

7. Values of Trigonometric Functions at Sum of Difference of Angles

Exercise 7.1

1. Question

If $\sin A = 4/5$ And $\cos B = 5/13$, where $0 < A, B < \pi/2$, find the values of the following:

(i) $\sin(A + B)$

(ii) $\cos(A + B)$

(iii) $\sin(A - B)$

(iv) $\cos(A - B)$

Answer

Given $\sin A = 4/5$ And $\cos B = 5/13$

We know that $\cos A = \sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$ where $0 < A, B < \pi/2$

$$\Rightarrow \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos A = \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\therefore \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Then,

(i) $\sin(A + B)$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{56}{65}$$

(ii) $\cos(A + B)$

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$= \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= -\frac{33}{65}$$

(iii) $\sin(A - B)$

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\begin{aligned} &= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13} \\ &= \frac{20}{65} - \frac{36}{65} \\ &= \frac{-16}{65} \end{aligned}$$

(iv) $\cos(A - B)$

We know that $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\begin{aligned} &= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} \\ &= \frac{15}{65} + \frac{48}{65} \\ &= \frac{63}{65} \end{aligned}$$

2 A. Question

If $\sin A = 12/13$ And $\sin B = 4/5$, where $\pi/2 < A < \pi$ And $0 < B < \pi/2$, find the following:

(i) $\sin(A + B)$ (ii) $\cos(A + B)$

Answer

Given $\sin A = 12/13$ And $\sin B = 4/5$ where $\pi/2 < A < \pi$ And $0 < B < \pi/2$

We know that $\cos A = -\sqrt{1 - \sin^2 A}$ and $\cos B = \sqrt{1 - \sin^2 B}$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{12}{13}\right)^2} \text{ and } \cos B = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{144}{169}} \text{ and } \cos B = \sqrt{1 - \frac{16}{25}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{25}{169}} \text{ and } \cos B = \sqrt{\frac{9}{25}}$$

$$\therefore \cos A = -\frac{5}{13} \text{ and } \cos B = \frac{3}{5}$$

(i) $\sin(A + B)$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\begin{aligned} &= \frac{12}{13} \times \frac{3}{5} + \frac{5}{13} \times \frac{4}{5} \\ &= \frac{36}{65} + \frac{20}{65} \\ &= \frac{56}{65} \end{aligned}$$

(ii) $\cos(A + B)$

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$\begin{aligned}
&= -\frac{5}{13} \times \frac{3}{5} - \frac{12}{13} \times \frac{4}{5} \\
&= -\frac{15}{65} - \frac{48}{65} \\
&= -\frac{63}{65}
\end{aligned}$$

2 B. Question

If $\sin A = 3/5$, $\cos B = -12/13$, where A And B Both lie in second quadrant, find the value of $\sin(A + B)$.

Answer

Given $\sin A = 3/5$ And $\cos B = -12/13$

A And B lie in the second quadrant.

So sine function is positive And cosine function is negative.

We know that $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{-12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{9}{25}} \text{ and } \sin B = \sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{16}{25}} \text{ and } \sin B = \sqrt{\frac{25}{169}}$$

$$\therefore \cos A = -\frac{4}{5} \text{ and } \sin B = \frac{5}{13}$$

Now consider $\sin(A + B)$,

$$\Rightarrow \sin(A + B) = \frac{3}{5} \times \frac{5}{13} + \frac{-4}{5} \times \frac{-12}{13}$$

$$= \frac{15}{65} + \frac{48}{65}$$

$$= \frac{63}{65}$$

3. Question

If $\cos A = -24/25$ And $\cos B = 3/5$, where $\pi < A < 3\pi/2$ And $3\pi/2 < B < 2\pi$, find the following:

(i) $\sin(A + B)$ (ii) $\cos(A + B)$

Answer

Given $\cos A = -24/25$ And $\cos B = 3/5$ where $\pi < A < 3\pi/2$ And $3\pi/2 < B < 2\pi$

A is in third quadrant And B is in fourth quadrant.

Here, sine function is negative.

We know that $\sin A = -\sqrt{1 - \cos^2 A}$ and $\sin B = -\sqrt{1 - \cos^2 B}$

$$\Rightarrow \sin A = -\sqrt{1 - \left(\frac{-24}{25}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin A = -\sqrt{1 - \frac{576}{625}} \text{ and } \sin B = -\sqrt{1 - \frac{9}{25}}$$

$$\Rightarrow \sin A = -\sqrt{\frac{49}{625}} \text{ and } \sin B = -\sqrt{\frac{16}{25}}$$

$$\therefore \sin A = -\frac{7}{25} \text{ and } \sin B = -\frac{4}{5}$$

Then,

(i) $\sin(A + B)$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$= \frac{-7}{25} \times \frac{3}{5} + \frac{-24}{25} \times \frac{-4}{5}$$

$$= \frac{-21}{125} + \frac{96}{125}$$

$$= \frac{75}{125}$$

$$= \frac{3}{5}$$

(ii) $\cos(A + B)$

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$= \frac{-24}{25} \times \frac{3}{5} - \frac{-7}{25} \times \frac{-4}{5}$$

$$= \frac{-72}{125} - \frac{28}{125}$$

$$= -\frac{100}{125}$$

$$= -\frac{4}{5}$$

4. Question

If $\tan A = 3/4$, $\cos B = 9/41$, where $\pi < A < 3\pi/2$ And $0 < B < \pi/2$, find $\tan(A + B)$.

Answer

Given $\tan A = 3/4$ And $\cos B = 9/41$ where $\pi < A < 3\pi/2$ And $0 < B < \pi/2$

A is in third quadrant And B is in first quadrant.

Tan function And sine function are positive.

We know that $\sin B = \sqrt{1 - \cos^2 B}$

$$\Rightarrow \sin B = \sqrt{1 - \left(\frac{9}{41}\right)^2}$$

$$= \sqrt{1 - \frac{81}{1681}}$$

$$= \sqrt{\frac{1600}{1681}}$$

$$\therefore \sin B = \frac{40}{41}$$

$$\text{We know that } \tan B = \frac{\sin B}{\cos B}$$

$$= \frac{\frac{40}{41}}{\frac{9}{41}}$$

$$= \frac{40}{9}$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan(A + B) = \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4} \times \frac{40}{9}}$$

$$= \frac{\frac{187}{36}}{\frac{84}{36}}$$

$$= -\frac{187}{84}$$

5. Question

If $\sin A = 1/2$, $\cos B = 12/13$, where $\pi/2 < A < \pi$ And $3\pi/2 < B < 2\pi$, find $\tan(A - B)$.

Answer

Given $\sin A = 1/2$ And $\cos B = 12/13$ where $\pi/2 < A < \pi$ And $3\pi/2 < B < 2\pi$

A is in second quadrant And B is in fourth quadrant.

In the second quadrant, the sine function is positive And cosine And tan functions negative.

In the fourth quadrant, sine And tan functions are negative, And cosine function are positive.

$$\text{We know that } \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = -\sqrt{1 - \cos^2 B}$$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = -\sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\therefore \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{5}{13}$$

$$\Rightarrow \tan A = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} \text{ and } \tan B = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{12}$$

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow \tan(A - B) = \frac{-\frac{1}{\sqrt{3}} - \left(-\frac{5}{12}\right)}{1 + \left(-\frac{1}{\sqrt{3}}\right)\left(-\frac{5}{12}\right)}$$

$$= \frac{-12 + 5\sqrt{3}}{12\sqrt{3}}$$
$$= \frac{12\sqrt{3} + 5}{12\sqrt{3}}$$

$$= \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}}$$

6. Question

If $\sin A = 1/2$, $\cos B = \frac{\sqrt{3}}{2}$, where $\pi/2 < A < \pi$ And $0 < B < \pi/2$, find the following:

(i) $\tan(A + B)$ (ii) $\tan(A - B)$

Answer

Given $\sin A = 1/2$ And $\cos B = \sqrt{3}/2$ where $\pi/2 < A < \pi$ And $0 < B < \pi/2$,

A is in second quadrant And B is in first quadrant.

In the second quadrant, the sine function is positive And cosine And tan functions are negative.

In first quadrant, All functions are positive.

We know that $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\therefore \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\Rightarrow \tan A = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} \text{ and } \tan B = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Then,

(i) $\tan(A + B)$

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \tan(A + B) = \frac{-\frac{1}{\sqrt{3}} + \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{0}{\frac{2}{3}}$$

$$= 0$$

(ii) $\tan(A - B)$

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow \tan(A - B) = \frac{-\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= -\sqrt{3}$$

7. Question

Evaluate the following:

(i) $\sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ$ (ii) $\cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ$

(iii) $\sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ$ (iv) $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$

Answer

(i) Given $\sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ$

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\Rightarrow \sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ = \sin(78 - 18)^\circ$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ = \frac{\sqrt{3}}{2}$$

(ii) Given $\cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ$

We know that $\cos A \cos B - \sin A \sin B = \cos(A + B)$

$$\Rightarrow \cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ = \cos(47 + 13)^\circ$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

$$\therefore \cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ = \frac{1}{2}$$

(iii) Given $\sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow \sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ = \sin(36 + 9)^\circ$$

$$= \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}}$$

$$\therefore \sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ = \frac{1}{\sqrt{2}}$$

(iv) Given $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$

We know that $\cos A \cos B + \sin A \sin B = \cos(A - B)$

$$\Rightarrow \cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ = \cos(80 - 20)^\circ$$

$$= \cos 60^\circ$$

$$= 1/2$$

$$\therefore \cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ = 1/2$$

8. Question

If $\cos A = -12/13$ and $\cot B = 24/7$, where A lies in the second quadrant and B in the third quadrant, find the values of the following:

(i) $\sin(A + B)$ (ii) $\cos(A + B)$ (iii) $\tan(A + B)$

Answer

Given $\cos A = -12/13$ And $\cot B = 24/7$

A lies in second quadrant And B in the third quadrant.

The sine function is positive in the second quadrant and in the third quadrant, Both sine And cosine functions are negative.

We know that $\sin A = \sqrt{1 - \cos^2 A}$ and $\sin B = -\frac{1}{\sqrt{1 + \cot^2 B}}$

$$\Rightarrow \sin A = \sqrt{1 - \left(-\frac{12}{13}\right)^2} \text{ and } \sin B = -\frac{1}{\sqrt{1 + \left(\frac{24}{7}\right)^2}}$$

$$\Rightarrow \sin A = \sqrt{1 - \frac{144}{169}} \text{ and } \sin B = -\frac{1}{\sqrt{1 + \frac{576}{49}}}$$

$$\Rightarrow \sin A = \sqrt{\frac{25}{169}} \text{ and } \sin B = -\frac{1}{\sqrt{\frac{625}{49}}}$$

$$\therefore \sin A = \frac{5}{13} \text{ and } \sin B = -\frac{7}{25}$$

$$\Rightarrow \cos B = -\sqrt{1 - \sin^2 B} = -\sqrt{1 - \left(-\frac{7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25}$$

Now,

(i) $\sin(A + B)$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$= \frac{5}{13} \times \frac{-24}{25} + \frac{-12}{13} \times \frac{-7}{25}$$

$$= \frac{-120}{325} + \frac{84}{325}$$

$$= \frac{-36}{325}$$

(ii) $\cos(A + B)$

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$= \frac{-12}{13} \times \frac{-24}{25} - \frac{5}{13} \times \frac{-7}{25}$$

$$= \frac{288}{325} + \frac{35}{325}$$

$$= \frac{323}{325}$$

(iii) $\tan(A + B)$

We know that $\tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)}$

$$\Rightarrow \tan(A + B) = \frac{\frac{-36}{325}}{\frac{323}{325}}$$

$$= -\frac{36}{323}$$

9. Question

Prove that: $\cos 7\pi/12 + \cos \pi/12 = \sin 5\pi/12 - \sin \pi/12$

Answer

$$\Rightarrow 7\pi/12 = 105^\circ, \pi/12 = 15^\circ; 5\pi/12 = 75^\circ$$

$$\text{LHS} = \cos 105^\circ + \cos 15^\circ$$

$$= \cos(90^\circ + 15^\circ) + \sin(90^\circ - 75^\circ)$$

$$= -\sin 15^\circ + \sin 75^\circ$$

$$= \sin 75^\circ - \sin 15^\circ = \text{RHS}$$

Hence proved.

10. Question

Prove that: $\frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A + B)}{\sin(A - B)}$

Answer

$$\text{LHS} = \frac{\tan A + \tan B}{\tan A - \tan B}$$

$$\Rightarrow \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}$$

$$= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}}$$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\Rightarrow \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}} = \frac{\sin(A+B)}{\sin(A-B)} = \text{RHS}$$

Hence, proved.

11. Question

Prove that:

$$(i) \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ \quad (ii) \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan 54^\circ$$

$$(ii) \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$$

Answer

$$(i) \text{LHS} = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

Dividing numerator And denominator by $\cos 11^\circ$,

$$\begin{aligned} \Rightarrow \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} &= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ} \\ &= \frac{1 + \tan 11^\circ}{1 - 1 \times \tan 11^\circ} \\ &= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \times \tan 11^\circ} \end{aligned}$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \times \tan 11^\circ} = \tan(45^\circ + 11^\circ)$$

$$= \tan 56^\circ = \text{RHS}$$

Hence proved.

$$(ii) \text{LHS} = \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

Dividing numerator And denominator by $\cos 9^\circ$,

$$\begin{aligned} \Rightarrow \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} &= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} \\ &= \frac{1 + \tan 9^\circ}{1 - 1 \times \tan 9^\circ} \\ &= \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \times \tan 9^\circ} \end{aligned}$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \times \tan 9^\circ} = \tan(45^\circ + 9^\circ)$$

$$= \tan 54^\circ = \text{RHS}$$

Hence proved.

$$(iii) \text{LHS} = \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

Dividing numerator And denominator by $\cos 8^\circ$,

$$\Rightarrow \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ}$$

$$= \frac{1 - \tan 8^\circ}{1 + 1 \times \tan 8^\circ}$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \times \tan 8^\circ}$$

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \times \tan 8^\circ} = \tan(45^\circ - 8^\circ)$$

$$= \tan 37^\circ = \text{RHS}$$

Hence proved.

12 A. Question

Prove that:

$$\sin\left(\frac{\pi}{3} - x\right) \cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{3} - x\right) \sin\left(\frac{\pi}{6} + x\right) = 1$$

Answer

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow \sin\left(\frac{\pi}{3} - x\right) \cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{3} - x\right) \sin\left(\frac{\pi}{6} + x\right) = \sin\left(\frac{\pi}{3} - x + \frac{\pi}{6} + x\right)$$

$$= \sin\left(\frac{2\pi + \pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$$= \sin 90^\circ$$

$$= 1 = \text{RHS}$$

Hence proved.

12 B. Question

Prove that:

$$\sin\left(\frac{4\pi}{9} + 7\right) \cos\left(\frac{\pi}{9} + 7\right) - \cos\left(\frac{4\pi}{9} + 7\right) \sin\left(\frac{\pi}{9} + 7\right) = \frac{\sqrt{3}}{2}$$

Answer

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\Rightarrow \sin\left(\frac{4\pi}{9} + 7\right) \cos\left(\frac{\pi}{9} + 7\right) - \cos\left(\frac{4\pi}{9} + 7\right) \sin\left(\frac{\pi}{9} + 7\right) = \sin\left(\frac{4\pi}{9} + 7 - \frac{\pi}{9} - 7\right)$$

$$= \sin\left(\frac{3\pi}{9}\right)$$

$$= \sin\left(\frac{\pi}{3}\right)$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2} = \text{RHS}$$

Hence, proved.

12 C. Question

Prove that:

$$\sin\left(\frac{3\pi}{8} - 5\right) \cos\left(\frac{\pi}{8} + 5\right) + \cos\left(\frac{3\pi}{8} - 5\right) \sin\left(\frac{\pi}{8} + 5\right) = 1$$

Answer

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow \sin\left(\frac{3\pi}{8} - 5\right) \cos\left(\frac{\pi}{8} + 5\right) + \cos\left(\frac{3\pi}{8} - 5\right) \sin\left(\frac{\pi}{8} + 5\right) = \sin\left(\frac{3\pi}{8} - 5 + \frac{\pi}{8} + 5\right)$$

$$= \sin\left(\frac{3\pi + \pi}{8}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$$= \sin 90^\circ$$

$$= 1 = \text{RHS}$$

Hence proved.

13. Question

Prove that: $\frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ} = -1$

Answer

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Here $A = 69^\circ$ And $B = 66^\circ$

$$\text{LHS} = \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ} = \tan(69 + 66)^\circ$$

$$= \tan 135^\circ$$

$$= -\tan 45^\circ$$

$$= -1 = \text{RHS}$$

Hence proved.

14 A. Question

If $\tan A = 5/6$ And $\tan B = 1/11$, prove that $A + B = \pi/4$.

Answer

Given $\tan A = \frac{5}{6}$; $\tan B = \frac{1}{11}$

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \tan(A + B) = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$$

$$= \frac{55 + 6}{66 - 5}$$

$$= \frac{61}{61}$$

$$= 1$$

$$\Rightarrow \tan(A + B) = \tan \pi/4$$

$$\therefore A + B = \pi/4$$

Hence proved.

14 B. Question

If $\tan A = m/m-1$ And $\tan B = 1/2m - 1$, then prove that $A - B = \pi/4$.

Answer

$$\text{Given } \tan A = \frac{m}{m-1}; \tan B = \frac{1}{2m-1}$$

$$\text{We know that } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \tan(A - B) = \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m}$$

$$= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1}$$

$$= 1$$

$$\Rightarrow \tan(A - B) = \tan \pi/4$$

$$\therefore A - B = \pi/4$$

Hence proved.

15 A. Question

prove that:

$$\cos^2 \pi/4 - \sin^2 \frac{\pi}{12} = \frac{\sqrt{3}}{4}$$

Answer

$$\text{LHS} = \cos^2 \frac{\pi}{4} - \sin^2 \frac{\pi}{12}$$

We know that $\cos^2 A - \sin^2 B = \cos(A + B) \cos(A - B)$

$$\Rightarrow \cos^2 \frac{\pi}{4} - \sin^2 \frac{\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right) \cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$$

$$= \cos \frac{4\pi}{12} \cos \frac{2\pi}{12}$$

$$= \cos \frac{\pi}{3} \cos \frac{\pi}{6}$$

$$= \frac{1}{2} \left(\frac{\sqrt{3}}{2} \right)$$

$$= \frac{\sqrt{3}}{4} = \text{RHS}$$

Hence, proved.

15 B. Question

prove that:

$$\sin^2(n+1)A - \sin^2nA = \sin(2n+1)A \sin A$$

Answer

We know that $\sin^2A - \sin^2B = \sin(A+B) \sin(A-B)$

Here $A = (n+1)A$ And $B = nA$

$$\Rightarrow \text{LHS: } \sin^2(n+1)A - \sin^2nA = \sin((n+1)A + nA) \sin((n+1)A - nA)$$

$$= \sin(nA + A + nA) \sin(nA + A - nA)$$

$$= \sin(2nA + A) \sin(A)$$

$$= \sin(2n+1)A \sin A = \text{RHS}$$

Hence proved.

16 A. Question

Prove that:

$$\frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} = \tan A$$

Answer

$$\text{LHS} = \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)}$$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ And $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\Rightarrow \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} = \frac{\sin A \cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B}{\cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B}$$

$$= \frac{2 \sin A \cos B}{2 \cos A \cos B}$$

$$= \tan A = \text{RHS}$$

Hence proved.

16 B. Question

Prove that:

$$\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$$

Answer

$$\text{LHS} = \frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A}$$

We know that $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$\begin{aligned} \Rightarrow \frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} \\ = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C - \cos B \sin C}{\cos B \cos C} \\ + \frac{\sin C \cos A - \cos C \sin A}{\cos C \cos A} \end{aligned}$$

$$= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C}{\cos B \cos C} - \frac{\cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A}{\cos C \cos A} - \frac{\cos C \sin A}{\cos C \cos A}$$

$$= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$$

$$= 0 = \text{RHS}$$

Hence proved.

16 C. Question

Prove that:

$$\frac{\sin(A - B)}{\sin A \sin B} + \frac{\sin(B - C)}{\sin B \sin C} + \frac{\sin(C - A)}{\sin C \sin A} = 0$$

Answer

$$\text{LHS} = \frac{\sin(A - B)}{\sin A \sin B} + \frac{\sin(B - C)}{\sin B \sin C} + \frac{\sin(C - A)}{\sin C \sin A}$$

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\begin{aligned} \Rightarrow & \frac{\sin(A - B)}{\sin A \sin B} + \frac{\sin(B - C)}{\sin B \sin C} + \frac{\sin(C - A)}{\sin C \sin A} \\ &= \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} \\ &= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A} \\ &= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \end{aligned}$$

$$= 0 = \text{RHS}$$

Hence proved.

16 D. Question

Prove that:

$$\sin^2 B = \sin^2 A + \sin^2(A - B) - 2 \sin A \cos B \sin(A - B)$$

Answer

$$\begin{aligned} \text{RHS} &= \sin^2 A + \sin^2(A - B) - 2 \sin A \cos B \sin(A - B) \\ &= \sin^2 A + \sin(A - B) [\sin(A - B) - 2 \sin A \cos B] \end{aligned}$$

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\begin{aligned} &= \sin^2 A + \sin(A - B) [\sin A \cos B - \cos A \sin B - 2 \sin A \cos B] \\ &= \sin^2 A + \sin(A - B) [-\sin A \cos B - \cos A \sin B] \\ &= \sin^2 A - \sin(A - B) [\sin A \cos B + \cos A \sin B] \end{aligned}$$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\begin{aligned} &= \sin^2 A - \sin(A - B) \sin(A + B) \\ &= \sin^2 A - \sin^2 A + \sin^2 B \end{aligned}$$

$$= \sin^2 B = \text{LHS}$$

Hence proved.

16 E. Question

Prove that:

$$\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A + B) = \sin^2(A + B)$$

Answer

$$\text{LHS} = \cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A + B)$$

$$= \cos^2 A + 1 - \sin^2 B - 2 \cos A \cos B \cos(A + B)$$

$$= 1 + \cos^2 A - \sin^2 B - 2 \cos A \cos B \cos(A + B)$$

$$\text{We know that } \cos^2 A - \sin^2 B = \cos(A + B) \cos(A - B)$$

$$= 1 + \cos(A + B) \cos(A - B) - 2 \cos A \cos B \cos(A + B)$$

$$= 1 + \cos(A + B) [\cos(A - B) - 2 \cos A \cos B]$$

$$\text{We know that } \cos(A - B) = \cos A \cos B + \sin A \sin B.$$

$$= 1 + \cos(A + B) [\cos A \cos B + \sin A \sin B - 2 \cos A \cos B]$$

$$= 1 + \cos(A + B) [-\cos A \cos B + \sin A \sin B]$$

$$= 1 - \cos(A + B) [\cos A \cos B - \sin A \sin B]$$

$$\text{We know that } \cos(A + B) = \cos A \cos B - \sin A \sin B.$$

$$= 1 - \cos^2(A + B)$$

$$= \sin^2(A + B) = \text{RHS}$$

Hence proved.

16 F. Question

Prove that:

$$\frac{\tan(A + B)}{\cot(A - B)} = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

Answer

$$\text{LHS} = \frac{\tan(A+B)}{\cot(A-B)} = \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}}$$

$$\text{We know that } \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\Rightarrow \frac{\tan(A + B)}{1} = \frac{\frac{\tan A + \tan B}{1 - \tan A \tan B}}{\frac{1}{\frac{\tan A - \tan B}{1 + \tan A \tan B}}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B} \times \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\text{We know that } (x + y)(x - y) = x^2 - y^2$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} \times \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B} = \text{RHS}$$

Hence, proved.

17 A. Question

Prove that:

$$\tan 8x - \tan 6x - \tan 2x = \tan 8x$$

$$\tan 6x \tan 2x$$

Answer

$$\text{We have } 8x = 6x + 2x$$

$$\Rightarrow \tan 8x = \tan(6x + 2x)$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan 8x = \frac{\tan 6x + \tan 2x}{1 - \tan 6x \tan 2x}$$

$$\Rightarrow \tan 8x (1 - \tan 6x \tan 2x) = \tan 6x + \tan 2x$$

$$\Rightarrow \tan 8x - \tan 8x \tan 6x \tan 2x = \tan 6x + \tan 2x$$

$$\therefore \tan 8x - \tan 6x - \tan 2x = \tan 8x \tan 6x \tan 2x$$

Hence, proved.

17 B. Question

Prove that:

$$\tan \frac{\pi}{12} + \tan \frac{\pi}{6} + \tan \frac{\pi}{12} \tan \frac{\pi}{6} = 1$$

Answer

$$\Rightarrow \pi/12 = 15^\circ \text{ And } \pi/6 = 30^\circ$$

$$\text{We have } 15^\circ + 30^\circ = 45^\circ$$

$$\Rightarrow \tan(15^\circ + 30^\circ) = \tan 45^\circ$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \frac{\tan 15^\circ + \tan 30^\circ}{1 - \tan 15^\circ \tan 30^\circ} = 1$$

$$\Rightarrow \tan 15^\circ + \tan 30^\circ = 1 - \tan 15^\circ \tan 30^\circ$$

$$\therefore \tan 15^\circ + \tan 30^\circ + \tan 15^\circ \tan 30^\circ = 1$$

Hence, proved.

17 C. Question

Prove that:

$$\tan 36^\circ + \tan 9^\circ + \tan 36^\circ \tan 9^\circ = 1$$

Answer

$$\text{We have } 36^\circ + 9^\circ = 45^\circ$$

$$\Rightarrow \tan(36^\circ + 9^\circ) = \tan 45^\circ$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \frac{\tan 36^\circ + \tan 9^\circ}{1 - \tan 36^\circ \tan 9^\circ} = 1$$

$$\Rightarrow \tan 36^\circ + \tan 9^\circ = 1 - \tan 36^\circ \tan 9^\circ$$

$$\therefore \tan 36^\circ + \tan 9^\circ + \tan 36^\circ \tan 9^\circ = 1$$

Hence proved.

17 D. Question

Prove that:

$$\tan 13x - \tan 9x - \tan 4x = \tan 13x \tan 9x \tan 4x$$

Tan 9x tan 4x

Answer

We have $13x = 9x + 4x$

$$\Rightarrow \tan 13x = \tan(9x + 4x)$$

$$\text{We know that } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan 13x = \frac{\tan 9x + \tan 4x}{1 - \tan 9x \tan 4x}$$

$$\Rightarrow \tan 13x(1 - \tan 9x \tan 4x) = \tan 9x + \tan 4x$$

$$\Rightarrow \tan 13x - \tan 13x \tan 9x \tan 4x = \tan 9x + \tan 4x$$

$$\therefore \tan 13x - \tan 9x - \tan 4x = \tan 13x \tan 9x \tan 4x$$

Hence proved.

18. Question

$$\text{Prove that: } \frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x - \tan^2 x} = \tan 3x \tan x$$

Answer

$$\text{LHS} = \frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x - \tan^2 x}$$

$$= \frac{(\tan 2x + \tan x)(\tan 2x - \tan x)}{1 - \tan^2 2x - \tan^2 x}$$

$$\Rightarrow \tan 3x = \tan(2x + x) \text{ And } \tan x = \tan(2x - x)$$

$$\Rightarrow \frac{(\tan 2x + \tan x)(\tan 2x - \tan x)}{1 - \tan^2 2x - \tan^2 x} = \frac{\tan 3x (1 - \tan 2x \tan x) \times \tan x (1 + \tan 2x \tan x)}{1 - \tan^2 2x - \tan^2 x}$$

$$= \frac{\tan 3x \tan x (1 - \tan^2 2x - \tan^2 x)}{1 - \tan^2 2x - \tan^2 x}$$

$$= \tan 3x \tan x = \text{RHS}$$

Hence, proved.

19. Question

$$\text{If } \frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}, \text{ show that } \frac{\tan x}{\tan y} = \frac{a}{b}.$$

Answer

$$\text{Given } \frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

$$\text{LHS} = \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)}$$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ And $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\Rightarrow \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} = \frac{\sin A \cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B}{\cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B}$$
$$= \frac{2 \sin A \cos B}{2 \cos A \cos B}$$

$$= \tan A = \text{RHS}$$

20. Question

If $\tan A = x \tan B$, prove that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$.

Answer

Given $\tan A = x \tan B$

$$\text{LHS} = \frac{\sin(A-B)}{\sin(A+B)}$$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\Rightarrow \frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B}$$

Dividing numerator And denominator by $\cos A \cos B$,

$$\Rightarrow \frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{\tan A - \tan B}{\tan A + \tan B}$$

$$= \frac{x \tan B - \tan B}{x \tan B + \tan B}$$

$$= \frac{(x-1) \tan B}{(x+1) \tan B}$$

$$= \frac{x-1}{x+1}$$

$$= \text{RHS}$$

Hence, proved.

21. Question

If $\tan(A+B) = x$ And $\tan(A-B) = y$, find the values of $\tan 2A$ And $\tan 2B$.

Answer

Given $\tan(A+B) = x$ And $\tan(A-B) = y$

Consider $\tan 2A = \tan(A+A)$

$$= \tan(A+B+A-B)$$

We know that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \tan(A+B+A-B) = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) \tan(A-B)}$$

$$= \frac{x+y}{1-xy}$$

Consider $\tan 2B = \tan(B+B)$

$$= \tan(B+A+B-A)$$

$$= \frac{\tan(B + A) + \tan(B - A)}{1 - \tan(B + A)\tan(B - A)}$$

We know that $\tan(-\theta) = -\tan \theta$

$$= \frac{\tan(A + B) - \tan(A - B)}{1 + \tan(A + B)\tan(A - B)}$$

$$= \frac{x - y}{1 + xy}$$

22. Question

If $\cos A + \sin B = m$ And $\sin A + \cos B = n$, prove that $2 \sin(A + B) = m^2 + n^2 - 2$.

Answer

Given $\cos A + \sin B = m$ And $\sin A + \cos B = n$

$$\text{RHS} = m^2 + n^2 - 2$$

$$= (\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2$$

$$= \cos^2 A + \sin^2 B + 2 \cos A \sin B + \sin^2 A + \cos^2 B + 2 \sin A \cos B - 2$$

$$= 1 + 1 + 2(\cos A \sin B + \sin A \cos B) - 2$$

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$= 2 \sin(A + B)$$

$$= \text{LHS}$$

Hence, proved.

23. Question

If $\tan A + \tan B = a$ And $\cot A + \cot B = b$, prove that: $\cot(A + B) = 1/a - 1/b$.

Answer

Given $\tan A + \tan B = a$ And $\cot A + \cot B = b$

Consider $\cot A + \cot B = b$

$$\Rightarrow \frac{1}{\tan A} + \frac{1}{\tan B} = b$$

$$\Rightarrow \frac{\tan A + \tan B}{\tan A \tan B} = b$$

Then,

$$\text{RHS} = \frac{1}{a} - \frac{1}{b}$$

$$= \frac{1}{\tan A + \tan B} - \frac{1}{\frac{\tan A + \tan B}{\tan A \tan B}}$$

$$= \frac{1}{\tan A + \tan B} - \frac{\tan A \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

We know that $\cot(A + B) = \frac{1 - \tan A \tan B}{\tan A + \tan B}$

$$= \cot(A + B) = \text{LHS}$$

Hence, proved.

24. Question

If x lies in the first quadrant And $\cos x = 8/17$, then prove that

$$\cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{4} - x\right) + \cos\left(\frac{2\pi}{3} - x\right) = \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right) \frac{23}{17}$$

Answer

Given x lies in the first quadrant i.e. $0 < x < \pi/2$ And $\cos x = 8/17$

We know that $\sin x = \sqrt{1 - \cos^2 x}$

$$\therefore \sin x = \sqrt{1 - \frac{64}{289}} = \frac{15}{17}$$

$$\text{LHS} = \cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{4} - x\right) + \cos\left(\frac{2\pi}{3} - x\right)$$

$$= \cos(30^\circ + x) + \cos(45^\circ - x) + \cos(120^\circ - x)$$

We know that $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$= \cos 30^\circ \cos x - \sin 30^\circ \sin x + \cos 45^\circ \cos x + \sin 45^\circ \sin x + \cos 120^\circ \cos x + \sin 120^\circ \sin x$$

$$= \cos x(\cos 30^\circ + \cos 45^\circ + \cos 120^\circ) + \sin x(-\sin 30^\circ + \sin 45^\circ + \sin 120^\circ)$$

$$= \frac{8}{17} \left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2} \right) + \frac{15}{17} \left(-\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \right)$$

$$= \frac{8}{17} \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right) + \frac{15}{17} \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right)$$

$$= \frac{23}{17} \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right)$$

= RHS

Hence, proved.

25. Question

If $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$, then prove that $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 1$.

Answer

$$\text{Given } \tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \tan x + \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} + \frac{\tan x + \tan \frac{2\pi}{3}}{1 - \tan x \tan \frac{2\pi}{3}} = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x} = 3$$

$$\Rightarrow \frac{\tan x (1 - 3 \tan^2 x) + \tan x + \sqrt{3} + \sqrt{3} \tan^2 x + 3 \tan x + \tan x - \sqrt{3} - \sqrt{3} \tan^2 x + 3 \tan x}{1 - 3 \tan^2 x}$$

$$= 3$$

$$\Rightarrow \frac{9 \tan x - 3 \tan^3 x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 1$$

Hence, proved.

26. Question

If $\sin(\alpha + \beta) = 1$ And $\sin(\alpha - \beta) = 1/2$, where $0 \leq \alpha, \beta \leq \frac{\pi}{2}$, then find the values of $\tan(\alpha + 2\beta)$ And $\tan(2\alpha + \beta)$.

Answer

Given $\sin(\alpha + \beta) = 1$ And $\sin(\alpha - \beta) = 1/2$

$$\Rightarrow \alpha + \beta = 90^\circ \dots(1) \text{ And } \alpha - \beta = 30^\circ \dots(2)$$

Adding(1) And(2),

$$\Rightarrow 2\alpha = 120^\circ$$

$$\therefore \alpha = 60^\circ$$

Subtracting(2) from(1),

$$\Rightarrow 2\beta = 60^\circ$$

$$\therefore \beta = 30^\circ$$

Then,

$$\therefore \tan(\alpha + 2\beta) = \tan(60^\circ + 2 \times 30^\circ) = \tan 120^\circ = -\sqrt{3}$$

$$\text{And } \tan(2\alpha + \beta) = \tan(2 \times 60^\circ + 30^\circ) = \tan 150^\circ = -(1/\sqrt{3})$$

27. Question

If α, β are two different values of x lying between 0 And 2π which satisfy the equation $6 \cos x + 8 \sin x = 9$, find the value of $\sin(\alpha + \beta)$.

Answer

Given $6 \cos x + 8 \sin x = 9$

Case 1:

$$\Rightarrow 6 \cos x = 9 - 8 \sin x$$

Squaring on both sides,

$$\Rightarrow 36 \cos^2 x = (9 - 8 \sin x)^2$$

We know that $\cos^2 x = 1 - \sin^2 x$.

$$\Rightarrow 36(1 - \sin^2 x) = 81 + 64 \sin^2 x - 144 \sin x$$

$$\Rightarrow 100 \sin^2 x - 144 \sin x + 45 = 0$$

$\therefore \cos \alpha$ And $\cos \beta$ are the roots of the above equation

$$\Rightarrow \sin \alpha \sin \beta = 45/100$$

Case 2:

$$\Rightarrow 8 \sin x = 9 - 6 \cos x$$

Squaring on both sides,

$$\Rightarrow 64 \sin^2 x = (9 - 6 \cos x)^2$$

We know that $\sin^2 x = 1 - \cos^2 x$

$$\Rightarrow 64(1 - \cos^2 x) = 81 + 36 \cos^2 x - 108 \cos x$$

$$\Rightarrow 100 \cos^2 x - 108 \cos x + 17 = 0$$

$\therefore \sin \alpha$ And $\sin \beta$ are the roots of the Above equation

$$\Rightarrow \cos \alpha \cos \beta = 17/100$$

Consider $\cos(\alpha + \beta)$,

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$\Rightarrow \cos(\alpha + \beta) = \frac{17}{100} - \frac{45}{100} = -\frac{28}{100} = -\frac{7}{25}$$

We know that $\sin x = \sqrt{1 - \cos^2 x}$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$= \sqrt{1 - \left(-\frac{7}{25}\right)^2}$$

$$= \sqrt{\frac{576}{625}}$$

$$= \frac{24}{25}$$

28. Question

If $\sin \alpha + \sin \beta = A$ And $\cos \alpha + \cos \beta = B$, show that

$$(i) \sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

$$(ii) \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

Answer

Given $\sin \alpha + \sin \beta = A$ And $\cos \alpha + \cos \beta = B$.

$$\Rightarrow A^2 + B^2 = (\sin \alpha + \sin \beta)^2 + (\cos \alpha + \cos \beta)^2$$

$$= \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta$$

$$= \sin^2 \alpha + \cos^2 \alpha + \sin^2 \beta + \cos^2 \beta + 2(\sin \alpha \sin \beta + \cos \alpha \cos \beta)$$

We know that $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\therefore A^2 + B^2 = 2 + 2 \cos(\alpha - \beta) \dots(1)$$

Then,

$$\Rightarrow B^2 - A^2 = (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2$$

$$= \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta - (\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta)$$

$$\begin{aligned}
&= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) - 2\cos(\alpha + \beta) \\
&= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\
&= \cos(\alpha + \beta)(2 + 2 \cos(\alpha - \beta)) \dots(2)
\end{aligned}$$

From(1) And(2),

$$\Rightarrow B^2 - A^2 = \cos(\alpha + \beta)(A^2 + B^2)$$

$$\therefore \frac{b^2 - a^2}{a^2 + b^2} = \cos(\alpha + \beta) \dots(ii)$$

$$\text{And } \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 + a^2}\right)^2}$$

$$= \sqrt{\frac{b^4 + a^4 - b^4 - a^4 + 4a^2b^2}{(b^2 + a^2)^2}}$$

$$\therefore \sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2} \dots(i)$$

29 A. Question

Prove that:

$$\frac{1}{\sin(x-a)\sin(x-b)} = \frac{\cot(x-a) - \cot(x-b)}{\sin(a-b)}$$

Answer

$$\text{RHS} = \frac{\cot(x-a) - \cot(x-b)}{\sin(a-b)}$$

$$= \frac{\frac{\cos(x-a)}{\sin(x-a)} - \frac{\cos(x-b)}{\sin(x-b)}}{\sin(a-b)}$$

$$= \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)\sin(a-b)}$$

We know that $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{\sin(x-b-x+a)}{\sin(x-a)\sin(x-b)\sin(a-b)}$$

$$= \frac{\sin(a-b)}{\sin(x-a)\sin(x-b)\sin(a-b)}$$

$$= \frac{1}{\sin(x-a)\sin(x-b)}$$

= LHS

Hence, proved.

29 B. Question

Prove that:

$$\frac{1}{\sin(x-a)\cos(x-b)} = \frac{\cot(x-a) + \tan(x-b)}{\cos(a-b)}$$

Answer

$$\text{RHS} = \frac{\cot(x-a) + \tan(x-b)}{\cos(a-b)}$$

$$= \frac{\frac{\cos(x-a)}{\sin(x-a)} + \frac{\sin(x-b)}{\cos(x-b)}}{\cos(a-b)}$$

$$= \frac{\cos(x-b) \cos(x-a) + \sin(x-b) \sin(x-a)}{\sin(x-a) \cos(x-b) \cos(a-b)}$$

We know that $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{\cos(x-b-x+a)}{\sin(x-a) \cos(x-b) \cos(a-b)}$$

$$= \frac{\cos(a-b)}{\sin(x-a) \cos(x-b) \cos(a-b)}$$

$$= \frac{1}{\sin(x-a) \cos(x-b)}$$

= LHS

Hence, proved.

29 C. Question

Prove that:

$$\frac{1}{\cos(x-a) \cos(x-b)} = \frac{\tan(x-b) - \tan(x-a)}{\sin(a-b)}$$

Answer

$$\text{RHS} = \frac{\tan(x-b) - \tan(x-a)}{\sin(a-b)}$$

$$= \frac{\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)}}{\sin(a-b)}$$

$$= \frac{\sin(x-b) \cos(x-a) - \cos(x-b) \sin(x-a)}{\cos(x-a) \cos(x-b) \sin(a-b)}$$

We know that $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{\sin(x-b-x+a)}{\cos(x-a) \cos(x-b) \sin(a-b)}$$

$$= \frac{\sin(a-b)}{\cos(x-a) \cos(x-b) \sin(a-b)}$$

$$= \frac{1}{\cos(x-a) \cos(x-b)}$$

= LHS

Hence, proved.

30. Question

If $\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$, prove that $1 + \cot \alpha \tan \beta = 0$.

Answer

$$\text{Given } \sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$$

$$\Rightarrow -(\cos \alpha \cos \beta - \sin \alpha \sin \beta) + 1 = 0$$

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$\Rightarrow -\cos(\alpha + \beta) + 1 = 0$$

$$\Rightarrow \cos(\alpha + \beta) = 1$$

We know that $\sin \theta = \sqrt{1 - \cos^2 \theta}$

$$\therefore \sin(\alpha + \beta) = 0 \dots(1)$$

Consider $1 + \cot \alpha \tan \beta$,

$$\Rightarrow 1 + \cot \alpha \tan \beta = 1 + \frac{\cos \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta}$$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\Rightarrow \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta} = \frac{\sin(\alpha + \beta)}{\sin \alpha \cos \beta}$$

$$= 0 = \text{RHS}$$

Hence, proved.

31. Question

If $\tan \alpha = x + 1$, $\tan \beta = x - 1$, show that $2 \cot(\alpha - \beta) = x^2$.

Answer

Given $\tan \alpha = x + 1$ And $\tan \beta = x - 1$

$$\text{LHS} = 2 \cot(\alpha - \beta)$$

We know that $\cot(A - B) = \frac{1 + \tan A \tan B}{\tan A - \tan B}$

$$\Rightarrow 2 \cot(\alpha - \beta) = \frac{2(1 + \tan A \tan B)}{\tan A - \tan B}$$

$$= \frac{2 + 2(x + 1)(x - 1)}{x + 1 - x + 1}$$

$$= \frac{2 + 2x^2 - 2}{2}$$

$$= x^2 = \text{RHS}$$

Hence, proved.

32. Question

If Angle θ is divided into two parts such that the tangents of the one part is λ times the tangent of other, And

ϕ is their difference, then show that $\sin \theta = \frac{\lambda + 1}{\lambda - 1} \sin \phi$.

Answer

Let α And β be the two parts of angle θ .

Then, given $\theta = \alpha + \beta$ And $\phi = \alpha - \beta$

Consider $\tan \alpha = \lambda \tan \beta$

$$\Rightarrow \frac{\tan \alpha}{\tan \beta} = \frac{\lambda}{1}$$

Applying componendo And dividendo,

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{\frac{\sin \alpha}{\cos \alpha} - \frac{\sin \beta}{\cos \beta}} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\lambda + 1}{\lambda - 1}$$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\Rightarrow \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\sin \theta}{\sin \phi} = \frac{\lambda + 1}{\lambda - 1}$$

$$\therefore \sin \theta = \frac{\lambda + 1}{\lambda - 1} \sin \phi$$

Hence, proved.

33. Question

If $\tan x = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then show that $\sin \alpha + \cos \alpha = \sqrt{2} \cos x$.

Answer

$$\text{Given } \tan x = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$$

Dividing numerator And denominator on RHS by $\cos \alpha$,

$$\Rightarrow \tan x = \frac{\tan \alpha - 1}{\tan \alpha + 1}$$

$$= \frac{\tan \alpha - \tan \frac{\pi}{4}}{1 + \tan \alpha \tan \frac{\pi}{4}}$$

$$\text{We know that } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \tan x = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow x = \alpha - \frac{\pi}{4}$$

$$\therefore \alpha = \frac{\pi}{4} + x$$

Consider $\sin \alpha + \cos \alpha$,

$$\Rightarrow \sin \alpha + \cos \alpha = \sin\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right)$$

We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ And $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$\begin{aligned} &\Rightarrow \sin\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) \\ &= \sin\frac{\pi}{4} \cos x + \cos\frac{\pi}{4} \sin x + \cos\frac{\pi}{4} \cos x - \sin\frac{\pi}{4} \sin x \\ &= \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x \\ &= \frac{2}{\sqrt{2}} \cos x \\ &= \sqrt{2} \cos x \end{aligned}$$

$$\therefore \sin \alpha + \cos \alpha = \sqrt{2} \cos x$$

Hence proved.

34. Question

If α and β are two solutions of the equation $A \tan x + B \sec x = c$, then find the values of $\sin(\alpha + \beta)$.

Answer

Given equation $A \tan x + B \sec x = c$

$$\Rightarrow c - A \tan x = B \sec x$$

Squaring on both sides,

$$\Rightarrow (c - A \tan x)^2 = (B \sec x)^2$$

$$\Rightarrow c^2 + A^2 \tan^2 x - 2ac \tan x = B^2 \sec^2 x$$

$$\Rightarrow c^2 + A^2 \tan^2 x - 2ac \tan x = B^2(1 + \tan^2 x)$$

$$\Rightarrow (a^2 - B^2) \tan^2 x - 2ac \tan x + (c^2 - B^2) = 0$$

There are two solutions $\tan \alpha$ and $\tan \beta$ in this quadratic.

$$\Rightarrow \tan \alpha + \tan \beta = \frac{2ac}{a^2 - b^2} \text{ and } \tan \alpha \times \tan \beta = \frac{c^2 - b^2}{a^2 - b^2}$$

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\frac{2ac}{a^2 - b^2}}{1 - \frac{c^2 - b^2}{a^2 - b^2}} = \frac{2ac}{a^2 - c^2}$$

$$\therefore \sin(\alpha + \beta) = \frac{2ac}{a^2 + c^2}$$

Exercise 7.2

1. Question

Find the maximum and minimum values of each of the following trigonometrical expressions:

(i) $12 \sin x - 5 \cos x$

(ii) $12 \cos x + 5 \sin x + 4$

(iii) $5 \cos x + 3 \sin\left(\frac{\pi}{6} - x\right) + 4$

(iv) $\sin x - \cos x + 1$

Answer

We know that the maximum value of $A\cos\alpha + B\sin\alpha + c$ is

$$c + \sqrt{A^2 + B^2}$$

And the minimum value is $c - \sqrt{A^2 + B^2}$.

(i) Given $f(x) = 12 \sin x - 5 \cos x$

Here $A = -5, B = 12$ and $c = 0$

$$\Rightarrow -\sqrt{(-5)^2 + 12^2} \leq 12 \sin x - 5 \cos x \leq \sqrt{(-5)^2 + 12^2}$$

$$\Rightarrow -\sqrt{25 + 144} \leq 12 \sin x - 5 \cos x \leq \sqrt{25 + 144}$$

$$\Rightarrow -\sqrt{169} \leq 12 \sin x - 5 \cos x \leq \sqrt{169}$$

$$\Rightarrow -13 \leq 12 \sin x - 5 \cos x \leq 13$$

Hence, the maximum and minimum values of $f(x)$ are 13 and -13 respectively.

(ii) Given $f(x) = 12 \cos x + 5 \sin x + 4$

Here $A = 12, B = 5$ and $c = 4$

$$\Rightarrow 4 - \sqrt{(12)^2 + 5^2} \leq 12 \cos x + 5 \sin x + 4 \leq 4 + \sqrt{(12)^2 + 5^2}$$

$$\Rightarrow 4 - \sqrt{144 + 25} \leq 12 \cos x + 5 \sin x + 4 \leq 4 + \sqrt{144 + 25}$$

$$\Rightarrow 4 - \sqrt{169} \leq 12 \cos x + 5 \sin x + 4 \leq 4 + \sqrt{169}$$

$$\Rightarrow -9 \leq 12 \cos x + 5 \sin x + 4 \leq 17$$

Hence, the maximum and minimum values of $f(x)$ are 17 and -9 respectively.

(iii) Given $f(x) = 5\cos x + 3\sin\left(\frac{\pi}{6} - x\right) + 4$

We know that $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\Rightarrow f(x) = 5\cos x + 3\left(\sin\frac{\pi}{6}\cos x - \cos\frac{\pi}{6}\sin x\right) + 4$$

$$= 5\cos x + \frac{3}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$$

$$= \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$$

Here $a = \frac{13}{2}; b = -\frac{3\sqrt{3}}{2}; c = 4$

$$\Rightarrow 4 - \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} \leq \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$$

$$\leq 4 + \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow 4 - \sqrt{\frac{169}{4} + \frac{27}{4}} \leq \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4 \leq 4 + \sqrt{\frac{169}{4} + \frac{27}{4}}$$

$$\Rightarrow 4 - 7 \leq \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4 \leq 4 + 7$$

$$\Rightarrow -3 \leq \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4 \leq 11$$

Hence, the maximum And minimum values of $f(x)$ are 11 And -3 respectively.

(iv) Given $f(x) = \sin x - \cos x + 1$

Here $A = -1, B = 1$ And $c = 1$

$$\Rightarrow 1 - \sqrt{(-1)^2 + 1^2} \leq \sin x - \cos x + 1 \leq 1 + \sqrt{(-1)^2 + 1^2}$$

$$\Rightarrow 1 - \sqrt{1+1} \leq \sin x - \cos x + 1 \leq 1 + \sqrt{1+1}$$

$$\Rightarrow 1 - \sqrt{2} \leq \sin x - \cos x + 1 \leq 1 + \sqrt{2}$$

Hence, the maximum And minimum values of $f(x)$ are $1 + \sqrt{2}$ And $1 - \sqrt{2}$ respectively.

2 A. Question

Reduce each of the following expressions to the Sine And Cosine of A single expression:

$$\sqrt{3} \sin x - \cos x$$

Answer

Let $f(x) = \sqrt{3} \sin x - \cos x$

Dividing and multiplying by $\sqrt{3 + 1} = 2$,

$$\Rightarrow f(x) = 2\left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x\right)$$

Sine of expression:

$$\Rightarrow f(x) = 2\left(\cos \frac{\pi}{6} \sin x - \sin \frac{\pi}{6} \cos x\right)$$

We know that $\sin A \cos B - \cos A \sin B = \sin(A - B)$

$$\therefore f(x) = 2\sin\left(x - \frac{\pi}{6}\right)$$

Cosine of the expression:

$$\Rightarrow f(x) = 2\left(\sin \frac{\pi}{3} \sin x - \cos \frac{\pi}{3} \cos x\right)$$

We know that $\cos A \cos B - \sin A \sin B = \cos(A + B)$

$$\therefore f(x) = -2\cos\left(\frac{\pi}{3} + x\right)$$

2 B. Question

Reduce each of the following expressions to the Sine And Cosine of A single expression:

$$\cos x - \sin x$$

Answer

Let $f(x) = \cos x - \sin x$

Dividing and multiplying by $\sqrt{1 + 1} = \sqrt{2}$,

$$\Rightarrow f(x) = \sqrt{2}\left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x\right)$$

Sine of expression:

$$\Rightarrow f(x) = \sqrt{2}\left(\sin \frac{\pi}{4} \cos x - \cos \frac{\pi}{4} \sin x\right)$$

We know that $\sin A \cos B - \cos A \sin B = \sin(A - B)$

$$\therefore f(x) = \sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$$

Cosine of the expression:

$$\Rightarrow f(x) = 2\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)$$

We know that $\cos A \cos B - \sin A \sin B = \cos(A + B)$

$$\therefore f(x) = \sqrt{2}\cos\left(\frac{\pi}{4} + x\right)$$

2 C. Question

Reduce each of the following expressions to the Sine And Cosine of A single expression:

$$24 \cos x + 7 \sin x$$

Answer

$$\text{Let } f(x) = 24 \cos x + 7 \sin x$$

Dividing and multiplying by $\sqrt{(24^2 + 7^2)} = \sqrt{625} = 25$,

$$\Rightarrow f(x) = 25\left(\frac{24}{25}\cos x + \frac{7}{25}\sin x\right)$$

Sine of expression:

$$\Rightarrow f(x) = 25(\sin \alpha \cos x + \cos \alpha \sin x) \text{ where } \sin \alpha = 24/25 \text{ And } \cos \alpha = 7/25$$

We know that $\sin A \cos B + \cos A \sin B = \sin(A + B)$

$$\therefore f(x) = 25 \sin(\alpha + x)$$

Cosine of the expression:

$$\Rightarrow f(x) = 25(\cos \alpha \cos x + \sin \alpha \sin x)$$

We know that $\cos A \cos B + \sin A \sin B = \cos(A - B)$

$$\therefore f(x) = 25 \cos(\alpha - x)$$

3. Question

Show that $\sin 100^\circ - \sin 10^\circ$ is positive.

Answer

$$\text{Let } f(x) = \sin 100^\circ - \sin 10^\circ$$

Dividing And multiplying By $\sqrt{(1 + 1)} = \sqrt{2}$,

$$\Rightarrow f(x) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 100^\circ - \frac{1}{\sqrt{2}}\sin 10^\circ\right)$$

$$\Rightarrow f(x) = \sqrt{2}\left(\cos\frac{\pi}{4}\sin(90 + 10)^\circ - \sin\frac{\pi}{4}\sin 10^\circ\right)$$

$$\Rightarrow f(x) = \sqrt{2}\left(\cos\frac{\pi}{4}\cos 10^\circ - \sin\frac{\pi}{4}\sin 10^\circ\right)$$

We know that $\cos A \cos B - \sin A \sin B = \cos(A + B)$

$$\Rightarrow f(x) = \sqrt{2}\cos\left(\frac{\pi}{4} + 10^\circ\right)$$

$$\therefore f(x) = \sqrt{2} \cos 55^\circ$$

4. Question

Prove that $(2\sqrt{3} + 3)\sin x + 2\sqrt{3}\cos x$ lies between $-(2\sqrt{3} + \sqrt{15})$ and $(2\sqrt{3} + \sqrt{15})$.

Answer

Let $f(x) = (2\sqrt{3} + 3)\sin x + 2\sqrt{3}\cos x$

Here $A = 2\sqrt{3}$, $B = 2\sqrt{3} + 3$ And $c = 0$

$$\Rightarrow -\sqrt{(2\sqrt{3})^2 + (2\sqrt{3} + 3)^2} \leq (2\sqrt{3} + 3)\sin x + 2\sqrt{3}\cos x \leq \sqrt{(2\sqrt{3})^2 + (2\sqrt{3} + 3)^2}$$

$$\Rightarrow -\sqrt{12 + 12 + 9 + 12\sqrt{3}} \leq (2\sqrt{3} + 3)\sin x + 2\sqrt{3}\cos x \leq \sqrt{12 + 12 + 9 + 12\sqrt{3}}$$

$$\Rightarrow -\sqrt{33 + 12\sqrt{3}} \leq (2\sqrt{3} + 3)\sin x + 2\sqrt{3}\cos x \leq \sqrt{33 + 12\sqrt{3}}$$

Hence proved.

Very Short Answer

1. Question

If $\alpha + \beta - \gamma = \pi$, and $\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma = \lambda \sin \alpha \sin \beta \cos \gamma$, then write the value of λ .

Answer

$$\alpha + \beta = \pi + \gamma$$

$$\sin(\alpha + \beta) = \sin(\pi + \gamma)$$

$$\sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha) = -\sin(\gamma)$$

Take square both side

$$[\sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)]^2 = \sin^2(\gamma)$$

$$\sin^2(\alpha)\cos^2(\beta) + \sin^2(\beta)\cos^2(\alpha) + 2\sin(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha) = \sin^2(\gamma)$$

$$\sin^2(\alpha)[1 - \sin^2(\beta)] + \sin^2(\beta)[1 - \sin^2(\alpha)] + 2\sin(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha) = \sin^2(\gamma)$$

$$\sin^2(\alpha) - \sin^2(\alpha)\sin^2(\beta) + \sin^2(\beta) - \sin^2(\beta)\sin^2(\alpha) - \sin^2(\gamma) = -2\sin(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha)$$

$$\sin^2(\alpha) + \sin^2(\beta) - \sin^2(\gamma) = 2\sin^2(\alpha)\sin^2(\beta) - 2\sin(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha)$$

$$\sin^2(\alpha) + \sin^2(\beta) - \sin^2(\gamma) = -2\sin(\alpha)\sin(\beta)[\cos(\beta)\cos(\alpha) - \sin(\alpha)\sin(\beta)]$$

$$\sin^2(\alpha) + \sin^2(\beta) - \sin^2(\gamma) = -2\sin(\alpha)\sin(\beta)\cos(\alpha + \beta)$$

$$\sin^2(\alpha) + \sin^2(\beta) - \sin^2(\gamma) = 2\sin(\alpha)\sin(\beta)\sin(\gamma)$$

2. Question

If $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$, then write the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

Answer

$$x \cos \theta = k = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$$

$$z \cos\left(\theta + \frac{4\pi}{3}\right) = -z \cos\left(\theta + \frac{\pi}{3}\right)$$

$$\frac{k}{x} = \cos \theta$$

$$\frac{k}{y} = -\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$$

$$\frac{k}{z} = -\left[\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta\right]$$

$$\frac{k}{x} + \frac{k}{y} + \frac{k}{z} = 0$$

3. Question

Write the maximum and minimum values of $3 \cos x + 4 \sin x + 5$.

Answer

the maximum value of $(a \cos x + b \sin x) = \sqrt{a^2 + b^2}$

So maximum value = $\sqrt{9 + 16}$

=5

the minimum value of $(a \cos x + b \sin x) = -\sqrt{a^2 + b^2}$

minimum value = -5

4. Question

Write the maximum values of $12 \sin x - 9 \sin^2 x$.

Answer

$$f(x) = 12 \sin x - 9(\sin x)^2$$

$$f(x) = -(3 \sin x - 2)^2 + 4$$

$$-1 \leq \sin x \leq 1$$

$$f(x) \in -(3[-1,1]-2)^2+4$$

$$f(x) \in -([-3,3]-2)^2+4$$

$$f(x) \in -([-5,1])^2+4$$

$$f(x) \in -[0,25]+4$$

$$f(x) \in [-25,0]+4$$

$$f(x) \in [-21,4]$$

5. Question

If $12 \sin x - 9 \sin^2 x$ attains its maximum value at $x = \alpha$, then write the value of $\sin \alpha$.

Answer

$$f(x) = 12 \sin(x) - 9 \sin^2(x)$$

$$f'(x) = 12 \cos(x) - 18 \sin(x) \cos(x)$$

$$f'(x) = 0 \text{ for the maximum value of } f(x)$$

$$12 \cos(x) - 18 \sin(x) \cos(x) = 0$$

$$\Rightarrow \sin x = \frac{2}{3}$$

6. Question

Write the interval in which the values of $5 \cos x + 3 \cos\left(x + \frac{\pi}{3}\right) + 3$ lie.

Answer

$$f(x) = 5 \cos x + 3 \cos\left(x + \frac{\pi}{3}\right) + 3$$

$$f(x) = 5 \cos x + \frac{3}{2}(\cos x - \sqrt{3} \sin x) + 3$$

$$f(x) = \frac{13}{2} \cos x - \frac{3}{2} \sqrt{3} \sin x + 3$$

$$f(x) \in \left[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}\right] + 3$$

$$f(x) \in [-7, 7] + 3$$

$$f(x) \in [-4, 10]$$

7. Question

If $\tan(A + B) = p$ and $\tan(A - B) = q$, then write the value of $\tan 2B$.

Answer

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan[(A + B) - (A - B)] = \frac{\tan(A + B) - \tan(A - B)}{1 + \tan(A - B) \tan(A + B)}$$

$$\tan(2B) = \frac{p - q}{1 + pq}$$

8. Question

If $\frac{\cos(x - y)}{\cos(x + y)} = \frac{m}{n}$, then write the value of $\tan x \tan y$.

Answer

use componendo and dividendo rule

$$\frac{\cos x - \cos y}{\cos x + \cos y} = \frac{m - n}{m + n}$$

$$\frac{\cos(x - y) - \cos(x + y)}{\cos(x - y) + \cos(x + y)} = \frac{m - n}{m + n}$$

$$\frac{\sin x \sin y}{\cos x \cos y} = \frac{m - n}{m + n}$$

$$\tan x \tan y = \frac{m - n}{m + n}$$

9. Question

If $a = b \cos \frac{2\pi}{3} = c \cos \frac{4\pi}{3}$, then write the value of $ab + bc + ca$.

Answer

$$a = b \cos \frac{2\pi}{3} = c \cos \frac{4\pi}{3} = k$$

$$a=k$$

$$b=2k$$

$$c=-2k$$

$$ab + bc + ca = 0$$

10. Question

If $A + B = C$, then write the value of $\tan A \tan B \tan C$.

Answer

$$A+B=C$$

$$\tan A + B = \tan C$$

$$\tan A + \tan B = \tan C(1 - \tan A \tan B)$$

$$\tan C - \tan A - \tan B = \tan A \tan B \tan C.$$

11. Question

If $\sin \alpha - \sin \beta = a$ and $\cos \alpha + \cos \beta = b$, then write the value of $\cos(\alpha + \beta)$.

Answer

$$\sin \alpha - \sin \beta = a$$

$$\cos \alpha + \cos \beta = b$$

$$(\sin \alpha - \sin \beta)^2 = a^2 \dots\dots 1$$

$$(\cos \alpha + \cos \beta)^2 = b^2 \dots\dots 2$$

add both equations

$$2 - 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = a^2 + b^2$$

$$\text{value of } \cos(\alpha + \beta) = \frac{a^2 + b^2 - 2}{2}$$

12. Question

If $\tan \alpha = \frac{1}{1+2^{-x}}$ and $\tan \beta = \frac{1}{1+2^{x+1}}$, then write the value of $\alpha + \beta$ lying in the interval $(0, \pi/2)$.

Answer

Assume $x=0$

$$\tan \alpha = \frac{1}{2}$$

$$\tan \beta = \frac{1}{3}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha + \beta) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

$$\tan(\alpha + \beta) = \frac{5}{5}$$

$$=1$$

$$(\alpha + \beta) = \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

MCQ

1. Question

Mark the correct alternative in the following:

The value of $\sin^2 \frac{5\pi}{12} - \sin^2 \frac{\pi}{12}$ is

- A. 1/2
- B. $\sqrt{3}/2$
- C. 1
- D. 0

Answer

$$f(x) = \left(\sin \frac{5\pi}{12}\right)^2 - \left(\sin \frac{\pi}{12}\right)^2$$

$$f(x) = \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) \sin\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

$$f(x) = \sin\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{3}\right)$$

$$f(x) = \frac{\sqrt{3}}{2}$$

2. Question

Mark the correct alternative in the following:

If $A + B + C = \pi$, then $\sec A (\cos B \cos C - \sin B \sin C)$ is equal to

- A. 0
- B. -1
- C. 1
- D. None of these

Answer

$$B + C = \pi - A$$

TAKE BOTH SIDE COS

$$\cos(B + C) = \cos(\pi - A)$$

$$(\cos B \cos C - \sin B \sin C) = -\cos(A)$$

$\sec A (\cos B \cos C - \sin B \sin C)$ is equal to -1

3. Question

Mark the correct alternative in the following:

$\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ$ is equal to

- A. $\frac{\sqrt{3}}{4}$

B. $\frac{\sqrt{3}}{2}$

C. $\sqrt{3}$

D. 1

Answer

$$\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ$$

$$\tan 60^\circ (1 - \tan 20^\circ \tan 40^\circ) + \sqrt{3} \tan 20^\circ \tan 40^\circ$$

$$= \sqrt{3}$$

4. Question

Mark the correct alternative in the following:

If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$, then the value of $A + B$ is

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

put $a=0$

$$\tan(A)=0$$

$$\tan(B)=1$$

$$\tan A + B = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan A + B = \frac{0 + 1}{1 - 0}$$

$$\tan A + B = 1$$

$$A+B=\tan^{-1} 1$$

$$A + B = \frac{\pi}{4}$$

5. Question

Mark the correct alternative in the following:

If $3 \sin x + 4 \cos x = 5$, then $4 \sin x - 3 \cos x =$

A. 0

B. 5

C. 1

D. None of these

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Answer

$$3\sin(x)+4\cos(x)=5$$

$$\frac{3}{5}\sin x + \frac{4}{5}\cos x = 1$$

$$\cos(37^\circ-x)=\cos 0^\circ (\because x=37^\circ)$$

$$4\sin(x)-3\cos(x)=k$$

$$4 \times \frac{3}{5} - 3 \times \frac{4}{5} = 0$$

6. Question

Mark the correct alternative in the following:

If in a ΔABC , $\tan A + \tan B + \tan C = 6$, then $\cot A \cot B \cot C =$

- A. 6
- B. 1
- C. $1/6$
- D. None of these

Answer

$$A+B=\pi-C$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\cot A \cot B \cot C = \frac{1}{6}$$

7. Question

Mark the correct alternative in the following:

$\tan 3A - \tan 2A - \tan A$ is equal to

- A. $\tan 3A \tan 2A \tan A$
- B. $-\tan 3A \tan 2A \tan A$
- C. $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$
- D. None of these

Answer

$$\tan(A)+\tan(B)+\tan(C)=\tan(A) \tan(B)\tan(C)$$

$$\tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$$

8. Question

Mark the correct alternative in the following:

If $A+B+C = \pi$, then $\frac{\tan A + \tan B + \tan C}{\tan A \tan B \tan C}$ is equal to

- A. $\tan A \tan B \tan C$
- B. 0
- C. 1

D. None of these

Answer

$$A+B=\pi-C$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\frac{\tan A + \tan B + \tan C}{\tan A \tan B \tan C} = 1$$

9. Question

Mark the correct alternative in the following:

If $\cos P = \frac{1}{7}$ and $\cos Q = \frac{13}{14}$, where P and Q both are acute angles. Then, the value of P - Q is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{5\pi}{12}$

Answer

$$\cos P = \frac{1}{7}, \sin P = \frac{\sqrt{48}}{7}$$

$$\cos Q = \frac{13}{14}, \sin Q = \frac{\sqrt{27}}{14}$$

$$\cos(p-q) = \cos(p)\cos(q) + \sin(p)\sin(q)$$

$$\cos P - Q = \frac{1}{2}$$

$$P - Q = \frac{\pi}{3}$$

10. Question

Mark the correct alternative in the following:

If $\cot(\alpha + \beta) = 0$, then $\sin(\alpha + 2\beta)$ is equal to

A. $\sin \alpha$

B. $\cos 2\beta$

C. $\cos \alpha$

D. $\sin 2\alpha$

Answer

$$\cot(\alpha + \beta) = 0$$

$$(\alpha + \beta) = \frac{\pi}{2}$$

$$\sin(\alpha + 2\beta) = \sin(\alpha + \beta)\cos(\beta) + \sin(\beta)\cos(\alpha + \beta)$$

$$\text{put } (\alpha + \beta) = \frac{\pi}{2}$$

$$\sin(\alpha + 2\beta) = \cos(\beta)$$

11. Question

Mark the correct alternative in the following:

$$\frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ} \text{ is equal to}$$

- A. $\tan 55^\circ$
- B. $\cot 55^\circ$
- C. $-\tan 35^\circ$
- D. $-\cot 35^\circ$

Answer

$$\frac{\cos(10^\circ) + \sin(10^\circ)}{\cos(10^\circ) - \sin(10^\circ)}$$

$$= \frac{\frac{1}{\sqrt{2}}\cos(10^\circ) + \frac{1}{\sqrt{2}}\sin(10^\circ)}{\frac{1}{\sqrt{2}}\cos(10^\circ) - \frac{1}{\sqrt{2}}\sin(10^\circ)}$$

$$= \frac{\sin(45^\circ)\cos(10^\circ) + \cos(45^\circ)\sin(10^\circ)}{\cos(45^\circ)\cos(10^\circ) - \sin(45^\circ)\sin(10^\circ)}$$

$$= \frac{\sin(55^\circ)}{\cos(55^\circ)}$$

$$= \tan 55^\circ$$

12. Question

Mark the correct alternative in the following:

$$\text{The value of } \cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right) \text{ is}$$

- A. $\frac{1}{2}\cos 2x$
- B. 0
- C. $-\frac{1}{2}\cos 2x$
- D. $\frac{1}{2}$

Answer

$$\cos^2 A - \sin^2 B$$

$$= \cos(A+B)\cos(A-B)$$

$$= \cos\left(\frac{\pi}{3}\right)\cos(2x)$$

$$= \frac{1}{2} \cos(2x)$$

13. Question

Mark the correct alternative in the following:

If $\tan \theta_1 \tan \theta_2 = k$, then $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)} =$

A. $\frac{1+k}{1-k}$

B. $\frac{1-k}{1+k}$

C. $\frac{k+1}{k-1}$

D. $\frac{k-1}{k+1}$

Answer

$$\frac{\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2}$$

$$= \frac{1 + \frac{\sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2}}{1 - \frac{\sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2}}$$

$$= \frac{1+k}{1-k}$$

14. Question

Mark the correct alternative in the following:

If $\sin(\pi \cos x) = \cos(\pi \sin x)$, then $\sin 2x =$

A. $\pm \frac{3}{4}$

B. $\pm \frac{4}{3}$

C. $\pm \frac{1}{3}$

D. None of these

Answer

$$\sin(\pi \cos x) = \cos(\pi \sin x)$$

$$\sin\left(\frac{\pi}{2} - \pi \cos x\right) = \cos(\pi \sin x)$$

$$2n\pi \pm \left(\frac{\pi}{2} - \pi \cos x\right) = (\pi \sin x)$$

$$\sin x = 2n \pm \left(\frac{1}{2} - \cos x\right)$$

Put $n=0$

$$\sin x = \pm\left(\frac{1}{2} - \cos x\right)$$

$$\sin x = \left(\frac{1}{2} - \cos x\right), \sin x = \left(\frac{1}{2} - \cos x\right)$$

$$\sin x + \cos x = \frac{1}{2}, \sin x - \cos x = -\frac{1}{2}$$

Take square both side

$$1 + \sin 2x = \frac{1}{4}, 1 - \sin 2x = \frac{1}{4}$$

$$\sin 2x = \frac{-3}{4}, \sin 2x = \frac{3}{4}$$

15. Question

Mark the correct alternative in the following:

If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

A. $\frac{\pi}{6}$

B. π

C. 0

D. $\frac{\pi}{4}$

Answer

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

$$\tan(\theta + \phi) = 1$$

$$(\theta + \phi) = \frac{\pi}{4}$$

16. Question

Mark the correct alternative in the following:

The value of $\cos(36^\circ - A) \cos(36^\circ + A) + \cos(54^\circ + A) \cos(54^\circ - A)$ is

A. $\sin 2A$

B. $\cos 2A$

C. $\cos 3A$

D. $\sin 3A$

Answer

$$\cos(54^\circ + A) = \sin(36^\circ - A)$$

$$\cos(54^\circ - A) = \sin(36^\circ + A)$$

$$\cos(36^\circ - A) \cos(36^\circ + A) + \sin(36^\circ - A) \sin(36^\circ + A) = \cos(2A)$$

17. Question

Mark the correct alternative in the following:

If $\tan(\pi/4 + x) + \tan(\pi/4 - x) = a$, then $\tan^2(\pi/4 + x) + \tan^2(\pi/4 - x) =$

- A. $a^2 + 1$
- B. $a^2 + 2$
- C. $a^2 - 2$
- D. None of these

Answer

$$\begin{aligned} [\tan(\frac{\pi}{4} + x)]^2 + [\tan(\frac{\pi}{4} - x)]^2 &= [\tan(\frac{\pi}{4} + x) + \tan(\frac{\pi}{4} - x)]^2 - 2 \tan(\frac{\pi}{4} + x) \tan(\frac{\pi}{4} - x) \\ &= a^2 - 2 \left(\frac{1 + \tan(x)}{1 - \tan(x)} \right) \left(\frac{1 - \tan(x)}{1 + \tan(x)} \right) \\ &= a^2 - 2 \end{aligned}$$

18. Question

Mark the correct alternative in the following:

If $\tan(A - B) = 1$, $\sec(A + B) = \frac{2}{\sqrt{3}}$, then the smallest positive value of B is

- A. $\frac{25\pi}{24}$
- B. $\frac{19\pi}{24}$
- C. $\frac{13\pi}{24}$
- D. $\frac{11\pi}{24}$

Answer

$$\begin{aligned} A - B &= \frac{\pi}{4} \\ A + B &= 2\pi - \frac{\pi}{6} \\ B &= \frac{19\pi}{24} \end{aligned}$$

19. Question

Mark the correct alternative in the following:

If $A - B = \pi/4$, then $(1 + \tan A)(1 - \tan B)$ is equal to

- A. 2
- B. 1
- C. 0

D. 3

Answer

$$1 = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$1 + \tan A \tan B = \tan A - \tan B$$

$$\tan A - \tan B - \tan A \tan B = 1$$

add both side 1

$$1 + \tan A - \tan B - \tan A \tan B = 1 + 1$$

$$(1 + \tan A)(1 + \tan B) = 2$$

Case2:

$$\text{put } A=0 \text{ AND } B = -\frac{\pi}{4}$$

$$(1 + \tan A)(1 - \tan A) = 1 \times 2$$

20. Question

Mark the correct alternative in the following:

The maximum value of $\sin^2\left(\frac{2\pi}{3} + x\right) + \sin^2\left(\frac{2\pi}{3} - x\right)$ is

A. 1/2

B. 3/2

C. 1/4

D. 3/4

Answer

$$1 - [\cos\left(\frac{2\pi}{3} + x\right)]^2 + [\sin\left(\frac{2\pi}{3} - x\right)]^2$$

$$1 - \cos\left(\frac{4\pi}{3}\right) \cos(2x)$$

$$1 - \frac{\cos(2x)}{2} [\cos 2x = -1]$$

$$= \frac{3}{2}$$

21. Question

Mark the correct alternative in the following:

If $\cos(A - B) = \frac{3}{5}$ and $\tan A \tan B = 2$, then

A. $\cos A \cos B = \frac{1}{5}$

B. $\cos A \cos B = -\frac{1}{5}$

C. $\sin A \sin B = -\frac{1}{5}$

$$D. \sin A \sin B = \frac{1}{5}$$

Answer

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\frac{3}{5} = \cos A \cos B + \sin A \sin B$$

$$\frac{3}{5 \cos A \cos B} = 1 + \frac{\sin A \sin B}{\cos A \cos B}$$

$$\frac{3}{5 \cos A \cos B} = 1 + 2$$

$$\frac{1}{5} = \cos A \cos B$$

22. Question

Mark the correct alternative in the following:

$$\text{If } \tan 69^\circ + \tan 66^\circ - \tan 69^\circ \tan 66^\circ = 2k, \text{ then } k =$$

- A. -1
- B. 1/2
- C. -1/2
- D. None of these

Answer

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(69^\circ + 66^\circ) = \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$$

$$-1 = \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$$

$$-1 + \tan 69^\circ \tan 66^\circ = \tan 69^\circ + \tan 66^\circ$$

$$\tan 69^\circ + \tan 66^\circ - \tan 69^\circ \tan 66^\circ = -1$$

$$2k = -1$$

$$k = -\frac{1}{2}$$

23. Question

Mark the correct alternative in the following:

$$\text{If } \tan \alpha = \frac{x}{x+1} \text{ and } \tan \beta = \frac{1}{2x+1}, \text{ then } \alpha + \beta \text{ is equal to}$$

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{3}$
- C. $\frac{\pi}{6}$

$$D. \frac{\pi}{4}$$

Answer

put $x=1$

$$\tan \alpha = \frac{1}{2}$$

$$\tan \beta = \frac{1}{3}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha + \beta) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1 \cdot 1}{2 \cdot 3}}$$

$$\tan(\alpha + \beta) = 1$$

$$(\alpha + \beta) = \frac{\pi}{4}$$

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