

40. Question

Factorize:

$2\sqrt{3}x^2 + x - 5\sqrt{3}$

Answer

We have,

$2\sqrt{3}x^2 + x - 5\sqrt{3}$

Now by using middle-term splitting, we get

= 2√3 * x * x + 6x - 5x - 5√3

= 2 √3 x (x + √3) - 5 (x+ √3)

= (x + √3)(2√3x - 5)

Hence,

The given expression can be factorized as:

$(x + \sqrt{3})(2\sqrt{3}x - 5)$

41. Question

Factorize:

$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Answer

We have,

$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Now by using middle-term splitting, we get

= 5√5 * x * x + 15x + 5x + 3√5

= 5x (√5x + 3) + √5 (√5x + 3)

= (√5x + 3)(5x + √5)

Hence,

The given expression can be factorized as:

$(\sqrt{5}x + 3)(5x + \sqrt{5})$

42. Question

Factorize:

$7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

Answer

We have,

$7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

Now by using middle-term splitting, we get

= 7√2 * x * x - 14x + 4x - 4√2

= 7√2x (x - √2) + 4 (x- √2)

= (x - √2) (7√2 x+4)

Hence,

The given expression can be factorized as:

$(x - \sqrt{2})(7\sqrt{2}x + 4)$

43. Question

Factorize:

$6\sqrt{3}x^2 - 47x + 5\sqrt{3}$

Answer

We have,

$6\sqrt{3}x^2 - 47x + 5\sqrt{3}$

Now by using middle-term splitting, we get

= 6√3 * x * x - 45x - 2x + 5√3

= 3√3 x (2x - 5√3) - 1 (2x- 5√3)

= (2x - 5√3)(3√3 x - 1)

Hence,

The given expression can be factorized as:

$(2x - 5\sqrt{3})(3\sqrt{3}x - 1)$

44. Question

Factorize:

$7x^2 + 2\sqrt{14}x + 2$

Answer

We have,

$7x^2 + 2\sqrt{14}x + 2$

Now by using middle-term splitting, we get

$= 7*x*x + \sqrt{2} \times \sqrt{7}x + \sqrt{2} \times \sqrt{7}\,x + 2$

$= \sqrt{7}x\,(\sqrt{7}x + \sqrt{2}) + \sqrt{2}\,(\sqrt{7}x + \sqrt{2})$

$= (\sqrt{7}x + \sqrt{2})(\sqrt{7}x + \sqrt{2})$

$= (\sqrt{7}x + \sqrt{2})^2$

Hence,

The given expression can be factorized as:

$$\left(\sqrt{7}x + \sqrt{2}\right)^2$$

45. Question

Factorize:

$2(x + y)^2 - 9(x + y) - 5$

Answer

We have,

$$2(x+y)^2 - 9(x+y) - 5$$

Let $x + y = z$

Then,

$2\,(x + y)^2 - 9\,(x + y) - 5$

Now by using middle-term splitting, we get

$= 2z^2 - 9z - 5$

$= 2z^2 - 10z + z - 5$

$= 2z\,(z - 5) + 1\,(z - 5)$

$= (z - 5)\,(2z + 1)$

Now,

Replacing z by (x + y), we get

$2\,(x + y)^2 - 9\,(x + y) - 5$

$= [(x + y) - 5]\,[2\,(x + y) + 1]$

$= (x + y - 5)\,(2x + 2y + 1)$

Hence,

The given expression can be factorized as:

$(x+y-5)(2x+2y+1)$

46. Question

Factorize:

$9(2a - b)^2 - 4(2a - b) - 13$

Answer

We have,

$9(2a-b)^2-4(2a-b)-13$

Let 2a − b = c

Then,

$9 (2a - b)^2 - 4 (2a - b) - 13$

Now by using middle-term splitting, we get

$= 9c^2 - 4c - 13$

$= 9c^2 - 13c + 9c - 13$

= c (9c − 13) + 1 (9c − 13)

= (c + 1) (9c - 13)

Now,

Replacing c by (2a - b) - 13, we get

$9 (2a - b)^2 - 4 (2a - b) - 13$

= (2a − b + 1) [9 (2a − b) − 13]

= (2a − b + 1) (18a − 9b − 13)

Hence,

The given expression can be factorized as:

$(18a-9b-13)(2a-b+1)$

47. Question

Factorize:

$7(x - 2y)^2 - 25(x - 2y) + 12$

Answer

We have,

$7(x-2y)^2-25(x-2y)+12$

Let x - 2y = z

Then,

$7(x - 2y)^2 - 25(x - 2y) + 12$

Now by using middle-term splitting, we get

$= 7z^2 - 25z + 12$

$= 7z^2 - 21z - 4z + 12$

$= 7z(z - 3) - 4(z - 3)$

$= (z - 3)(7z - 4)$

Now,

Replacing z by (x - 2y), we get

$7(x - 2y)^2 - 25(x - 2y) + 12$

$= (x - 2y - 3)[7(x - 2y) - 4]$

$= (x - 2y - 3)(7x - 14y - 4)$

Hence,

The given expression can be factorized as:

$(x-2y-3)(7x-14y-4)$

48. Question

Factorize:

$4x^4 + 7x^2 - 2$

Answer

We have,

$4x^4 + 7x^2 - 2$

Now by using middle-term splitting, we get

$= 4y^2 + 7y - 2$

$= 4y^2 + 8y - y - 2$

$= 4y(y + 2) - (y + 2)$

$= (y + 2)(4y - 1)$

Now,

Replacing y by x^2, we get

$4x^4 + 7x^2 - 2$

$= (x^2 + 2)(4x^2 - 1)$ [Therefore, $a^2 - b^2 = (a - b)(a + b)$]

$= (x^2 + 2)(2x + 1)(2x - 1)$

Hence,

The given expression can be factorized as:

$(x^2 + 2)(2x - 1)(2x + 1)$

Exercise 2H

1. Question

Expand:

(i) $(a + 2b + 5c)^2$

(ii) $(2a - b + c)^2$

(iii) $(a - 2b - 3c)^2$

Answer

(i) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (a)^2 + (2b)^2 + (5c)^2 + 2(a)(2b) + 2(2b)(5c) + 2(5c)(a)$

$= a^2 + 4b^2 + 25c^2 + 4ab + 20bc + 10ac$

(ii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (2a)^2 + (-b)^2 + (c)^2 + 2(2a)(-b) + 2(-b)(c) + 2(c)(2a)$

$= 4a^2 + b^2 + c^2 - 4ab - 2bc + 4ac$

(iii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (a)^2 + (-2b)^2 + (-3c)^2 + 2(a)(-2b) + 2(-2b)(-3c) + 2(-3c)(a)$

$= a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ac$

2. Question

Expand

(i) $(2a - 5b - 7c)^2$

(ii) $(-3a + 4b - 5c)^2$

(iii) $\left(\frac{1}{2}a - \frac{1}{4}b + 2\right)^2$

Answer

(i) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (2a)^2 + (-5b)^2 + (-7c)^2 + 2(2a)(-5b) + 2(-5b)(-7c) + 2(-7c)(2a)$

$= 4a^2 + 25b^2 + 49c^2 - 20ab + 70bc - 28ac$

(ii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (-3a)^2 + (4b)^2 + (-5c)^2 + 2(-3a)(4b) + 2(4b)(-5c) + 2(-5c)(-3a)$

$= 9a^2 + 16b^2 + 25c^2 - 24ab - 40bc + 30ac$

(iii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (1/2a)^2 + (-1/4b)^2 + (2)^2 + 2(1/2a)(-1/4b) + 2(-1/4b)(2) + 2(2)(1/2a)$

$= \frac{a^2}{4} + \frac{b^2}{16} + 4 - \frac{ab}{4} - b + 2a$

3. Question

Factorize:

$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Answer

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$

Using this formula, we get

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2$

4. Question

Factorize:

$9x^2 + 16y^2 + 4z^2 - 24xy + 16yz - 12xz$

Answer

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$

Using this formula, we get

$= (-3x)^2 + (4y)^2 + (2z)^2 + 2(-3x)(4y) + 2(4y)(2z) + 2(2z)(-3x)$

$= (-3x + 4y + 2z)^2$

5. Question

Factorize:

$25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$

Answer

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$

Using this formula, we get

$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$

$= (5x - 2y + 3z)^2$

6. Question

Evaluate:

(i) $(99)^2$

(ii) $(998)^2$

Answer

(i) We know that,

$(a - b)^2 = a^2 - 2ab + b^2$

Using this formula, we get

$= (100 - 1)^2$

$= (100)^2 - 2(100)(1) + (1)^2$

$= 10000 - 200 + 1$

$= 9801$

(ii) We know that,

$(a - b)^2 = a^2 - 2ab + b^2$

Using this formula, we get

$= (1000 - 2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$

$= 1000000 - 4000 + 4$

$= 996004$

Exercise 2I

1. Question

Expand:

(i) $(3x + 2)^2$

(ii) $(3a - 2b)^2$

(iii) $\left(\frac{2}{3}x+1\right)^3$

Answer

(i) We know that,

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Using this formula, we get

$= (3x)^3 + (2)^3 + 3 \times 3x \times 2(3x + 2)$

$= 27x^3 + 8 + 18x(3x + 2)$

$= 27x^3 + 8 + 54x^2 + 36x$

(ii) We know that,

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Using this formula, we get

$= (3a)^3 - (2b)^3 - 3 \times 3a \times 2b(3a - 2)$

$= 27a^3 - 8b^3 - 18ab(3a - 2b)$

$= 27a^3 - 8b^3 - 54a^2b + 36ab^2$

(iii) We know that,

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

Using this formula, we get

$= (2/3x)^3 + (1)^3 + 3 \times 2/3x \times 1 (2/3x + 1)$

$= 8/27x^3 + 1 + 2x (2/3x + 1)$

$= \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$

2. Question

Expand:

(i) $\left(2x - \frac{2}{x}\right)^3$

(ii) $\left(3a + \frac{1}{4b}\right)^3$

(iii) $\left(\frac{4}{5}x - 2\right)^3$

Answer

(i) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

Using this formula, we get

$= (2x)^3 - (2/x)^3 - 3 \times 2x \times 2/x (2x - 2/x)$

$= 8x^3 - 8/x^3 - 12 (2x - 2/x)$

$= 8x^3 - \frac{8}{x^3} - 24x + \frac{24}{x}$

(ii) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

Using this formula, we get

$= (3a)^3 - (1/4b)^3 + 3 \times 3a \times 1/4b (3a + 1/4b)$

$= 27a^3 + 1/64b^3 + 9a/4b (3a + 1/4b)$

$= 27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}$

(iii) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

Using this formula, we get

$= (4/5x)^3 - (2)^3 - 3 \times 4x/5 \times 2 (3a + 1/4b)$

$= 64/125a^3 - 8 - 24/5x (4/5x - 2)$

$= \frac{64}{125}x^3 - 8 - \frac{96}{25}x^2 + \frac{48}{5}x$

3. Question

Evaluate:

(i) $(95)^3$

(ii) $(999)^3$

Answer

(i) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

Using this formula, we get

$= (100 - 5)^3$

$= (100)^3 - (5)^3 - 3 \times 100 \times 5 (100 - 5)$

$= 1000000 - 125 - (1500 \times 95)$

$= 857375$

(ii) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

Using this formula, we get

$= (1000 - 1)^3$

$= (1000)^3 - (1)^3 - 3 \times 1000 \times 1 (1000 - 1)$

$= 1000000000 - 1 - 3000 (1000 - 1)$

$= 1000000000 - 1 - 3000 \times 1000 + 3000 \times 1$

$= 997002999$

Exercise 2J

1. Question

Factorize:

$x^3 + 27$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= x^3 + 3^3$

$= (x + 3)(x^2 - 3x + 9)$

2. Question

Factorize:

$8x^3 + 27y^3$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (2x)^3 + (3y)^3$

$= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2]$

$= (2x + 3y)(4x^2 - 6xy + 9y^2)$

3. Question

Factorize:

$343 + 125\ b^3$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (7)^3 + (5b)^3$

$= (7 + 5b)[(7)^2 - (7)(5b) + (5b)^2]$

$= (7 + 5b)(49 - 35b + 25b^2)$

4. Question

Factorize:

$1 + 64x^3$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (1)^3 + (4x)^3$

$= (1 + 4x)[(1)^2 - 1(4x) + (4x)^2]$

$= (1 + 4x)(1 - 4x + 16x^2)$

5. Question

Factorize:

$$125a^3 + \frac{1}{8}$$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (5a)^3 + (\frac{1}{2})^3$

$= (5a + \frac{1}{2})[(5a)^2 - 5a \times \frac{1}{2} + (\frac{1}{2})^2]$

$= (5a + \frac{1}{2})(25a^2 - \frac{5a}{2} + \frac{1}{4})$

6. Question

Factorize:

$$216x^3 + \frac{1}{125}$$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (6x)^3 + (\frac{1}{5})^3$

$= (6x + \frac{1}{5}) [(6x)^2 - 6x \times \frac{1}{5} + (\frac{1}{5})^2]$

$= (6x + \frac{1}{5}) (36x^2 - \frac{6x}{5} + \frac{1}{25})$

7. Question

Factorize:

$16x^4 + 54x$

Answer

We know that,

$a^3 + b^3 = (a + b) (a^2 - a \times b + b^2)$

Using this formula, we get

$= 2x (8x^3 + 27)$

$= 2x [(2x)^3 + (3)^3]$

$= 2x (2x + 3) [(2x)^2 - 2x(3) + 3^2]$

$= 2x (2x + 3) (4x^2 - 6x + 9)$

8. Question

Factorize:

$7a^3 + 56b^3$

Answer

We know that,

$a^3 + b^3 = (a + b) (a^2 - a \times b + b^2)$

Using this formula, we get

$= 7 (a^3 + 8b^3)$

$= 7 (a + 2b) (a^2 - a \times 2b + (2b)^2]$

$= 7 (a + 2b) (a^2 - 2ab + 4b^2)$

9. Question

Factorize:

$x^5 + x^2$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= x^2 (x^3 + 1)$

$= x^2 (x + 1) [(x^2) - x (1) + (1)^2]$

$= x^2 (x + 1) (x^2 - x + 1)$

10. Question

Factorize:

$a^3 + 0.008$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (a)^3 + (0.2)^3$

$= (a + 0.2) [(a)^2 - a (0.2) + (0.2)^2]$

$= (a + 0.2) (a^2 - 0.2a + 0.04)$

11. Question

Factorize:

$x^6 + y^6$

Answer

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (x^2)^3 + (y^2)^3$

$= (x^2 + y^2) [(x^2)^2 - x^2(y^2) + (y^2)^2]$

$= (x^2 + y^2) (x^4 - x^2y^2 + y^4)$

12. Question

Factorize:

$2a^3 + 16b^3 - 5a - 10b$

Answer

We know that,

$a^3 + b^3 = (a + b) (a^2 - a \times b + b^2)$

Using this formula, we get

$= 2 (a^3 + 8b^3) - 5 (a + 2b)$

$= 2 [(a)^3 + (2b)^3] - 5 (a + 2b)$

$= 2 (a + 2b) [(a)^2 - a (2b) + (2b)^2] - 5 (a + 2b)$

$= (a + 2b) [2 (a^2 - 2ab + 4b^2) - 5]$

13. Question

Factorize:

$x^3 + 512$

Answer

We know that,

$a^3 + b^3 = (a + b) (a^2 - a \times b + b^2)$

Using this formula, we get

$= (x)^3 - (8)^3$

$= (x - 8) [(x)^2 + x (8) + (8)^2]$

$= (x - 8) (x^2 + 8x + 64)$

14. Question

Factorize:

$64x^3 - 343$

Answer

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= (4x)^3 - (7)^3$

$= (4x - 7) [(4x)^2 + 4x (7) + (7)^2)$

$= (4x - 7) (16x^2 + 28x + 49)$

15. Question

Factorize:

$1 - 27x^3$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (1)^3 - (3x)^3$

$= (1 - 3x)[(1)^2 + 1(3x) + (3x)^2)$

$= (1 - 3x)(1 + 3x + 9x^2)$

16. Question

Factorize:

$x^3 - 125y^3$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (x)^3 - (5y)^3$

$= (x - 5y)[(x)^2 + x(5y) + (5y)^2$

$= (x - 5y)(x^2 + 5xy + 25y^2)$

17. Question

Factorize:

$8x^3 - \frac{1}{27y^3}$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (2x)^3 - (\frac{1}{3y})^3$

$= (2x - \frac{1}{3y})[(2x)^2 + 2x \times \frac{1}{3y} + (\frac{1}{3y})^2]$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

18. Question

Factorize:

$a^3 - 0.064$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (a)^3 - (0.4)^3$

$= (a - 0.4)[(a)^2 + a(0.4) + (0.4)^2]$

$= (a - 0.4)(a^2 + 0.4a + 0.16)$

19. Question

Factorize:

$(a + b)^3 - 8$

Answer

$(a + b)^3 - (2)^3$

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (a + b - 2)[(a + b)^2 + (a + b)2 + (2)^2]$

$= (a + b - 2)[a^2 + b^2 + 2ab + 2(a + b) + 4]$

20. Question

Factorize:

$x^6 - 729$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (x^2)^3 - (9)^3$

$= (x^2 - 9) [(x^2)^2 + x^2 9 + (9)^2]$

$= (x^2 - 9) (x^4 + 9x^2 + 81)$

$= (x + 3) (x - 3) [(x^2 + 9)^2 - (3x)^2]$

$= (x + 3) (x - 3) (x^2 + 3x + 9) (x^2 - 3x + 9)$

21. Question

Factorize:

$(a + b)^3 - (a - b)^3$

Answer

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= [a + b - (a - b)] [(a + b)^2 + (a + b) (a - b) + (a - b)^2]$

$= (a + b - a + b) [a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab]$

$= 2b (3a^2 + b^2)$

22. Question

Factorize:

$x^6 - 729$

Answer

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= x (1 - 8y^3)$

$= x [(1)^3 - (2y)^3]$

$= x (1 - 2y) [(1)^2 + 1 (2y) + (2y)^2]$

$= x (1 - 2y) (1 + 2y + 4y^2)$

23. Question

Factorize:

$32x^4 - 500x$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= 4x (8x^3 - 125)$

$= 4x [(2x)^3 - (5)^3]$

$= 4x [(2x - 5) [(2x)^2 + 2x (5) + (5)^2]$

$= 4x (2x - 5)(4x^2 + 10x + 25)$

24. Question

Factorize:

$3a^7b - 81a^4b^4$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= 3a^4b (a^3 - 27b^3)$

$= 3a^4b [(a)^3 - (3b)^3]$

$= 3a^4b (a - 3b) [(a)^2 + a (3b) + (3b)^2]$

$= 3a^4b (a - 3b)(a^2 + 3ab + 9b^2)$

25. Question

Factorize:

$$a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$$

Answer

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= a^3 - 1/a^3 - 2 (a - 1/a)$

$= (a - 1/a)(a^2 + a \times 1/a + 1/a^2) - 2 (a - 1/a)$

$= (a - 1/a) (a^2 + 1 + 1/a^2 - 2)$

$= \left(a - \frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right)$

26. Question

Factorize:

$8a^3 - b^3 - 4ax + 2bx$

Answer

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= 8a^3 - b^3 - 2x (2a - b)$

$= (2a)^3 - (b)^3 - 2x (2a - b)$

$= (2a - b) [(2a)^2 + 2a (b) + (b)^2] - 2x (2a - b)$

$= (2a - b) (4a^2 + 2ab + b^2) - 2x (2a - b)$

$= (2a - b) (4a^2 + 2ab + b^2 - 2x)$

27. Question

Factorize:

$a^3 + 3a^2b + 3ab^2 + b^3 - 8$

Answer

We know that,

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

Using this formula, we get

$= (a + b)^3 - 8$

$= (a + b)^3 - (2)^3$

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= (a + b - 2) [(a+ b)^2 + 2 (a + b) + 4)]$

Exercise 2K

1. Question

Factorize:

$125a^3 + b^3 + 64c^3 - 60abc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (5a)^3 + (b)^3 + (4c)^3 - 3 (5a) (b) (4c)$

$= (5a + b + 4c) [(5a)^2 + b^2 + (4c)^2 - (5a) (b) - (b) (4c) - (5a) (4c)]$

$= (5a + b + 4c) (25a^2 + b^2 + 16c^2 - 5ab - 4bc - 20ac)$

2. Question

Factorize:

$a^3 + 8b^3 + 64c^3 - 24abc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (a)^3 + (2b)^3 + (4c)^3 - 3 (a) (2b) (4c)$

$= (a + 2b + 4c) (a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$

3. Question

Factorize:

$1 + b^3 + 8c^3 - 6bc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (1)^3 + (b)^3 + (2c)^3 - 3 (1) (b) (2c)$

$= (1 + b + 2c) [(1)^2 + (b)^2 + (4c)^2 - (1) (b) - (2b) (c) - (2c) (1)]$

$= (1 + b + 2c) (1 + b^2 + 4c^2 - b - 2bc - 2c)$

4. Question

Factorize:

$216 + 27b^3 + 8c^3 - 108bc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (6)^3 + (3b)^3 + (2c)^3 - 3 (6) (3b) (2c)$

$= (6 + 3b + 2c) [(6)^2 + (3b)^2 + (2c)^2 - (6) (3b) - (3b) (2c) - (2c) (6)]$

$= (6 + 3b + 2c) (36 + 9b^2 + 4c^2 - 18ab - 6bc - 12ac)$

5. Question

Factorize:

$27a^3 - b^3 + 8c^3 + 18abc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (3a)^3 + (-b)^3 + (2c)^3 - 3 (3a) (-b) (2c)$

$= [3a + (-b) + 2c] [(3a)^2 + (-b)^2 + (2c)^2 - (3a) (-b) - (-b) (2c) - (2c) (3a)]$

$= (3a - b + 2c) (9a^2 + b^2 + 4c^2 + 3ab + 2bc - 6ca)$

6. Question

Factorize:

$8a^3 + 125b^3 - 64c^3 + 120abc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (2a)^3 + (5b)^3 + (-4c)^3 - 3 (2a) (5b) (-4c)$

$= (2a + 5b - 4c) [(2a)^2 + (5b)^2 + (-4c)^2 - (2a) (5b) - (5b) (-4c) - (-4c) (2a)]$

$= (2a + 5b - 4c) (4a^2 + 25b^2 + 16c^2 - 10ab + 20bc + 8ca)$

7. Question

Factorize:

$8 - 27b^3 - 343c^3 - 126bc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (2)^3 + (-3b)^3 + (-7c)^3 - 3 (2) (-3b) (-7c)$

$= (2 - 3b - 7c) [(2)^2 + (-3b)^2 + (-7c)^2 - (2) (-3b) - (-3b) (-7c) - (-7c) (2)]$

$= (2 - 3b - 7c) (4 + 9b^2 + 49c^2 + 6b - 21bc + 14c)$

8. Question

Factorize:

$125 - 8x^3 - 27y^3 - 90xy$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (5)^3 + (-2x)^3 + (-3y)^3 - 3 (5) (-2x) (-3y)$

$= (5 - 2x - 3y) [(5)^2 + (-2x)^2 + (-3y)^2 - (5) (-2x) - (-2x) (-3y) - (-3y) (5)]$

$= (5 - 2x - 3y) (25 + 4x^2 + 9y^2 + 10x - 6xy + 15y)$

9. Question

Factorize:

$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + (c)^3 - 3 (\sqrt{2}a) (2\sqrt{2}b) (c)$

$= (\sqrt{2}a + 2\sqrt{2}b + c) [(\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + (c)^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)(c) - (c)(\sqrt{2}a)]$

$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$

10. Question

Factorize:

$x^3 + y^3 - 12xy + 64$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

$= (x)^3 + (y)^3 + (4)^3 - 3(x)(y)(4)$

$= (x + y + 4)[(x)^2 + (y)^2 + (4)^2 - (x)(y) - (y)(4) - (4)(x)]$

$= (x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$

11. Question

Factorize:

$(a - b)^3 + (b - c)^3 + (c - a)^3$

Answer

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

Using this formula, we get

Putting $(a - b) = x$, $(b - c) = y$ and $(c - a) = z$, we get

$(a - b)^3 + (b - c)^3 + (c - a)^3 = x^3 + y^3 + z^3$

Where $(x + y + z) = (a - b) + (b - c) + (c - a) = 0$

$= 3xyz$ [Since, $(x + y + z) = 0$ so $(x^3 + y^3 + z^3) = 3xyz$]

$= 3(a - b)(b - c)(c - a)$

12. Question

Factorize:

$(3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3$

Answer

We know that,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

We have,

$(3a - 2b)(2b - 5c) + (5c - 3a) = 0$

So,

$(3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3 = 3(3a - 2b)(2b - 5c)(5c - 3a)$

13. Question

Factorize:

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

Answer

We have,

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

$= [a(b - c)]^3 + [b(c - a)]^3 + [c(a - b)]^3$

Since,

$a(b - c) + b(c - a) + c(a - b) = ab - ac + bc - ba + ca - bc = 0$

So,

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

$= 3a(b - c)\,b(c - a)\,c(a - b)$

$= 3abc(a - b)(b - c)(c - a)$

14. Question

Factorize:

$(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3$

Answer

We have,

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Since,

$(5a - 7b) + (9c - 5a) + (7b - 9c) = 5a - 7b + 9c - 5a + 7b - 9c = 0$

Therefore,

$(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = 3 (5a - 7b) (9c - 5a) (7b - 9c)$

15. Question

Find the product:

$(x + y - z)(x^2 + y^2 + z^2 - xy + yz + zx)$

Answer

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Using this, we get

$= [x + y + (-z)] [(x)^2 + (y)^2 + (-z)^2 - (x) (y) - (y) (-z) - (-z) (x)]$

$= x^3 + y^3 - z^3 + 3xyz$

16. Question

Find the product:

$(x -2y - 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9)$

Answer

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Using this, we get

$= [x + (-2y) + 3] [(x)^2 + (-2y)^2 + (3) - (x) (-2y) - (-2y) (3) - (3) (x)]$

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^\# + b^3 + c^3 - 3abc$

Where,

$x = a$, $b = -2y$ and $c = 3$

$(x - 2y + 3) (x^2 + 4y^2 + 2xy - 3x + 6y + 9)$

$= (x)^3 + (-2y)^3 + (3)^3 - 3 (x) (-2y) (3)$

$= x^3 - 8y^3 + 27 + 18xy$

17. Question

Find the product:

$(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + zx + 2yz)$

Answer

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Using this, we get

$= [x + (-2y) + (-z)] [(x)^2 + (-2y)^2 + (-z)^2 - (x) (-2y) - (-2y) (-z) - (-z) (x)$

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc$

Where,

$x = a$, $b = -2y$ and $c = -z$

$(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + zx - 2yz)$

$= (x)^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z)$

$= x^3 - 8y^3 - z^3 - 6xyz$

18. Question

If $x + y + 4 = 0$, find the value of $(x^3 + y^3 - 12xy + 64)$.

Answer

We have,

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Given,

$x + y + 4 = 0$

We have,

$(x^3 + y^3 - 12xy + 64)$

$= (x)^3 + (y)^3 + (4)^3 - 3(x)(y)(4) = 0$

19. Question

If $x = 2y + 6$, find the value of $(x^3 - 8y^3 - 36xy - 216)$.

Answer

We have,

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Given, x = 2y + 6

Or, x − 2y − 6 = 0

We have,

$(x^3 - 8y^3 - 36xy - 216)$

$= (x^3 - 8y^3 - 216 - 36xy)$

$= (x)^3 + (-2y)^3 + (-6)^3 - 3\,(x)\,(-2y)\,(-6)$

$= (x - 2y - 6)\,[(x)^2 + (-2y)^2 + (-6)^2 - (x)\,(-2y) - (-2y)\,(-6) - (-6)\,(x)]$

$= (x - 2y - 6)\,(x^2 + 4y^2 + 36 + 2xy - 12y + 8x)$

$= 0\,(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$

= 0

CCE Questions

1. Question

Which of the following expressions is a polynomial in one variable?

A. $x+\frac{2}{x}+3$

B. $3\sqrt{x}+\frac{2}{\sqrt{x}}+5$

C. $\sqrt{2}x^2-\sqrt{3}x+6$

D. $x^{10}+y^5+8$

Answer

Polynomials in one variable are algebraic expressions that consist of terms having same variable all through

∴ since in the expression $\sqrt{2}x^2 - \sqrt{3}x + 6$ the only polynomial used is x.

Hence, option C is correct

2. Question

Which of the following expressions is a polynomial?

A. $\sqrt{x}-1$

B. $\frac{x-1}{x+1}$

C. $x^2 - \frac{2}{x^2} + 5$

D. $x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6$

Answer

*Note: *A polynomial is an expression having variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables.*

Now,

$\sqrt{x} - 1$ and $\frac{x-1}{x+1}$ is not a polynomial because it does not contain integer power of the variable "x".

And,

$x^2 - \frac{2}{x^2} + 5$ is not a polynomial because it contains a negative power of the variable, which is not the criteria for a polynomial.

∴ $x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6$ is a polynomial it contains only integral powers of the variable (x) i.e. x^2

Hence, option D is correct

3. Question

Which of the following is a polynomial?

A. $\sqrt[3]{y+4}$ B. $\sqrt{y-3}$

C. y

D. $\frac{1}{\sqrt{y}} + 7$

Answer

*Note: *A polynomial is an expression having variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables*

$\sqrt[3]{y+4}$ and $\sqrt{y-3}$ is not a polynomial because it does not contain integer power of the variable "y"

And, $\frac{1}{\sqrt{y}} + 7$ is not a polynomial because it contains a negative power of the variable, which is not the criteria for a polynomial.

∴ y is a polynomial as it follows the criteria of polynomial and all the other do not follow these criteria.

Hence, option C is correct

4. Question

Which of the following is a polynomial?

A. $x-\frac{1}{x}+2$

B. $\frac{1}{x}+5$

C. $\sqrt{x}+3$

D. -4

Answer

As we know that,

A polynomial is an expression having variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables

Out of the 4 given options, we have:

$x-\frac{1}{x}+2$ and $\frac{1}{x}+5$ are not polynomials as they have negative exponent.

$\sqrt{x}+3$ is not a polynomial as it has non integral exponent.

And -4 can be written as $-4x^0$, thus it is a polynomial with integer power.

Thus, -4 is the correct option.

5. Question

Which of the following is a polynomial?

A. $x^{-2}+x^{-1}+3$

B. $x+x^{-1}+2$

C. x^{-1}

D. 0

Answer

Out of the 4 given options, D is the correct option because all the other has negative exponential value which shows that they aren't a polynomial

6. Question

Which of the following is a quadratic polynomial?

A. $x+4$

B. x^3+x

C. x^3+2x+6

D. x^2+5x+4

Answer

A quadratic polynomial is a polynomial of degree 2 which means that it must have a variable with degree 2.

∴ $x^2 + 5x + 4$ is a quadratic polynomial

Hence, option D is correct

7. Question

Which of the following is a linear polynomial?

A. $x+x^2$

B. $x+1$

C. $5x^2-x+3$

D. $x+\frac{1}{x}$

Answer

x + 1 is a linear polynomial because it has degree one which means that highest power of the variable must be 1

Hence, option B is correct

8. Question

Which of the following is a binomial?

A. x^2+x+3

B. x^2+4

C. $2x^2$

D. $x+3+\frac{1}{x}$

Answer

In algebra, a binomial is a polynomial which is the sum of two terms, each of which is a monomial. It is the simplest kind of polynomial after the monomials

∴ $x^2 + 4$ is a binomial

Hence, option B is correct

9. Question

$\sqrt{3}$ is a polynomial of degree

A. $\frac{1}{2}$

B. 2

C. 1

D. 0

Answer

$\sqrt{3}$ is a polynomial of degree 0 this is because it do not have any variable

Hence, option D is correct

10. Question

Degree of the zero polynomial is

A. 1

B. 0

C. not defined

D. none of these

Answer

The degree of the zero polynomial is left undefined

Hence, option C is correct

11. Question

Zero of the polynomial p(x) = 2x + 3 is

A. $\frac{3}{2}$

B. $\frac{-3}{2}$

C. $\frac{-2}{3}$

D. $\frac{1}{2}$

Answer

We have,

p (x) = 2x + 3

So, zero of the given polynomial can be calculated as follows:

$0 = 2x + 3$

$2x = -3$

$x = -\frac{3}{2}$

Hence, option B is correct

12. Question

Zero of the polynomial p(x) = 2 - 5x is

A. $\frac{2}{5}$

B. $\frac{5}{2}$

C. $\frac{-2}{5}$

D. $\frac{-5}{2}$

Answer

We have,

p (x) = 2 – 5x

So, zero of the given polynomial can be calculated as follows:

0 = 2 – 5x

5x = 2

$x = \frac{2}{5}$

Hence, option A is correct

13. Question

Zero of the zero polynomial is

A. 0

B. 1

C. every real number

D. not defined

Answer

The zero of the zero polynomial is not defined

Hence, option D is correct

14. Question

If p(x) = x + 4, then p(x) + (-x) =?

A. 0

B. 4

C.2x

D. 8

Answer

We have,

p (x) = x + 4

p (-x) = - x + 4

Then, the value of p (x) + (-x) and it can be calculated as follows:

p (x) + p (-x) = x + 4 – x + 4

= 4 + 4

= 8

Hence, option D is correct

15. Question

If $p(x) = x^2 - 2\sqrt{2}x + 1,$ then $p(2\sqrt{2}) = ?$

A. 0

B. 1

C. $4\sqrt{2}$

D. -1

Answer

We have,

p (x) = x^2 - $2\sqrt{2}$x + 1

$$\therefore p\left(2\sqrt{2}\right) = \left(2\sqrt{2}\right)^2 - \left(2\sqrt{2}\right) \times \left(2\sqrt{2}\right) + 1$$

= 8 – 8 + 1

= 1

Hence, option B is correct

16. Question

The zeroes of the polynomial p (x) = $x^2 + x - 6$ are

A. 2, 3

B. -2, 3

C. 2, -3

D. -2, -3

Answer

We have,

p (x) = $x^2 + x - 6$

So, zero of the given polynomial can be calculated as follows:

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x (x + 3) - 2 (x + 3) = 0$

$(x + 3) (x - 2) = 0$

Now, x + 3 = 0

x = - 3

And, x − 2 = 0

x = 2

Hence, the zeros of the given polynomial are -3 and 2

∴ Option C is correct

17. Question

The zeroes of the polynomial p (x) = $2x^2 + 5x - 3$ are

A. $\frac{1}{2}, 3$

B. $\frac{1}{2}, -3$

C. $\frac{-1}{2}, 3$

D. $1, \frac{-1}{2}$

Answer

We have,

p (x) = $2x^2 + 5x - 3$

So, zero of the given polynomial can be calculated as follows:

$2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x (x + 3) - 1 (x + 3) = 0$

$(2x - 1) (x + 3) = 0$

Now, $(2x - 1) = 0$

$2x = 1$

$x = \frac{1}{2}$

Also, $(x + 3) = 0$

$x = -3$

Hence, zeros of the given polynomial are $\frac{1}{2}$ and -3

∴ Option B is correct

18. Question

If $(x^2 + kx - 3) = (x - 3)(x + 1)$ then k =?

A. 2

B. -2

C. 3

D. -1

Answer

We have, $(x^2 + kx - 3) = (x - 3)(x + 1)$

So, the value of k can be calculated as follows:

$(x^2 + kx - 3) = x^2 - 3x + x - 3$

$x^2 + kx - 3 = x^2 - 2x - 3$

On comparing the coefficients, we get,

$kx = -2x$

$k = -2$

Thus, the value of $k = -2$

Hence, option B is correct

19. Question

If (x + 1) is a factor of $2x^2 + kx$, then k =?

A. -3

B. -2

C. 2

D. 4

Answer

Let p (x) = $2x^2 + kx$

It is given that, (x + 1) is a factor of $(2x^2 + kx)$

Thus, x = -1 is a factor of $(2x^2 + kx)$

∴ p (-1) = 0

$2(-1)^2 + k(-1) = 0$

2 – k = 0

k = 2

Thus, the value of k = 2

Hence, option C is correct

20. Question

The coefficient of the highest power of x in the polynomial $2x^2 - 4x^4 + 5x^2 - x^5 + 3$ is:

A. 2

B. -4

C. 3

D. -1

Answer

We have,

$2x^2 - 4x^4 + 5x^2 - x^5 + 3$

From the given polynomial,

The highest power of x = 5

Coefficient of x^5 = - 1

Hence, option D is correct

21. Question

When $(x^{31} + 31)$ is divided by $(x + 1)$, the remainder is

A. 0

B. 1

C. 30

D. 31

Answer

Let, $p(x) = (x^{31} + 31)$

And, $x + 1 = 0$

$x = -1$

It is given that, $(x + 1)$ is a factor of $p(x)$ so the remainder is equal to $p(-1)$

$\therefore p(-1) = (-1)^{31} + 31$

$= -1 + 31$

$= 30$

Hence, option C is correct

22. Question

When $p(x) = x^3 - ax^2 + x$ is divided by $(x - a)$, the remainder is

A. 0

B. a

C. 2a

D. 3a

Answer

We have,

$x^3 - ax^2 + x$

Let, $p(x) = x^3 - ax^2 + x$

And, $x - a = 0$

$x = a$

It is given that, $(x - a)$ is a factor of $p(x)$ so the remainder is equal to $p(a)$

$\therefore p(a) = (a)^3 - a(a)^2 + a$

$= a^3 - a^3 + a$

$= a$

Hence, option B is correct

23. Question

When p (x) = $(x^3 + ax^2 + 2x + a)$ is divided by (x + a), the remainder is

A. 0

B. a

C. −a

D. 2a

Answer

We have,

$(x^3 + ax^2 + 2x + a)$

Let, p (x) = $(x^3 + ax^2 + 2x + a)$

And, x + a = 0

x = - a

It is given that, (x + a) is a factor of p (x) so the remainder is equal to p (-a)

∴ p (-a) = $(-a)^3 + a(-a)^2 + 2(-a) + a$

= $-a^3 + a^3 - 2a + a$

= - a

Hence, option C is correct

24. Question

When p (x) = $x^4 + 2x^3 - 3x^2 + x - 1$ is divided by (x − 2), the remainder is

A. 0

B. -1

C. −15

D. 21

Answer

We have,

$x^4 + 2x^3 - 3x^2 + x - 1$

Let, p (x) = $x^4 + 2x^3 - 3x^2 + x - 1$

And, x - 2 = 0

x = 2

It is given that, (x - 2) is a factor of p (x) so the remainder is equal to p (2)

∴ $p(2) = (2)^4 + 2(2)^3 - 3(2)^2 + 2 - 1$

= 16 + 16 − 12 + 2 − 1

= 34 − 13

= 21

Hence, option D is correct

25. Question

When $p(x) = x^3 - 3x^2 + 4x + 32$ is divided by (x + 2), the remainder is

A. 0

B. 32

C. 36

D. 4

Answer

We have,

$x^3 - 3x^2 + 4x + 32$

Let, $p(x) = x^3 - 3x^2 + 4x + 32$

And, x + 2 = 0

x = - 2

It is given that, (x + 2) is a factor of p (x) so the remainder is equal to p (- 2)

∴ $p(-2) = (-2)^3 - 3(-2)^2 + 4(-2) + 32$

= - 8 - 12 − 8 + 32

= - 28 + 32

= 4

Hence, option D is correct

26. Question

When $p(x) = 4x^3 - 12x^2 + 11x - 5$ is divided by (2x − 1), the remainder is

A. 0

B. -5

C. -2

D. 2

Answer

We have,

$4x^3 - 12x^2 + 11x - 5$

Let, p (x) = $4x^3 - 12x^2 + 11x - 5$

And, (2x – 1) = 0

2x = 1

x = $\frac{1}{2}$

It is given that, (2x - 1) is a factor of p (x) so the remainder is equal to p($\frac{1}{2}$)

∴ p (2) = 4 $(\frac{1}{2})^3$ - 12 $(\frac{1}{2})^2$ + 11 $(\frac{1}{2})$ - 5

= 4 × $\frac{1}{8}$ - 12 × $\frac{1}{4}$ + 11 × $\frac{1}{2}$ - 5

= $\frac{1}{2}$ - 3 + $\frac{11}{2}$ - 5

= $\frac{12}{2}$ – 8

= 6 – 8

= - 2

Hence, option C is correct

27. Question

(x +1) is a factor of the polynomial:

A. $x^3 - 2x^2 + x + 2$

B. $x^3 - 2x^2 + x - 2$

C. $x^3 - 2x^2 - x - 2$

D. $x^3 - 2x^2 - x + 2$

Answer

We have, (x + 1) = 0

x = - 1

Firstly, putting (x = - 1) in $x^3 - 2x^2 + x + 2$ we get:

= $(- 1)^3 - 2 (-1)^2 + (-1) + 2$

= - 1 – 2 – 1 + 2

= - 2

∴ (x + 1) is not a factor of $x^3 - 2x^2 + x + 2$

Secondly, putting (x = - 1) in $x^3 + 2x^2 + x - 2$ we get:

= $(-1)^3 + 2(-1)^2 + (-1) - 2$

= - 1 + 2 − 1 − 2

= - 2

∴ (x + 1) is not a factor of $x^3 + 2x^2 + x - 2$

Thirdly, putting (x = - 1) in $x^3 + 2x^2 - x - 2$ we get:

= $(-1)^3 + 2(-1)^2 - (-1) - 2$

= - 1 + 2 + 1 − 2

= 0

Hence, (x + 1) is a factor of $x^3 + 2x^2 + x - 2$

Thus, option C is correct

28. Question

$4x^2 + 4x - 3$ = ?

A. (2x − 1) (2x - 3)

B. (2x + 1) (2x - 3)

C. (2x − 1) (2x + 3)

D. none of these

Answer

We have,

$4x^2 + 4x - 3$

= $4x^2 - 2x + 6x - 3$

= 2x (2x − 1) + 3 (2x − 1)

= (2x − 1) (2x + 3)

Hence, option D is correct

29. Question

$6x^2 + 17x + 5$

A. (2x - 1) (3x + 5)

B. (2x + 5) (3x - 1)

C. (6x + 5) (x + 1)

D. none of these

Answer

We have,

$6x^2 + 17x + 5$

$= 6x^2 + 2x + 15x + 5$

= 2x (3x + 1) + 5 (3x + 1)

= (2x + 5) (3x + 1)

Hence, option D is correct

30. Question

$x^2 - 4x - 21 = ?$

A. (x - 3) (x - 7)

B. (x - 3) (x + 7)

C. (x + 3) (x - 7)

D. none of these

Answer

We have,

$x^2 - 4x - 21$

$= x^2 + 3x - 7x - 21$

= x (x + 3) - 7 (x + 3)

= (x + 3) (x - 7)

Hence, option C is correct

31. Question

If (x + 5) is a factor of p (x) = $x^3 - 20x + 5k$, then k =?

A. -5

B. 5

C. 3

D. -3

Answer

We have,

$p(x) = x^3 - 20x + 5k$

It is given in the question that, $(x + 5)$ is a factor of $p(x)$ so:

$p(-5) = 0$

$(-5)^3 - 20(-5) + 5k = 0$

$-125 + 100 + 5k = 0$

$-25 + 5k = 0$

$5k = 25$

$k = \frac{25}{5}$

$k = 5$

Hence, option B is correct

32. Question

$3x^3 + 2x^2 + 3x + 2 = ?$

A. $(3x - 2)(x^2 - 1)$

B. $(3x - 2)(x^2 + 1)$

C. $(3x + 2)(x^2 - 1)$

D. $(3x + 2)(x^2 + 1)$

Answer

We have,

$3x^3 + 2x^2 + 3x + 2$

$= x^2(3x + 2) + 1(3x + 2)$

$= (x^2 + 1)(3x + 2)$

Hence, option D is correct

33. Question

If $\frac{x}{y} + \frac{y}{x} = -1$, where $x \neq 0$ and $y \neq 0$, then the value of $(x^3 - y^3)$ is

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer

We have,

$\frac{x}{y} + \frac{y}{x} = -1$

$\frac{x^2+y^2}{xy} = -1$

$x^2 + y^2 = -xy$

$x^2 + y^2 + xy = 0$

∴ Value of $(x^3 - y^3) = (x - y)(x^2 + y^2 + xy)$

$= (x - y) \times 0$

$= 0$

Hence, option C is correct

34. Question

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = ?$

A. 0

B. abc

C. 2abc

D. 3abc

Answer

The correct answer is D

It is given that: $a + b + c = 0$

We know,

$(a + b + c)^3 = a^3 + b^3 + c^3 + 2(ab + bc + ac)(a + b + c)$

Put $a + b + c = 0$ in the above equation we get,

Then, $a^3 + b^3 + c^3 = 3\ abc$

Hence, option D is correct

35. Question

$(x + 2)$ and $(x - 1)$ are factors of $(x^3 + 10x^2 + mx + n)$ then

A. m = 5, n = -3

B. m = 7, n = -18

C. m = 17, n = -8

D. m = 23, n = -19

Answer

We have,

$(x^3 + 10x^2mx + n)$

Let, $p(x) = x^3 + 10x^2 + mx + n$

It is given in the question that (x + 2) and (x – 1) are the factors of p (x)

∴ p (-2) = 0

$(-2)^3 + 10(-2)^2 + m(-2) + n = 0$

- 8 + 40 – 2m + n = 0

32 – 2m + n = 0

2m – n – 32 = 0 ……….(i)

And,

p (1) = 0

$(1)^3 + 10(1)^2 + m(1) n$

1 + 10 + m + n = 0

11 + m + n = 0

m + n + 11 = 0 ………. (ii)

Now, adding equation (i) and (ii) we get

2m – n - 32 + m + n + 11 = 0

3m - 21 = 0

3m = 21

$m = \frac{21}{3}$

m = 7

Now, putting the value of m in (ii) we get:

7 + n + 11 = 0

18 + n = 0

n = - 18

∴ the value of m is 7 and that of n is – 18

Hence, option B is correct

36. Question

The value of $(369)^2 - (368)^2$ = ?

A. 1^2

B. 81

C. 37

D. 737

Answer

We have,

$(369)^2 - (368)^2$

We know that,

$(a^2 - b^2) = (a + b)(a - b)$

Using this identity, we get:

$(369)^2 - (368)^2 = (369 + 368)(369 - 368)$

$= 737 \times 1$

$= 737$

Hence, option D is correct

37. Question

104 x 96 =?

A. 9894

B. 9984

C. 9684

D. 9884

Answer

We have,

$104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= 9984$

Hence, option B is correct

38. Question

$4a^2 + b^2 + 4ab + 8a + 4b + 4$ = ?

A. $(2a + b + 2)^2$

B. $(2a - b + 2)^2$

C. $(a + 2b + 2)^2$

D. none of these

Answer

We have,

$4a^2 + b^2 + 4ab + 8a + 4b + 4$

We know that,

$x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$

$= (2a)^2 + (b)^2 + (2)^2 + 2\,(2a)\,(b) + 2\,(b)\,(2) + 2 \times 2\,(2a)$

$= (2a + b + 2)^2$

Hence, option A is correct

39. Question

The coefficient of x in the expansion of $(x + 3)^3$ is

A. 1

B. 9

C. 18

D. 27

Answer

The coefficient of x in the expansion of $(x+3)^2$ can be calculated as follows:

$(x + 3)^3 = x^3 + (3)^3 + 3\,(x)\,(3)\,(x + 3)$

$= x^3 + 27 + 9x\,(x + 3)$

$= x^3 + 27 + 9x^2 + 27x$

∴ Coefficient of x is 27

Hence, option D is correct

40. Question

If a + b + c = 0, then $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right) = ?$

A. 1

B. 0

C. -1

D. 3

Answer

It is given in the question that,

a + b + c = 0

So, $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right) = \left(\frac{a^3 + b^3 + c^3}{abc}\right)$

$= \frac{3abc}{abc}$

= 3

Hence, option D is correct

41. Question

If x + y + z = 9 and xy + yz + zx = 23, then the value of $(x^3 + y^3 + z^3 - 3xyz)$ = ?

A. 108

B. 207

C. 669

D. 729

Answer

It is given that,

x + y + z = 9

And, xy + yz + zx = 23

As we know that,

$(x + y + z)^2 = (x^2 + y^2 + z^2 + 2xy + 2yz + 2zx)$

$\therefore (9)^2 = [x^2 + y^2 + z^2 + 2(xy + yz + zx)]$

$x^2 + y^2 + z^2 = 81 - 2 \times 23$

$x^2 + y^2 + z^2 = 81 - 46$

$x^2 + y^2 + z^2 = 35$

We also know that:

$(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)[x^2 + y^2 + z^2 - (xy + yz + zx)]$

= 9 (35 − 23)

= 9 × 12

= 108

Hence, option A is correct

42. Question

If ($x^{100} + 2x^{99} + k$) is divisible by (x + 1), then the value of k is

A. 1

B. 2

C. -2

D. -3

Answer

Let p (x) = $x^{100} + 2x^{99} + k$

It is given in the question that, (x + 1) is divisible by (x + 1)

So, p (-1) = 0

$(-1)^{100} + 2(-1)^{99} + k = 0$

1 + 2 (-1) + k = 0

1 − 2 + k = 0

- 1 + k = 0

k = 1

Thus, the value of k = 1

Hence, option A is correct

43. Question

In a polynomial in x, the indices of x must be

A. Integers

B. Positive integers

C. Non-negative integers

D. Real numbers

Answer

We know that,

In any polynomial in x, the indices of x must be a non-negative integer

Hence, option C is correct

44. Question

For what value of k is the polynomial p (x) = $2x^3 - kx^2 + 3x + 10$ exactly divisible by (x + 2)?

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. 3

D. -3

Answer

We have,

p (x) = $2x^3 - kx^2 + 3x + 10$

It is given in the question that (x + 2) is exactly divisible by p (x)

∴ p (-2) = 0

$2(-2)^3 - k(-2)^2 + 3(-2) + 10 = 0$

2 × (-8) – k × (4) – 6 + 10 = 0

- 16 – 4k – 6 + 10 = 0

- 22 – 4k + 10 = 0

- 12 – 4k = 0

- 12 = 4k

k = $-\frac{12}{4}$

k = - 3

Hence, option D is correct

45. Question

207 x 193 =?

A. 39851

B. 39951

C. 39961

D. 38951

Answer

We have,

207 × 193 = (200 + 7) (200 – 7)

= $(200)^2 - (7)^2$

= 40000 – 49

= 39951

Hence, option B is correct

46. Question

305 x 308 =?

A. 94940

B. 93840

C. 93940

D. 94840

Answer

We have,

305 × 308 = 305 × (300 + 8)

= 305 × 300 + 305 × 8

= 91500 + 2440

= 93940

Hence, option C is correct

47. Question

The zeroes of the polynomial p (x) = $x^2 - 3x$ are

A. 0, 0

B. 0, 3

C. 0, -3

D. 3, -3

Answer

We have,

p (x) = $x^2 - 3x$

∴ Zeros of p (x) are:

p (x) = 0

$x^2 - 3x = 0$

x (x − 3) = 0

So, x = 0

And, x − 3 = 0

x = 3

Thus, zeros of the given polynomial are 0 and 3

Hence, option B is correct

48. Question

The zeroes of the polynomial p (x) = $3x^2 - 1$ are

A. $\frac{1}{3}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{-1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$

Answer

We have,

p (x) = $3x^2 - 1$

∴ Zeros of p (x) are:

p (x) = 0

$3x^2 - 1 = 0$

$3x^2 = 1$

So, $x^2 = \frac{1}{3}$

And, x = $\pm\sqrt{\frac{1}{3}}$

Thus, zeros of the given polynomial are $\sqrt{\frac{1}{3}}$ and $-\sqrt{\frac{1}{3}}$

Hence, option D is correct

49. Question

The question consists of two statements, namely, Assertion (A) and Reason (R), Please select the correct answer.

Assertion (A)	**Reason (R)**
If (x – 1) is a factor of $p(x) = x^2 + kx + 1$, then k = - 2	If (x – a) is a factor of p(x), then p(a) = 0.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct ex

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The correct answer is: (a)

We have,

$p(x) = x^2 + kx + 1$

(x − 1) is a factor of p (x)

∴ p (1) = 0

$(1)^2 + k(1) + 1 = 0$

1 + k + 1 = 0

2 + k = 0

k = - 2

Hence, both Assertion (A) and Reason (R) are true also Reason (R) is the correct explanation of Assertion (A)

∴ Option A is correct

50. Question

The question consists of two statements, namely, Assertion (A) and Reason (R), Please select the correct answer.

Assertion (A)	Reason (R)
If $p(x)=x^3-ax^2+6x-a$ is divided by (x – a), then the remainder is 5a.	If p(x) is divided by (x – a), then the remainder is p(a).

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct ex

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

We have,

p (x) = $x^3 - ax^2 + 6x - a$

(x − a) is divided by p (x)

∴ p (a) = $(a)^3 - a\,(a)^2 + 6\,(a) - a$

= $a^3 - a^3 + 6a - a$

= 5a

Hence, both Assertion (A) and Reason (R) are true also Reason (R) is the correct explanation of Assertion (A)

∴ Option A is correct

51. Question

The question consists of two statements, namely, Assertion (A) and Reason (R), Please select the correct answer.

Assertion (A)	Reason (R)
If (x – 2) is a factor of $p(x) = x^3 - 2x + 3k$, then $\frac{-4}{3}$.	If p(x) is divided by (x – a), then the remainder is p(a).

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct ex

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The correct answer is: (B)

We have,

p (x) = x^3 – 2x + 3k

(x – 2) is a factor of p (x)

∴ p (2) = 0

$(2)^3$ – 2 (2) + 3k = 0

8 – 4 + 3k = 0

4 + 3k = 0

3k = - 4

k = $-\frac{4}{3}$

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

∴ Option B is correct

52. Question

The question consists of two statements, namely, Assertion (A) and Reason (R), Please select the correct answer.

Assertion (A)	**Reason (R)**
The value of $(25)^3+(-16)^3+(-9)^3$ is 10800.	If a + b + c = 0, then $a^3+b^3+c^3=3abc$.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct ex

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The correct answer is: (a)

We have,

$(25)^3 + (-16)^3 + (-9)^3$

Since, 25 – 16 – 9 = 25 – 25

= 0

∴ 3 (-25) (-16) (-9)

= 75 × 144

= 10800

Hence, both Assertion (A) and Reason (R) are true also Reason (R) is the correct explanation of Assertion (A)

∴ Option A is correct

53. Question

Match the following columns:

Column I	Column II
(a) If p (x) = $x^3 - 2x^2 + 3x + 1$ is divided by (x + 1), then remainder = ……	(p) $\frac{3}{8}$
(b) If (2x – 1) is a factor of q (x) = $x^3 + 2x^2 + 3x + k$ then k = ……	(q) 50
(c) The degree of the constant polynomial (-5) is = ……	(r) – 1
(d) When $x^{51} + 51$ is divided by (x + 1), the remainder is = ……	(s) 0

The correct answer is:

(a)-……., (b)-……., (c)-……., (d)-…….,

Answer

The correct match for the above is as follows:

Column I	Column II
(a) If p (x) = $x^3 - 2x^2 + 3x + 1$ is divided by (x + 1), then remainder = ...	(r) – 1
(b) If (2x – 1) is a factor of q (x) = $x^3 + 2x^2 + 3x + k$ then k = ...	(p) $\frac{3}{8}$
(c) The degree of the constant polynomial (-5) is = ...	(s) 0
(d) When $x^{51} + 51$ is divided by (x + 1), the remainder is = ...	(q) 50

Hence, the correct answer is:

(a) – (r)

(b) – (p)

(c) – (s)

(d) – (q)

54. Question

Match the following columns:

Column I	Column II
(a) If p (x) = $81x^4 + 54x^3 - 9x^2 - 3x + 2$ is divided by (3x + 2), then remainder = ……	(p) 1
(b) p (x) = $ax^3 + 3x^2 - 3$ and q (x) = $2ax^3 - 5x + a$ divided by (x – 4) leave the same remainder. Then, a = …..	(q) -7
(c) If p (x) = $7x^2 - 4\sqrt{2}x + c$ is completely divisible by $(x - \sqrt{2})$, then c = …..	(r) 0
(d) If q (x) = $2x^3 + bx^2 + 11x + b + 3$ is divisible by (2x – 1), then b = …..	(s) – 6

The correct answer is:

(a)-……., (b)-……., (c)-……., (d)-…….,

Answer

The correct match for the above is as follows:

Column I	Column II
(a) If p (x) = $81x^4 + 54x^3 - 9x^2 - 3x + 2$ is divided by $(3x + 2)$, then remainder = ……	(r) 0
(b) p (x) = $ax^3 + 3x^2 - 3$ and q (x) = $2ax^3 - 5x + a$ divided by $(x - 4)$ leave the same remainder. Then, a = …..	(p) 1
(c) If p (x) = $7x^2 - 4\sqrt{2}x + c$ is completely divisible by $(x - \sqrt{2})$, then c = …..	(s) – 6
(d) If q (x) = $2x^3 + bx^2 + 11x + b + 3$ is divisible by $(2x - 1)$, then b = …..	(q) – 7

Hence, the correct answer is:

(a) – (r)

(b) – (p)

(c) – (s)

(d) – (q)

Formative Assessment (Unit Test)

1. Question

Let $p(x) = 3x^3 + 4x^2 - 5x + 8$. Find p(-2).

Answer

We have,

$p(x) = 3x^3 + 4x^2 - 5x + 8$

So, $p(-2) = 3(-2)^3 + 4(-2)^2 - 5(-2) + 8$

$= 3 \times -8 + 4 \times 4 + 10 + 8$

$= -24 + 16 + 10 + 8$

$= -24 + 34$

$= 10$

2. Question

Find the remainder when $p(x) = 4x^3 + 8x^2 - 17x + 10$ is divided by $(2x - 1)$.

Answer

We have,

$p(x) = 4x^3 + 8x^2 - 17x + 10$

Also, $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$\therefore p(\frac{1}{2}) = 4(\frac{1}{2})^3 + 8(\frac{1}{2})^2 - 17(\frac{1}{2}) + 10$

$= 4 \times \frac{1}{8} + 8 \times \frac{1}{4} - 17 \times \frac{1}{2} + 10$

$= \frac{1}{2} + 2 - \frac{17}{2} + 10$

$= \frac{1-17}{2} + 12$

$= -\frac{16}{2} + 12$

$= -8 + 12$

$= 4$

Hence, remainder = 4

3. Question

If $(x - 2)$ is a factor of $2x^3 - 7x^2 + 11x + 5a$, find the value of a.

Answer

It is given in the question that (x – 2) is a factor of $2x^3 - 7x^2 + 11x + 5a$

Let $f(x) = 2x^3 - 7x^2 + 11x + 5a$

Now, x – 2 = 0

x = 2

∴ f (2) = 0

$2(2)^3 - 7(2)^2 + 11(2) + 5a = 0$

16 – 28 + 22 + 5a = 0

38 – 28 + 5a = 0

10 + 5a = 0

5a = - 10

$a = -\frac{10}{5}$

a = - 2

Hence, the value of a = - 2

4. Question

For what value of m, $p(x) = (x^3 - 2mx^2 + 16)$ is divisible by (x + 2)?

Answer

It is given in the question that (x + 2) is a factor of $x^3 - 2mx^2 + 16$

$p(x) = x^3 - 2mx^2 + 16$

Now, x + 2 = 0

x = - 2

∴ f (-2) = 0

$(-2)^3 - 2m(-2)^2 + 16 = 0$

- 8 – 8m + 16 = 0

- 8 + 8m = 0

- 8 = - 8m

$m = \frac{8}{8}$

m = 1

Hence, the value of m = 1

5. Question

If (a + b + c) = 8 and (ab + bc + ca) = 19, find ($a^2 + b^2 + c^2$)

Answer

It is given in the question that,

(a + b + c) = 8

And, (ab + bc + ca) = 19

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$

$\therefore (8)^2 = a^2 + b^2 + c^2 + 2 \times 19$

$64 = a^2 + b^2 + c^2 + 38$

$64 - 38 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 26$

6. Question

Expand: $(3a + 4b + 5c)^2$.

Answer

We have,

$(3a + 4b + 5c)^2$

We know that,

$(a + b + c)^2 = a^2 + y^2 + c^2 + 2 (xy + yz + zx)$

$\therefore (3a + 4b + 5c)^2 = (9a^2 + 16b^2 + 25c^2 + 2 \times 3a \times 4b + 2 \times 4b \times 5c + 2 \times 5c \times 3a)$

$= 9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ac$

7. Question

Expand: $(3x + 2)^2$.

Answer

We have,

$(3x + 2)^2$

We know that,

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

$\therefore (3x + 2)^2 = (3x)^3 + (2)^3 + 3 (3x) (2) (3x + 2)$

$= 27x^3 + 8 + 18x (3x + 2)$

$= 27x^3 + 8 + 54x^2 + 36x$

$= 27x^3 + 54x^2 + 36x + 8$

8. Question

Evaluate: $\{(28)^3 + (-15)^3 + (-13)^3\}$.

Answer

We have,

$[(28)^3 + (-15)^3 + (-13)^3]$

Now putting a = 28, b = - 15 and c = - 13

Now, a + b + c = 28 – 15 – 13

= 28 – 28

= 0

So, $x^3 + y^3 + z^3 = 3xyz$

$(28)^3 + (-15)^3 + (-13)^3 = 3 \times (28) \times (-15) \times (-13)$

$= 84 \times 195$

= 16380

9. Question

If $(x^{60} + 60)$ is divided by (x + 1), the remainder is

A. 0

B. 59

C. 61

D. 2

Answer

It is given in the question that (x + 1) is divided by $(x^{60} + 60)$

Let $f(x) = x^{60} + 60$

And, (x + 1) = 0

x = - 1

$\therefore f(x) = x^{60} + 60$

$f(-1) = (-1)^{60} + 60$

= 1 + 60

= 61

Hence, remainder = 61

Thus, option C is correct

10. Question

One of the factors of $(36x^2 - 1) + (1 + 6x)^2$ is

A. (6x – 1)

B. (6x + 1)

C. 6x

D. 6-x

Answer

We have,

$(36x^2 - 1) + (1 + 6x)^2$

$= [(6x)^2 - (1)^2] + (1 + 6x)^2$

= (6x – 1) (6x + 1) + (6x + 1) (6x + 1)

= (6x + 1) (6x – 1 + 1 + 6x)

= (6x + 1) (12x)

∴ (6x + 1) is a factor of $(36x^2 - 1) + (6x + 1)^2$

Hence, option B is correct

11. Question

If $\frac{a}{b} + \frac{b}{a} = -1$ then $(a^3 - b^3) = ?$

A. -1

B. -3

C. -2

D. 0

Answer

It is given in the question that,

$$\frac{a}{b} + \frac{b}{a} = -1$$

$$\frac{a \times a + b \times b}{ab} = -1$$

$a^2 + b^2 = - ab$

$a^2 + b^2 + ab = 0$ (i)

We know that,

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

From (i), we have $a^2 + b^2 + ab = 0$

$\therefore a^3 - b^3 = (a - b) \times 0$

$a^3 - b^3 = 0$

Hence, option D is correct

12. Question

The coefficient of x in the expansion of $(x + 5)^3$ is

A. 1

B. 15

C. 45

D. 75

Answer

We have,

$(x + 5)^3 = x^3 + (5)^3 + 3 \times x \times 5 (x + 5)$

$= x^3 + 125 + 15x (x + 5)$

$= x^3 + 125 + 15x^2 + 75x$

$\therefore$ Coefficient of x = 75

Hence, option D is correct

13. Question

$\sqrt{5}$ is a polynomial of degree

A. $\frac{1}{2}$

B. 2

C. 0

D. 1

Answer

We know that,

$\sqrt{5}$ is a polynomial of degree 0

Hence, option C is correct

14. Question

One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

A. 2

B. $\frac{1}{2}$

C. -2

D. $\frac{-1}{2}$

Answer

Let $f(x) = 2x^2 + 7x - 4$

$= 2x^2 + 8x - x - 4$

$= 2x (x + 4) - 1 (x + 4)$

$= (2x - 1) (x + 4)$

So, $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

And, $x + 4 = 0$

$x = - 4$

Thus, one of zero of the given polynomial is $\frac{1}{2}$

Hence, option B is correct

15. Question

Zero of the zero polynomial is

A. 0

B. 1

C. every real number

D. not defined

Answer

We know that,

The zero of the zero polynomial is not defined

Hence, option D is correct

16. Question

If (x + 1) and (x − 1) are factors of p (x) = $ax^3 + x^2 - 2x + b$, find the values of a and b.

Answer

We have,

p (x) = $ax^3 + x^2 - 2x + b$

It is given in the question that,

(x + 1) and (x − 1) are the factors of p (x)

∴ p (-1) = p (1)

So, p (-1) = $a(-1)^3 + (-1)^2 - 2(-1) + b$

0 = - a + 1 + 2 + b

0 = - a + 3 + b (i)

Also, p (1) = $a(1)^3 + (1)^2 - 2(1) + b$

0 = a + 1 − 2 + b

0 = a + b − 1 (ii)

As, p (-1) = p (1)

- a + 3 + b = a + b - 1

- 2a = - 4

a = 2

Now, putting the value of a in (ii), we get:

2 + b − 1 = 0

1 + b = 0

b = - 1

Hence, the value of a is 2 and that of b is -1

17. Question

If (x + 2) is a factor of p (x) = $ax^3 + bx^2 + x - 6$ and p(x) when divided by (x − 2) leaves a remainder 4, prove that a = 0 and b = 2.

Answer

We have,

$p(x) = ax^3 + bx^2 + x - 6$

It is given in the question that, (x + 1) is a factor of p (x)

$x + 2 = 0$

$x = -2$

∴ $f(-2) = 0$

$a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$

$-8a + 4b - 2 - 6 = 0$

$-8a + 4b - 8 = 0$

$-4(2a - b + 2) = 0$

$2a - b + 2 = 0$ (i)

Also, it is given in the question that when p (x) is divided by (x − 2) then it leaves a remainder 4

∴ $p(2) = 4$

$a(2)^3 + b(2)^2 + (2) - 6 = 4$

$8a + 4b + 2 - 6 = 4$

$8a + 4b - 4 - 4 = 0$

$4(2a + b - 2) = 0$

$2a + b - 2 = 0$ (ii)

Now, adding (i) and (ii) we get:

$2a - b + 2 + 2a + b - 2 = 0$

$4a = 0$

$a = 0$

Putting the value of a in (ii), we get:

$2(0) + b - 2 = 0$

$b - 2 = 0$

$b = 2$

Hence, it is proved that the value of a is 0 and that of b is 2

18. Question

The expanded form of $(3x - 5)^3$ is

A. $27x^3 + 135x^2 + 225x - 125$

B. $27x^3 + 135x^2 - 225x - 125$

C. $27x^3 - 135x^2 + 225x - 125$

D. none of these

Answer

We have,

$(3x - 5)^3 = (3x)^3 - (5)^3 - 3\ (3x)\ (5)\ (3x - 5)$

$= 27x^3 - 125 - 45x\ (3x - 5)$

$= 27x^3 - 125 - 135x^2 + 225x$

$= 27x^3 - 135x^2 + 225x - 125$

Hence, option D is correct

19. Question

If a + b + c = 5 and ab + bc + ca = 10, prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Answer

It is given in the question that,

a + b + c = 5

And, ab + bc + ca = 0

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2\ (ab + bc + ca)$

Putting the given values, we get:

$(5)^2 = a^2 + b^2 + c^2 + 20$

$25 - 20 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 5$ (i)

Also, we know that

$(a^3 + b^3 + c^3 - 3abc) = (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)\ [(a^2 + b^2 + c^2) - (ab + bc + ca)]$

Now, putting the values we get:

$= (5) \times (5 - 10)$

$= 5 \times (-5)$

$= -25$

Hence, it is proved that

$(a^3 + b^3 + c^3 - 3abc) = -25$

20. Question

If p (x) = $2x^3 + ax^2 + 3x - 5$ and q (x) = $x^3 + x^2 - 4x + a$ leave the same remainder when divided by (x − 2), show that $a = \frac{-13}{3}$.

Answer

We have,

$p(x) = 2x^3 + ax^2 + 3x - 5$

$q(x) = x^3 + x^2 - 4x + a$

It is given in the question that, when p (x) and q (x) is divided by (x − 2) it leaves same remainder

$\therefore p(2) = q(2)$

$2(2)^3 + a(2)^2 + 3(2) - 5 = (2)^3 + (2)^2 - 4(2) + a$

$2 \times 8 + a \times 4 + 3 \times 2 - 5 = 8 + 4 - 4 \times 2 + a$

$16 + 4a + 6 - 5 = 12 - 8 + a$

$16 + 4a + 1 = 4 + a$

$4a - a = 4 - 17$

$3a = -13$

$a = -\frac{13}{3}$

Hence proved