Properties of Triangles
Exercise 15A

Q1

Answer :

Sum of the angles of a triangle is 180°.

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$
$$72^\circ + 63^\circ + \angle C = 180^\circ$$
$$\angle C = 45^\circ$$

Hence, ∠C measures 45°.

Q2

Answer :

Sum of the angles of any triangle is 180°.
In Δ*DEF*:

$$\angle D + \angle E + \angle F = 180^\circ$$
$$\angle D + 105^\circ + 40^\circ = 180^\circ$$
$$\text{or } \angle D = 180^\circ - (105^\circ + 40^\circ)$$
$$\text{or } \angle D = 35^\circ$$

Q3

Answer :

Sum of the angles of any triangle is 180°.
In Δ*XYZ*:

$$\angle X + \angle Y + \angle Z = 180^\circ$$
$$90^\circ + \angle Y + 48^\circ = 180^\circ$$
$$=> \angle Y = 180^\circ - 138^\circ = 42^\circ$$

Q4

Answer :

Suppose the angles of the triangle are $(4x)^o$, $(3x)^o$ and $(2x)^o$.

Sum of the angles of any triangle is 180^o.

$\therefore 4x + 3x + 2x = 180$
$9x = 180$
$x = 20$

Therefore, the angles of the triangle are $(4 \times 20)^\circ$, $(3 \times 20)^\circ$ and $(2 \times 20)^\circ$, i.e. 80°, 60° and 40°.

Q5

Answer :

Sum of the angles of a triangle is 180°.
Suppose the other angle measures x.

It is a right angle triangle. Hence, one of the angle is 90°.

$\therefore 36^\circ + 90^\circ + x = 180^\circ$
$x = 54^\circ$

Hence, the other angle measures 54°.

Q6

Answer :

Suppose the acute angles are $(2x)^\circ$ and $(x)^\circ$
Sum of the angles of any triangle is 180°

$\therefore 2x+x+ 90 = 180$
$\Rightarrow (3x) = 180-90$
$\Rightarrow (3x) = 90$
$\Rightarrow x = 30$
So, the angles measure $(2\times30)^\circ$ and 30° i.e. 60° and 30°

Q7

Answer :

The other two angles are equal. Let one of these angles be x°.

Sum of angles of any triangle is 180°.

$\therefore x + x + 100 = 180$
$2x = 80$
$x = 40$

Hence, the equal angles of the triangle are 40° each.

Q8

Answer :

Suppose the third angle of the isosceles triangle is x^o.
Then, the two equal angles are $(2x)^o$ and $(2x)^o$.
Sum of the angles of any triangle is 180^o.

$\therefore 2x + 2x + x = 180$
$5x = 180$
$x = 36$

Hence, the angles of the triangle are 36°, $(2 \times 36)^\circ$ and $(2 \times 36)^\circ$, i.e. 36°, 72° and 72°.

Q9

Answer :

Suppose the angles are $\angle A$, $\angle B$ and $\angle C$. (Sum of the angles of a triangle is 180°)

Given :

$\angle A = \angle B + \angle C$

$Also, \angle A + \angle B + \angle C = 180^\circ$

$\therefore \angle A + \angle A = 180^\circ$

$\Rightarrow 2\angle A = 180^\circ$

$\Rightarrow \angle A = 90^\circ$

Hence, the triangle ABC is right angled at $\angle$A.

Q10

Answer :

Suppose: $2\angle A = 3\angle B = 6\angle C = x^\circ$

Then, $\angle A = \left(\frac{x}{2}\right)^o$

$\angle B = \left(\frac{x}{3}\right)^o$ and $\angle C = \left(\frac{x}{6}^\circ\right)$

Sum of the angles of any triangle is 180°.

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \frac{x}{2} + \frac{x}{3} + \frac{x}{6} = 180^\circ$

$\Rightarrow \frac{3x+2x+x}{6} = 180^\circ$

$\Rightarrow \frac{6x}{6} = 180^\circ$

$\Rightarrow x = 180$

$\therefore \angle A = \left(\frac{180}{2}\right)^o = 90^\circ$

$\angle B = \left(\frac{180}{3}\right)^o = 60^o$

$\angle C = \left(\frac{180}{6}\right)^o = 30^\circ$

Q11

Answer :

We know that the angles of an equilateral triangle are equal.
Let the measure of each angle of an equilateral triangle be x°.

$\therefore x + x + x = 180$

$x = 60$

Hence, the measure of each angle of an equilateral triangle is 60°.

Q12

Answer :

(i)

$DE \parallel BC$

$\therefore \angle ABC = \angle ADE = 55^\circ$

(Corresponding angles)

(ii) Sum of the angles of any triangle is 180°.

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$\angle C = 180^\circ - (65^\circ + 55^\circ) = 60^\circ$

DE || BC

$\therefore \angle AED = \angle ACB = 60^\circ$ (corresponding angles)

(iii) We have found in point (ii) that $\angle C$ is equal to 60°.

Q13

Answer :

(i) No. This is because the sum of all the angles is 180°.
(ii) No. This is because a triangle can only have one obtuse angle.
(iii) Yes
(iv) No. This is because the sum of the angles cannot be more than 180°.
(v) No. This is because one angle has to be more than 60° as the sum of all angles is always 180°.
(vi) Yes, it will be an equilateral triangle.

Q14

Answer :

(i) Yes, it will be an isosceles right triangle.

(ii) Yes, a right triangle can have all sides of different measures. For example, 3, 4 and 5 are the sides of a scalene right triangle.

(iii) No, it cannot be an equilateral triangle since the hypotenuse square will be the sum of the square of the other two sides.

(iii) Yes, if an obtuse triangle has an obtuse angle of 120° and the other two angles of 30° each, then it will be an isosceles triangle.

Q15

Answer :

(i) obtuse (since the sum of the other two angles of the right triangle is 90°)
(ii) equal to the sum of 90°
(iii) 45° (since their sum is equal to 90°)
(iv) 60°
(v) a hypotenuse
(vi) perimeter

Properties of Triangles
Exercise 15B

Q1

Answer :

We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\angle ACD = \angle CAB + \angle CBA$
$\angle ACD = 75^\circ + 45^\circ = 120^\circ$

Q2

Answer :

We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\therefore \angle BAC + \angle ABC = \angle ACD$
$x + 68 = 130$
$x = 62$

Sum of the angles in any triangle is 180°.

$\therefore \angle BAC + \angle ABC + \angle ACB = 180^\circ$
$62 + 68 + y = 180$
$y = 50$

Q3

Answer :

We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\therefore \angle BAC + \angle CBA = \angle ACD$
$32 + x = 65$
$x = 33$

Also, sum of the angles in any triangle is 180°.

$\therefore \angle BAC + \angle CBA + \angle ACB = 180^\circ$
$32 + 33 + y = 180$
$y = 115$

$\therefore x = 33$
$y = 115$

Q4

Answer :

Suppose the interior opposite angles are $(2x)°$ and $(3x)°$.
We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\therefore 3x + 2x = 110$
$x = 22$

The interior opposite angles are $(2 \times 22)^\circ$ and $(3 \times 22)^\circ$, i.e. 44° and 66°.
Suppose the third angle of the triangle is $y°$.
Now, sum of the angles in any triangle is 180°.

$\therefore 44 + 66 + y = 180$
$y = 70$

Hence, the angles of the triangle are 44°, 66° and 70°.

Q5

Answer :

Suppose the interior opposite angles of an exterior angle 100^o are x^o and x^o.
We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\therefore x + x = 100$
$2x = 100$
$x = 50$

Also, sum of the angles of any triangle is 180°.
Let the measure of the third angle be y°.

$\therefore x + x + y = 180$
$50 + 50 + y = 180$
$y = 80$

Hence, the angles are of the measures 50°, 50° and 80°.

Q6

Answer :

We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

In △ABC:
$\angle ACD = \angle BAC + \angle ABC = 25^\circ + 45^\circ$
$\angle ACD = 70^\circ$
(ii) $In\ \triangle ECD:$
$\angle AED = \angle ECD + \angle EDC = 70^\circ + 40^\circ$
$=> \angle AED = 110^\circ$

Q7

Answer :

Sum of the angles of a triangle is 180°.

$In \triangle ABC:$
$\angle BAC + \angle CBA + \angle ACB = 180^\circ$
$\angle BAC = 180^\circ - (40^\circ + 100^\circ)$
$=> \angle BAC = 40^\circ$

We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\angle ACD = \angle BAC + \angle CBA = 40^\circ + 40^\circ = 80^\circ$
$(i)\ \angle ACD = 80^\circ$
(ii) In $\triangle\ ACD:$
$\angle CAD + \angle ACD + \angle ADC = 180^\circ$
$=> \angle ADC = 180^\circ - (50^\circ + 80^\circ)$
$=> \angle ADC = 50^\circ$
$\therefore\ \angle ADC = 50^\circ$
$(iii)\ \angle DAB + \angle DAE = 180^\circ$ (since BE is a straight line)
$\angle DAE = 180^\circ - (\angle DAC + \angle CAB)$
$\angle DAE = 180^\circ - (50^\circ + 40^\circ)$
$\angle DAE = 90^\circ$

Q8

Answer :

$\frac{x}{y} = \frac{2}{3}$
$\Rightarrow 3x = 2y$
$\Rightarrow x = \frac{2}{3}y$

We know that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

$\therefore \angle A + \angle B = \angle ACD$

$x^\circ + y^\circ = 130^\circ$

$\Rightarrow \frac{2y}{3} + y = 130$
$\Rightarrow 5y = 130 \times 3$
$\Rightarrow 5y = 390$
$\Rightarrow y = 78$
$\Rightarrow x = \frac{2}{3} \times 78$
$\Rightarrow x = 52$

Also, sum of the angles in any triangle is 180°
$\therefore$ x+ y + z = 180
z= 180- 78 - 52
z= 50

Properties of Triangles
Exercise 15C

Q1

Answer :

(i) Consider numbers 1, 1 and 1.

Clearly, 1 + 1 >1
1 + 1 >1
1 + 1 >1

Thus, the sum of any two sides is greater than the third side.
Hence, it is possible to draw a triangle having sides 1 cm, 1 cm and 1 cm.

(ii)
Clearly, 2 + 3 >4
3 + 4 >2
2+ 4 >3
Thus, the sum of any two sides is greater than the third side.
Hence, it is possible to a draw triangle having sides 2 cm, 3 cm and 4 cm.

(iii)
Clearly, 7 + 8 = 15

Thus, the sum of these two numbers is not greater than the third number.
Hence, it is not possible to draw a triangle having sides 7 cm, 8 cm and 15 cm.

(iv) Consider the numbers 3.4, 2.1 and 5.3.

Clearly: 3.4 + 2.1 >5.3
5.3 + 2.1 > 3.4
5.3 + 3.4 > 2.1

Thus, the sum of any two sides is greater than the third side.
Hence, it is possible to draw a triangle having sides 3.4 cm, 2.1 cm and 5.3 cm.

(v) Consider the numbers 6, 7 and 14.
Clearly, 6+7 $\ngtr$ 14

Thus, the sum of these two numbers is not greater than the third number.
Hence, it is not possible to draw a triangle having sides 6 cm, 7 cm and 14 cm.

Q2

Answer :

Let the length of the third side be *x* cm.

Sum of any two sides of a triangle is greater than the third side.

∴ 5 + 9 > x
$\Rightarrow x < 14$

Hence, the length of the third side must be less than 14 cm.

Q3

Answer :

(i) >
(ii) >
(iii) <

The reason for the above three is that the sum of any two sides of a triangle is greater than the third side.

Q4

Answer :

Sum of any two sides of a triangle is greater than the third side.

In △AMB:
AB + BM >AM........(i)

In △AMC:
AC + CM >AM.........(ii)

Adding the above two equation:
AB + BM + AC + CM >AM + AM
AB + BC + AC > 2AM

Hence, proved.

Q5

Answer :

Sum of any two sides of a triangle is greater than the third side.

$$
\begin{aligned}
&In\ \triangle\ APB:\\
&AB + BP > AP\\
&In\ \triangle\ APC:\\
&AC + PC > AP\\
&Adding\ the\ correspondong\ sides:\\
&AB + BP + AC + PC > AP + AP\\
&AB + AC + BC > 2AP
\end{aligned}
$$

Hence, proved.

Q6

Answer :

Sum of any two sides of a triangle is greater than the third side.

In △ABC:
AB + BC > AC

In △ADC:
CD + DA > AC

Adding the above two:

AB + BC + CD + DA > 2 AC ... (i)

In △ADB:
AD + AB > BD

In △BDC:
CD + BC > BD

Adding the above two:

AB + BC + CD + DA >2 BD ... (ii)

Adding equation (i) and (ii):

AB + BC + CD + DA+AB + BC + CD + DA> 2(AC+BD)
=> 2(AB + BC + CD + DA)>2(AC+BD)
=> AB + BC + CD + DA > AC+BD

Q7

Answer :

We know that the sum of any two sides of a triangle is greater than the third side.

In △AOB:
OA + OB > AB...........(1)

In △BOC:
OB + OC > BC........................... (2)

In △AOC:
OA + OC > CA.............................(3)

Adding (1), (2) and (3):

OA + OB + OB + OC + OA + OC > AB + BC + CA

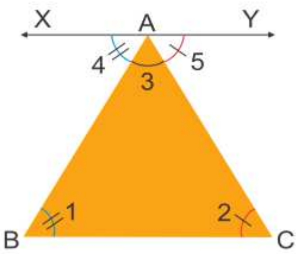
2(OA + OB + OC) > AB +BC + CA

Hence, proved.

Properties of Triangles
Exercise 15D

Properties of Triangles

Angle Sum Property of a Triangle



The sum of the interior angles of a triangle is 180°.

Proof:

Draw XY || BC

$\angle 1 = \angle 4$

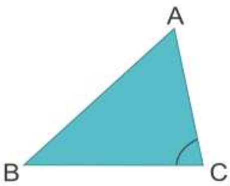
$\angle 2 = \angle 5$ { *Alternate Interior angles are equal* }

$\Rightarrow \angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 3$

But, $\angle 4 + \angle 5 + \angle 3 = 180^\circ$ *(By linearity property)*

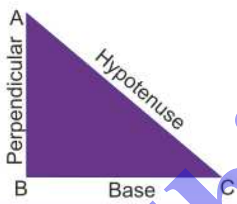
$\therefore \angle 1 + \angle 2 + \angle 3 = 180^\circ$

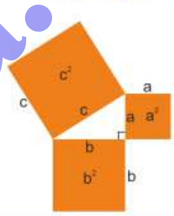
Triangle Inequality Property



- The sum of any two sides of a triangle is always greater than its third side. $AB + BC > AC$
- The angle opposite to the longest side is the largest angle.
- The angle opposite to the smallest side is the smallest angle

Pythagoras Theorem





Pythagoras' Theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of squares of its remaining two sides. $(AC)^2 = (AB)^2 + (BC)^2$ or $c^2 = a^2 + b^2$

Pythagorean triplets

Three positive numbers a, b, c in this order, are said to form a Pythagorean triplet, if $c^2 = a^2 + b^2$

Converse of Pythagoras' Theorem

If a triangle has sides of length a, b and c units such that $a^2 + b^2 = c^2$, then the triangle is right angled

Q1

Answer :

Suppose the length of the hypotenuse is *a* cm.

Then, by Pythagoras theorem:

$a^2 = 9^2 + 12^2$
=> a^2 = 81 + 144
=> a^2 =225
=> a = $\sqrt{225}$
=> a= 15

Hence, the length of the hypotenuse is 15 cm.

Q2

Answer :

Suppose the length of the other side is *a* cm.

Then, by Pythagoras theorem:

$26^2 = 10^2 + a^2$
$\Rightarrow a^2 = 676 - 100$
$\Rightarrow a^2 = 576$
$\Rightarrow a = \sqrt{576}$
$\Rightarrow a = 24$

Hence, the length of the other side is 24 cm.

Q3

Answer :

Suppose the length of the other side is *a* cm.

Then, by Pythagoras theorem:

$4.5^2 + a^2 = 7.5^2$
$\Rightarrow a^2 = 56.25 - 20.25$
$\Rightarrow a^2 = 36$
$\Rightarrow a = \sqrt{36}$
$\Rightarrow a = 6$

Hence, the length of the other side of the triangle is 6 cm.

Q4

Answer :

Suppose the length of the two legs of the right triangle are *a* cm and *a* cm.
Then, by Pythagoras theorem:
$a^2 + a^2 = 50$
$\Rightarrow 2a^2 = 50$
$\Rightarrow a^2 = 25$
$\Rightarrow a = \sqrt{25}$
$\Rightarrow a = 5$

Hence, the length of each leg is 5 cm.

Q5

Answer :

The largest side of the triangle is 39 cm.

$15^2 + 36^2$
$= 225 + 1296 = 1521$

Also, $39^2 = 1521$
$\therefore 15^2 + 36^2 = 39^2$

Sum of the square of the two sides is equal to the square of the third side.

Hence, the triangle is right angled.

Q6

Answer :

Suppose the length of the hypotenuse is c cm.
Then, by Pythagoras theorem:

$a^2 + b^2 = c^2$
$\Rightarrow c^2 = 6^2 + 4.5^2$
$\Rightarrow c^2 = 36 + 20.25$
$\Rightarrow c^2 = 56.25$
$\Rightarrow c = \sqrt{56.25}$
$\Rightarrow c = 7.5$

Hence, the length of its hypotenuse is 7.5 cm.

Q7

Answer :

(i) Largest side, c = 25 cm

We have:
$a^2 + b^2$ = 225 + 400 = 625

Also, c^2 = 625

$\therefore a^2 + b^2 = c^2$
Hence, the given triangle is right angled using the Pythagoras theorem.

(ii) Largest side, c = 16 cm
We have:
$a^2 + b^2$ = 81 + 144 = 225

Also, c^2 = 256

Here, $a^2 + b^2 \neq c^2$

Therefore, the given triangle is not right angled.

Q8

Answer :

We have:
$\angle B$ = 35° and $\angle C$ = 55°

$\therefore \angle B$ = 180 - 35 -55 = 90° (since sum of the angles of any triangle is 180°)

We know that the side opposite to the right angle is the hypotenuse.

By Pythagoras theorem:
$BC^2 = AB^2 + AC^2$

Hence, (iii) is true.

Q9

Answer :

By Pythagoras theorem in △ABC:
AB2 = AC2 + BC2
152 = x2 +122⇒x2 = 225 - 144 ⇒x2= 81⇒x2=92⇒x=9
$\therefore x$ = 9 cm

Hence, the distance of the foot of the ladder from the wall is 9 cm.

Q10

Answer :

Suppose the foot of the ladder is *x* m far from the wall.

Let the ladder is represented by AB, the height at which it reaches the wall be AC and the distance between the foot of ladder and wall be BC.

Then, by Pythagoras theorem:

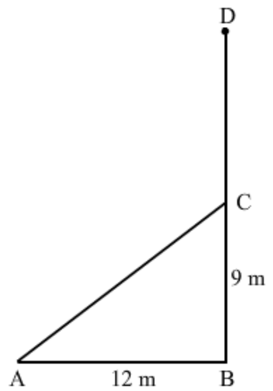
$$AB^2 = AC^2 + BC^2$$
$$\Rightarrow 5^2 = 4.8^2 + x^2$$
$$\Rightarrow x^2 = 25 - 23.04$$
$$\Rightarrow x^2 = 1.96$$
$$\Rightarrow x^2 = \left(1.4\right)^2$$
$$\Rightarrow x = 1.4$$

Hence, the foot of the ladder is 1.4 m far from the wall.

Q11

Answer :

Let BD be the height of the tree broken at point C and suppose CD take the position CA



Now as per given conditions we have AB = 9 m , BC = 12 m

By Pythagoras theorem:

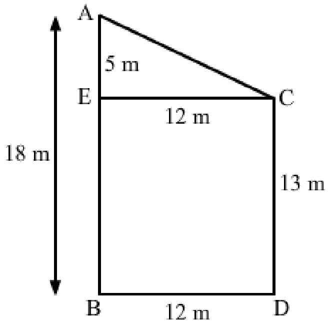
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow AC^2 = 12^2 + 9^2$$
$$\Rightarrow AC^2 = 144 + 81$$
$$\Rightarrow AC^2 = 225$$
$$\Rightarrow AC^2 = 15^2$$
$$\Rightarrow AC = 15$$

Length of the tree before it broke = AC + AB
= 15 + 9
= 24 m

Q12

Answer :

Suppose, the two poles are AB and CD, having the length of 18 m and 13 m, respectively.
Distance between them, BD, is equal to 12 m.
We need to find AC.



From C, draw CE⊥AB.

AE=AB-EB
= AB-CD (CD = EB)
= 18-13
= 5 cm
EC = BD = 12 m

Now, by Pythagoras theorem in $\triangle AEC$:

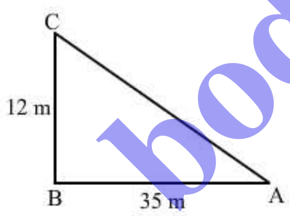
$$\begin{aligned}
&AC^2 = AE^2 + EC^2\\
&\Rightarrow AC^2 = 5^2 + 12^2\\
&\Rightarrow AC^2 = 25 + 144\\
&\Rightarrow AC^2 = 169\\
&\Rightarrow AC^2 = 13^2\\
&\Rightarrow AC = 13
\end{aligned}$$

Hence, the distance between their tops is 13 m.

Q13

Answer :

Suppose the man starts at point A and goes 35 m towards west, say AB. He then goes 12 m north, say BC.



We need to find AC.

By Pythagoras theorem:

$$\begin{aligned}
&AC^2 = BC^2 + AB^2\\
&\Rightarrow AC^2 = 35^2 + 12^2\\
&\Rightarrow AC^2 = 1225 + 144\\
&\Rightarrow AC^2 = 1369\\
&\Rightarrow AC^2 = 37^2\\
&\Rightarrow AC = 37\ m
\end{aligned}$$

Hence, the man is 37 m far from the starting point.

Q14

Answer :

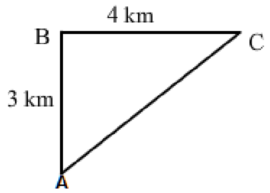
Suppose the man starts from A and goes 3 km north and reaches B.
He then goes 4 km towards east and reaches C.

∴ AB = 3 km
BC = 4 km
We have to find AC.

By Pythagoras theorem:

$\Rightarrow AC^2 = AB^2 + BC^2$
$\Rightarrow AC^2 = 3^2 + 4^2$
$\Rightarrow AC^2 = 25$
$\Rightarrow AC^2 = 5^2$
$\Rightarrow AC = 5 \text{ km}$

Hence, he is 5 km far from the initial position.



Q15

Answer :

Suppose the sides are *x* and *y* of lengths 16 cm and 12 cm, respectively.

Let the diagonal be *z* cm.

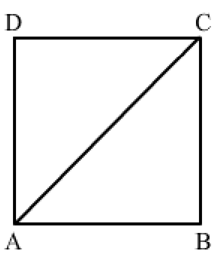
Clearly, the diagonal is the hypotenuse of the right triangle with legs *x* and *y*.

By Pythagoras theorem:
$z^2 = x^2 + y^2$
$\Rightarrow z^2 = 16^2 + 12^2$
$\Rightarrow z^2 = 256 + 144$
$\Rightarrow z^2 = 400$
$\Rightarrow z^2 = 20^2$
$\Rightarrow z = 20$

Hence, the length of the diagonal is 20 cm.

Q16

Answer :



AB = 40 cm
Diagonal, AC = 41 cm

Then, by Pythagoras theorem in right $\triangle ABC$:
$AC^2 = AB^2 + BC^2$
$\Rightarrow BC^2 = 41^2 - 40^2$
$\Rightarrow BC^2 = 1681 - 1600$
$\Rightarrow BC^2 = 81$
$\Rightarrow BC^2 = 9^2$
$\Rightarrow BC = 9 \text{ cm}$

∴ Length = 40 cm
Breadth = 9 cm

∴ Perimeter of the rectangle = 2(length + breadth)
= 2(40+9)
= 98 cm

Q17

Answer :

We know that the diagonals of a rhombus bisect each other at right angles.
Therefore, in right triangle AOB, we have:
AO = 8 cm
BO = 15 cm

By Pythagoras theorem in ΔAOB:

$$AB^2 = AO^2 + BO^2$$
$$\Rightarrow AB^2 = 8^2 + 15^2$$
$$\Rightarrow AB^2 = 64 + 225$$
$$\Rightarrow AB^2 = 289$$
$$\Rightarrow AB^2 = 17^2$$
$$\Rightarrow AB = 17 \text{ cm}$$

Now, as we know that all sides of a rhombus are equal.
∴ Perimeter of the rhombus = 4(side)
= 4(17)
= 68 cm

Q18

Answer :

(i) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
(ii) If the square of one side of a triangle s equal to the sum of the squares of the other two sides then the triangle is right angled.
(iii) Of all the line segments that can be drawn to a given line from a given point outside it, the perpendicular is the shortest.