1. Real Number

Exercise 1A

1. Question

What are rational numbers? Give ten examples of rational numbers.

Answer

Rational number is a number which can be written in the form $\frac{p}{q}$,

where p and q both are integers but q is not equals to zero.

Examples = $0, \frac{1}{9}, \frac{4}{7}, \frac{3}{5}, \frac{6}{11}, \frac{13}{12}, \frac{23}{16}, \frac{4}{5}, \frac{56}{57}, 1$

2. Question

Represent each of the following rational numbers on the number line:

(i) 5 (ii) -3

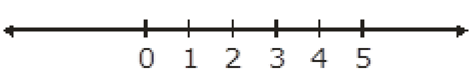
(iii) $\frac{5}{7}$ (iv) $\frac{8}{3}$ (v) 1.3

(vi) -2.4 (vii) $\frac{23}{6}$

Answer

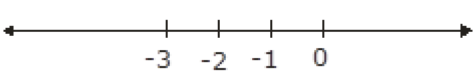
(i) Given number = 5

First draw a line and mark the origin Zero. The given number 5 is positive so we are going to locate it on the right side of the zero.



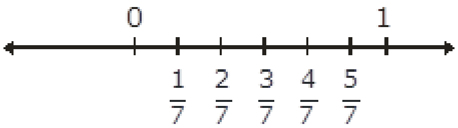
(ii) Given = -3

First draw a line and mark the origin Zero. The given number is negative (-3) so we are going to locate it on the left side of the zero.



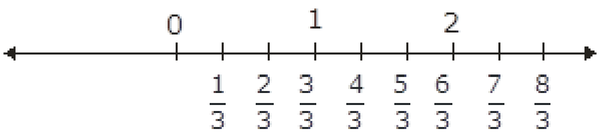
(iii) Given = 5/7

First draw a line and mark the origin Zero. The given number $\frac{5}{7}$ is positive so we are going to locate it on the right side of the zero.



(iv) Given = 8/3

First draw a line and mark the origin Zero. The given number $\frac{8}{3}$ is positive so we are going to locate it on the right side of the zero.



(v) Given = 1.3

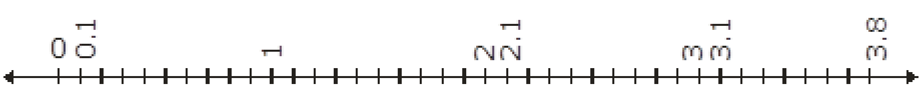
First draw a line and mark the origin Zero. The given number 1.3 is positive so we are going to locate it on the right side of the zero.

(vi) Given = -2.4

First draw a line and mark the origin Zero. The given number -2.4 is negative so we are going to locate it on the left side of the zero.

(vii) Given = 23/6

First draw a line and mark the origin Zero. The given number $\frac{23}{6} = 3.8$ is positive so we are going to locate it on the right side of the zero.



3. Question

Find a rational number lying between

(i) $\frac{1}{4}$ *and* $\frac{1}{3}$ (ii) $\frac{3}{8}$ *and* $\frac{2}{5}$

(iii) 1.3 and 1.4 (iv) 0.75 and 1.2

(v) -1 *and* $\frac{1}{2}$ (vi) $-\frac{3}{4}$ *and* $-\frac{2}{5}$

Answer

(i) $\frac{1}{4} and \frac{1}{3}$

Let suppose,

$$a = \frac{1}{4}\ and\ b = \frac{1}{3}$$

As we can see that a < b

So, the rational number lying between a and b

$$= \frac{1}{2}(a + b)$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right)$$

$$= \frac{1}{2} \times \left(\frac{3+4}{12}\right)$$

$$= \frac{1}{2} \times \left(\frac{7}{12}\right) = \frac{7}{24}$$

So, we can say that $\frac{7}{24}$ is a rational number lying between 1/4 and 1/3.

(ii) $\frac{3}{8}\ and\ \frac{2}{5}$

Let's take $a = \frac{3}{8}\ and\ b = \frac{2}{5}$

As we can see a<b

A rational number lying between $\frac{3}{8}\ and\ \frac{2}{5}$

$$= \frac{1}{2}\ (a + b)$$

$$= \frac{1}{2}\left(\frac{3}{8} + \frac{2}{5}\right)$$

$$= \frac{1}{2}\left(\frac{15 + 16}{40}\right)$$

$$= \frac{1}{2}\left(\frac{31}{40}\right) = \frac{31}{80}$$

$\frac{31}{80}$ is the number lying between $\frac{3}{8}\ and\ \frac{2}{5}$

(iii) 1.3 and 1.4

By taking a = 1.3 and b = 1.4

As we can see a < b

The rational number between a and b,

$=\frac{1}{2}(a + b)$

$=\frac{1}{2}(1.3 + 1.4)$

$=\frac{1}{2}(2.7)$

$=\frac{1}{2} \times 2.7 = 1.35$

1.35 is the number lying between 1.3 and 1.4

(iv) 0.75 and 1.2

Let's take a = 0.75 and b = 1.2

As we can see a<b,

The rational number between a and b,

$=\frac{1}{2}(a + b)$

$=\frac{1}{2}(0.75 + 1.2)$

$=\frac{1}{2}(1.95)$

$=\frac{1}{2} \times 1.95 = 0.975$

0.975 is the number between 0.75 and 1.2

(v) $-1 \; and \; \frac{1}{2}$

Let's take $a = -1 \; and \; b = \frac{1}{2}$

The rational number between a and b,

$=\frac{1}{2}(a + b)$

$=\frac{1}{2}\left(-1 + \frac{1}{2}\right)$

$$= \frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$$

$-\frac{1}{4}$ is the number between -1 and $\frac{1}{2}$

(vi) $-\frac{3}{4}$ *and* $-\frac{2}{5}$

Let's take $a = -\frac{3}{4}$ *and* $b = -\frac{2}{5}$

The rational number between a and b,

$$= \frac{1}{2}(a + b)$$

$$= \frac{1}{2}\left(-\frac{3}{4} + \left(-\frac{2}{5}\right)\right)$$

$$= \frac{1}{2}\left(-\frac{3}{4} - \frac{2}{5}\right)$$

$$= \frac{1}{2}\left(\frac{-15 - 8}{20}\right)$$

$$= \frac{1}{2}\left(\frac{-23}{20}\right) = -\frac{23}{40}$$

So, the rational number between $-\frac{3}{4}$ *and* $-\frac{2}{5}$ *is* $-\frac{23}{40}$

4. Question

Find three rational numbers lying between $\frac{1}{5}$ *and* $\frac{1}{4}$.

Answer

Let's take $a = \frac{1}{5}, b = \frac{1}{4}$ and n = 3

n= numbers required to find out

So, $d = \frac{b-a}{n+1} = \frac{\left(\frac{1}{4}-\frac{1}{5}\right)}{3+1} = \frac{\frac{5-4}{20}}{4} = \frac{1}{80}$

Thus, three rational numbers are:

(a + d), (a + 2d), and (a + 3d)

$$(a + d) = \left(\frac{1}{5} + \frac{1}{80}\right) = \left(\frac{16 + 1}{80}\right) = \frac{17}{80}$$

$$(a + 2d) = \left(\frac{1}{5} + 2 \times \frac{1}{80}\right) = \left(\frac{1}{5} + \frac{2}{80}\right) = \frac{16 + 2}{80} = \frac{18}{80}$$

$$(a + 3d) = \left(\frac{1}{5} + \left(3 \times \frac{1}{80}\right)\right) = \left(\frac{1}{5} + \frac{3}{80}\right) = \left(\frac{16 + 3}{80}\right) = \frac{19}{80}$$

Hence, three rational numbers lying between 1/5 and 1/4 are $\frac{17}{80}, \frac{18}{80}, \frac{19}{80}$

5. Question

Find five rational numbers lying between $\frac{2}{5} and \frac{3}{4}$.

Answer

Let's take $a = \frac{2}{5}, b = \frac{3}{4}$ $and\ n = 5$

n= numbers required to be find out

So, $d = \frac{b-a}{n+1} = \frac{\left(\frac{3}{4}-\frac{2}{5}\right)}{5+1} = \frac{\frac{15-8}{20}}{6} = \frac{7}{6 \times 20} = \frac{7}{120}$

Thus, five rational numbers are:

(a + d), (a + 2d), (a + 3d), (a + 4d) and (a + 5d)

$$(a + d) = \left(\frac{2}{5} + \frac{7}{120}\right) = \left(\frac{48+7}{120}\right) = \frac{55}{120}$$

$$(a + 2d) = \left(\frac{2}{5} + 2 \times \frac{7}{120}\right) = \left(\frac{2}{5} + \frac{14}{120}\right) = \frac{48+14}{120} = \frac{62}{120}$$

$$(a + 3d) = \left(\frac{2}{5} + \left(3 \times \frac{7}{120}\right)\right) = \left(\frac{2}{5} + \frac{21}{120}\right) = \left(\frac{48+21}{80}\right) = \frac{69}{120}$$

$$(a + 4d) = \left(\frac{2}{5} + \left(4 \times \frac{7}{120}\right)\right) = \left(\frac{2}{5} + \frac{28}{120}\right) = \left(\frac{48+28}{80}\right) = \frac{76}{120}$$

$$(a + 5d) = \left(\frac{2}{5} + \left(5 \times \frac{7}{120}\right)\right) = \left(\frac{2}{5} + \frac{35}{120}\right) = \left(\frac{48 + 35}{80}\right) = \frac{83}{120}$$

Hence, five rational numbers lying between $\frac{2}{5}\ and\ \frac{3}{4}\ are\ \frac{55}{120}, \frac{62}{120}, \frac{69}{120}, \frac{76}{120}\ and\ \frac{83}{120}$

6. Question

Insert six rational numbers between 3 and 4.

Answer

Let's take a = 3, b = 4 and n = 6

n= numbers required to be find out

So, $d = \frac{b-a}{n+1} = \frac{(4-3)}{6+1} = \frac{1}{7}$

Thus, six rational numbers are:

(a + d), (a + 2d), (a + 3d), (a + 4d), (a + 5d) and (a + 6d)

$$(a + d) = \left(3 + \frac{1}{7}\right) = \left(\frac{21+1}{7}\right) = \frac{22}{7}$$

$$(a + 2d) = \left(3 + \left(2 \times \frac{1}{7}\right)\right) = \left(3 + \frac{2}{7}\right) = \left(\frac{21+2}{7}\right) = \frac{23}{7}$$

$$(a + 3d) = \left(3 + \left(3 \times \frac{1}{7}\right)\right) = \left(3 + \frac{3}{7}\right) = \left(\frac{21+3}{7}\right) = \frac{24}{7}$$

$$(a + 4d) = \left(3 + \left(4 \times \frac{1}{7}\right)\right) = \left(3 + \frac{4}{7}\right) = \left(\frac{21+4}{7}\right) = \frac{25}{7}$$

$$(a + 5d) = \left(3 + \left(5 \times \frac{1}{7}\right)\right) = \left(3 + \frac{5}{7}\right) = \left(\frac{21+5}{7}\right) = \frac{26}{7}$$

$$(a + 6d) = \left(3 + \left(6 \times \frac{1}{7}\right)\right) = \left(3 + \frac{6}{7}\right) = \left(\frac{21+6}{7}\right) = \frac{27}{7}$$

Hence, six rational numbers lying between $3 and\ 4\ are \frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}\ and\ \frac{27}{7}$

7. Question

Insert 16 rational numbers between 2.1 and 2.2.

Answer

Let's take $a = 2.1, b = 2.2\ and\ n = 16$

n= numbers required to be find out

So, $d = \frac{b-a}{n+1} = \frac{2.2-2.1}{16+1} = \frac{0.1}{17} = \frac{1}{170} = 0.005$

Thus, 16 rational numbers are:

(a + d), (a + 2d), (a + 3d), (a + 4d), (a + 5d), (a + 6d), (a + 7d), (a + 8d), (a + 9d), (a + 10d), (a + 11d), (a + 12d), (a + 13d), (a + 14d), (a + 15d) and (a + 16d)

So,

(a + d) = (2.1 + 0.005) = 2.105

(a + 2d) = [2.1 + (2 × 0.005)] = 2.110

(a + 3d) = [2.1 + (3 × 0.005)] = 2.115

(a + 4d) = [2.1 + (4 × 0.005)] = 2.120

(a + 5d) = [2.1 + (5 × 0.005)] = 2.125

(a + 6d) = [2.1 + (6 × 0.005)] = 2.130

(a + 7d) = [2.1 + (7 × 0.005)] = 2. 135

(a + 8d) = [2.1 + (8 × 0.005)] = 2. 140

(a + 9d) = [2.1 + (9 × 0.005)] = 2. 145

(a + 10d) = [2.1 + (10 × 0.005)] = 2. 150

(a + 11d) = [2.1 + (11 × 0.005)] = 2. 155

(a + 12d) = [2.1 + (12 × 0.005)] = 2. 160

(a + 13d) = [2.1 + (13 × 0.005)] = 2. 165

(a + 14d) = [2.1 + (14 × 0.005)] = 2. 170

(a + 15d) = [2.1 + (15 × 0.005)] = 2. 175

(a + 16d) = [2.1 + (16 × 0.005)] = 2. 180

Thus, the rational numbers between 2.1 and 2.2 are 2.105, 2.110, 2.115, 2.120, 2.125, 2.130, 2.135, 2.140, 2.145, 2.150, 2.155, 2.160, 2.165, 2.170, 2.175, 2.180,

Exercise 1B

1. Question

Without actual division, find which of the following rationals are terminating decimals.

(i) $\frac{13}{80}$ (ii) $\frac{7}{24}$ (iii) $\frac{5}{12}$

(iv) $\frac{8}{35}$ (v) $\frac{16}{125}$

Answer

First we have to know what is terminating decimal. Terminating decimal is the number which has digits that do not go on forever.

(i) $\frac{13}{80}$

Denominator 80 has factors = 2 × 2 × 2 × 2 × 5

So, 80 has no prime factors other than 2 and 5, thus $\frac{13}{80}$ is terminating decimal.

(ii) $\frac{7}{24}$

Denominator 24 has factors = 2 × 2 × 2 × 3

So, 24 has factors other than 2 and 5, thus $\frac{7}{24}$ is not a terminating decimal.

(iii) $\frac{5}{12}$

Denominator 12 has factors = 2 × 2 × 3

So, 12 has factors other than 2 and 5, thus $\frac{5}{12}$ is not a terminating decimal.

(iv) $\frac{8}{35}$

Denominator 35 has factors = 5 × 7

So, 35 has factors other than 2 and 5, thus $\frac{8}{35}$ is not a terminating decimal.

(v) $\frac{16}{125}$

Denominator 24 has factors = 5 × 5 × 5

So, 125 has no prime factors other than 2 and 5, thus $\frac{16}{125}$ is a terminating decimal.

2. Question

Convert each of the following into a decimal.

(i) $\frac{5}{8}$ (ii) $\frac{9}{16}$ (iii) $\frac{7}{25}$

(iv) $\frac{11}{24}$ (v) $2\frac{5}{12}$

Answer

(i) Given $\frac{5}{8}$

By actual division method, we get

```
   0.625
8)5.000
  48
   20
   16
    40
    40
     ×
```

Thus $\frac{5}{8} = 0.625$

(ii) Given $\frac{9}{16}$

By actual division method we get:

```
     0.5625
16)9.0000
   80
   100
    96
     40
    32
      80
      80
       ×
```

Thus $\frac{9}{16} = 0.5625$

(iii) $\frac{7}{25}$

By actual division method we get:

```
    0.28
25)7.00
   50
   200
   200
    ×
```

Thus $\frac{7}{25} = 0.28$

(iv) $\frac{11}{24}$

By actual division method we get:

```
     0.45833..
24) 11.00
    96
    140
    120
     200
     192
       80
       72
       80
```

Thus $\frac{11}{24} = 0.45833$

(v) $2\frac{5}{12} = \frac{29}{12}$

By actual division method we get:

```
     2.4166....
12)  29.0
     24
     --
      50
      48
      --
       20
       12
       --
        80
        72
        --
         8
```

Thus $\frac{29}{12} = 2.4166$

3. Question

Express each of the following as a fraction in simplest form.

(i) $0.\overline{3}$ (ii) $1.\overline{3}$ (iii) $0.\overline{34}$

(iv) $3.\overline{14}$ (v) $0.\overline{324}$ (vi) $0.1\overline{7}$

(vii) $0.5\overline{4}$ (viii) $0.1\overline{63}$

Answer

(i) Given $0.\overline{3}$

Let x equals to the repeating decimal = $0.\overline{3}$

As we can see the repeating digit is 3

x = 0.3333333……… (i)

10x = 3.333333……… (ii)

Subtracting (i) from (ii), we get

10x - x = 3.3333 – 0.3333

9x = 3

$$x = \frac{3}{9} = \frac{1}{3}$$

So, we can say that 0.3333333333…. is equals to the $\frac{1}{3}$.

(i) Given $1.\overline{3}$

Let x equals to the repeating decimal = $1.\overline{3}$

As we can see the repeating digit is 3

x = 1.3333333……… (i)

10x = 13.333333……… (ii)

Subtracting (i) from (ii), we get

10x-x = 13.3333 – 1.3333

9x = 12

$$x = \frac{12}{9} = \frac{4}{3}$$

So, we can say that 1.3333333333…. is equals to the $\frac{4}{3}$.

(ii) $0.\overline{34}$

Let x equals to the repeating decimal = $0.\overline{34}$

As we can see the repeating digit is 34

X = 0.3434343434……… (i)

10x = 3.434343434……… (ii)

100x = 34.3434343434….. (iii)

Subtracting (i) from (iii), we get

100x-x = 0.34343434 – 34.34343434

99x = 34

$$x = \frac{34}{99} =$$

So, we can say that 0.34343434343434…. is equals to the $\frac{34}{99}$.

(iii) $3.\overline{14}$

Let x equals to the repeating decimal = $3.\overline{14}$

As we can see the repeating digit is 14

X = 3.1414141414……… (i)

10x = 31.41414141414……… (ii)

100x = 314.1414141414….. (iii)

Subtracting (i) from (iii), we get

100x - x = 3.1414141414 – 314.14141414

99x = 311

$$x = \frac{311}{99}$$

So, we can say that 3.1414141414…. is equals to the $\frac{311}{99}$.

(iv) $0.\overline{324}$

Let x equals to the repeating decimal = $0.\overline{324}$

As we can see the repeating digit is 324

X = 0.324324324324324……… (i)

10x = 3.24324324324324……… (ii)

100x = 32.4324324324324….. (iii)

1000x = 324.324324324324…….(iv)

Subtracting (i) from (iv), we get

1000x - x = 0.324324324324 – 324.324324324324

999x = 324

$$x = \frac{324}{999}$$

So, we can say that 0.324324324324324324324…. is equals to the $\frac{324}{999}$.

(v) $0.1\overline{7}$

Let x equals to the repeating decimal = $0.1\overline{7}$

As we can see the repeating digit is 7

x = 0.17777777777……… (i)

10x = 1.777777777……… (ii)

100x = 17.77777777….. (iii)

Subtracting (ii) from (iii), we get

100x - 10x = 17.777777-1.777777

90x = 16

$$x = \frac{16}{90} = \frac{8}{45}$$

So, we can say that 0.17777777…. is equals to the $\frac{8}{45}$

(vi) $0.5\overline{4}$

Let x equals to the repeating decimal = $0.5\overline{4}$

As we can see the repeating digit is 4

X = 0.5444444444……… (i)

10x = 5.44444444……… (ii)

100x = 54.444444….. (iii)

Subtracting (ii) from (iii), we get

100x-10x = 54.44444-5.44444

90x = 49

$$x = \frac{49}{90}$$

So, we can say that 0.5444444…. is equals to the $\frac{49}{90}$

(vii) $0.1\overline{63}$

Let x equals to the repeating decimal = $0.1\overline{63}$

As we can see the repeating digit is 63

X = 0.16363636363……… (i)

10x =1.6363636363……… (ii)

100x = 16.363636363….. (iii)

1000x = 163.63636363…. (iv)

Subtracting (ii) from (iv), we get

1000x-10x = 163.636363-1.63636363

990x = 162

$$x = \frac{162}{990} = \frac{18}{110} = \frac{9}{55}$$

So, we can say that 0.16363636363…. is equals to the $\frac{9}{55}$

4. Question

Write, whether the given statement is true or false. Give reasons.

(i) Every natural number is a whole number.

(ii) Every whole number is a natural number.

(iii) Every integer is a rational number.

(iv) Every rational number is a whole number.

(v) Every terminating decimal is a rational number.

(vi) Every repeating decimal is a rational number.

(vii) 0 is a rational number.

Answer

(i) True: every natural number is the whole number because natural number starts with 1 and whole number start with 0. So, every natural number will automatically fall in the category of whole number.

(ii) False: every whole number can't be natural number as natural number starts from 1 and whole number starts with 0.

(iii) True: Integers includes all whole numbers and their negative counterparts. Rational numbers can be expressed in the form of fractions where denominator is not equals to the zero but both, numerator and denominator are integers.

(iv) False: Rational number is the number which can be expressed in the form of fraction where denominator is not equals to zero. But whole numbers are natural numbers including zero and they can't be written in fractional form.

(v) True: Rational number is the number which can be expressed in the form of fraction where denominator is not equals to zero and terminating decimal can also be written in fraction form.

(vi) True: Yes, every repeating decimal is also the rational number because it also written in the form of fraction.

(vii) True: yes, 0 is also the rational number because it can be written in the form of fraction.

Exercise 1C

1. Question

What are irrational numbers? How do they differ from rational numbers? Give examples.

Answer

Irrational number:- A number which can't be expressed as an terminating decimal or recurring decimal and fractional form is called irrational.

Ex: п, (3.1415926535), $\sqrt{7}$

Irrational number is different from Rational number because Rational number is a number which can be expressed as fractional form or terminating decimal form is called rational number. It is exactly the opposite of Irrational number.

Ex:- 5 , $\frac{7}{8}$, $.21$

2. Question

Classify the following numbers as rational or irrational. Give reasons to support your answer.

(i) $\sqrt{4}$ (ii) $\sqrt{196}$ (iii) $\sqrt{21}$

(iv) $\sqrt{43}$ (v) $3+\sqrt{3}$ (vi) $\sqrt{7}-2$

(vii) $\frac{2}{3}\sqrt{6}$ (viii) $0.\bar{6}$ (ix) 1.232332333...

(x) 3.040040004... (xi) 3.2576

(xii) 2.3565656... (xiii) π (xiv) $\frac{22}{7}$

Answer

(i) $\sqrt{4}$

= $\sqrt{4} = \sqrt{2 \times 2} = 2$

$\because$ we can express 2 as $\frac{2}{1}$ which is the quotient of the integer 2 and 1

Hence, it is a rational number.

(ii) $\sqrt{196}$

= $\sqrt{196} = \sqrt{14 \times 14} = 14.$

$\because$ we can express 14 as $\frac{14}{1}$ which is the quotient of the integer 14 and 1

Hence, it is a rational number.

(iii) $\sqrt{21}$

= $\sqrt{21} = \sqrt{3 \times 7} = \sqrt{3} \times \sqrt{7}$

$\because$we can not simplify $\sqrt{3}$ *and* $\sqrt{7}$, in the form $\frac{p}{q}$, $q \neq 0$

Hence, it is an irrational number.

(iv) $\sqrt{43}$

We know that 43 is a prime number so we can not get prime factors of it and neither we can write $\sqrt{43}$ in fractional form.

Hence, it is an irrational number.

(v) $3+\sqrt{3}$

$\because$ $\sqrt{3}$ is irrational number and addition of an irrational number to any real number always gives irrational number.

Hence, it is an irrational number.

(vi) $\sqrt{7}-2$

$\because$ $\sqrt{7}$ is irrational number and addition of an irrational number to any real number always gives irrational number.Hence, it is an irrational number.

(vii) $\frac{2}{3}\sqrt{6}$

$= \frac{2}{3} \times \sqrt{3 \times 2} = \frac{2}{3} \times \sqrt{3} \times \sqrt{2}$

∵ As, $\sqrt{3}$ and $\sqrt{2}$ are irrational numbers and multiplication of an irrational number to a non zero rational number gives irrational number.

Hence, it is an irrational number.

(viii) $0.\overline{6}$

∵ we know that all repeating decimals are rational,

Hence, it is a rational number.

(ix) 1.232332333...

∵ The decimal expansion here is non terminating and non repeating,

Hence, it is an irrational number.

(x) 3.040040004...

∵ The decimal expansion here is non terminating and non repeating,

Hence, it is an irrational number.

(xi) 3.2576

∵ It is a terminating decimal fraction and can be expressed in form $\frac{32576}{10000}$.

Hence it is a rational number.

(xii) 2.3565656...

∵ it is a non terminating but repeating decimal form that can be written as $2.35\overline{65}$.

Hence, it is a rational number.

(xiii) π

∵ We know that π is a non terminating Decimal fraction,

Hence it is an irrational number.

(xiv) $\frac{22}{7}$

∵ it is an fractional form,

Hence it is rational.

3. Question

Represent $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$ on the real line.

Answer

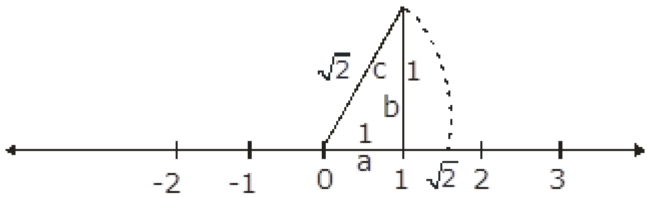
By using Pythagoras theorem,

$a^2 + b^2 = c^2$

$1^2 + 1^2 = c^2$

$c^2 = 2$

$c = \sqrt{2}$



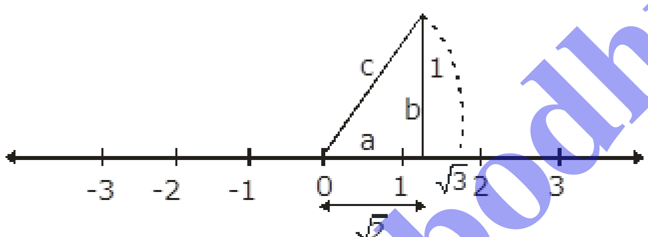
By using Pythagoras theorem,

$a^2 + b^2 = c^2$

$\left(\sqrt{2}\right)^2 + 1^2 = c^2$

$c^2 = 2 + 1 = 3$

$c = \sqrt{3}$

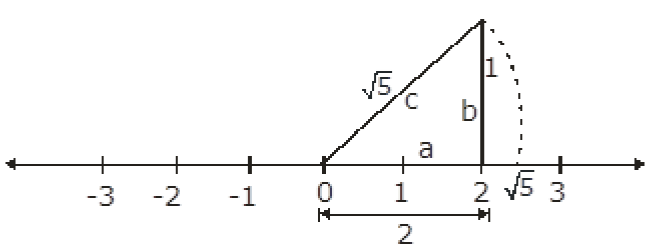


By using Pythagoras theorem,

$a^2 + b^2 = c^2$

$2^2 + 1^2 = c^2$

$c^2 = 5\ c = \sqrt{5}$



4. Question

Represent $\sqrt{6}$ and $\sqrt{7}$ on the real line.

Answer

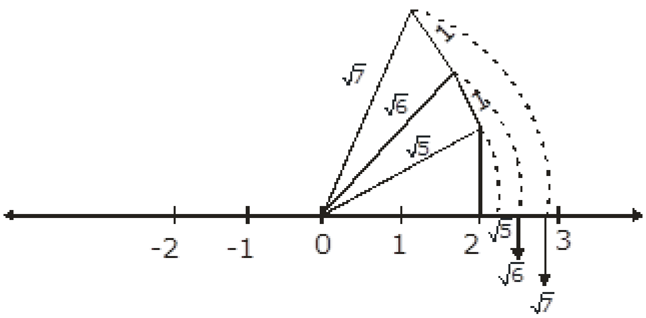
By using Pythagoras theorem,

(i) $(AB)^2 + (BC)^2 = (AC)^2$

$(AC)^2 = (2)^2 + (1)^2$

$(AC)^2 = 5$

$AC = \sqrt{5}$



(ii) $(AC)^2 + (DC)^2 = (AD)^2$

$(AD)^2 = (\sqrt{5})^2 + (1)^2$

$(AD)^2 = 5 + 1 = 6$

$AD = \sqrt{6}$

(iii) $(AD)^2 + (ED)^2 = (AE)^2$

$(AE)^2 = (\sqrt{6})^2 + (1)^2$

$(AE)^2 = 6 + 1$

$AE = \sqrt{7}$

5. Question

Giving reason in each case, show that each of the following numbers is irrational.

(i) $4+\sqrt{5}$ (ii) $(-3+\sqrt{6})$ (iii) $5\sqrt{7}$

(iv) $-3\sqrt{8}$ (v) $\frac{2}{\sqrt{5}}$ (vi) $\frac{4}{\sqrt{3}}$

Answer

(i) $4+\sqrt{5}$

We cannot simplify $\sqrt{5}$,

Hence 4 + $\sqrt{5}$ is an irrational number.

(ii) $(-3+\sqrt{6})$

= $-3 + \sqrt{2 \times 3} = -3 + \sqrt{2} \times \sqrt{3}$

We can't simplify $\sqrt{2} and \sqrt{3}$.

Hence it is an irrational number.

(iii) $5\sqrt{7}$

We can't simplify $\sqrt{7}$,

Hence it is an irrational number.

(iv) $-3\sqrt{8}$

= $-3 \times \sqrt{4 \times 2} = -3 \times 2 \times \sqrt{2} = -6\sqrt{2}$

We can't simplify $\sqrt{2}$,

Hence it is an irrational number.

(v) $\frac{2}{\sqrt{5}}$

We cannot simplify $\sqrt{5}$,

Hence it is an irrational number.

(vi) $\frac{4}{\sqrt{3}}$

We cannot simplify $\sqrt{3}$,

Hence it is an irrational number.

6. Question

State in each case, whether the given statement is true or false.

(i) The sum of two rational numbers is rational.

(ii) The sum of two irrational numbers is irrational.

(iii) The product of two rational numbers is rational.

(iv) The product of two irrational numbers is irrational.

(v)The sum of a rational number and an irrational number is irrational.

(vi) The product of a nonzero rational number and an irrational number is a rational number.

(vii)Every real number is rational.

(viii) Every real number is either rational or irrational.

(ix) π is irrational and $\frac{22}{7}$ is rational.

Answer

(i) True: = $\frac{2}{3}+\frac{1}{3}=3$, always a rational number.

(ii) False: = $\sqrt{11}+\left(-\sqrt{11}\right)=0$, which is a rational number.

(iii) True: =$\frac{5}{8}\times\frac{2}{3}=\frac{10}{24}=\frac{5}{12}$, always a rational number.

(iv) False: = $\sqrt{3}\times\sqrt{3}=3$, which is a rational number.

(v) True : = $2+\sqrt{3}$, is always irrational.

(vi) False: = $5\times\sqrt{3}=5\sqrt{3}$, is always an irrational number.

(vii) False : As rational numbers are on number line and all numbers on number line is real. Hence, every rational number is also Real.

(viii) True: As both rational and irrational numbers can be presented at number line are real. Hence they may be rational or irrational.

(ix) True: π = 3.141592653...... non terminating decimal form and $\frac{22}{7}$ is a fractional form.

Exercise 1D

1. Question

Add:

(i) $(2\sqrt{3}-5\sqrt{2})\, and\, (\sqrt{3}+2\sqrt{2})$

(ii) $(2\sqrt{2}+5\sqrt{3}-7\sqrt{5})\, and\, (3\sqrt{3}-\sqrt{2}+\sqrt{5})$

(iii) $\left(\frac{2}{3}\sqrt{7}-\frac{1}{2}\sqrt{2}+6\sqrt{11}\right) and \left(\frac{1}{3}\sqrt{7}+\frac{3}{2}\sqrt{2}-\sqrt{11}\right)$

Answer

(i) $\left(2\sqrt{3}-5\sqrt{2}\right) and \left(\sqrt{3}+2\sqrt{2}\right)$

Adding by making pairs,

= $2\sqrt{3}+\sqrt{3}-5\sqrt{2}+2\sqrt{2}=\left(3\sqrt{3}-3\sqrt{2}\right)$

(ii) $\left(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5}\right) and \left(3\sqrt{3} - \sqrt{2} + \sqrt{5}\right)$

Adding by making pairs,

$= 2\sqrt{2} - \sqrt{2} + 5\sqrt{3} + 3\sqrt{3} - 7\sqrt{5} + \sqrt{5} = \left(\sqrt{2} + 8\sqrt{3} - 6\sqrt{5}\right)$

(iii) $\left(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}\right) and \left(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11}\right)$

Adding by making pairs,

$= \frac{2}{3}\sqrt{7} + \frac{1}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + \frac{3}{2}\sqrt{2} + 6\sqrt{11} - \sqrt{11} = \left(\frac{3}{3}\sqrt{7} + \frac{2}{2}\sqrt{2} + 5\sqrt{11}\right)$

$= \sqrt{7} + \sqrt{2} + 5\sqrt{11}.$

2. Question

Multiply:

(i) $3\sqrt{5}\ by\ 2\sqrt{5}$ (ii) $6\sqrt{15}\ by\ 4\sqrt{3}$

(iii) $2\sqrt{6}\ by\ 3\sqrt{3}$ (iv) $3\sqrt{8}\ by\ 3\sqrt{2}$

(v) $\sqrt{10}\ by\ \sqrt{40}$ (vi) $3\sqrt{28}\ by\ 2\sqrt{7}$

Answer

(i) $3\sqrt{5}\ by\ 2\sqrt{5}$

$= 3\sqrt{5} \times 2\sqrt{5} = (3 \times 2)\left(\sqrt{5} \times \sqrt{5}\right) = 6 \times 5 = 30.$

(ii) $6\sqrt{15}\ by\ 4\sqrt{3}$

$= 6\sqrt{15} \times 4\sqrt{3} = (6 \times 4)\left(\sqrt{15} \times \sqrt{3}\right)$

$= 24 \times \sqrt{45} = 24 \times \sqrt{9 \times 5}$

$= 24 \times 3\sqrt{5} = 72\sqrt{5}.$

(iii) $2\sqrt{6}\ by\ 3\sqrt{3}$

$= 2\sqrt{6} \times 3\sqrt{3} = (2 \times 3)\left(\sqrt{18}\right) = 6 \times \sqrt{9 \times 2}$

$= 6 \times 3\sqrt{2} = 18\sqrt{2}.$

(iv) $3\sqrt{8}\ by\ 3\sqrt{2}$

$= 3\sqrt{8} \times 3\sqrt{2} = (3 \times 3)\left(\sqrt{16}\right) = 9 \times 4 = 36.$

(v) $\sqrt{10}\ by\ \sqrt{40}$

= $\sqrt{10} \times \sqrt{40} = \sqrt{10 \times 40} = \sqrt{400} = 20.$

(vi) $3\sqrt{28}\ by\ 2\sqrt{7}$

= $3\sqrt{28} \times 2\sqrt{7} = (3 \times 2)\left(\sqrt{4 \times 7 \times 7}\right) = 6 \times 2 \times 7 = 84.$

3. Question

Divide:

(i) $16\sqrt{6}\ by\ 4\sqrt{2}$ (ii) $12\sqrt{15}\ by\ 4\sqrt{3}$

(iii) $18\sqrt{21}\ by\ 6\sqrt{7}$

Answer

(i) $16\sqrt{6}\ by\ 4\sqrt{2}$

= $\frac{(16\sqrt{6})}{4\sqrt{2}} = \left(\frac{16}{4}\right)\left(\frac{\sqrt{6}}{\sqrt{2}}\right) = 4\sqrt{3}.$

(ii) $12\sqrt{15}\ by\ 4\sqrt{3}$

= $\frac{12\sqrt{15}}{4\sqrt{3}} = \left(\frac{12}{4}\right)\left(\frac{\sqrt{15}}{\sqrt{3}}\right) = 3\sqrt{5}.$

(iii) $18\sqrt{21}\ by\ 6\sqrt{7}$

= $\frac{18\sqrt{21}}{6\sqrt{7}} = \left(\frac{18}{6}\right)\left(\frac{\sqrt{21}}{\sqrt{7}}\right) = 3\sqrt{3}$

4. Question

Simplify:

(i) $(4+\sqrt{2})(4-\sqrt{2})$

(ii) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

(iii) $(6-\sqrt{6})(6+\sqrt{6})$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{2}-\sqrt{3})$

(v) $(\sqrt{5}-\sqrt{3})^2$ (vi) $(3-\sqrt{3})^2$

Answer

(i) $\left(4 + \sqrt{2}\right)\left(4 - \sqrt{2}\right)$

∵ $(a + b)(a - b) = a^2 - b^2$

$= (4 + \sqrt{2})(4 - \sqrt{2}) = 4^2 - \sqrt{2}^2 = 16 - 2 = 14.$

(ii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

$\because$ (a + b) (a - b) = $a^2 - b^2$

$= (\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = \sqrt{5}^2 - \sqrt{3}^2 = 5 - 3 = 2.$

(iii) $(6 - \sqrt{6})(6 + \sqrt{6})$

$\because$ (a + b) (a - b) = $a^2 - b^2$

$= (6 - \sqrt{6})(6 + \sqrt{6}) = 6^2 - \sqrt{6}^2 = 36 - 6 = 30.$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{2} - \sqrt{3})$

$= (\sqrt{5} - \sqrt{2})(\sqrt{2} - \sqrt{3}) = (\sqrt{10} - \sqrt{15} - \sqrt{4} + \sqrt{6})$

$= \sqrt{10} - \sqrt{15} - 2 + \sqrt{6}$

(v) $(\sqrt{5} - \sqrt{3})^2$

$\because (a - b)^2 = a^2 + b^2 - 2ab$

$= (\sqrt{5} - \sqrt{3})^2 = (\sqrt{5}^2 + \sqrt{3}^2 - 2 \times \sqrt{5} \times \sqrt{3} = 8 - 2\sqrt{15}.$

(vi) $(3 - \sqrt{3})^2$

$\because (a - b)^2 = a^2 + b^2 - 2ab$

$= (3 - \sqrt{3})^2 = (3^2 + \sqrt{3}^2 - 2 \times 3 \times \sqrt{3}) = 12 - 6\sqrt{3}$

5. Question

Represent $\sqrt{3.2}$ geometrically on the number line.

Answer

Let's draw a line AB = 3.2 units

Extend this line from B to C by 1 unit.

Now find the mid-point M of AC.

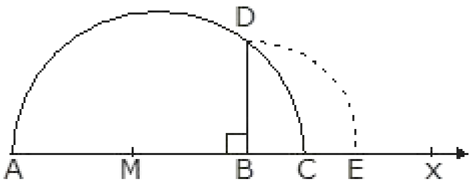
Take M as the center and MA as radius draw a semicircle.

Draw BD ⊥ AC intersecting the semi circle at D.

Then BD = $\sqrt{3.2}\, units$

With B as center and BD as radius, draw an arc, meeting AC produced at E.

Then BE = BD = $\sqrt{3.2}\,units.$



6. Question

Represent $\sqrt{7.28}$ geometrically on the number line.

Answer

Lets draw a line AB = 7.28 units

Extend this line from B to C by 1 unit.

Now find the mid-point M of AC.

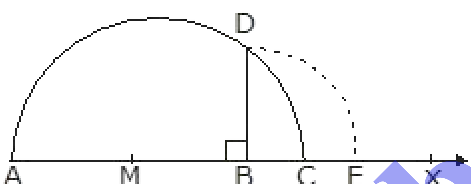
Take M as the center and MA as radius draw a semicircle.

Draw BD ⊥ AC intersecting the semi circle at D.

Then BD = $\sqrt{7.28}\,units$

With B as center and BD as radius, draw an arc, meeting AC produced at E.

Then BE = BD = $\sqrt{7.28}\,units.$



7. Question

Mention the closure property, associative law, commutative law, existence of identity, existence of inverse of each real number for each of the operations

(i) addition (ii) multiplication on real numbers.

Answer

Closure property of addition of rational numbers:

The sum of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b} + \frac{c}{d}\right)$ is also a rational number.

Example:

Consider the rational numbers $\frac{1}{3}$ and $\frac{3}{4}$ Then,

$= \frac{1}{3} + \frac{3}{4} = \frac{4+9}{12} = \frac{13}{12}$, is a rational number

Commutative property of addition of rational numbers:

Two rational numbers can be added in any order.

Thus for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$= \left(\frac{a}{b} + \frac{c}{d}\right) = \left(\frac{c}{d} + \frac{a}{b}\right)$

Example:

$= \frac{1}{2} + \frac{3}{4} = \frac{3}{4} + \frac{1}{2}$

$= \frac{2+3}{4} = \frac{3+2}{4}$

$= \frac{5}{4} = \frac{5}{4}$

Existence of additive identity property of addition of rational numbers:

0 is a rational number such that the sum of any rational number and 0 is the rational number itself.

Thus,

$= \frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$, for every rational number $\frac{a}{b}$

0 is called the additive identity for rationals.

Example:

$= \left(\frac{2}{5} + 0\right) = \left(\frac{2}{5} + \frac{0}{5}\right) = \frac{2+0}{5} = \frac{2}{5}$,

Existence of additive inverse property of addition of rational numbers:

For every rational number $\frac{a}{b}$, there exists a rational number $-\frac{a}{b}$

such that $\left(\frac{a}{b} + \left(-\frac{a}{b}\right)\right) = \frac{\{a + (-a)\}}{b} = \frac{0}{b} = 0$ and similarly, $\left(-\frac{a}{b} + \frac{a}{b}\right) = 0$.

Thus, $\left(\frac{a}{b} + \left(-\frac{a}{b}\right)\right) = \left(-\frac{a}{b} + \frac{a}{b}\right) = 0$

$= -\frac{a}{b}$ is called the additive inverse of $\frac{a}{b}$

Example:

= $\left(\frac{3}{5} + \left(-\frac{3}{5}\right)\right) = \frac{\{3 + (-3)\}}{5} = \frac{0}{5} = 0$ and similarly, $\left(-\frac{3}{5} + \frac{3}{5}\right) = 0$

Thus, $\frac{3}{5}$ and $-\frac{3}{5}$ are additive inverses of each other.

Associative property of addition of rational numbers:

While adding three rational numbers, they can be grouped in any order.

Thus, for any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$, we have

= $\left(\frac{a}{b} + \frac{c}{d}\right) + \left(\frac{e}{f}\right) = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$

Example:

Consider three rational numbers, $-\frac{2}{3}, \frac{5}{7}$ *and* $\frac{1}{6}$ Then,

= $\left(-\frac{2}{3} + \frac{5}{7}\right) + \frac{1}{6} = -\frac{2}{3} + \left(\frac{5}{7} + \frac{1}{6}\right)$

= $\frac{-14 + 15}{21} + \frac{1}{6} = -\frac{2}{3} + \frac{30 + 7}{42}$

= $\frac{2 + 7}{42} = \frac{-28 + 37}{42}$

= $\frac{9}{42} = \frac{9}{42}$.

Closure property of multiplication of rational numbers:

The product of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\left(\frac{a}{b} \times \frac{c}{d}\right)$ is also a rational number.

Example:

Consider the rational numbers $\frac{1}{3}$ and $\frac{2}{7}$. Then,

Commutative property of multiplication of rational numbers:

Two rational numbers can be multiplied in any order.

Thus, for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have:

= $\left(\frac{a}{b} \times \frac{c}{d}\right) = \left(\frac{c}{d} \times \frac{a}{b}\right)$

Example:

Let us consider the rational numbers $\frac{4}{5}$ and $\frac{2}{7}$ Then,

$= \left(\frac{4}{5} \times \frac{2}{7}\right) = \left(\frac{2}{7} \times \frac{4}{5}\right) = \frac{8}{35}$ and $\left(\frac{4}{5} \times \frac{2}{7}\right) = \frac{4 \times 2}{5 \times 7} = \frac{8}{35}$.

Therefore, $\left(\frac{4}{5} \times \frac{2}{7}\right) = \left(\frac{2}{7} \times \frac{4}{5}\right)$

Associative property of multiplication of rational numbers:

While multiplying three or more rational numbers, they can be grouped in any order.

Thus, for any rationals $\frac{a}{b}, \frac{c}{d}, and \frac{e}{f}$ we have:

$= \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$

Example:

Consider the rationals $-\frac{5}{2}, -\frac{7}{4}$ and $\frac{1}{3}$, we have

$=\left(-\frac{5}{2} \times -\frac{7}{4}\right) \times \frac{1}{3} = \left\{\left(-\frac{5}{7}\right) \times \left(-\frac{7}{4} \times \frac{1}{3}\right)\right\}$

$= \left(\frac{35}{8}\right) \times \frac{1}{3} = \left(-\frac{5}{7}\right)\left(-\frac{7}{12}\right)$

$= \frac{35}{24} = \frac{35}{24}$.

Existence of multiplicative identity property:

For any rational number $\frac{a}{b}$, we have $\left(\frac{a}{b} \times 1\right) = \left(1 \times \frac{a}{b}\right)$

1 is called the multiplicative identity for rationals.

Example:

Consider the rational number $\frac{3}{4}$. Then, we have

$= \left(\frac{3}{4} \times 1\right) = \left(\frac{3}{4} \times \frac{1}{1}\right) = \frac{3 \times 1}{4 \times 1} = \frac{3}{4}$

Existence of multiplicative inverse property:

Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.

Thus, $\left(\frac{a}{b} \times \frac{b}{a}\right) = \left(\frac{b}{a} \times \frac{a}{b}\right) = 1$

$= \frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

Clearly, zero has no reciprocal.

Reciprocal of 1 is 1 and the reciprocal of (-1) is (-1)

Exercise 1E

1. Question

Rationalise the denominator of each of the following :

$$\frac{1}{\sqrt{7}}$$

Answer

$$\frac{1}{\sqrt{7}}$$

By rationalization the denominator, we get,

$$= \frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

2. Question

Rationalise the denominator of each of the following :

$$\frac{\sqrt{5}}{2\sqrt{3}}$$

Answer

$$\frac{\sqrt{5}}{2\sqrt{3}}$$

By rationalization the denominator, we get,

$$= \frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{2 \times \sqrt{5} \times \sqrt{3}}{(2\sqrt{3})^2} = \frac{2\sqrt{15}}{12} = \frac{\sqrt{15}}{6}.$$

3. Question

Rationalise the denominator of each of the following :

$$\frac{1}{(2+\sqrt{3})}$$

Answer

$$\frac{1}{(2+\sqrt{3})}$$

By rationalization the denominator, we get,

$$= \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}.$$

4. Question

Rationalise the denominator of each of the following :

$$\frac{1}{(\sqrt{5}-2)}$$

Answer

$$\frac{1}{(\sqrt{5}-2)}$$

By rationalization the denominator, we get,

$$=\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{\sqrt{5}^2-2^2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2.$$

5. Question

Rationalise the denominator of each of the following :

$$\frac{1}{(5+3\sqrt{2})}$$

Answer

$$\frac{1}{(5+3\sqrt{2})}$$

By rationalization the denominator, we get,

$$=\frac{1}{5+3\sqrt{2}}=\frac{1}{5+3\sqrt{2}}\times\frac{(5-3\sqrt{2})}{5-3\sqrt{2}}=\frac{5-3\sqrt{2}}{5^2-(3\sqrt{2})^2}=\frac{(5-3\sqrt{2})}{25-18}=\frac{5-3\sqrt{2}}{7}.$$

6. Question

Rationalise the denominator of each of the following :

$$\frac{1}{(\sqrt{6}-\sqrt{5})}$$

Answer

$$\frac{1}{(\sqrt{6}-\sqrt{5})}$$

By rationalization the denominator, we get,

$$=\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}^2-\sqrt{5}^2}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}+\sqrt{5}.$$

7. Question

Rationalise the denominator of each of the following :

$$\frac{4}{(\sqrt{7}+\sqrt{3})}$$

Answer

$$\frac{4}{(\sqrt{7}+\sqrt{3})}$$

By rationalization the denominator, we get,

$$= \frac{4}{\sqrt{7}+\sqrt{3}} = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \left(\frac{4(\sqrt{7}-\sqrt{3})}{\sqrt{7}^2-\sqrt{3}^2}\right) = \frac{4(\sqrt{7}-\sqrt{3})}{4} = \sqrt{7}-\sqrt{3}.$$

8. Question

Rationalise the denominator of each of the following :

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Answer

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

By rationalization the denominator, we get,

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1^2} = \frac{(3+1-2\sqrt{3})}{3-1} = \frac{2(2-\sqrt{3})}{2} = 2-\sqrt{3}.$$

9. Question

Rationalise the denominator of each of the following :

$$\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$$

Answer

$$\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$$

By rationalization the denominator, we get,

$$= \frac{(3-2\sqrt{2})}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{(3-2\sqrt{2})^2}{3^2-(2\sqrt{2})^2} = \frac{9+8-12\sqrt{2}}{9-8} = \frac{17-12\sqrt{2}}{1} = 17-12\sqrt{2}.$$

10. Question

Find the values of a and b in each of the following.

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = a+b\sqrt{3}$$

Answer

By rationalizing the L.H.S we get,

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{\sqrt{3}^2-1^2} = \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2} = \left(\frac{2(2+\sqrt{3})}{2}\right) = 2+\sqrt{3}$$

Putting LHS = RHS, we get,

= $2 + \sqrt{3} = a + b\sqrt{3}$

Clearly, a = 2 and b = 1.

11. Question

Find the values of a and b in each of the following.

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$$

Answer

By rationalizing the L.H.S we get,

$$= \frac{(3+\sqrt{2})}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{(3+\sqrt{2})^2}{3^2-\sqrt{2}^2} = \frac{9+2+6\sqrt{2}}{9-2} = \frac{11+6\sqrt{2}}{7}$$

Putting LHS = RHS, we get,

$$= \frac{11+6\sqrt{2}}{7} = a + b\sqrt{2}$$

$$= \frac{11}{7} + \frac{6}{7}\sqrt{2} = a + b\sqrt{2}$$

Clearly, a = $\frac{11}{7}$ $and\ b = \frac{6}{7}$.

12. Question

Find the values of a and b in each of the following.

$$\frac{5-\sqrt{6}}{5+\sqrt{6}} = a - b\sqrt{6}$$

Answer

By rationalizing the L.H.S we get,

$$= \frac{5-\sqrt{6}}{5+\sqrt{6}} \times \frac{5-\sqrt{6}}{5-\sqrt{6}} = \frac{(5-\sqrt{6})^2}{5^2-\sqrt{6}^2} = \frac{25+6-10\sqrt{6}}{25-6} = \frac{31-10\sqrt{6}}{19}$$

Putting LHS = RHS, we get,

$$= \frac{31-10\sqrt{6}}{19} = a - b\sqrt{6}$$

$$= \frac{31}{19} - \frac{10}{19}\sqrt{6} = a - b\sqrt{6}$$

Clearly, $a = \frac{31}{19}$ $and\ b = \frac{10}{19}$.

13. Question

Find the values of a and b in each of the following.

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a-b\sqrt{3}$$

Answer

By rationalizing the L.H.S we get,

$$=\frac{5+2\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}=\frac{35-20\sqrt{3}+14\sqrt{3}-24}{7^2-(4\sqrt{3})^2}=\frac{11-6\sqrt{3}}{49-48}=11-6\sqrt{3}.$$

Putting LHS = RHS, we get,

$= 11 - 6\sqrt{3} = a - b\sqrt{3}$

Clearly a = 11 and b = 6.

14. Question

Simplify: $\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}+\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)$.

Answer

By taking LCM,

$$\frac{\{(\sqrt{5}-1)(\sqrt{5}-1)+(\sqrt{5}+1)(\sqrt{5}+1)\}}{\sqrt{5}^2-1^2}$$

$$=\frac{\{(\sqrt{5}-1)^2+(\sqrt{5}+1)^2\}}{4}$$

$$=\frac{\{(5+1-2\sqrt{5})+(5+1+2\sqrt{5})\}}{4}$$

$$=\frac{12}{4}=3.$$

15. Question

Simplify: $\left(\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}\right)$.

Answer

By taking LCM,

$$=\left(\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}\right)=\frac{\{(4+\sqrt{5})^2+(4-\sqrt{5})^2\}}{4^2-\sqrt{5}^2}=\frac{\{21+8\sqrt{5}+21-8\sqrt{5}\}}{11}=\frac{42}{11}.$$

16. Question

If $x = (4 - \sqrt{15})$, find the value of $\left(x + \frac{1}{x}\right)$.

Answer

Given that, $x = (4 - \sqrt{15})\ so, \frac{1}{x} = \frac{1}{4 - \sqrt{15}}$

By rationalizing $\frac{1}{x}$, we get,

$$= \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}} = \frac{4+\sqrt{15}}{4^2-\sqrt{15}^2} = \frac{4+\sqrt{15}}{16-15} = 4 + \sqrt{15}$$

Hence, $x + \frac{1}{x} = 4 - \sqrt{15} + 4 + \sqrt{15} = 4 + 4 = 8.$

17. Question

If $x = (2 + \sqrt{3})$, find the value of $\left(x^2 + \frac{1}{x^2}\right)$.

Answer

Given that, $x = (2 + \sqrt{3})\ so, \frac{1}{x} = \frac{1}{2 + \sqrt{3}}$

By Rationalizing $\frac{1}{x}$ *we get,*

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{1} = 2 - \sqrt{3}$$

Now, $x^2 + \frac{1}{x^2} = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$

= 7 + 7 = 14

18. Question

Show that $\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)} = 5.$

Answer

By rationalizing LHS we get,

$$= \left\{\left(\frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}}\right) - \left(\frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right) + \left(\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}\right) - \left(\frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right) + \left(\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}\right)\right\}$$

$$= \frac{3+\sqrt{8}}{(9-8)} - \frac{\sqrt{8}+\sqrt{7}}{(8-7)} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

= 3 + √8 - √8 - √7 + √7 + √6 - √6 - √5 + √5 + 2 = 3 + 2 = 5

Clearly, LHS = RHS,

Hence, proved.

Exercise 1F

1. Question

Simplify:

(i) $(6^{2/5} \times 6^{3/5})$ (ii) $(3^{1/2} \times 3^{1/3})$ (iii) $(7^{5/6} \times 7^{2/3})$

Answer

(i) $(6^{2/5} \times 6^{3/5})$

We know that powers get added in multiplication, so,

$= \left(6^{\frac{2}{5}} \times 6^{\frac{3}{5}}\right)$

$= 6^{\left(\frac{2}{5}\right) + \left(\frac{3}{5}\right)}$

$= 6^{\frac{2+3}{5}}$

$= 6^{\frac{5}{5}} = 6^1 = 6.$

(ii) $(3^{1/2} \times 3^{1/3})$

We know that powers get added in multiplication, so,

$= 3^{\frac{1}{2}} \times 3^{\frac{1}{3}}$

$= 3^{\frac{3+2}{6}}$

$= 3^{\frac{5}{6}}.$

(iii) $(7^{5/6} \times 7^{2/3})$

We know that powers get added in multiplication, so,

$= 7^{\frac{5}{6}} \times 7^{\frac{2}{3}}$

$= 7^{\left(\frac{5}{6}\right) + \left(\frac{2}{3}\right)}$

$= 7^{\frac{5+4}{6}}$

$= 7^{\frac{9}{6}}$

$= 7^{\frac{3}{2}}$

2. Question

Simplify:

(i) $\frac{6^{1/4}}{6^{1/5}}$ (ii) $\frac{8^{1/2}}{8^{2/3}}$ (iii) $\frac{5^{6/7}}{5^{2/3}}$

Answer

(i) $\frac{6^{1/4}}{6^{1/5}}$

We know that powers get subtracted in dividing, so,

$\frac{6^{\frac{1}{4}}}{6^{\frac{1}{5}}}$

$= 6^{\frac{1}{4}-\frac{1}{5}}$

$= 6^{\left(\frac{5-4}{20}\right)}$

$= 6^{\frac{1}{20}}$

(ii) $\frac{8^{1/2}}{8^{2/3}}$

We know that powers get subtracted in dividing, so,

$= \frac{8^{\frac{1}{2}}}{8^{\frac{2}{3}}}$

$= 8^{\frac{1}{2}-\frac{2}{3}}$

$= 8^{\frac{3-4}{6}}$

$= 8^{-\frac{1}{6}}$

(iii) $\frac{5^{6/7}}{5^{2/3}}$

We know that powers get subtracted in dividing, so,

$= \frac{5^{\frac{6}{7}}}{5^{\frac{2}{3}}}$

$= 5^{\frac{6}{7}-\frac{2}{3}}$

$= 5^{\frac{(18-14)}{21}}$

$= 5^{\frac{4}{21}}$

3. Question

Simplify:

(i) $3^{1/4} \times 5^{1/4}$ (ii) $2^{5/8} \times 3^{5/8}$ (iii) $6^{1/2} \times 7^{1/2}$

Answer

(i) $3^{1/4} \times 5^{1/4}$

We know that when the powers are same then only numbers get multiplied, so,

$= 3^{\frac{1}{4}} \times 5^{\frac{1}{4}} = (3 \times 5)^{\frac{1}{4}} = 15^{\frac{1}{4}}$

(ii) $2^{5/8} \times 3^{5/8}$

We know that when the powers are same then only numbers get multiplied, so,

$= 2^{\frac{5}{8}} \times 3^{\frac{5}{8}} = (2 \times 3)^{\frac{5}{8}} = 6^{\frac{5}{8}}$

(iii) $6^{1/2} \times 7^{1/2}$

We know that when the powers are same then only numbers get multiplied, so,

$= 6^{\frac{1}{2}} \times 7^{\frac{1}{2}} = (6 \times 7)^{\frac{1}{2}} = 42^{\frac{1}{2}}$

4. Question

Simplify:

(i) $(3^4)^{1/4}$ (ii) $(3^{1/3})^{1/4}$ (iii) $\left(\frac{1}{3^4}\right)^{1/2}$

Answer

(i) $(3^4)^{1/4}$

$= (3^4)^{\frac{1}{4}} = 3^{4 \times \frac{1}{4}} = 3^1 = 3.$

(ii) $(3^{1/3})^{1/4}$

$= \left(3^{\frac{1}{3}}\right)^{\frac{1}{4}} = (3)^{\frac{1}{3} \times \frac{1}{4}} = 3^{\frac{1}{12}}.$

(iii) $\left(\frac{1}{3^4}\right)^{1/2}$

$= \left(\frac{1}{3^4}\right)^{\frac{1}{2}} = (3^{-4})^{\frac{1}{2}} = (3)^{-4 \times \frac{1}{2}} = 3^{-2}.$

5. Question

Evaluate:

(i) $(i)\ (49)^{1/2}\quad (ii)\ (125)^{1/3}\quad (iii)(64)^{1/6}$

Answer

(i) $(49)^{\frac{1}{2}} = (7^2)^{\frac{1}{2}} = (7)^{2 \times \frac{1}{2}} = 7^1 = 7.$

(ii) $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = (5)^{3 \times \frac{1}{3}} = 5^1 = 5.$

(iii) $(64)^{\frac{1}{6}} = (2^6)^{\frac{1}{6}} = (2)^{6 \times \frac{1}{6}} = 2^1 = 2.$

6. Question

Evaluate:

$(i)\ (25)^{3/2}\quad (ii)\ (32)^{2/5}\quad (iii)(81)^{3/4}$

Answer

(i) $(25)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = (5)^{2 \times \frac{3}{2}} = 5^3 = 125.$

(ii) $(32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = (2)^{5 \times \frac{2}{5}} = 2^2 = 4.$

(iii) $(81)^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = (3)^{\left(4 \times \frac{3}{4}\right)} = 3^3 = 27.$

7. Question

Evaluate:

$(i)(64)^{-1/2}\quad (ii)(8)^{-1/3}\quad (iii)(81)^{-1/4}$

Answer

(i) $(64)^{-\frac{1}{2}} = (8^2)^{-\frac{1}{2}} = (8)^{2 \times -\frac{1}{2}} = 8^{-1} = \frac{1}{8}.$

(ii) $(8)^{-\frac{1}{3}} = (2^3)^{-\frac{1}{3}} = (2)^{3 \times -\frac{1}{3}} = 2^{-1} = \frac{1}{2}.$

(iii) $(81)^{-\frac{1}{4}} = (3^4)^{-\frac{1}{4}} = (3)^{4 \times -\frac{1}{4}} = 3^{-1} = \frac{1}{3}.$

CCE Questions

1. Question

Which of the following is an irrational number?

A. 3.14

B. $3.\overline{14}$

C. $3.1\overline{4}$

D. 3.141141114…

Answer

A number which cannot be written as simple fraction is called Irrational Number.

3.141141114… is an irrational number because in this decimal is going forever without repeating.

2. Question

Which of the following is an irrational number?

A. $\sqrt{49}$

B. $\sqrt{\frac{9}{16}}$

C. $\sqrt{5}$

D. $\frac{\sqrt{20}}{\sqrt{5}}$

Answer

$\sqrt{49}$ = 7 = $\frac{7}{1}$

$\sqrt{\frac{9}{16}}$ = $\frac{3}{4}$

$\frac{\sqrt{20}}{\sqrt{5}}$ = $\sqrt{\frac{20}{5}}$ = $\sqrt{4}$ = 2 = $\frac{2}{1}$

$\sqrt{5}$ = 2.236067977…..

$\sqrt{5}$ is an irrational number because in this decimal is going forever without repeating.

3. Question

Which of the following is an irrational number?

A. $0.\overline{32}$

B. $0.3\overline{21}$

C. $0.\overline{321}$

D. 0.3232232223...

Answer

0.3232232223... is an irrational number because decimal is going forever without repeating.

4. Question

Which of the following is a rational number?

A. $\sqrt{2}$

B. $\sqrt{23}$

C. $\sqrt{225}$

D. 0.1010010001...

Answer

$\sqrt{225} = 15 = \frac{15}{1}$

So, $\sqrt{225}$ is a rational number.

5. Question

Every rational number is

A. a natural number

B. a whole number

C. an integer

D. a real number

Answer

Every rational number is a real number.

6. Question

Between any two rational numbers there

A. is no rational number

B. is exactly one rational number

C. are infinitely many rational numbers

D. is no irrational number

Answer

Between any two rational numbers there are infinitely many rational numbers.

7. Question

The decimal representation of a rational number is

A. always terminating

B. either terminating or repeating

C. either terminating or non-repeating

D. neither terminating nor repeating

Answer

The decimal representation of a rational number is neither terminating nor repeating

8. Question

The decimal representation of an irrational number is

A. always terminating

B. either terminating or repeating

C. either terminating or non-repeating

D. neither terminating nor repeating

Answer

The decimal representation of an irrational number is non-terminating and non-repeating. Ex. Value of pie.

9. Question

Decimal expansion of $\sqrt{2}$ is:

A. a finite decimal

B. a terminating or repeating decimal

C. a non-terminating and non-repeating decimal

D. none of these

Answer

$\sqrt{2}$ = 1.41421356.......

So, Decimal expansion of $\sqrt{2}$ is a non-terminating and non-repeating decimal

10. Question

The product of two irrational number is

A. always irrational B. always rational

C. always an integer

D. sometimes rational and sometimes irrational

Answer

The product of two irrational number is sometimes rational and sometimes irrational

Case 1: $\sqrt{2}$ X $\sqrt{2}$ = 2 (this is rational)

Case 2: $\sqrt{5}\times\sqrt{2}=\sqrt{10}$ (this is irrational)

11. Question

Which of the following is a true statement?

A. The sum of two irrational numbers is an irrational number

B. The product of two irrational numbers is an irrational number

C. Every real number is always rational

D. Every real number is either rational or irrational

Answer

Every real number is either rational or irrational. If any real number can be written as fraction then it would be rational number otherwise it will be irrational number.

12. Question

Which of the following is a true statement?

A. π and $\frac{22}{7}$ are both rationals

B. π and $\frac{22}{7}$ are both irrationals

C. π is rational and $\frac{22}{7}$ is irrational

D. π is irrational and $\frac{22}{7}$ is rational

Answer

π is irrational and $\frac{22}{7}$ is rational

π = 3.14159265358979…….. (this is non-terminating and non-repeating)

$\frac{22}{7}$ = 3.142857142857142857 (this is repeating)

13. Question

A rational number between $\sqrt{2}$ *and* $\sqrt{3}$ is

A. $\frac{1}{2}(\sqrt{2}+\sqrt{3})$

B. $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

C. 2.5

D. 1.5

Answer

$\sqrt{2}$ = 1.4142…

$\sqrt{3}$ = 1.7321…..

1.5 is a rational number between $\sqrt{2}$ *and* $\sqrt{3}$.

Others would be 1.45, 1.55, 1.6 etc.

14. Question

Solve the equation and choose the correct answer: $(125)^{-1/3} = ?$

A. 5

B. -5

C. $\frac{1}{5}$

D. $\frac{-1}{5}$

Answer

$(125)^{-1/3} = (5)^{3*\frac{-1}{3}} = 5^{-1}$

$= \frac{4}{2} = \frac{1}{5}$

15. Question

Solve the equation and choose the correct answer: $\frac{(\sqrt{32}+\sqrt{48})}{(\sqrt{8}+\sqrt{12})} = ?$

A. $\sqrt{2}$

B. 2

C. 4

D. 8

Answer

$$\frac{(\sqrt{32}+\sqrt{48})}{(\sqrt{8}+\sqrt{12})} = \frac{\sqrt{16*2}+\sqrt{16*3}}{\sqrt{4*2}+\sqrt{4*3}}$$

$$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})}$$

$= \frac{4}{2} = 2$

16. Question

Solve the equation: $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32} = ?$

A. 2

B. $\sqrt{2}$

C. $2\sqrt{2}$

D. $4\sqrt{2}$

Answer

$$\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32} = (2)^{\frac{1}{3}} \times (2)^{\frac{1}{4}} \times (2^5)^{\frac{1}{12}}$$

$= (2)^{\frac{1}{3}} \times (2)^{\frac{1}{4}} \times (2)^{\frac{5}{12}}$

$= (2)^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}}$

$= (2)^{\frac{12}{12}} = 2^1 = 2$

17. Question

Solve the equation and choose the correct answer: $\left(\frac{81}{16}\right)^{3/4} = ?$

A. $\frac{4}{9}$

B. $\frac{9}{4}$

C. $\frac{27}{8}$

D. $\frac{8}{27}$

Answer

$\left(\frac{81}{16}\right)^{3/4} = \frac{(81)^{\frac{3}{4}}}{(16)^{\frac{3}{4}}} = \frac{(3^4)^{\frac{3}{4}}}{(2^4)^{\frac{3}{4}}}$

$= \frac{3^3}{2^3} = \frac{27}{8}$

18. Question

Solve the equation and choose the correct answer: $\sqrt[4]{(64)^2} = ?$

A. 4

B. $\frac{1}{4}$

C. 8

D. $\frac{1}{8}$

Answer

$\sqrt[4]{(64)^2}$

$= \sqrt[4]{\left((8)^2\right)^2}$

$= \sqrt[4]{(8)^4} = 8$

19. Question

Solve the equation and choose the correct answer: $\frac{1}{(\sqrt{4}-\sqrt{3})} = ?$

A. $(2+\sqrt{3})$

B. $(2-\sqrt{3})$

C. 1

D. none of these

Answer

$$\frac{1}{(\sqrt{4}-\sqrt{3})} = \frac{1}{(\sqrt{4}-\sqrt{3})} \times \frac{\sqrt{4}+\sqrt{3}}{\sqrt{4}+\sqrt{3}}$$

$$= \frac{1\left(\sqrt{4}+\sqrt{3}\right)}{\left(\sqrt{4}\right)^2 - \left(\sqrt{3}\right)^2} = \frac{1\left(\sqrt{4}+\sqrt{3}\right)}{4-3}$$

$$= \frac{1\left(\sqrt{4}+\sqrt{3}\right)}{1}$$

$$= 2+\sqrt{3}$$

20. Question

Solve the equation and choose the correct answer: $\frac{1}{(3+2\sqrt{2})} = ?$

A. $\frac{3-2\sqrt{2}}{17}$

B. $\frac{(3-2\sqrt{2})}{13}$

C. $(3-2\sqrt{2})$

D. none of these

Answer

$$\frac{1}{(3+2\sqrt{2})}$$

$$=\frac{1}{(3+2\sqrt{2})}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$=\frac{3-2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}=\frac{3-2\sqrt{2}}{9-8}$$

$$=3-2\sqrt{2}$$

$$=\frac{1}{(3+2\sqrt{2})}$$

21. Question

Solve the equation and choose the correct answer: $If\ x=(7+4\sqrt{3}), then \left(x+\frac{1}{x}\right)=?$

A. $8\sqrt{3}$

B. 14

C. 49

D. 48

Answer

$$\left((7+4\sqrt{3})+\frac{1}{(7+4\sqrt{3})}\right)=\left(\frac{(7+4\sqrt{3})^2+1}{(7+4\sqrt{3})}\right)$$

$$= \left(\frac{(7)^2 + \left(4\sqrt{3}\right)^2 + 2 \times 7 \times 4\sqrt{3} + 1}{(7 + 4\sqrt{3})} \right)$$

$$= \left(\frac{49 + 48 + 2 \times 7 \times 4\sqrt{3} + 1}{(7 + 4\sqrt{3})} \right)$$

$$= \left(\frac{98 + 56\sqrt{3}}{(7 + 4\sqrt{3})} \right)$$

$$= \left(\frac{14(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})} \right)$$

= 14

22. Question

Solve the equation and choose the correct answer: If $\sqrt{2} = 1.41$, then $\frac{1}{\sqrt{2}} = ?$

A. 0.075

B. 0.75

C. 0.705

D. 7.05

Answer

$\frac{1}{\sqrt{2}} = \frac{1}{1.41}$ (When $\sqrt{2} = 1.41$ only)

= 0.709

23. Question

Solve the equation and choose the correct answer: If $\sqrt{7} = 2.646$, then $\frac{1}{\sqrt{7}} = ?$

A. 0.375

B. 0.378

C. 0.441

D. None of these

Answer

$$\frac{1}{\sqrt{7}} = 0.378$$

(When $\sqrt{7} = 2.646$ only)

24. Question

Solve the equation: $\sqrt{10} \times \sqrt{15} = ?$

A. $\sqrt{25}$

B. $5\sqrt{6}$

C. $6\sqrt{5}$

D. none of these

Answer

$$\sqrt{10} \times \sqrt{15} = \sqrt{150}$$

$$= \sqrt{25 \times 6}$$

$$= 5\sqrt{6}$$

25. Question

$(625)^{0.16} \times (625)^{0.09} = ?$

A. 5

B. 25

C. 125

D. 625.25

Answer

$$= (625)^{0.16+0.09}$$

$$= (625)^{0.25}$$

$$= (5^4)^{0.25}$$

$= 5^{4\times0.25}$

= 5

26. Question

Solve the equation and choose correct answer: If $\sqrt{2} = 1.414$, then $\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}} = ?$

A. 0.207

B. 2.414

C. 0.414

D. 0.621

Answer

$$\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}} = \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{(2-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{1}}$$

$$= \sqrt{(\sqrt{2}-1)^2}$$

$$= \sqrt{(1.414-1)^2}$$

$$= \sqrt{(0.414)^2}$$

= 0.414

27. Question

The simplest form of $1.\overline{6}$ *is*

A. $\frac{833}{500}$

B. $\frac{8}{5}$

C. $\frac{5}{3}$

D. none of these

Answer

Let x = $1.\overline{6}$ then

X = 1.666 …………………(i)

10x = 16.666 ……………(ii)

On subtracting (i) from (ii) we get,

9x = 15

x= $\frac{15}{9}$

x= $\frac{5}{3}$

28. Question

The simplest form of $0.\overline{54}$

A. $\frac{27}{50}$

B. $\frac{6}{11}$

C. $\frac{4}{7}$

D. none of these

Answer

Let x = $0.\overline{54}$ then

X = 0.545454………………(i)

10x = 5.45454 ……………(ii)

100x = 54.545454 ……….(iii)

On subtracting (i) from (iii) we get,

99x = 54

$x = \frac{54}{99}$

$x = \frac{6}{11}$

19. Question

The simplest form of $0.3\overline{2}$ *is*

A. $\frac{16}{45}$

B. $\frac{32}{99}$

C. $\frac{29}{90}$

D. none of these

Answer

Let x = $0.3\overline{2}$ then

X = 0.32222 ………………(i)

10x = 3.2222 ……………(ii)

100x = 32.222 ……….(iii)

On subtracting (ii) from (iii) we get,

90x = 29

$x = \frac{29}{90}$

30. Question

28. The simplest form of $0.12\overline{3}$ is

A. $\frac{41}{330}$

B. $\frac{37}{330}$

C. $\frac{41}{333}$

D. none of these

Answer

Let x = $0.12\overline{3}$ then

X = 0.123333 ………………(i)

10x = 1.23333 …………….(ii)

100x = 12.3333 ……….(iii)

1000x = 123.3333………(iv)

On subtracting (iii) from (iv) we get,

900x = 111

x= $\frac{111}{900}$

x = $\frac{37}{300}$

31. Question

An irrational number between 5 and 6 is

A. $\frac{1}{2}(5+6)$

B. $\sqrt{5+6}$

C. $\sqrt{5\times6}$

D. none of these

Answer

$\sqrt{5\times6}$ = 5.477225575051…..

$\sqrt{5+6}$ = 3.316624790……..

The decimal representation of $\sqrt{5\times6}$ is non-terminating and non-repeating.

So, $\sqrt{5\times6}$ is an irrational number between 5 and 6.

32. Question

An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is

A. $(\sqrt{2}+\sqrt{3})$

B. $\sqrt{2}\times\sqrt{3}$

C. $5^{1/4}$

D. $6^{1/4}$

Answer

$\sqrt{2}=1.414$

$\sqrt{3}=1.732$

Now, $(\sqrt{2}+\sqrt{3})$ > $\sqrt{3}$

And, $\left(\sqrt{2}\times\sqrt{3}\right)$ > $\sqrt{3}$

And, $5^{1/4}$ = 1.49534878122

And, $6^{1/4}$ = 1.56508458007

Both the options C & D are non-terminating and non-ending. So, both could be the answer.

33. Question

An irrational number between $\frac{1}{7}$ and $\frac{2}{7}$ is

A. $\frac{1}{2}\left(\frac{1}{7}+\frac{2}{7}\right)$

B. $\left(\frac{1}{7}\times\frac{2}{7}\right)$

C. $\sqrt{\frac{1}{7}\times\frac{2}{7}}$

D. none of these

Answer

$\frac{1}{7}$ = 0.142857142857

$\frac{2}{7}$ = 0.2857142857142857

$\sqrt{\frac{1}{7}\times\frac{2}{7}}$ = 0.2020305089104421……. It is non-terminating and non-ending.

34. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
The rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$ are: $\frac{9}{20}, \frac{10}{20}$ *and* $\frac{11}{20}$.	A rational number between two rational numbers p and q is $\frac{1}{2}(p+q)$.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

A rational number between $\frac{2}{5}$ and $\frac{3}{5}$ is : $\frac{1}{2}\left(\frac{2}{5}+\frac{3}{5}\right) = \frac{5}{10}$

Thus, the rational number between $\frac{2}{5}$ and $\frac{5}{10}$ is : $\frac{1}{2}\left(\frac{2}{5}+\frac{5}{10}\right) = \frac{9}{20}$

Thus, the rational number between $\frac{3}{5}$ and $\frac{5}{10}$ is : $\frac{1}{2}\left(\frac{3}{5}+\frac{5}{10}\right) = \frac{11}{20}$

Thus, three rational numbers between $\frac{2}{5}$ *and* $\frac{3}{5}$ *are* : $\frac{9}{10}, \frac{11}{20}$ *and* $\frac{10}{20}$

35. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
$\sqrt{3}$ is irrational number.	Square root of a positive integer which is not a perfect square is an irrational number.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The square roots of numbers that are not a perfect square are members of the irrational numbers.

Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

36. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
e is an irrational number.	π is an irrational number.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

e may or may not be π .

Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

37. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
$\sqrt{3}$ is an irrational number.	The sum of a rational number and an irrational number is an irrational number.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The square roots of 3 is not a perfect square. So, it is an irrational number.

Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

38. Question

Match the following columns:

Column I	Column II
A. $6.\overline{54}$ *is*.... .	(p) 14
B. π *is*.... .	(q) 6
C. The length of period of $\frac{1}{7}=$...... .	(r) a rational number
D. If $x=(2-\sqrt{3})$, then $\left(x^2+\frac{1}{x^2}\right)=$...... .	(s) an irrational number

The correct answer is:

(a)-......., (b)-......., (c)-......., (d)-.......,

Answer

(a)-(r)

$6.\overline{54}$ *is* rational number.

(b)-(s)

$\frac{1}{7}$ an irrational number.

(c)-(q)

The value of $\frac{1}{7}$ is 0.142857142857142857........ So length of period is 6.

(d)-(p)

If $x=(2-\sqrt{3})$, then $\left(x^2+\frac{1}{x^2}\right)=14$

39. Question

Match the following columns:

Column I	Column II
A. $\sqrt[4]{(81)^{-2}} = \ldots.$	(p) 4
B. $If \left(\frac{a}{b}\right)^{x-2} = \left(\frac{b}{a}\right)^{x-4}$, then x = ……. ,	(q) $\frac{2}{9}$
C. $if\ x = (9+4\sqrt{5})$, then $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \ldots\ldots$	(r) $\frac{1}{9}$
	(s) 3
D. $\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{64}{27}\right)^{-1/3} = ?$	

The correct answer is:

(a)-……., (b)-……., (c)-……., (d)-…….,

Answer

(a)-(r)

$$\sqrt[4]{(81)^{-2}} = (81)^{\frac{-2}{4}}$$

$$= (3^4)^{\frac{-2}{4}}$$

$$= 3^{-2}$$

$$= \frac{1}{3^2} = \frac{1}{9}$$

(b)-(s)

$$\left(\frac{a}{b}\right)^{x-2} = \left(\frac{b}{a}\right)^{x-4}$$

$$\left(\frac{a}{b}\right)^{x-2} = \left(\frac{a}{b}\right)^{-(x-4)}$$

x -2 = - x + 4

x + x = 4 + 2

2x = 6

X = 3

(c)-(p)

$if\ x = (9+4\sqrt{5}),$ then $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 4.$

(d)-(q)

$$\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{64}{27}\right)^{-1/3} = \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left(\frac{4^3}{3^3}\right)^{-1/3}$$

$$= \left(\frac{3}{2}\right)^{4\times -3/4} \times \left(\frac{4}{3}\right)^{3\times -1/3}$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{4}{3}\right)^{-1}$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{2}{9}$$

40. Question

Give an example of two irrational numbers whose sum as well as the product is rational.

Answer

Let two irrational numbers be **10 + 2√5 and 5 - 2√5** (i) sum is rational**= 10 + 2√5 + 5 - 2√5**

= 15 (a rational number)(ii) product is rational are

= (10 + 2√5) (10 - 2√5)= $(10)^2$ - $(2\sqrt{5})^2$= 100 - 20 = 80 (a rational number)

41. Question

If x is rational and y is irrational, then show that (x + y) is always irrational.

Answer

Let x = 10 and y = $2\sqrt{5}$

X + y = $10+2\sqrt{5}$ is irrational number.

42. Question

Is the product of a rational and an irrational number always irrational?

Give an example.

Answer

No

If you multiply any irrational number by the rational number zero, the result will be zero, which is rational.

$0\times 2\sqrt{5}=0$ is rational number.

43. Question

Given an example of a number x such that x^2 is an irrational number and x^4 is a rational number.

Answer

$Take\, x=\sqrt[4]{3}$

Let x = $\sqrt[4]{2}$

Then $x^2=(\sqrt[4]{2})^2=\sqrt{2}$ is an irrational number

And $x^4=(\sqrt[4]{2})^4=2$ is a rational number.

44. Question

The number $4.\overline{17}$ expressed as a vulgar fraction is

A. $\frac{417}{100}$

B. $\frac{417}{99}$

C. $\frac{413}{99}$

D. $\frac{413}{90}$

Answer

Let x = $4.\overline{17}$ then

X = 4.17171717………………(i)

10x = 41.7171717 …………….(ii)

100x = 417.171717 ………..(iii)

On subtracting (i) from (iii) we get,

99x = 413

X= $\frac{413}{99}$

45. Question

if $x=(2+\sqrt{3})$, find the value of $\left(x^2+\frac{1}{x^2}\right)^2$.

Answer

$$x=(2+\sqrt{3})$$

$$\left(x^2+\frac{1}{x^2}\right)^2=\left((2+\sqrt{3})^2+\frac{1}{(2+\sqrt{3})^2})\right)^2$$

$$=\left((4+3+4\sqrt{3})+\frac{1}{(4+3+4\sqrt{3})}\right)^2$$

$$=\left(7+4\sqrt{3}+\frac{1}{7+4\sqrt{3}}\right)^2$$

$$=\left(\frac{(7+4\sqrt{3})^2+1}{7+4\sqrt{3}}\right)^2$$

$$=\left(\frac{49+48+56\sqrt{3}+1}{7+4\sqrt{3}}\right)^2$$

$$=\left(\frac{49+48+56\sqrt{3}+1}{7+4\sqrt{3}}\right)^2$$

$$=\left(\frac{14(7+4\sqrt{3})}{7+4\sqrt{3}}\right)^2$$

$$=14^2=196$$

46. Question

If $\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}=(a-b\sqrt{3})$, find the values of a and b.

Answer

$$\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$$

$$=\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1^2}=\frac{3+1-2\sqrt{3}}{2}$$

$$=\frac{4-2\sqrt{3}}{2}=\frac{2(2-\sqrt{3})}{2}$$

$$=2-\sqrt{3}$$

$$(a-b\sqrt{3}) = 2-\sqrt{3}$$

So, a =2 and b =1

47. Question

If $\frac{(4+\sqrt{5})}{(4-\sqrt{5})}=(a+b\sqrt{5})$, find the values of a and b.

Answer

$$\frac{(4+\sqrt{5})}{(4-\sqrt{5})}\times\frac{(4+\sqrt{5})}{(4+\sqrt{5})}$$

$$= \frac{(4+\sqrt{5})^2}{4^2-(\sqrt{5})^2} = \frac{16+5+8\sqrt{5}}{16-5}$$

$$= \frac{21+8\sqrt{5}}{11}$$

$(a+b\sqrt{5})$ = $\frac{21}{11}+\frac{8\sqrt{5}}{11}$

So, a = $\frac{21}{11}$ and b = $\frac{8}{11}$

48. Question

If $\frac{(\sqrt{5}-1)}{(\sqrt{5}+1)}+\frac{(\sqrt{5}+1)}{(\sqrt{5}-1)}=(a+b\sqrt{5})$, find the values of a and b.

Answer

$$\frac{(\sqrt{5}-1)}{(\sqrt{5}+1)}\times\frac{(\sqrt{5}-1)}{(\sqrt{5}-1)}+\frac{(\sqrt{5}+1)}{(\sqrt{5}-1)}\times\frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$$

$$= \frac{(\sqrt{5}-1)^2}{(\sqrt{5})^2-1^2}+\frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-1^2}$$

$$= \frac{5+1-2\sqrt{5}}{4}+\frac{5+1+2\sqrt{5}}{4}$$

$$= \frac{12}{4} = 3$$

$(a+b\sqrt{5})$ =3

So, a = 3 and b=0.

49. Question

If $\frac{(\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}=(a+b\sqrt{6})$, find the values of a and b.

Answer

$$\frac{(\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}=\frac{(\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}\times\frac{(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})}$$

$$=\frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$=\frac{6+3\sqrt{6}+2\sqrt{6}+6}{18-12}$$

$$=\frac{12+5\sqrt{6}}{6}$$

So, a = $\frac{12}{6}=2$ and b = $\frac{5}{6}$

50. Question

If $x=\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$ and $y=\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$, find (x^2+y^2).

Answer

$$x=\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}=\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}\times\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$$=\frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3+2+2\sqrt{6}}{3-2}$$

$$=5+2\sqrt{6}$$

$$y=\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})}=\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})}\times\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$$

$$=\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3+2-2\sqrt{6}}{3-2}$$

$= 5 - 2\sqrt{6}$

Now, $(x^2 + y^2) = (5 + 2\sqrt{6})^2 + (5 - 2\sqrt{6})^2$

$= 25 + 24 + 20\sqrt{6} + 25 + 24 - 20\sqrt{6}$

= 98

51. Question

If $x = \frac{1}{(2 - \sqrt{3})}$, show that the value of $(x^3 - 2x^2 - 7x + 5)$ is 3.

Answer

$x = \frac{1}{(2 - \sqrt{3})} \times \frac{(2 + \sqrt{3})}{(2 + \sqrt{3})}$

$= \frac{(2 + \sqrt{3})}{(2)^2 - (\sqrt{3})^2}$

$= \frac{(2 + \sqrt{3})}{4 - 3}$

Then, x $= 2 + \sqrt{3}$

Then, $x^2 = (2 + \sqrt{3})^2 = 4 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}$

Ans, $x^3 = (2 + \sqrt{3})(7 + 4\sqrt{3}) = 14 + 7\sqrt{3} + 8\sqrt{3} + 12 = 26 + 15\sqrt{3}$

Now,

$x^3 - 2x^2 - 7x + 5$ =

$= 26 + 15\sqrt{3} - 2(7 + 4\sqrt{3}) - 7(2 + \sqrt{3}) + 5$

$= 26 + 15\sqrt{3} - 14 - 8\sqrt{3} - 14 - 7\sqrt{3} + 5$

$= 26 - 28 + 5$

= 3

52. Question

if $x = (3+\sqrt{8})$, show that $\left(x^2 + \frac{1}{x^2}\right) = 34.$

Answer

We have, $x = (3+\sqrt{8})$

$x^2 = (3+\sqrt{8})^2 = 9+8+6\sqrt{8}$

$= 17+6\sqrt{8}$

Then, $x^2 + \frac{1}{x^2} = \left((17+6\sqrt{8}) + \frac{1}{17+6\sqrt{8}}\right)$

$= \left(\frac{\left(17+6\sqrt{8}\right)^2 + 1}{17+6\sqrt{8}}\right)$

$= \frac{289+288+204\sqrt{8}+1}{17+6\sqrt{8}}$

$= \frac{578+204\sqrt{8}}{17+6\sqrt{8}} = \frac{34\left(17+6\sqrt{8}\right)}{17+6\sqrt{8}}$

= 34

53. Question

if $x = (2+\sqrt{3})$, show that $\left(x^3 + \frac{1}{x^3}\right) = 52.$

Answer

$x = (2+\sqrt{3})$

$x^2 = (2+\sqrt{3})^2 = 4+3+4\sqrt{3} = 7+4\sqrt{3}$

$x^3 = (2+\sqrt{3})(7+4\sqrt{3}) = 14+7\sqrt{3}+8\sqrt{3}+12 = 26+15\sqrt{3}$

Now,

$x^3 + \frac{1}{x^3} = (26+15\sqrt{3}) + \frac{1}{26+15\sqrt{3}}$

$$= \frac{(26+15\sqrt{3})^2+1}{26+15\sqrt{3}}$$

$$= \frac{676+675+780\sqrt{3}+1}{26+15\sqrt{3}}$$

$$= \frac{1352+780\sqrt{3}}{26+15\sqrt{3}}$$

$$= \frac{52(26+15\sqrt{3})}{26+15\sqrt{3}}$$

$$= 52$$

54. Question

if $x=(3-2\sqrt{2})$, show that $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=\pm 2.$

Answer

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=\frac{x-1}{\sqrt{x}}$$

$$= \frac{3-2\sqrt{2}-1}{\sqrt{3-2\sqrt{2}}}=\frac{2-2\sqrt{2}}{\sqrt{3-2\sqrt{2}}}$$

$$= \frac{2(1-\sqrt{2})}{\sqrt{3-2\sqrt{2}}}=\frac{2(1-\sqrt{2})}{\sqrt{\sqrt{2}^2+1^2-2\sqrt{2}}}=\frac{2(1-\sqrt{2})}{\sqrt{(1-\sqrt{2})^2}}$$

$$= \frac{2(1-\sqrt{2})}{1-\sqrt{2}}=2$$

55. Question

if $x=(5+2\sqrt{6})$, show that $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=\pm 2\sqrt{3}.$

Answer

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=\frac{x+1}{\sqrt{x}}$$

$$= \frac{5+2\sqrt{6}+1}{\sqrt{5+2\sqrt{6}}} = \frac{6+2\sqrt{6}}{\sqrt{5+2\sqrt{6}}} = \frac{6+2\sqrt{6}}{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{6}}}$$

$$= \frac{6+2\sqrt{6}}{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{6}}} = \frac{6+2\sqrt{6}}{\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}}$$

$$= \frac{6+2\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{\sqrt{3}+\sqrt{2}}$$

$$= 2\sqrt{3}$$

Formative Assessment (Unit Test)

1. Question

Find two rational numbers lying between $\frac{1}{3} \text{ and } \frac{1}{2}$.

Answer

If x and y are two rational numbers such that x<y then $\frac{1}{2}(x+y)$ is a rational number between x and y.

So, rational numbers will be:

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{2+3}{6}\right) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

$$\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right) = \frac{1}{2}\left(\frac{4+5}{12}\right) = \frac{1}{2} \times \frac{9}{12} = \frac{9}{24} = \frac{3}{8}$$

$\frac{5}{12} \text{ and } \frac{3}{8}$ are two rational numbers lying between $\frac{1}{3} \text{ and } \frac{1}{2}$.

2. Question

Find four rational numbers between $\frac{3}{5} \text{ and } \frac{4}{5}$.

Answer

If x and y are two rational numbers such that x<y then $\frac{1}{2}(x+y)$ is a rational number between x and y.

So, rational numbers will be:

$$\frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{3+4}{5}\right)=\frac{1}{2}\times\frac{7}{5}=\frac{7}{10}$$

$$\frac{1}{2}\left(\frac{3}{5}+\frac{7}{10}\right)=\frac{1}{2}\left(\frac{6+7}{10}\right)=\frac{1}{2}\times\frac{13}{10}=\frac{13}{20}$$

$$\frac{1}{2}\left(\frac{13}{20}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{13+16}{20}\right)=\frac{1}{2}\times\frac{29}{10}=\frac{29}{40}$$

$$\frac{1}{2}\left(\frac{4}{5}+\frac{29}{40}\right)=\frac{1}{2}\left(\frac{32+29}{40}\right)=\frac{1}{2}\times\frac{61}{40}=\frac{61}{80}$$

$\frac{7}{10},\frac{13}{20},\frac{29}{40}\,and\,\frac{61}{80}$ are two rational numbers lying between $\frac{3}{5}\,and\,\frac{4}{5}$.

3. Question

Write four irrational numbers between 0.1 and 0.2.

Answer

Four irrational numbers are

0.1010010001…,

0.1212212221…,

0.13113313331…, and

0.1414414441…As they all have non terminating and non repeating decimal

4. Question

Express $\sqrt[4]{1250}$ in its simplest form.

Answer

$$\sqrt[4]{1250}=\sqrt[4]{625\times 2}$$

$$=\sqrt[4]{5^4\times 2}$$

$$=5\sqrt[4]{2}$$

5. Question

Express $\frac{2}{3}.\sqrt{18}$ as a pure surd.

Answer

$$\frac{2}{3}.\sqrt{18} = \sqrt{\frac{2}{3}\times\frac{2}{3}\times 18}$$

$$= \sqrt{\frac{2\times 2\times 18}{3\times 3}}$$

$$= \sqrt{2\times 2\times 2}$$

$$= \sqrt{8}$$

6. Question

Divide $16\sqrt{75}\ by\ 5\sqrt{12}.$

Answer

$16\sqrt{75}\ \ by\ \ 5\sqrt{12}$ is given as:

$$= \sqrt{\frac{16\times 16\times 75}{5\times 5\times 12}}$$

$$= \sqrt{\frac{16\times 16\times 3}{12}}$$

$$= \sqrt{\frac{4\times 16\times 3}{3}}$$

$$= \sqrt{4\times 16}$$

$$= \sqrt{64} = 8$$

7. Question

Express $0.1\overline{23}$ as a rational number in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0.$

Answer

Let x = $0.1\overline{23}$ then

X = 0.123232323……………..(i)

10x = 1.23232323 …………….(ii)

100x = 123.23232323 ………..(iii)

On subtracting (ii) from (iii) we get,

90x = 122

$$X=\frac{122}{90}$$

$$X=\frac{61}{45}$$

8. Question

If $\frac{6}{(3\sqrt{2}-2\sqrt{3})}=(a\sqrt{2}+b\sqrt{3})$, find the values of a and b.

Answer

$$\frac{6}{(3\sqrt{2}-2\sqrt{3})}$$

$$=\frac{6}{(3\sqrt{2}-2\sqrt{3})}\times\frac{(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})}$$

$$=\frac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$=\frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$$

$$=\frac{6(3\sqrt{2}+2\sqrt{3})}{6}$$

$$=3\sqrt{2}+2\sqrt{3}$$

So, a = 3 and b =2

9. Question

The simplest form of $\left(\frac{64}{729}\right)^{-1/6}$ is

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer

$\left(\frac{64}{729}\right)^{-1/6}$

$=\left(\frac{2^6}{3^6}\right)^{-1/6}$

$=\left(\frac{2}{3}\right)^{6\times\frac{-1}{6}}$

$=\left(\frac{2}{3}\right)^{-1}$

$=\frac{3}{2}$

10. Question

Which of the following is irrational?

A. $0.\overline{14}$

B. $0.14\overline{16}$

C. $0.\overline{1416}$

D. 0.1401401400014…..

Answer

0.1401401400014….. is an irrational number because it is non-ending and non-terminating.

11. Question

Between two rational numbers

A. there is no rational number

B. there is exactly one rational number

C. there are infinitely many irrational numbers

D. there is no irrational number

Answer

There are infinitely many irrational numbers between two rational numbers.

12. Question

Decimal representation of an irrational number is

A. always a terminating decimal

B. either a terminating or a repeating decimal

C. either a terminating or a non-terminating or a non-repeating decimal

D. always non-terminating and non-repeating decimal

Answer

Decimal representation of an irrational number is always non-terminating and non-repeating decimal.

13. Question

If $x = (7+5\sqrt{2}), then \left(x^2 + \frac{1}{x^2}\right) = ?$

A. 160

B. 198

C. 189

D. 156

Answer

We have, $x = (7+5\sqrt{2})$

Then,

$x^2 = (7+5\sqrt{2})^2 = 49 + 50 + 70\sqrt{2}$

$= 99+70\sqrt{2}$

$= x^2+\frac{1}{x^2}=\left((99+70\sqrt{2})+\frac{1}{99+70\sqrt{2}}\right)$

$=\left(\frac{\left(99+70\sqrt{2}\right)^2+1}{99+70\sqrt{2}}\right)$

$=\frac{9801+9800+13860\sqrt{2}+1}{99+70\sqrt{2}}$

$=\frac{19602+13860\sqrt{2}}{99+70\sqrt{2}}$

$=\frac{198\left(99+70\sqrt{2}\right)}{99+70\sqrt{2}}$

$=198$

14. Question

Rationalize the denominator of $\left(\frac{5\sqrt{3}-4\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\right)$.

Answer

We have, $\left(\frac{5\sqrt{3}-4\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\right)$

$=\left(\frac{5\sqrt{3}-4\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\right)\times\left(\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}\right)$

$=\left(\frac{\left(5\sqrt{3}-4\sqrt{2}\right)\left(4\sqrt{3}-3\sqrt{2}\right)}{\left(4\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2}\right)$

$=\left(\frac{60-16\sqrt{6}-15\sqrt{6}+24}{48-18}\right)$

$$= \left(\frac{84 - 31\sqrt{6}}{30} \right)$$

15. Question

Simplify: $\frac{1}{(27)^{-1/3}} + \frac{1}{(625)^{-1/4}}$.

Answer

Now, we have,

$$\frac{1}{(27)^{-1/3}} + \frac{1}{(625)^{-1/4}} = \frac{1}{(3^3)^{-1/3}} + \frac{1}{(5^4)^{-1/4}}$$

$$= \frac{1}{(3)^{-1}} + \frac{1}{(5)^{-1}}$$

= 3 + 5

= 8

16. Question

Find the smallest of the numbers $\sqrt[3]{6}, \sqrt[6]{24}, and\ \sqrt[4]{8}$.

Answer

$\sqrt[3]{6}, \sqrt[6]{24}, and\ \sqrt[4]{8}$ can be written as:

$$(6)^{\frac{1}{3}}, (24)^{\frac{1}{6}}\ and\ (8)^{\frac{1}{4}}$$

Equalizing powers by multiplying and multiplying and dividing by 12, we get,

$$= (6)^{(12)/(3\times12)}, (24)^{12/(2\times12)}, (8)^{(12)/(4\times12)}$$

$$= (6^4)^{1/12}, (24^6)^{1/12}, (8^3)^{1/12}$$

$$= (1296)^{1/12}, (191102976)^{1/12}, (512)^{1/12}$$

Now, in ascending order,

$$= (512)^{1/12}, (1296)^{1/12}, (191102976)^{1/12}$$

So, $\sqrt[4]{8}$ is the smallest number.

17. Question

Match the following columns:

Column I	Column II
A. π *is*.... .	(p) a rational number
B. $3.\overline{1416}$ *is*.... .	(q) an irrational number
C. $0.\overline{23} = ...$.	(r) $\frac{7}{30}$
D. $0.2\overline{3} = ...$.	(s) $\frac{23}{99}$

The correct answer is:

(a)-......., (b)-......., (c)-......., (d)-.......,

Answer

(a)-(q)

π is an irrational number.

(b)-(p)

$3.\overline{1416}$ is a rational number.

(c)-(s) $\frac{23}{99}$ = 0.23232323...... = $0.\overline{23}$

(d)-(r) $\frac{7}{30}$ = 0.23333... = $0.2\overline{3}$

18. Question

If $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, find the value of (x^2+y^2).

Answer

We have,

$$x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

On rationalizing we get,

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{5+3+2\sqrt{15}}{5-3}$$

$$=\frac{8+2\sqrt{15}}{2}$$

$$=4+\sqrt{15}$$

Now,

$$x^2=\left(4+\sqrt{15}\right)^2$$

$$=16+15+8\sqrt{15}$$

$$x^2=31+8\sqrt{15}$$

Now,

$$y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{5+3-2\sqrt{15}}{5-3}$$

$$= \frac{8-2\sqrt{15}}{2}$$

$$= 4-\sqrt{15}$$

Now, $y^2 = \left(4-\sqrt{15}\right)^2$

$$y^2 = 16+15-8\sqrt{15}$$

$$y^2 = 31-8\sqrt{15}$$

$$(x^2+y^2) = 31+8\sqrt{15}+31-8\sqrt{15}$$
$$= 62$$

19. Question

If $\sqrt{2} = 1.41 \, and \, \sqrt{5} = 2.24$, find the value of $\frac{3}{(8\sqrt{2}+5\sqrt{5})} + \frac{2}{(8\sqrt{2}-5\sqrt{5})}$.

Answer

We have,

$$\frac{3}{(8\sqrt{2}+5\sqrt{5})} + \frac{2}{(8\sqrt{2}-5\sqrt{5})}$$

$$= \frac{24\sqrt{2}-15\sqrt{5}+16\sqrt{2}+10\sqrt{5}}{(8\sqrt{2})^2-(5\sqrt{5})^2}$$

$$= \frac{40\sqrt{2}-5\sqrt{5}}{128-125}$$

$$= \frac{40\sqrt{2}-5\sqrt{5}}{3}$$

$= \frac{(40 \times 1.41) - (5 \times 2.24)}{3}$

$= \frac{56.4 - 11.2}{3}$

$= \frac{45.2}{3}$

= 15.7

20. Question

Prove that $\left(\frac{81}{16}\right)^{-3/4} \times \left\{\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right\} = 1.$

Answer

We have, $\left(\frac{81}{16}\right)^{-3/4} \times \left\{\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right\}$

$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{5^2}{3^2}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right\}$

$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{5}{3}\right)^{2 \times -\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right\}$

$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right\}$

$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right\}$

$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}\right\}$

$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \frac{27}{8}$

$$=\left(\frac{3}{2}\right)^{-3}\times\frac{27}{8}$$

$$=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{27}{8}$$

= 1

Hence, Proved.