23. The Straight Lines

Exercise 23.1

1 A. Question

Find the slopes of the lines which make the following angles with the positive direction of x - axis :

$$-\frac{\pi}{4}$$

Answer

Given $-\frac{\pi}{4}$

To Find: Slope of the line

Angle made with the positive x - axis is $-\frac{\pi}{4}$

The Slope of the line is m

Formula Used: m = tanθ

So, The slope of Line is m = tan $\left(-\frac{\pi}{4}\right)$ = – 1

Hence, The slope of the line is – 1.

1 B. Question

Find the slopes of the lines which make the following angles with the positive direction of x - axis :

$$\frac{2\pi}{3}$$

Answer

Given $\frac{2\pi}{3}$

To Find: Slope of the line

Angle made with the positive x - axis is $\frac{2\pi}{3}$

The Slope of the line is m

Formula Used: m = tanθ

So, The slope of Line is m = tan $\left(\frac{2\pi}{3}\right)$

$$\Rightarrow \tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow \tan\left(\frac{2\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right)$$

$$\Rightarrow \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

Hence, The slope of the line is $-\sqrt{3}$.

1 C. Question

Find the slopes of the lines which make the following angles with the positive direction of x - axis :

$\frac{3\pi}{4}$

Answer

Given $\frac{3\pi}{4}$

To Find: Slope of the line

Angle made with positive x - axis is $\frac{3\pi}{4}$

The Slope of the line is m

Formula Used: m = tanθ

So, The slope of Line is m = tan $\left(\frac{3\pi}{4}\right)$

$\Rightarrow \tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right)$

$\Rightarrow \tan\left(\frac{3\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

$\Rightarrow \tan\left(\frac{3\pi}{4}\right) = -1$

Hence, The slope of the line is – 1.

1 D. Question

Find the slopes of the lines which make the following angles with the positive direction of x - axis :

$\frac{\pi}{3}$

Answer

Given $\frac{\pi}{3}$

To Find: Slope of the line

Angle made with positive x - axis is $\frac{\pi}{3}$

The Slope of the line is m

Formula Used: m = tan θ

So, The slope of Line is m = tan $\left(\frac{\pi}{3}\right)$

$\Rightarrow \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

Hence, The slope of the line is √3.

2 A. Question

Find the slopes of a line passing through the following points :

(– 3, 2) and (1, 4)

Answer

Given (– 3, 2) and (1, 4)

To Find The slope of the line passing through the given points.

Here,

The formula used: Slope of line = $\frac{y_2-y_1}{x_2-x_1}$

So, The slope of the line, m = $\frac{4-2}{1-(-3)}$

m = $\frac{2}{4}$ = $\frac{1}{2}$

Hence, The slope of the line is $\frac{1}{2}$

2 B. Question

Find the slopes of a line passing through the following points :

($at^2{}_1$, $2at_1$) and ($at^2{}_2$, $2at_2$)

Answer

Given ($at^2{}_1$, $2at_1$) and ($at^2{}_2$, $2at_2$)

To Find: The slope of the line passing through the given points.

The formula used: Slope of line = $\frac{y_2-y_1}{x_2-x_1}$

So, The slope of the line, m = $\frac{2at_2-2at_1}{at_2^2-at_1^2}$

m = $\frac{2a(t_2-t_1)}{a(t_2^2-t_1^2)}$

m = $\frac{2a(t_2-t_1)}{a(t_2-t_1)t_2+t_1)}$

[Since, ($a^{2\ -}$ b^2 = (a – b)(a + b)]

m = $\frac{2}{t_2+t_1}$

Hence, The slope of the line is $\frac{2}{t_2+t_1}$

2 C. Question

Find the slopes of a line passing through the following points :

(3, – 5) and (1, 2)

Answer

Given (3, – 5) and (1, 2)

To Find: The slope of line passing through the given points.

Here,

The formula used: Slope of line = $\frac{y_2-y_1}{x_2-x_1}$

So, The slope of the line, m = $\frac{2-(-5)}{1-3}$

m = $\frac{7}{-2}$

Hence, The slope of the line is $\frac{7}{-2}$

3 A. Question

State whether the two lines in each of the following are parallel, perpendicular or neither :

Through (5, 6) and (2, 3); through (9, – 2) and (6, – 5)

Answer

We have given Coordinates off two lines.

Given: (5, 6) and (2, 3); (9, – 2) and 96, – 5)

To Find: Check whether Given lines are perpendicular to each other or parallel to each other.

Concept Used: If the slopes of this line are equal the lines are parallel to each other. Similarly, If the product of the slopes of this two line is – 1, then lines are perpendicular to each other.

The formula used: Slope of a line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of the line whose Coordinates are (5, 6) and (2, 3)

$$\Rightarrow m_1 = \frac{3-6}{2-5}$$

$$\Rightarrow m_1 = \frac{-3}{-3}$$

So, $m_1 = 1$

Now, The slope of the line whose Coordinates are (9, – 2) and (6, – 5)

$$\Rightarrow m_2 = \frac{-5-(-2)}{6-9}$$

$$\Rightarrow m_2 = \frac{-3}{-3}$$

So, $m_2 = 1$

Here, $m_1 = m_2 = 1$

Hence, The lines are parallel to each other.

3 B. Question

State whether the two lines in each of the following are parallel, perpendicular or neither :

Through (9, 5) and (– 1, 1); through (3, – 5) and 98, – 3)

Answer

We have given Coordinates off two line.

Given: (9, 5) and (– 1, 1); through (3, – 5) and (8, – 3)

To Find: Check whether Given lines are perpendicular to each other or parallel to each other.

Concept Used: If the slopes of this line are equal the the lines are parallel to each other. Similarly, If the product of the slopes of this two line is – 1, then lines are perpendicular to each other.

The formula used: Slope of a line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of the line whose Coordinates are (9, 5) and (– 1, 1)

$$\Rightarrow m_1 = \frac{1-5}{-1-9}$$

$$\Rightarrow m_1 = \frac{-4}{-10}$$

So, $m_1 = \frac{2}{5}$

Now, The slope of the line whose Coordinates are (3, – 5) and (8, – 3)

$$\Rightarrow m_2 = \frac{-3-(-5)}{8-3}$$

$$\Rightarrow m_2 = \frac{2}{5}$$

So, $m_2 = \frac{2}{5}$

Here, $m_1 = m_2 = \frac{2}{5}$

Hence, The lines are parallel to each other.

3 C. Question

State whether the two lines in each of the following are parallel, perpendicular or neither :

Through (6, 3) and (1,1); through (– 2, 5) and (2, – 5)

Answer

We have given Coordinates off two line.

Given: (6, 3) and (1,1) and (– 2, 5) and (2, – 5)

To Find: Check whether Given lines are perpendicular to each other or parallel to each other.

Concept Used: If the slopes of this line are equal the the lines are parallel to each other. Similarly, If the product of the slopes of this two line is – 1, then lines are perpendicular to each other.

The formula used: Slope of a line, $m = \frac{y_2-y_1}{x_2-x_1}$

Now, The slope of the line whose Coordinates are (6, 3) and (1, 1)

$$\Rightarrow m_1 = \frac{1-3}{1-6}$$

$$\Rightarrow m_1 = \frac{-2}{-5}$$

So, $m_1 = \frac{2}{5}$

Now, The slope of the line whose Coordinates are (– 2, 5) and (2, – 5)

$$\Rightarrow m_2 = \frac{-5-5}{2+2}$$

$$\Rightarrow m_2 = \frac{-10}{4}$$

So, $m_2 = \frac{-5}{2}$

Here, $m_1m_2 = \frac{2}{5} \times -\frac{5}{2}$

$m_1m_2 = -1$

Hence, The line is perpendicular to other.

3 D. Question

State whether the two lines in each of the following are parallel, perpendicular or neither :

Through (3, 15) and (16, 6); through (– 5, 3) and (8, 2)

Answer

We have given Coordinates off two line.

Given: (3, 15) and (16, 6) and (– 5, 3) and (8, 2)

To Find: Check whether Given lines are perpendicular to each other or parallel to each other.

Now,

Concept Used: If the slopes of these line are equal the the lines are parallel to each other. Similarly, If the product of the slopes of these two line is – 1, then lines are perpendicular to each other.

The formula used: Slope of a line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of the line whose Coordinates are (3, 15) and (16, 6)

$$\Rightarrow m_1 = \frac{6-15}{16-3}$$

$$\Rightarrow m_1 = \frac{-9}{13}$$

So, $m_1 = \frac{-9}{13}$

Now, The slope of the line whose Coordinates are (– 5, 3) and (8, 2)

$$\Rightarrow m_2 = \frac{2-3}{8-(-5)}$$

$$\Rightarrow m_2 = \frac{-1}{13}$$

So, $m_2 = \frac{-5}{2}$

Here, $m_1 \neq m_2$ nor $m_1 m_2$ = – 1

Hence, The lines are neither perpendicular and nor parallel to each other.

4. Question

Find the slopes of a line

(i) which bisects the first quadrant angle

(ii) which makes an angle of 30^0 with the positive direction of y - axis measured anticlockwise.

Answer

(i) Given, Line bisects the first quadrant

To Find: Find the slope of the line.

Here, If the line bisects in the first quadrant, then the angle must be between line and the positive direction of x - axis .

Since, Angle = $\frac{90}{2} = 45°$

The formula used: The slope of the line, m = tan θ

Similarly, The slope of the line for a given angle is m = tan 45

m = 1

Hence, The slope of the line is 1.

(ii) To Find: Find the slope of the line.

Here, The line makes an angle of 30^o with the positive direction of y - axis (Given)

Since Angle between line and positive side of axis = $90^o + 30^o = 120^o$

The formula used: The slope of the line, m = tan θ

Similarly, The slope of the line for a given angle is m = tan 120^o

m = – √3

Hence, The slope of the line is – √3 .

5 A. Question

Using the method of slopes show that the following points are collinear:

A(4, 8), b(5, 12), C(9, 28)

Answer

We have three points given A(4, 8), b(5, 12), C(9, 28)

To Prove: Given Points are collinear

Proof: A(4, 8), B(5, 12), C(9, 28)

The formula used: The slope of the line = $\frac{y_2-y_1}{x_2-x_1}$

The slope of line AB = $\frac{12-8}{5-4}$

AB = $\frac{4}{1}$

The slope of line BC = $\frac{28-12}{9-5}$

BC = $\frac{16}{4}$ = 4

The slope of line CA = $\frac{8-28}{4-9}$

$$CA = \frac{-20}{-5} = 4$$

Here, AB = BC = CA

Hence, The Given points are collinear.

5 B. Question

Using the method of slopes show that the following points are collinear:

A(16, – 18), B(3, – 6), C(– 10, 6)

Answer

We have three points given A(16, – 18), B(3, – 6), C(– 10, 6)

To Prove: Given Points are collinear

Proof: AB[(16, – 18),(3, – 6)], BC[(3, – 6),(– 10, 6)], CA[(– 10, 6),(16, – 18)]

Formula used: The slope of the line = $\frac{y_2-y_1}{x_2-x_1}$

The slope of line AB = $\frac{-6-(-18)}{3-16}$

AB = $\frac{12}{-13}$

The slope of line BC = $\frac{6-(-6)}{-10-3}$

BC = $\frac{12}{-13}$

The slope of line CA = $\frac{6-(-18)}{-10-16}$

CA = $\frac{12}{-13}$

Here, AB = BC = CA

Hence, The Given points are collinear.

6. Question

What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (– 1, 4) and (0, 6) ?

Answer

We have given coordinates of two lines (3, y) and (2, 7), (– 1, 4) and (0, 6)

To Find: Value of y?

The concept used: Slopes of the parallel line are always equal.

The formula used: The slope of line = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of the line whose coordinates are (3, y) and (2, 7).

$M_1 = \frac{7-y}{2-3}$ ...... (1)

And, Now, The slope of the line whose coordinates are (– 1, 4) and (0, 6).

$M_2 = \frac{6-4}{0-(-1)}$

$M_2 = \frac{2}{1}$ ...... (2)

On equating the equation (1) and (2), we get

$$\frac{7-y}{2-3} = \frac{2}{1}$$

7 – y = 2(– 1)

– y = – 2 – 7

Y = 9

Hence, The value of y is 9.

7. Question

What can be said regarding a line if its slope is

(i) zero

(ii) positive

(iii) negative

Answer

(i) If the slope of the line is zero it means

M = tan θ

M = tan 0

Since, m = 0

So, The line is parallel to x - axis .

(ii) If the slope of the line is positive it means $0 < \theta < \frac{\pi}{2}$

Since θ is an acute

So, The line makes an acute angle with the positive x - axis .

(iii) If the slope of the line is positive it means $\theta > \frac{\pi}{2}$

Since, θ is an obtuse

So, The line makes an obtuse angle with positive x - axis .

8. Question

Show that the line joining (2, – 3) and (– 5, 1) is parallel to the line joining (7, – 1) and (0, 3).

Answer

To Prove: The given line is parallel to another line.

Proof: Let Assume the coordinate A(2, – 3) and B(– 5, 1), C(7, – 1) and D(0,3).

The concept used: Slopes of the parallel lines are equal.

The formula used: The slope of the line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of AB = $\frac{1-(-3)}{-5-2}$

The Slope of AB = $\frac{4}{-7}$

Now, The slope of CD = $\frac{3-(-1)}{0-7}$

The Slope of AB = $\frac{4}{-7}$

So, The slope of AB = The slope of CD

Hence, The given Lines are parallel to each other.

9. Question

Show that the line joining (2, – 5) and (– 2, 5) is perpendicular to the line joining (6, 3) and (1,1).

Answer

To Prove: The Given line is perpendicular to each other.

Proof: Let Assume the coordinate A(2, – 5) and B(– 2, 5) joining the line AB, C(6,3) and D(1,1) joining the line CD.

The concept used: The product of the slopes of lines always – 1.

The formula used: The slope of the line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of AB = $\frac{5-(-5)}{-2-2}$

The Slope of AB = $\frac{10}{-4}$

Now, The slope of CD = $\frac{1-3}{1-6}$

The Slope of AB = $\frac{2}{5}$

So, AB × CD = $\frac{10}{-4} \times \frac{2}{5}$

AB × CD = – 1

Hence, The given Lines are perpendicular to each other.

10. Question

Without using Pythagoras theorem, show that the points A(0, 4), B(1, 2), C(3, 3) are the vertices of a right – angled triangle.

Answer

We have given three points of a triangle.

Given: A(0, 4), B(1, 2), C(3, 3)

To Prove: Given points are the vertices of Right – angled Triangle.

Proof: We have A(0, 4), B(1, 2), C(3, 3)

The concept used: If the two lines are perpendicular to each other then it will be a right – angled triangle.

Now, Joining the points to make a line as AB, BC, and CA

The formula used: The slope of the line, m = $\frac{y_2 - y_1}{x_2 - x_1}$

Now, The slope of line $m_{AB} = \frac{2-4}{1-0}$

The slope of $m_{AB} = \frac{-2}{1}$

and, The slope of line BC = $\frac{3-2}{3-1}$

The slope of $m_{Bc} = \frac{1}{2}$

Now, $m_{AB} \times m_{Bc} = -\frac{2}{1} \times \frac{1}{2}$

$m_{AB} \times m_{Bc}$ = – 1

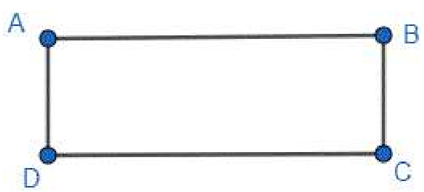
Since, AB is perpendicular to BC, it means B = $\frac{\pi}{2}$

Hence, ABC is a right angle Triangle.

11. Question

Prove that the points (– 4, – 1), (– 2, – 4), (4, 0) and (2, 3) are the vertices of a rectangle.

Answer



To Prove: Given vertices are of the rectangle.

Explanation: We have given points A(– 4, – 1), B(– 2, – 4), C(4, 0) and D(2, 3)

The points are joining in the form of AB, BC, CD, and AD

The formula used: The slope of the line, m = $\frac{y_2 - y_1}{x_2 - x_1}$

Now, The slope of Line AB, $m_{AB} = \frac{-4-(-1)}{-2-(-4)}$

$m_{AB} = \frac{-3}{2}$

The slope of BC, $m_{BC} = \frac{0-(-4)}{4-(-2)}$

$m_{BC} = \frac{4}{6} = \frac{2}{3}$

Now, The slope of Line CD, $m_{CD} = \frac{3-0}{2-4}$

$m_{CD} = \frac{3}{-2}$

The slope of AD, $m_{AD} = \frac{3-(-1)}{2-(-4)}$

$m_{AD} = \frac{4}{6} = \frac{2}{3}$

Here, We can see that, $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$

i.e, AB ∥ CD and BC ∥ AD

And, $m_{AB} \times m_{BC} = -\frac{3}{2} \times \frac{2}{3} = -1$

$M_{CD} \times m_{AD} = -\frac{3}{2} \times \frac{2}{3} = -1$

So, that AB⊥BC and CD⊥AD

Hence, ABCD is a Rectangle.

12. Question

If three points A(h, 0), P(a, b) and B(0, k) lie on a line, show that:

$$\frac{a}{h} + \frac{b}{k} = 1.$$

Answer

If these three points lie on a line, the slope will be equal.

So, slope of A(h, 0) and P(a, b) = Slope of A(h, 0) and B(0, k)

Slope of AP $= \left(\frac{b-a}{a-h}\right)$

Slope of AB $= \left(\frac{k-0}{0-h}\right)$

Now,

$$\left(\frac{b-a}{a-h}\right) = \left(\frac{k-0}{0-h}\right)$$

$$\frac{b}{a-h} = -\frac{k}{h}$$

bh = - ka + kh

ak + bh = kh

Dividing both sides by kh, we get,

$$\frac{a}{h} + \frac{b}{k} = 1$$

13. Question

The slope of a line is double of the slope of another line. If tangents of the angle between them is $\frac{1}{3}$, find the slopes of the other line.

Answer

Given, The tangent of the angle between them is $\frac{1}{3}$

To Find Slope of the other line.

Assumption: The slope of line m_1 = x, and m_2 = 2x

Formula used: $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Explanation: We have $\tan\theta = \frac{1}{3}$ given, then

$$\frac{1}{3} = \left|\frac{x - 2x}{1 + 2x^2}\right|$$

Case 1:

$$\frac{1}{3} = \frac{x - 2x}{1 + 2x^2}$$

$2x^2 + 1 = 3x - 6x$

$2x^2 + 3x + 1 = 0$

$2x^2 + 2x + x + 1 = 0$

$2x(x + 1) + 1(x + 1) = 0$

$(2x + 1)(x + 1) = 0$

$x = -1, -\frac{1}{2}$

Case 2:

$$\frac{1}{3} = \frac{x}{1 + 2x^2}$$

$2x^2 + 1 = 3x$

$2x^2 - 3x + 1 = 0$

$2x^2 - 2x - x + 1 = 0$

$2x(x - 1) - 1(x - 1) = 0$

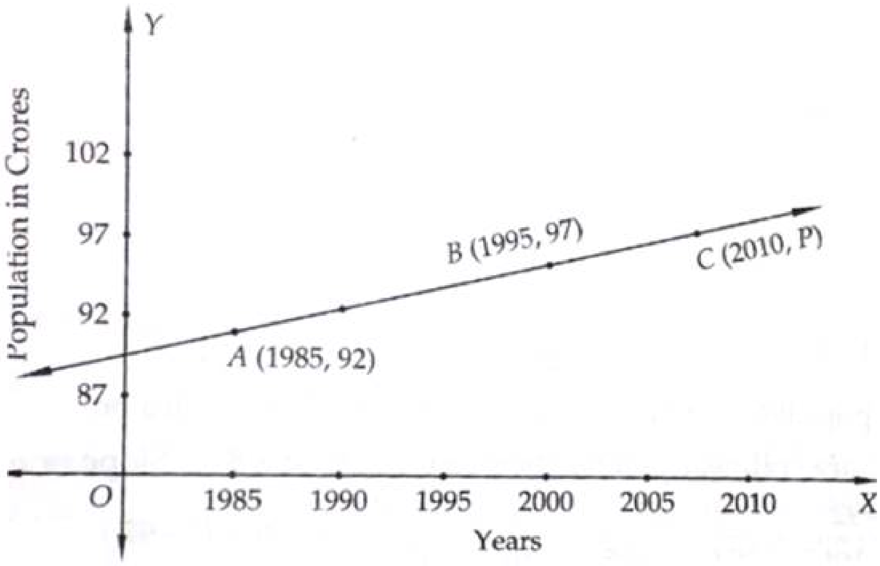
$(2x - 1)(x - 1) = 0$

$x = 1, \frac{1}{2}$

Hence, The slope of other line is either $1, \frac{1}{2}$ or $-1, -\frac{1}{2}$.

14. Question

Consider the following population and year graph:



Find the slope of the line AB and using it, find what will be the population in the year 2010.

Answer

For the given graph,

Slope of line AB $= \left(\frac{97-92}{1995-1985}\right)$

Slope of line AB $= \left(\frac{1}{5}\right)$

Now, Slope of AB = Slope of AC

Therefore,

Slope of AC $= \left(\frac{(P-92)}{2010-1985}\right) = \frac{1}{5}$

5p – 460 = 25

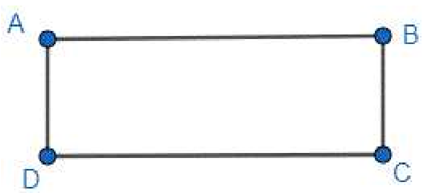
5p = 485

P = 97 Crores.

15. Question

Without using the distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (– 3, 2) are the vertices of a parallelogram.

Answer



To Prove: Given points are of Parallelogram.

Explanation: Let us Assume that we have points, A (– 2, – 1), B(4, 0), C(3, 3) and D(– 3, 2), are joining the sides as AB, BC, CD, and AD.

The formula used: The slope of the line, m $= \frac{y_2-y_1}{x_2-x_1}$

Now, The slope of Line AB, $m_{AB} = \frac{0-(-1)}{4-(-2)}$

$m_{AB} = \frac{1}{6}$

The slope of BC, $m_{BC} = \frac{3-0}{3-4}$

$m_{BC} = \frac{3}{-1}$

Now, The slope of Line CD, $m_{CD} = \frac{2-3}{-3-3}$

$m_{CD} = \frac{1}{6}$

The slope of AD, $m_{AD} = \frac{2-(-1)}{-3-(-2)}$

$m_{AD} = \frac{3}{-1}$

Here, We can see that, $m_{AB} = m_{CD}$and $m_{BC} = m_{AD}$

i.e, AB ∥ CD and BC ∥ AD

We know, If opposite side of a quadrilateral are parallel that it is parallelogram.

Hence, ABCD is a Parallelogram.

16. Question

Find the angle between the X - axis and the line joining the points (3, – 1) and (4, – 2).

Answer

Given, (3, – 1) and (4, – 2)

To find: Find the angle between x - axis and the line.

Explanation: We have two points A(3, – 1) and B(4, – 2).

The formula used: The slope of the line, $m = \frac{y_2-y_1}{x_2-x_1}$

Now, The slope of line AB, $m_{AB} = \frac{-2-(-1)}{4-3}$

$m_{AB} = -1$

and, we know, The slope of x - axis is always 0

Now, the angle between x - axis and slope of line AB is,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-1 - 0}{1 + (-1)(0)}\right|$$

$$\tan\theta = -\frac{1}{1}$$

$\theta = \tan^{-1} - 1$

$\theta = 135^o$

Hence, The angle between the x - axis and the line is 135^0.

17. Question

The line through the points (– 2, 6) and 94, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x.

Answer

To Find: Find the value of x ?

The concept used: If two line is perpendicular then, the product of their slopes is – 1.

Explanation: We have two lines having point A(– 2,6) and B(4,8) and other line having points C(8,12) and D(x,24).

The formula used: The slope of the line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of Line AB is, m_{AB} = $\frac{4-(-2)}{8-6}$

$m_{AB} = \frac{6}{2}$

and, The slope of Line CD is, m_{CD} = $\frac{x-8}{24-12}$

$m_{CD} = \frac{x-8}{12}$

We know the product of the slopes of perpendicular line is always – 1. Then,

$m_{AB} \times m_{CD} = -1$

$\frac{6}{2} \times \frac{x-8}{12} = -1$

$\frac{x-8}{4} = -1$

x – 8 = – 4

x = – 4 + 8

x = 4

Hence, The value of x is 4.

18. Question

Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

Answer

The given points (x, – 1), (2, 1) and (4, 5) are collinear.

To Find: The value of x.

Concept Used: It is given that points are collinear, SO the area of the triangle formed by the points must be zero.

Formula used: The area of triangle = $x_1(y_1 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$

Explanation: Let be points of triangle A(x, – 1), B(2, 1) and C(4, 5)

Now, The points are collinear than, Area of a triangle is zero.

Here, Put the given values in formula and we get,

x(1 – 5) + (2)(5 – (– 1)) + 4(– 1 – 1) = 0

x – 5x + 12 – 8 = 0

– 4x + 4 = 0

4x = 4

x = 1

Hence, The value of x is 1.

19. Question

Find the angle between X - axis and the line joining the points (3, – 1) and (4, – 2).

Answer

Given, (3, – 1) and (4, – 2)

To find: Find the angle between x - axis and the line.

Explanation: We have two points A(3, – 1) and B(4, – 2).

The formula used: The slope of the line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of line AB, m_{AB} = $\frac{-2-(-1)}{4-3}$

m_{AB} = – 1

and, we know, The slope of x - axis is always 0

Now, the angle between x - axis and slope of line AB is,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-1 - 0}{1 + (-1)(0)}\right|$$

$$\tan\theta = -\frac{1}{1}$$

θ = $\tan^{-1} - 1$

θ = 135^o

Hence, The angle between the x – axis and the line is 135^0.

20. Question

By using the concept of slope, show that the points (– 2, – 1), (4, 0), (3, 3) and (– 3, 2) vertices of a parallelogram.

Answer

To Prove: Given points are of Parallelogram.

Explanation: Let us Assume that we have points, A (– 2, – 1), B(4, 0), C(3, 3) and D(– 3, 2), are joining the sides as AB, BC, CD, and AD.

The formula used: The slope of the line, m = $\frac{y_2-y_1}{x_2-x_1}$

Now, The slope of Line AB, m_{AB} = $\frac{0-(-1)}{4-(-2)}$

$m_{AB} = \frac{1}{6}$

The slope of BC, m_{BC} = $\frac{3-0}{3-4}$

$m_{BC} = \frac{3}{-1}$

Now, The slope of Line CD, m_{CD} = $\frac{2-3}{-3-3}$

$m_{CD} = \frac{1}{6}$

The slope of AD, m_{AD} = $\frac{2-(-1)}{-3-(-2)}$

$m_{AD} = \frac{3}{-1}$

Here, We can see that, $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$

i.e, AB ∥ CD and BC ∥ AD

We know, If opposite side of a quadrilateral are parallel that it is a parallelogram.

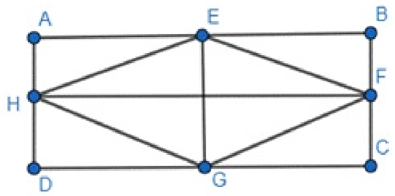
Hence, ABCD is a Parallelogram.

21. Question

A quadrilateral has vertices (4, 1), (1, 7), (– 6, 0) and (– 1, – 9). Show that the mid – points of the sides of this quadrilateral form a parallelogram.

Answer

Given, A quadrilateral has vertices (4, 1), (1, 7), (– 6, 0) and (– 1, – 9).



To Prove: Mid – Points of the quadrilateral form a parallelogram.

The formula used: Mid point formula = $\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$

Explanation: Let ABCD is a quadrilateral

E is the midpoint of AB

F is the midpoint of BC

G is the midpoint of CD

H is the midpoint of AD

Now, Find the Coordinates of E, F,G and H using midpoint Formula

Coordinate of E = $\left[\frac{4+1}{2}, \frac{1+7}{2}\right] = \left[\frac{5}{2}, 4\right]$

Coordinate of F = $\left[\frac{1-6}{2}, \frac{7+0}{2}\right] = \left[-\frac{5}{2}, \frac{7}{2}\right]$

Coordinate of G = $\left[\frac{-6-1}{2}, \frac{0-9}{2}\right] = \left[\frac{-7}{2}, -\frac{9}{2}\right]$

Coordinate of H = $\left[\frac{-1+4}{2}, \frac{-9+1}{2}\right] = \left[\frac{3}{2}, -4\right]$

Now, EFGH is a parallelogram if the diagonals EG and FH have the same mid – point

Coordinate of mid – point of EG = $\left[\frac{\frac{5-7}{2}}{2}, \frac{4-\frac{9}{2}}{2}\right] = \left[-\frac{1}{2}, -\frac{1}{4}\right]$

Coordinate of mid – point of FH = $\left[\frac{\frac{-5+3}{2}}{2}, \frac{\frac{7-8}{2}}{2}\right] = \left[-\frac{1}{2}, -\frac{1}{4}\right]$

Since Diagonals are equals then EFGH is a parallelogram.

Hence, EFGH is a parallelogram.

Exercise 23.2

1. Question

Find the equation of the parallel to x–axis and passing through (3, – 5).

Answer

Given, A line which is parallel to x–axis and passing through (3, – 5)

To Find: The equation of the line.

Formula used: The equation of line is [$y - y_1 = m(x - x_1)$]

Explanation: Here, The line is parallel to the x–axis,

So, The parallel lines have equal slopes,

And, the slope of x–axis is always 0, then

The slope of line, m = 0

Coordinates of line are $(x_1, y_1) = (3, -5)$

The equation of line = $y - y_1 = m(x - x_1)$

By putting the values, we get

$y - (-5) = 0(x - 3)$

$y + 5 = 0$

Hence, The equation of line is $y + 5 = 0$

2. Question

Find the equation of the line perpendicular to x–axis and having intercept – 2 on x–axis.

Answer

Given, A line which is perpendicular to x–axis and having intercept – 2.

To Find: The equation of the line.

Formula used: The equation of line is [$y - y_1 = m(x - x_1)$]

Explanation: Here, The line is perpendicular to the x–axis, then x is 0 and y is – 1.

So, The slope of line is, m = $\frac{y}{x}$

m = $\frac{-1}{0}$

Since, It is given that x–intercept is – 2, so, y is 0.

Coordinates of line are $(x_1, y_1) = (-2, 0)$

The equation of line = $y - y_1 = m(x - x_1)$

By putting the values, we get

$y - 0 = \frac{-1}{0}(x - (-2))$

$x + 2 = 0$

Hence, The equation of line is $x + 2 = 0$

3. Question

Find the equation of the line parallel to x–axis and having intercept – 2 on y – axis.

Answer

Given, A line which is parallel to x–axis and having intercept – 2 on y – axis.

To Find: The equation of the line.

Formula used: The equation of line is [$y - y_1 = m(x - x_1)$]

Explanation Here, The line is parallel to the x–axis,

So, The parallel lines have equal slopes,

And, the slope of x–axis is always 0, then

The slope of line, m = 0

Since, It is given that intercept is – 2, on y – axis then

Coordinates of line are $(x_1, y_1) = (0, -2)$

The equation of line is $y - y_1 = m(x - x_1)$ – – – – (1)

By putting the values in equation (1), we get

$y - (-2) = 0\ (x - 0)$

$y + 2 = 0$

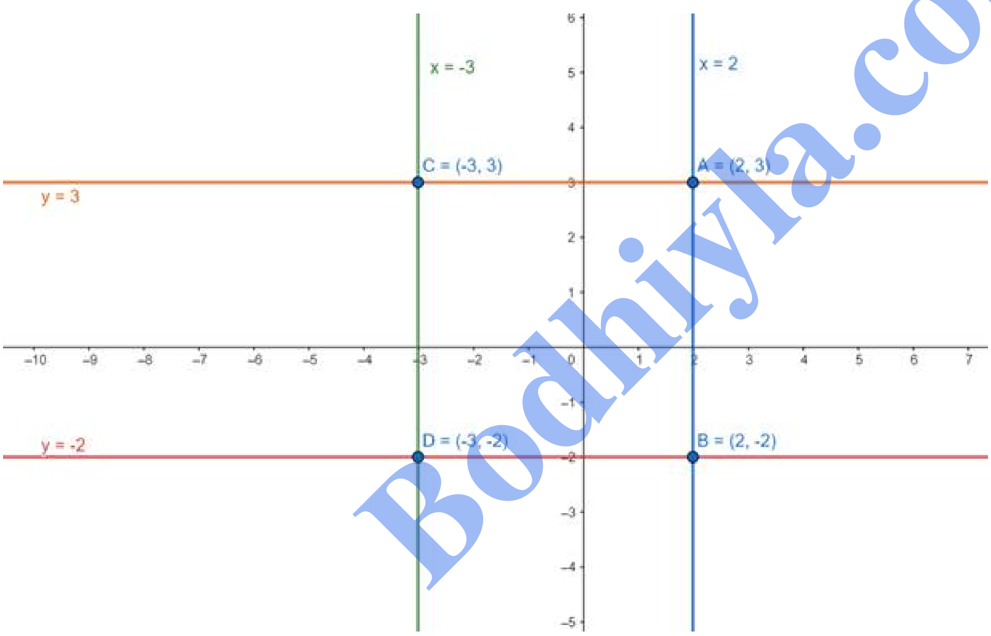
Hence, The equation of line is $y + 2 = 0$

4. Question

Draw the lines x = – 3, x = 2, y = – 2, y = 3 and write the coordinates of the vertices of the square so formed.

Answer

Given, x = – 3, x = 2, y = – 2 and y = 3



Coordinates of the square are : A(2, 3), B(2, –2), C(–3, 3), and D(–3, –2).

5. Question

Find the equations of the straight lines which pass through (4, 3) and are respectively parallel and perpendicular to the x–axis.

Answer

Given, A line which is perpendicular and parallel to x–axis respectively and passing through (4, 3)

To Find: Find the equation of that line.

Formula used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation:

Case 1 : When Line is parallel to x–axis

So, The parallel lines have equal slopes, And, the slope of x–axis is always 0, then

The slope of line, m = 0

Coordinates of line are $(x_1, y_1) = (4, 3)$

The equation of line is $y - y_1 = m(x - x_1)$ – – – – (1)

By putting the values in equation (1), we get

$y - (3) = 0(x - 4)$

$y - 3 = 0$

Case 2: when line is perpendicular to x–axis

Here, The line is perpendicular to the x–axis, then x is 0 and y is – 1.

So, The slope of the line is, $m = \frac{y}{x}$

$m = \frac{-1}{0}$

Coordinates of line are $(x_1, y_1) = (4, 3)$

The equation of line = $y - y_1 = m(x - x_1)$

By putting the values, we get

$$y - 3 = \frac{-1}{0}(x - 4)$$

$x = 4$

Hence, The equation of line when it is parallel to x –axis is y = 3 and it is perpendicular is x = 4.

6. Question

Find the equation of the line which is equidistant from the lines x = –2 and x = 6.

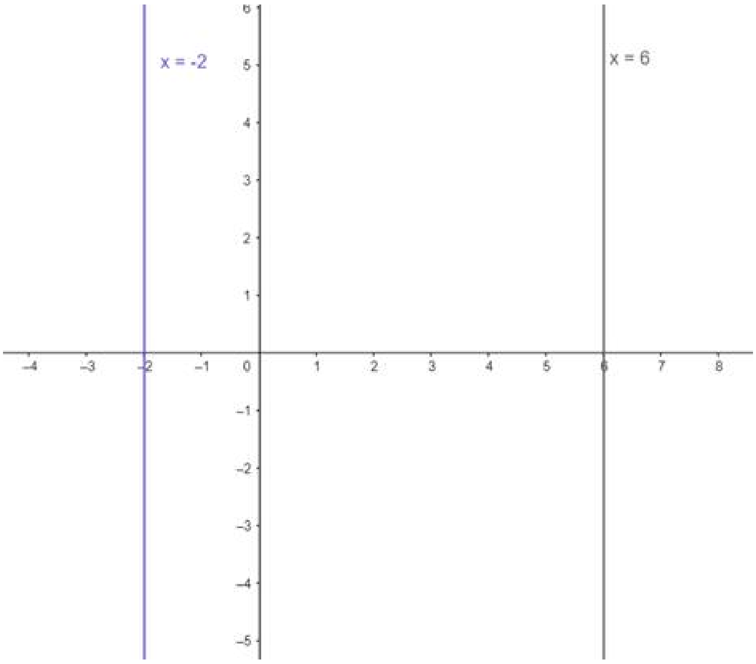
Answer

To Find: The equation of the line

Formula Used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation:

Let us plot the lines



Now the line must be at the centre of the lines (–2, 0) and (6, 0).

The Midpoint formula = $\left[\frac{x+x_1}{2}, \frac{y+y_1}{2}\right]$

Therefore,

Midpoint = $\left[\frac{-2+6}{2}, \frac{0+0}{2}\right]$

Midpoint = (2, 0)

Hence, the equation of the line is x = 2.

7. Question

Find the equation of a line equidistant from the lines y = 10 and y = – 2.

Answer

A line which is equidistant from the lines y = 10 and y = – 2

To Find: The equation of the line

Formula used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation: A line which is equidistant from, two other lines,

So, the slopes must be the same .

Therefore, The slope of line y = 10 and y = – 2 is 0, because lines are parallel to the x–axis.

Since, The required line will pass from the midpoint of the line joining (0, – 2) and (0, 10)

The Midpoint formula = $\left[\frac{x+x_1}{2}, \frac{y+y_1}{2}\right]$

So, The coordinates of the point will be $\left[0, \frac{10-2}{2}\right] = (0, 4)$

Since The equation of the line is :

y – 4 = 0(x – 0)

y = 4

Hence, The equation of the line is y = 4

Exercise 23.3

1. Question

Find the equation of a line making an angle of 150° with the x–axis and cutting off an intercept 2 from y–axis.

Answer

A line which makes an angle of 150^o with the x–axis and cutting off an intercept at 2.

To Find: The equation of that line.

Formula used: The equation of a line is y = mx + c

Explanation: Here, angle, θ = 150^o

SO, The slope of the line, m = tan θ

m = tan 150^o

m = $-\frac{1}{\sqrt{3}}$

Coordinate of y–intercept is (0, 2)

The required equation of the line is y = mx + c

$$y = -\frac{x}{\sqrt{3}} + 2$$

$$\sqrt{3}y - 2\sqrt{3} + x = 0$$

$$x + \sqrt{3}y = 2\sqrt{3}$$

Hence, The equation of line is $+\sqrt{3}y = 2\sqrt{3}$.

2. Question

Find the equation of a straight line:

(i) with slope 2 and y – intercept 3;

(ii) with slope – 1/ 3 and y – intercept – 4.

(iii) with slope – 2 and intersecting the x–axis at a distance of 3 units to the left of origin.

Answer

(i) Here, The slope is 2 and the coordinates are (0, 3)

Now, The required equation of line is

y = mx + c

y = 2x + 3

(ii) Here, The slope is – 1/3 and the coordinates are (0, – 4)

Now, The required equation of line is

y = mx + c

y = $-\frac{1}{3}$x – 4

3y + x = – 12

(iii) Here, The slope is – 2 and the coordinates are (– 3, 0)

Now, The required equation of line is y – y_1 = m (x – x_1)

y – 0 = – 2(x + 3)

y = – 2x – 6

2x + y + 6 = 0

3. Question

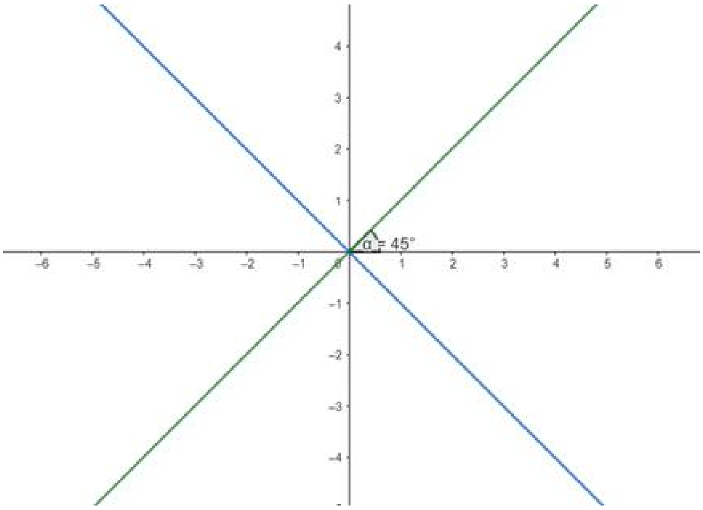
Find the equations of the bisectors of the angles between the coordinate axes.

Answer

To Find: Equations of bisectors of the angles between coordinate axes.

Formula Used: The equation of line is y = mx + c

Diagram:



Explanation:

Co–ordinate axes make an angle of 90° with each other.

So the bisector of angles between co–ordinate axes will subtend $= \frac{90^{\circ}}{2} = 45^{\circ}$

Now, we can see that there are two bisectors.

Angles subtended from x–axis are: 90° and 135°

And there is no intercept, c = 0

Equations are:

y = tan45°x and y = tan135°x

y = x and y = –x

Hence, the equations of bisectors of angle between coordinate axis are y = x and y = –x

4. Question

Find the equation of a line which makes an angle of $\tan^{-1}(3)$ with the x–axis and cuts off an intercept of 4 units on the negative direction of y–axis.

Answer

Given: The equation which makes an angle of $\tan^{-1}(3)$ with the x–axis and cuts off an intercept of 4 units on the negative direction of y–axis.

To Find: The equation of the line?

The formula used: The equation of the line is y = mx + c

Explanation: Here, angle θ = $\tan^{-1}(3)$

So, tan θ = 3

The slope of the line is, m = 3

And, Intercept in the negative direction of y–axis is (0, -4)

Now, The required equation of the line is y = mx + c

y = 3x – 4

Hence, The equation of the line is y = 3x – 4.

5. Question

Find the equation of a line that has y – intercept – 4 and is parallel to the line joining (2, – 5) and (1, 2).

Answer

Given, A line segment joining (2, – 5) and (1, 2) if it cuts off an intercept – 4 from y–axis.

To Find: The equation of that line.

Formula used: The equation of line is y = mx + C

Explanation: Here, The required equation of line is y = mx + c

Now, c = – 4 (Given)

Slope of line joining (x_1 – x_2) and (y_1 – y_2) , $m = \frac{y_2 - y_1}{x_2 - x_1}$

So, Slope of line joining (2, – 5) and (1, 2), $m = \frac{2-(-5)}{1-2} = \frac{7}{-1}$

Therefore, m = – 7

Now, The equation of line is y = mx + c

y = –7x – 4

y + 7x + 4 = 0

Hence, The equation of line is y + 7x + 4 = 0.

6. Question

Find the equation of a line which is perpendicular to the line joining (4, 2) and (3 5) and cuts off an intercept of length 3 on y – axis.

Answer

Given, A line segment joining (4, 2) and (3, 5) if it cuts off an intercept 3 from y–axis.

To Find: The equation of that line.

Formula used: The equation of line is y = mx + C

Explanation: Here, The required equation of line is y = mx + c

Now, c = 3 (Given)

Let m be slope of given line = – 1

Slope of line joining (x_1 – x_2) and (y_1 – y_2) , $m = \frac{y_2 - y_1}{x_2 - x_1}$

So, Slope of line joining (4, 2) and (3, 5), $m = \frac{5-2}{3-4} = \frac{3}{-1}$

Therefore, $m = \frac{1}{3}$

Now, The equation of line is y = mx + c

$$y = \frac{1}{3}x + 3$$

x – 3y + 9 = 0

Hence, The equation of line is 2y + 5x + 6 = 0.

7. Question

Find the equation of the perpendicular to the line segment joining (4, 3) and (– 1 1) if it cuts off an intercept – 3 from y – axis.

Answer

Given, A line segment joining (4, 3) and (– 1, 1) if it cuts off an intercept – 3 from y–axis.

To Find: The equation of that line.

Formula used: The equation of line is y = mx + C

Explanation: Here, The required equation of line is y = mx + c

Now, c = – 3 (Given)

Let m be slope of given line = – 1

Slope of line joining (x_1 – x_2) and (y_1 – y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$

So, Slope of line joining (4, 3) and (– 1, 1), $m = \frac{1-3}{-1-4} = \frac{-2}{-5}$

Therefore, $m = -\frac{2}{5}$

Now, The equation of the line is y = mx + c

$$y = -\frac{2}{5}x - 3$$

$$y + 3 = -\frac{5x}{2}$$

2y + 5x + 6 = 0

Hence, The equation of line is 2y + 5x + 6 = 0.

8. Question

Find the equation of the straight line intersecting y – axis at a distance of 2 units above the origin and making an angle of 30^0 with the positive direction of the x–axis.

Answer

Given, A line which intersects at y–axis at a distance of 2 units and makes an angle of 30^0 with the positive direction of x–axis.

To Find: The equation of that line.

Formula used: The equation of line is [y – y_1 = m(x – x_1)]

Explanation: Here, Angle = 30^o (Given)

So, The slope of the line, m = tan θ

m = tan 30^o

$$m = \frac{1}{\sqrt{3}}$$

Now, The coordinates are (x_1, y_1) = (0, 2)

The equation of line = y – y_1 = m(x – x_1)

y – 2 = $\frac{1}{\sqrt{3}}$ (x – 0)

$\sqrt{3y} + 2\sqrt{3} = x$

$\sqrt{3y} + 2\sqrt{3} - x = 0$

Hence, The equation of line is $\sqrt{3y} + 2\sqrt{3} - x = 0$.

Exercise 23.4

1. Question

Find the equation of the straight line passing through the point (6, 2) and having slope – 3.

Answer

Given, A straight line passing through the point (6,2) and the slope is – 3

To Find: The equation of the straight line.

Formula used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation: Here, The line is passing through (6,2)

The slope of line, m = – 3 (Given)

Coordinates of line are $(x_1,y_1) = (6,2)$

The equation of line = $y - y_1 = m(x - x_1)$

By putting the values, we get

$y - 2 = -3(x - 6)$

$y - 2 = -3x + 18$

$y + 3x - 20 = 0$

Hence, The equation of line is $y + 3x - 20 = 0$

2. Question

Find the equation of the straight line passing through (– 2, 3) and indicated at an angle of 45° with the x – axis.

Answer

A line which is passing through (– 2,3), the angle is 45^o.

To Find: The equation of a straight line.

Formula used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation: Here, angle, $\theta = 45^o$

SO, The slope of the line, $m = \tan\theta$

$m = \tan 45^o$

$m = 1$

The line passing through $(x_1,y_1) = (-2,3)$

The required equation of line is $y - y_1 = m(x - x_1)$

$y - 3 = 1(x - (-2))$

$y - 3 = x + 2$

$x - y + 5 = 0$

Hence, The equation of line is $x - y + 5 = 0$

3. Question

Find the equation of the line passing through (0, 0) with slope m

Answer

Given, A straight line passing through the point (0,0) and slope is m.

To Find: The equation of the straight line.

Formula used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation: Here, The line is passing through (0,0)

The slope of line, m = m (Given)

Coordinates of line are $(x_1,y_1) = (0,0)$

The equation of line = $y - y_1 = m(x - x_1)$

By putting the values, we get

$y - 0 = m(x - 0)$

$y = mx$

Hence, The equation of line is $y = mx$.

4. Question

Find the equation of the line passing through $(2, 2\sqrt{3})$ and inclined with x – axis at an angle of 75^0.

Answer

A line which is passing through $(2,2\sqrt{3})$, the angle is 75^o.

To Find: The equation of a straight line.

Formula used: The equation of line is $[y - y_1 = m(x - x_1)]$

Explanation: Here, angle, $\theta = 75^o$

SO, The slope of the line, $m = \tan\theta$

$m = \tan 75^o$

$m = 3.73 = 2 + \sqrt{3}$

The line passing through $(x_1,y_1) = (2,2\sqrt{3})$

The required equation of the line is $y - y_1 = m(x - x_1)$

$y - 2\sqrt{3} = 2 + \sqrt{3}\,(x - 2)$

$y - 2\sqrt{3} = (2 + \sqrt{3})x - 7.46$

$(2 + \sqrt{3})x - y - 4 = 0$

Hence, The equation of the line is $(2 + \sqrt{3})x - y - 4 = 0$

5. Question

Find the equation of the straight line which passes through the point (1,2) and makes such an angle with the positive direction of x – axis whose sine is $\frac{3}{5}$.

Answer

A line which is passing through (1,2)

To Find: The equation of a straight line.

Formula used: The equation of line is [$y - y_1 = m(x - x_1)$]

Explanation: Here, $\sin\theta = 3/5$

We know, $\sin\theta = \frac{\text{perpendicular}}{\text{Hypotenues}} = \frac{3}{5}$

According to Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$(5)^2 = (\text{Base})^2 + (3)^2$

$(\text{Base}) = \sqrt{25 - 9}$

$(\text{Base})^2 = \sqrt{16}$

Base = 4

Hence, $\tan\theta = \frac{\text{perpendicular}}{\text{Base}} = \frac{3}{4}$

SO, The slope of the line, $m = \tan\theta$

$m = \frac{3}{4}$

The line passing through $(x_1, y_1) = (1,2)$

The required equation of line is $y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{4}(x - 1)$

$4y - 8 = 3x - 3$

$3x - 4y + 5 = 0$

Hence, The equation of line is $3x - 4y + 5 = 0$

6. Question

Find the equation of the straight line passing through (3, – 2) and making an angle of 60° with the positive direction of y – axis.

Answer

A line which is passing through (3, – 2), the angle is 60^o with the positive direction of the y – axis.

To Find: The equation of a straight line.

Formula used: The equation of line is [$y - y_1 = m(x - x_1)$]

Explanation: Here, angle, $\theta = 60^o$ with the y – axis. So, it makes 30^0 with the positive direction of the x – axis.

SO, The slope of the line, $m = \tan\theta$

$m = \tan 30^o$

$m = \frac{1}{\sqrt{3}}$

The line passing through $(x_1, y_1) = (3, -2)$

The required equation of line is $y - y_1 = m(x - x_1)$

$y - (-2) = \frac{1}{\sqrt{3}}(x - 3)$

$\sqrt{3}y + 2\sqrt{3} = x - 3$

$x - \sqrt{3}y - 3 - 2\sqrt{3} = 0$

Hence, The equation of the line is $x - \sqrt{3y} - 3 - 2\sqrt{3} = 0$

7. Question

Find the lines through the point (0, 2) making angles $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the x–axis. Also, find the lines parallel to them cutting the y–axis at a distance of 2 units below the origin.

Answer

We know that equation of line having angle θ from x–axis and passing through (x_1, y_1) is given by,

$y - y_1 = \tan\theta\,(x - x_1)$

Therefore,

Equation of first line,

$(y - 2) = \tan\left(\frac{\pi}{3}\right)(x - 0)$

$y - 2 = \sqrt{3}x$

$\mathbf{y - \sqrt{3}x - 2 = 0}$

The equation of line parallel to this line and passing through (0, –2),

$(y + 2) = \sqrt{3}x$

$\mathbf{y - \sqrt{3}x + 2 = 0}$

Equation of second line,

$(y - 2) = \tan\left(\frac{2\pi}{3}\right)(x - 0)$

$y - 2 = -\sqrt{3}x$

$\mathbf{y + \sqrt{3}x - 2 = 0}$

The equation of line parallel to this line and passing through (0, –2),

$(y + 2) = -\sqrt{3}x$

$\mathbf{y + \sqrt{3}x + 2 = 0}$

8. Question

Find the equations of the straight lines which cut off an intercept 5 from the y – axis and are equally inclined to the axes.

Answer

A line which cut off an intercept 5 from the y – axis are equally to axes.

To Find: Find the equation

The formula used: The equation of the line is y = mx + c

Explanation: If a line is equally inclined to the axis, then

Angle $\theta = 45^o$ and $\theta = (180 - 45) = 135^o$

Then, The slope of the line, m = tan θ

m = 1

Since, the intercept is 5, C = 5

Now, The equation of the line is y = mx + c

$y = 1x + 5$

$y - x = 5$

Hence, The equation of the line is $y - x = 5$.

9. Question

Find the equation of the line which intercepts a length 2 on the positive direction of the x – axis and is inclined at an angle of 135° with the positive direction of the y – axis.

Answer

Given, A line which cut off an intercept a length w from the x – axis.

To Find: Find the equation

Formula used: The equation of line is $[(y - y_1) = m(x - x_1)]$

Explanation: If a line is inclined at angle 135^o on y – axis, then angle on the x – axis is

Angle $\theta = 135^o$ and $\theta = (180 - 135) = 45^o$

Then, The slope of the line, $m = \tan\theta$

$m = 1$

Since the line passes through the point (2,0)

Now, The equation of line is $(y - y_1) = m(x - x_1)$

$(y - 0) = 1(x - 2)$

$y = x - 2$

$x - y - 2 = 0$

Hence, The equation of the line is $x - y - 2 = 0$.

10. Question

Find the equation of the straight line which divides the join of the points (2, 3) and (– 5, 8) in the ratio 3 : 4 and is also perpendicular to it.

Answer

Given, A line which divides the join of the points (2,3) and (– 5,8) in the ratio 3:4

To Find : The equation of the line.

Explanation: The coordinates of the point which divides the join of the points (2,3) and (– 5,8) in the ratio 3:4 is given by (x,y).

Coordinate of x when line divides in ratio m:n = $\frac{m(x_2) + n(x_1)}{m + n}$

$x = \frac{3(-5) + 4(-2)}{3 + 4}$

$x = -\frac{9}{7}$

Coordinate of y when line divides in ratio m:n = $\frac{m(y_2) + n(y_1)}{m + n}$

$y = \frac{3(8) + 4(3)}{3 + 4}$

$y = \frac{36}{7}$

The slope of the line with two points is, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Now, The slope of joining the points (2,3) and (– 5,8) = $\frac{8-3}{-5-2}$

m = $\frac{5}{-7}$

The equation of the line is

$y-\frac{36}{7} = \frac{7}{5}\left(x-\left(-\frac{9}{7}\right)\right)$

$y-\frac{36}{7} = \frac{7}{5}\left(x + \frac{9}{7}\right)$

$\frac{7y-36}{7} = \frac{7}{5}\left(\frac{7x + 9}{7}\right)$

$\frac{7y-36}{7} = \frac{49x + 63}{35}$

$\frac{7y-36}{1} = \frac{49x + 63}{5}$

35y – 180 = 49x + 63

49x – 35y + 229 = 0

Hence, The equation of line is 49x – 35y + 229 = 0

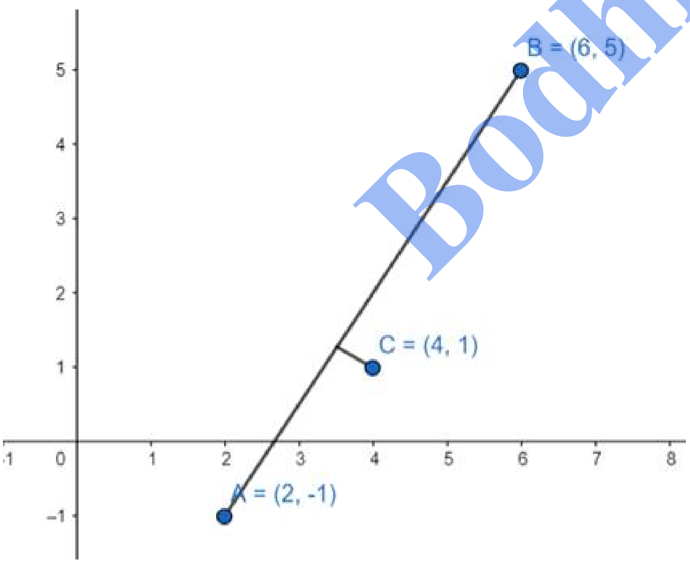
11. Question

Prove that the perpendicular drawn from the point (4, 1) on the join of (2, – 1) and (6 5) divides it in the ratio 5:8.

Answer

Given, A perpendicular drawn from the point (4,1) on the join of (2, – 1) and (6,5)

To Prove: The perpendicular divides the line in the ratio 5:8.



Explanation: Let us Assume,The perpendicular drawn from point C(4,1) on a line joining A(2, – 1) and B(6,5) divide in the ratio k:1 at the point R.

Now, The coordinates of R are:

By using Sectional Formula, (x,y) = $\frac{m(x_2) + n(x_1)}{m + n}, \frac{m(y_2) + n(y_1)}{m + n}$

R(x,y) = $\frac{6k + 2}{k + 1}, \frac{5k-1}{k + 1}$ – – – (1)

The slope of the line with two points is, m = $\frac{y_2-y_1}{x_2-x_1}$

The slope of AB = $\frac{5+1}{6-2}$

The slope of CR = $\frac{y-1}{x-4}$

And, PR is perpendicular to AB

Since, (Slope of CR)×(Slope of AB) = – 1

$$\left(\frac{y-1}{x-4}\right) \times \left(\frac{5 + 1}{6-2}\right) = -1$$

$$\frac{\left\{\frac{6k + 2}{k + 1}\right\} - 1}{\left\{\frac{5k-1}{k + 1}\right\} - 4} \times \frac{6}{4} = -1$$

$$\frac{5k - 1 - k - 1}{6k + 2 - 4k - 4} = -\frac{4}{6}$$

$$\frac{4k - 2}{2k - 2} = -\frac{2}{3}$$

3(4k – 2) = – 2(2k – 2)

12k – 6 = – 4k + 4

16k = 10

K = $\frac{5}{8}$

So, The ratio is 5:8

Hence, R divides AB in the ratio 5:8.

12. Question

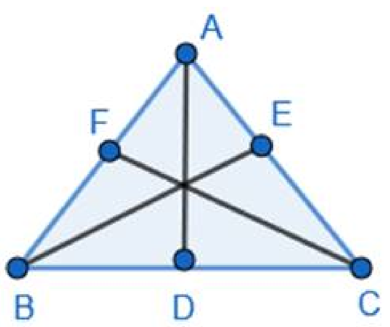
Find the equations to the altitudes of the triangle whose angular points are A (2, – 2), B(1, 1), and C (– 1, 0).

Answer

A triangle is given with three angular points A (2, – 2), B(1, 1), and C (– 1, 0)

To Find: Find the equation.

Formula Used: The equation of line is $(y - y_1) = m(x - x_1)$



Explanation: Here, AD, BE and CF are the three altitudes of the triangle.

Now,

We know, The slope of the line with two points is, m = $\frac{y_2 - y_1}{x_2 - x_1}$

So, The slope of BC = $\frac{0-1}{-1-1} = -\frac{1}{-2}$

The slope of AC = $\frac{0-(-2)}{-1-2} = \frac{2}{-3}$

The slope of AB = $\frac{1+2}{1-2} = \frac{3}{-1}$

and, The product of two slopes of the perpendicular line is always – 1

So, (slope of AB) × (slope of CF) = – 1

The slope of CF = $\frac{1}{\frac{3}{-1}} = \frac{1}{3}$

(slope of BE)×(slope of AC) = – 1

The slope of BE = $-\frac{1}{-\frac{2}{3}} = \frac{3}{2}$

(slope of AD)×(slope of BC) = – 1

The slope of AD = $-\frac{1}{\frac{1}{2}} = 2$

So, The equation of line is $(y - y_1) = m(x - x_1)$

The equation of Line AD is

y – (– 2) = – 2(x – 2)

y + 2 = – 2x + 2

2x + y – 2 = 0

The equation of Line BE is

$$y - 1 = \frac{3}{2}(x - 1)$$

2y – 2 = 3x – 3

2y – 3x + 1 = 0

The equation of Line CF is

$$y - 0 = \frac{1}{3}(x + 1)$$

x – 3y + 1 = 0

Hence, The equation of the three equation is calculated.

13. Question

Find the equation of the right bisector of the line segment joining the points (3, 4) and (– 1, 2).

Answer

Given, The line segment joining the points (3,4) and (– 1,2)

To Find: Find the equation of the line

Formula used: The equation of line is $(y - y_1) = m(x - x_1)$

Explanation: Here, The right bisector PQ of AB at C and is perpendicular to AB

Now, The coordinate of the mid – points = $\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$

The coordinates of point C are = $\left[\frac{3-1}{2}, \frac{4+2}{2}\right]$ = (1,3)

And, The slope of PQ = $-\frac{1}{\text{Slope of AB}}$

The slope of PQ, m = $-\frac{1}{2-4}(-1-3) = \frac{4}{-2}$

SO, The slope of PQ, m = – 2

The required equation of PQ is $(y - y_1) = m(x - x_1)$

y – 3 = – 2(x – 1)

y – 3 = – 2x + 2

y + 2x = 5

Hence, The equation of line is y + 2x = 5

14. Question

Find the equation of the line passing through the point (– 3, 5) and perpendicular to the line joining (2, 5) and (– 3, 6).

Answer

Given, A line which passes through the point (– 3,5) and perpendicular to the line joining (2,5) and (– 3,6)

To Find: Find the equation

Formula Used: The equation of line is $(y - y_1) = m(x - x_1)$

Explanation: Here, The line passes through the point (– 3,5), Given

So, The coordinate (x_1,y_1) = (– 3,5)

Now, The line is perpendicular to the line joining (2,5) and (– 3,6),

We know, The slope of the line with two points is, m = $\frac{y_2-y_1}{x_2-x_1}$

So, the slope of line joining (2, 5) and (– 3,6) is = $\frac{6-5}{-3-2}$

m = $-\frac{1}{5}$

Therefore, The slope of the required line is, m = $\frac{1}{\text{Slope of joining line }(2,5)\text{and}(-3,6)}$

So, m = $-\frac{1}{-\frac{1}{5}}$

m = 5

Now, The equation of straight line is $(y - y_1) = m(x - x_1)$

y – 5 = 5 (x – (– 3)

y – 5 = 5x + 15

5x – y + 20 = 0

Hence, The equation of line is 5x – y + 20 = 0

15. Question

Find the equation of the right bisector of the line segment joining the points A(1, 0) and B(2, 3).

Answer

Given, The line segment joining the points (1,0) and (2,3)

To Find: Find the equation of line

Formula used: The equation of line is $(y - y_1) = m(x - x_1)$

Explanation: Here, The right bisector PQ of AB at C and is perpendicular to AB

So, The slope of the line with two points is, $m = \frac{y_2 - y_1}{x_2 - x_1}$

The slope of the line AB $= \frac{3-0}{2-1} = 3$

We know, The product of two slopes of the perpendicular line is always – 1

Therefore, (slope of AB) × (slope of PQ) = – 1

Since Slope of PQ $= -\frac{1}{3}$

Now, The coordinate of the mid – points $= \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$

The coordinates of point C are $= \left[\frac{1+2}{2}, \frac{3+0}{2}\right] = \left[\frac{3}{2}, \frac{3}{2}\right]$

The required equation of PQ is $(y - y_1) = m(x - x_1)$

$$\left(y - \frac{3}{2}\right) = -\frac{1}{3}\left(x - \frac{3}{2}\right)$$

6y – 9 = – 2x + 3

x + 3y = 6

Hence, The equation of line is x + 3y = 6

Exercise 23.5

1 A. Question

Find the equation of the straight lines passing through the following pair of points:

(0, 0) and (2, - 2)

Answer

Given:

$(x_1, y_1) = (0, 0), (x_2, y_2) = (2, -2)$

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of the straight line passing through a pair of points.

Explanation:

So, the equation of the line passing through the two points (0, 0) and (2, −2) is

The formula used: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - 0 = \frac{-2-0}{2-0}(x - 0)$

⇒ y = -x

Hence, equation of line is y = -x

1 B. Question

Find the equation of the straight lines passing through the following pair of points:

(a, b) and (a + c sin α, b + c cos α)

Answer

Given:

$(x_1, y_1) = (a, b)$, $(x_2, y_2) = (a + c \sin\alpha, b + c \cos\alpha)$

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of the straight line passing through a pair of points.

Explanation:

So, the equation of the line passing through the two points (0, 0) and (2, −2) is

The formula used: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - b = \frac{b + c\cos\alpha - b}{a + c\sin\alpha - a}(x - a)$

$\Rightarrow y - b = \cot\alpha(x - a)$

Hence, equation of line is $y - b = \cot\alpha(x - a)$

1 C. Question

Find the equation of the straight lines passing through the following pair of points:

(0, - a) and (b, 0)

Answer

Given:

$(x_1, y_1) = (0, -a)$, $(x_2, y_2) = (b, 0)$

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2)

To find:

Equation of straight line passing through pair of points.

Explanation:

So, the equation of the line passing through the two points is

The formula used: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - a = \frac{0 + a}{b - 0}(x - 0)$

$\Rightarrow ax - by = c$

Hence, the equation of line is $ax - by = c$

1 D. Question

Find the equation of the straight lines passing through the following pair of points:

(a, b) and (a + b, a - b)

Answer

Given: $(x_1, y_1) = (a, b)$, $(x_2, y_2) = (a+b, a-b)$

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of the straight line passing through a pair of points.

Explanation:

So, the equation of the line passing through the two points is

The formula used: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - b = \frac{a-b-b}{a+b-a}(x - a)$

$\Rightarrow$ by – b^2 = (a – 2b)x – a^2 + 2ab

$\Rightarrow$ (a – 2b)x – by + b^2 + 2ab – a^2 = 0

Hence, the equation of line is (a – 2b)x – by + b^2 + 2ab – a^2 = 0

1 E. Question

Find the equation of the straight lines passing through the following pair of points:

(at_1, a/t_1) and (at_2, a/t_2)

Answer

Given: (x_1,y_1) = $\left(at_1, \frac{a}{t_1}\right)$, (x_2, y_2) = $\left(at_2, \frac{a}{t_2}\right)$

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of straight line passing through a pair of points.

Explanation:

So, the equation of the line passing through the two points is

The formula used: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - a/t_1 = \frac{\frac{a}{t_2} - \frac{a}{t_1}}{at_2 - at_1}(x - at_1)$

$\Rightarrow y - a/t_1 = \frac{-1}{t_2 t_1}(x - at_1)$

$\Rightarrow x + t_2 t_1 y = a(t_2 + t_1)$

Hence, the equation of the line is $x + t_2 t_1 y = a(t_2 + t_1)$

1 F. Question

Find the equation of the straight lines passing through the following pair of points:

(a cos α, a sin α) and (a cos β, a sin β)

Answer

Given: (x_1,y_1) = (a cos α, a sin α), (x_2,y_2) = (a cos β, a sin β)

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of straight line passing through a pair of points.

Explanation:

So, the equation of the line passing through the two points is

The formula used: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$

$\Rightarrow y(\cos\beta - \cos\alpha) - x(\sin\beta - \sin\alpha) - a\sin\alpha\cos\beta + a\sin\alpha\cos\alpha + a\cos\alpha\sin\beta - a\cos\alpha\sin\alpha = 0$

$\Rightarrow y(\cos\beta - \cos\alpha) - x(\sin\beta - \sin\alpha) = a\sin\alpha\cos\beta - a\cos\alpha\sin\beta$

$\Rightarrow 2y\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) - 2x\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) = a\sin(\alpha - \beta)$

$\Rightarrow 2y\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) + 2x\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) = a\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Dividing by $\sin\left(\frac{\alpha-\beta}{2}\right)$

$\Rightarrow 2y\sin\left(\frac{\alpha+\beta}{2}\right) + 2x\cos\left(\frac{\alpha+\beta}{2}\right) = a\cos\left(\frac{\alpha-\beta}{2}\right)$

Hence, the equation of the line is $2y\sin\left(\frac{\alpha+\beta}{2}\right) + 2x\cos\left(\frac{\alpha+\beta}{2}\right) = a\cos\left(\frac{\alpha-\beta}{2}\right)$

2 A. Question

Find the equations to the sides of the triangles the coordinates of whose angular points are respectively:

(1,4), (2, - 3) and (-1, - 2)

Answer

Given:

Points A (1, 4), B(2, -3) and C(-1, -2).

Assuming:

m_1, m_2, and m_3 be the slope of the sides AB, BC and CA, respectively.

Concept Used:

The slope of the line passing through the two points (x_1, y_1) and (x_2, y_2).

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of sides of the triangle.

Explanation:

$m_1 = \frac{-3-4}{2-1}$, $m_2 = \frac{-2+3}{-1-2}$, $m_3 = \frac{4+2}{1+1}$

$m_1 = -7$, $m_2 = -\frac{1}{3}$ and $m_3 = 3$

So, the equation of the sides AB, BC and CA are

Formula used: $y - y_1 = m(x - x_1)$

$y - 4 = -7(x - 1)$, $y + 3 = -\frac{1}{3}(x - 2)$ and $y + 2 = 3(x+1)$

$\Rightarrow 7x + y = 11$, $x + 3y + 7 = 0$ and $3x - y + 1 = 0$

Hence, equation of sides are $7x + y = 11$, $x + 3y + 7 = 0$ and $3x - y + 1 = 0$

2 B. Question

Find the equations to the sides of the triangles the coordinates of whose angular points are respectively:

(0,1), (2, 0) and (-1, - 2)

Answer

Given:

Points A (0, 1), B(2, 0) and C(-1, -2).

Assuming:

m_1, m_2 and m_3 be the slope of the sides AB, BC and CA, respectively.

Concept Used:

The slope of the line passing through the two points (x_1, y_1) and (x_2, y_2).

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of sides of the triangle.

Explanation:

$m_1 = \frac{0-1}{2-0}$, $m_2 = \frac{-2-0}{-1-2}$, $m_3 = \frac{1+2}{1+0}$

$m_1 = -\frac{1}{2}$, $m_2 = -\frac{2}{3}$ and $m_3 = 3$

So, the equation of the sides AB, BC and CA are

Formula used: $y - y_1 = m (x - x_1)$

$y - 1 = \frac{1}{2}(x - 0)$, $y - 0 = -\frac{2}{3}(x - 2)$ and $y + 2 = 3(x+1)$

$\Rightarrow x + 2y = 2$, $2x - 3y = 4$ and $3x - y + 1 = 0$

Hence, equation of sides are $x + 2y = 2$, $2x - 3y = 4$ and $3x - y + 1 = 0$

3. Question

Find the equations of the medians of a triangle, the coordinates of whose vertices are (-1, 6), (-3,-9) and (5, -8).

Answer

Given:

A (−1, 6), B (−3, −9) and C (5, −8) be the coordinates of the given triangle.

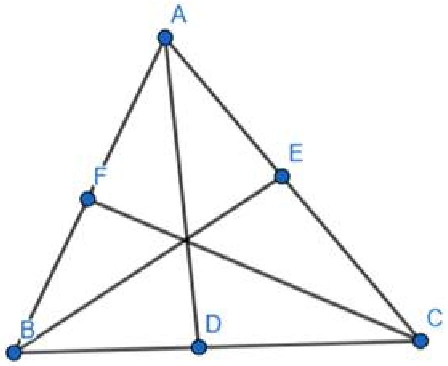
Assuming:

D, E, and F be midpoints of BC, CA and AB, respectively. So, the coordinates of D, E and F are

To find:

The equation of median of a triangle.

Explanation:



Median AD passes through A (-1, 6) and D (1, -17/2)

So, its equation is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$y - 6 = \frac{-\frac{17}{2} - 6}{1 + 1}(x + 1)$$

4y – 24 = -29x – 29

29x + 4y + 5 = 0

Median BE passes through B (-3,-9) and E (2,-1)

So, its equation is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$y + 9 = \frac{-1+9}{2+3}(x + 3)$$

5y + 45 = 8x + 24

8x – 5y – 21=0

Median CF passes through C (5,-8) and F(-2,-3/2)

So, its equation is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$\Rightarrow\ y + 9 = \frac{-\frac{3}{2}+8}{-2-5}(x - 5)$$

⇒ -14y – 112 = 13x – 65

⇒ 13x + 14y + 47 = 0

Hence, the equation of line is 13x + 14y + 47 = 0

4. Question

Find the equations to the diagonals of the rectangle the equations of whose sides are x = a, x = a', y = b and y = b'.

Answer

Given: The rectangle formed by the lines x = a, x = a’, y = b and y = b’

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2)

To find:

The equation of diagonal of the rectangle.

Explanation:

Clearly, the vertices of the rectangle are A(a, b), B(a’, b), C(a,’ b’) and D(a, b’) .

The diagonal passing through A (a, b) and C (a’, b’) is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$y - b = \frac{b' - b}{a' - a}(x - a)$$

⇒ (a’ – a)y – b(a’ – a) = (b’ – b)x – a(b’ – b)

⇒ (a’ – a) – (b’ – b)x = ba’ – ab’

And, the diagonal passing through B(a’, b) and D(a, b’) is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$y - b = \frac{b' - b}{a - a'}(x - a')$$

⇒ (a’ – a)y – b(a – a’) = (b’ – b)x – a’(b’ – b)

⇒ (a’ – a) – (b’ – b)x = a’b’ – ab

Hence, the equation of diagonals are (a’ – a) – (b’ – b)x = ba’ – ab’ and (a’ – a) – (b’ – b)x = a’b’ – ab

5. Question

Find the equation of the side BC of the triangle ABC whose vertices are A (-1, -2), B (0, 1) and C (2, 0) respectively. Also, find the equation of the median through A (-1, - 2).

Answer

Given: The vertices of triangle ABC are A (-1, -2), B(0, 1) and C(2, 0).

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2)

To find:

Equation of side BC of triangle ABC.

The equation of median through A.

Explanation:

So, the equation of BC is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$y - 1 = \frac{0-1}{2-0}(x - 0)$

$\Rightarrow y - 1 = \frac{-1}{2}(x - 0)$

⇒ x + 2y – 2 = 0

Let D be the midpoint of median AD is

So, $D\left(\frac{0+2}{2}, \frac{1+0}{2}\right) = \left(1, \frac{1}{2}\right)$

So, the equation of the median AD is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$$y + 2 = \frac{\frac{1}{2}+2}{1+1}(x + 1)$$

⇒ 4y + 8 = 5x + 5

⇒ 5x – 4y – 3 = 0

The equation of line BC is x + 2y – 2 = 0

Hence, the equation of median is 5x – 4y – 3 = 0

6. Question

By using the concept of the equation of a line, prove that the three points (- 2, - 2), (8, 2) and (3, 0) are collinear.

Answer

Given: points be A (-2, 2), B (8, 2) and C(3,0).

To prove:

Points (- 2, - 2), (8, 2) and (3, 0) are collinear.

Explanation:

The equation of the line passing through A (-2,-2) and B (8, 2) is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$$y + 2 = \frac{2+2}{8+2}(x + 2)$$

⇒ 5y + 10 = 2x + 4

⇒ 2x – 5y – 6 = 0

Clearly, point C (3, 0) satisfies the equation 2x – 5y – 6 = 0

Hence Proved, the given points are collinear.

7. Question

Prove that the line y - x + 2 = 0 divides the join of points (3,-1) and (8, 9) in the ratio 2:3

Answer

Assuming:

y – x +2 = 0 divides the line joining the points (3, -1) and (8, 9) at the point P in the ratio k : 1

To prove:

Line y - x + 2 = 0 divides the join of points (3,-1) and (8, 9) in the ratio 2:3

Explanation:

$P = \left(\frac{3+8k}{k+1}, \frac{-1+9k}{k+1}\right)$

P lies on the y – x +2 = 0

Therefore,

$$\left(\frac{-1+9k}{k+1}\right) - \left(\frac{3+8k}{k+1}\right) + 2 = 0$$

⇒ -1 + 9k – 3 – 8k +2k + 2 = 0

$\Rightarrow$ 3k = 2

$\Rightarrow k = \frac{2}{3}$

Hence Proved, the line y – x +2 = 0 divides the line joining the points (3, -1) and (8, 9) in the ratio 2 : 3

8. Question

Find the equation to the straight line which bisects the distance between the points (a, b), (a', b') and also bisects the distance between the points (-a, b) and (a', - b').

Answer

Given: points be A (a, b), B(a', b'),C(-a, b) and D(a', -b')

Assuming:

P and Q be the mid points of AB and CD, respectively.

$P = \left(\frac{a+a'}{2}, \frac{b+b'}{2}\right)$

$Q = \left(\frac{a'-a}{2}, \frac{b'-b}{2}\right)$

Explanation:

The equation of the line passing through P and Q is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$$y - \frac{b+b'}{2} = \frac{\frac{b'-b}{2} - \frac{b+b'}{2}}{\frac{a'-a}{2} - \frac{a'+a}{2}}\left(x - \frac{a'+a}{2}\right)$$

$\Rightarrow 2y - b - b' = \frac{b'}{a}(2x - a - a')$

$\Rightarrow$ 2ay - 2b'x = ab – a'b'

Hence, the equation of the required straight line is 2ay - 2b'x = ab – a'b'

9. Question

In what ratio is the line joining the points (2, 3) and (4, -5) divided by the line passing through the points (6, 8) and (- 3, -2).

Answer

Given: the equation of the line joining the points (6, 8) and (-3, -2) is

To find: In what ratio line joining the points divided by a line.

Assuming: 10x – 9y +12 = 0 divide the line joining the points (2, 3) and (4, 5) at points P in the ratio k : 1

Explanation:

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$\Rightarrow y - 8 = \frac{-2-8}{-3-6}(x - 6)$

$\Rightarrow$ 10x – 9y +12 = 0

$P = \left(\frac{4k+2}{k+1}, -\frac{5k+3}{k+1}\right)$

P lies on the 10x – 9y +12 = 0

Therefore,

$$10\left(\frac{4k+2}{k+1}\right) - 9\left(\frac{5k+3}{k+1}\right) + 12 = 0$$

⇒ 40k + 20 + 45k – 27 +12k + 12 = 0

⇒ 97k + 5 = 0

⇒ $K = -\frac{5}{97}$

Hence, the joining the points (2, 3) and (4, 5) is divided by the line passing through the points (6, 8) and (-3, -2) in the ratio 5: 97 externally.

10. Question

The vertices of a quadrilateral are A (-2, 6), B (1, 2), C (10, 4) and D (7, 8). Find the equations of its diagonals.

Answer

Given: the two diagonals of the quadrilateral with vertex A (-2, 6), B(1, 2), C(10, 4) and D(7, 8) are

AC and BD.

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).

To find:

The equation of diagonal of the quadrilateral.

Explanation:

The equation of AC passing through A (-2, 6) and C (10, 4) is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$y - 6 = \frac{4-6}{10+2}(x + 2)$

⇒ x + 6y – 34 = 0

And the equation of AC passing through, B (1, 2) and D(7, 8) is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$y - 2 = \frac{8-2}{7-1}(x - 1)$

⇒ x – y + 1 = 0

Hence, the equation of the diagonal are x + 6y – 34 = 0 and x – y + 1 = 0

11. Question

The length L (in centimeters) of a copper rod is a linear function of its Celsius temperature C. In an experiment if L =124.942 when C =20 and L =125.134 when C =110, express L in terms of C.

Answer

Assuming:

C along the x-axis and L along the y-axis

Given:

Points (20, 124.942) and (110, 125.134) in CL plane.

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2)

To find:

The equation of L in term of C.

Explanation:

L is a linear function of C, the equation of the line passing through (20, 124.942) and(110, 125.134) is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$L - 124.942 = \frac{125.134 - 124.942}{110 - 20}(C - 20)$$

$\Rightarrow L - 124.942 = \frac{0.192}{90}(C - 20)$

$\Rightarrow L - 124.942 = \frac{0.032}{15}(C - 20)$

$\Rightarrow L = \frac{0.032}{15}C + 124.942 - 20 \times \frac{0.032}{15}$

$\Rightarrow L = \frac{0.032}{15}C + 124.942 - 0.04267$

$\Rightarrow L = \frac{4}{1875}C + 124.899$

Hence, the equation of L in term of C is $L = \frac{4}{1875}C + 124.899$

12. Question

The owner of a milk store finds that he can sell 980 liters milk each week at Rs. 14 per liter and 1220 liters of milk each week at Rs. 16 per liter. Assuming a linear relationship between selling price and demand, how many liters could he sell weekly at Rs. 17 per liter.

Answer

Assuming:

x denotes the price per liter, and y denote the quality of the milk sold at this price.

Since there is a linear relationship between the price and the quality, the line representing this

Given:

Relationship passes through (14, 980) and (16, 1220).

To find:

How many liters could he sell weekly at Rs. 17 per liter.

Explanation:

So, the equation of the line passing through these points is

Formula used: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$\Rightarrow y - 980 = 120(x - 14)$

$\Rightarrow 120x - y - 700 = 0$

When x = 17 then we have,

$120(17) - y - 700 = 0$

$\Rightarrow y = 1340$

Hence, the owner of the milk store can shell 1340 litres of milk at Rs. 17 per litre.

13. Question

Find the equation of the bisector of angle A of the triangle whose vertices are A (4, 3), B (0, 0) and C (2,3).

Answer

Given: the vertices of triangle ABC are A (4, 3), B (0, 0) and C (2, 3).

To find:

The equation of bisector of angle A.

Explanation:

Let us find the lengths of sides AB and AC.

AB = $\sqrt{(4-0)^2+(3-0^2)} = 5$

AC = $\sqrt{(4-2)^2+(3-3^2)} = 2$

We know that the internal bisector AD of angle BAC divides BC in the ratio AB: AC, i.e. 5: 2

D $\left(\frac{2\times0+5\times2}{5+2}, \frac{2\times0+5\times3}{5+2}\right) = \left(\frac{10}{7}, \frac{15}{7}\right)$

Thus, the equation of AD is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$$y - 3 = \frac{3-\frac{15}{7}}{4-\frac{10}{7}}(x - 4)$$

$\Rightarrow y - 3 = \frac{1}{4}(x - 4)$

⇒ x – 3y +5 = 0

Hence, the equation of line is x – 3y +5 = 0

14. Question

Find the equations to the straight lines which go through the origin and trisect the portion of the straight line 3 x + y = 12 which is intercepted between the axes of coordinates.

Answer

To find:

The equation of the required line.

Assuming:

The line 3x + y = 12 intersect the x-axis and the y-axis at A and B, respectively.

y =m_1x and m_2x be the lines passing through the origin and trisect the line 3x + y =12 at P and Q.

Explanation:

At x = 0

0 + y =12

⇒ y =12

At y = 0

3x + 0 =12

⇒ x = 4

∴ A (4, 0) and B (0,12)

y =m_1x and m_2x be the lines passing through the origin and trisect the line 3x + y =12 at P and Q.

AP = PQ = QB

Let us find the coordinates of P and Q.

P = $\left(\frac{2\times4+1\times0}{2+1}, \frac{2\times0+1\times12}{2+1}\right) = \left(\frac{8}{3}, 4\right)$

Q = $\left(\frac{1\times4+2\times0}{2+1}, \frac{1\times0+2\times12}{2+1}\right) = \left(\frac{4}{3}, 8\right)$

Clearly, P and Q lie on y = m_1x and y = m_2x, respectively.

∴ $4 = m_1 \times \frac{8}{3}$ and $8 = m_2 \times \frac{4}{3}$

⇒ $m_1 = \frac{3}{2}$ and m_2 = 6

Hence, the required lines are $y = \frac{3}{2}x$

⇒ 2y = 3x and y = 6x

Hence, the equation of line is 2y = 3x and y = 6x

15. Question

Find the equations of the diagonals of the square formed by the lines x = 0, y = 0, x = 1 and y = 1.

Answer

Given:

The square formed by the lines x = 0, x = 1, y = 0 and y = 1.

Concept Used:

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2)

To find:

The equation of diagonal of the square.

Explanation:

Clearly, the vertices of the square are A(0, 0), B(1, 0), C(1, 1) and D(0,1) .

The diagonal passing through A (0, 0) and C(1, 1) is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$$y - 0 = \frac{1-0}{1-0}(x - 0)$$

⇒ y = x

And, the diagonal passing through B(1, 0) and D(0, 1) is

Formula used: $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

$$y - 0 = \frac{1-0}{0-1}(x - 1)$$

⇒ y = -x + 1

⇒ x + y = 1

Hence, the equation of diagonals are y = x and x + y = 1.

Exercise 23.6

1 A. Question

Find the equation to the straight line

cutting off intercepts 3 and 2 from the axes.

Answer

Given:

Here, a = 3, b = 2

To find:

The equation of line cutoff intercepts from the axes.

Explanation:

So, the equation of the line is

Formula used: $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{3}+\frac{y}{2}=1$

⇒ 2x + 3y =6

Hence the equation of line cut off intercepts 3 and 2 from the axes is 2x + 3y = 6

1 B. Question

Find the equation to the straight line

cutting off intercepts -5 and 6 from the axes.

Answer

Given:

Here, a = -5, b=6

To find:

The equation of line cutoff intercepts from the axes.

Explanation:

So, the equation of the line is

Formula used: $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{-5}+\frac{y}{6}=1$

⇒ 6x -5y =-30

Hence, the equation of line cut off intercepts -5 and 6 from the axes is 6x – 5y = -30

2. Question

Find the equation of the straight line which passes through (1, -2) and cuts off equal intercepts on the axes.

Answer

Given:

A line passing through (1, -2)

Assuming:

The equation of the line cutting equal intercepts at coordinates of length ‘ a ‘ is

Explanation:

Formula used: $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{a}+\frac{y}{a}=1$

⇒ x + y =a

The line x + y =a passes through (1, -2) So the point satisfy the equation

1 -2 =a

⇒ a = -1

Hence the equation of the line is x+ y = -1

3 A. Question

Find the equation to the straight line which passes through the point (5, 6) and has intercepts on the axes

Equal in magnitude and both positive

Answer

Given:

Here, a = b

To find:

The equation of line cutoff intercepts from the axes.

Explanation:

So, the equation of the line is

Formula used: $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{a}+\frac{y}{a}=1$

⇒ x + y = a

The line passes through the point (5, 6) So equation satisfy the points,

⇒ 5 + 6 = a

⇒ a = 11

Hence the equation of the line is x + y = 11

3 B. Question

Find the equation to the straight line which passes through the point (5, 6) and has intercepts on the axes

Equal in magnitude but opposite in sign

Answer

Given:

Here, b = -a

To find:

The equation of line cutoff intercepts from the axes.

Explanation:

So, the equation of the line is

Formula used: $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{a}+\frac{y}{-a}=1$$

⇒ x – y =a

The line passes through the point (5, 6) So equation satisfy the points,

⇒5 – 6 = a

⇒ a = -1

The equation of the line is x – y = -1

4. Question

For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x – 3y + 6 = 0 on the axes.

Answer

Given:

Intercepts cut off on the coordinate axes by the line ax + by +8 = 0 ......(i)

And are equal in length but opposite in sign to those cut off by the line

2x – 3y +6 = 0 ......(ii)

Explanation:

The slope of two lines are equal

The slope of the line (i) is $-\frac{a}{b}$

The slope of the line (ii) is $\frac{2}{3}$

$$\therefore\ -\frac{a}{b}=\frac{2}{3}$$

$a=-\frac{2b}{3}$

The length of the perpendicular from the origin to the line (i) is

The formula used: $d=\left|\frac{ax+by+d}{\sqrt{a^2+b^2}}\right|$

$d_1=\left|\frac{a(0)+b(0)+8}{\sqrt{a^2+b^2}}\right|$

$d_1=\frac{8\times 3}{\sqrt{13b^2}}$

The length of the perpendicular from the origin to the line (ii) is

The formula used: $d=\left|\frac{ax+by+d}{\sqrt{a^2+b^2}}\right|$

$d_2=\left|\frac{2(0)-3(0)+6}{\sqrt{2^2+3^2}}\right|$

Given: $d_1=d_2$

$$\frac{8 \times 3}{\sqrt{13b^2}} = \frac{6}{\sqrt{13}}$$

$\Rightarrow$ b = 4

$$\therefore \; a = -\frac{2b}{3} = -\frac{8}{3}$$

Hence the value of a and b is $-\frac{8}{3}, 4.$

5. Question

Find the equation to the straight line which cuts off equal positive intercepts on the axes and their product is 25

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

To find:

The equation of the line which cutoff intercepts on the axes.

Given:

Here a = b and ab = 25

Explanation:

$\therefore a^2 = 25$

$\Rightarrow$ a = 5 since we are to take only positive value of intercepts

Hence, the equation of the required line is

Formula used: $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{5} + \frac{y}{5} = 1$$

$\Rightarrow$ x + y = 5

Hence, the equation of line is x + y = 5

6. Question

Find the equation of the line which passes through the point (-4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5: 3 by this point.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

Given:

The line $\frac{x}{a} + \frac{y}{b} = 1$ intersects the axes (a,0) and (0,b).

Explanation:

So, (-4,3) divides the line segment AB and the ratio 5:3

$-4 = \frac{5+3a}{5+3}, 3 = \frac{5b}{5+3}$

$\Rightarrow$ a $= -\frac{32}{3}$, b $= \frac{24}{5}$

So, the equation of the line is $\frac{x}{-\frac{32}{3}} + \frac{y}{\frac{24}{5}} = 1$

⇒ 9x – 20y = -96

Hence, the equation of line is 9x – 20y = -96

7. Question

A straight line passes through the point (α, β) and this point bisects the portion of the line intercepted between the axes. Show that the equation of the straight line is $\frac{x}{2\alpha} + \frac{y}{2\beta} = 1$.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

Given:

The line intersects the axis at A (a, 0) and B (b, 0)

Explanation:

Here, (α, β) is the midpoint of AB

⇒ $\alpha = \frac{a+0}{2}$, $\beta = \frac{0+b}{2}$

⇒ $\alpha = \frac{a}{2}$, $\beta = \frac{b}{2}$

Hence, The equation is $\frac{x}{2\alpha} + \frac{y}{2\beta} = 1$

8. Question

Find the equation of the line which passes through the point (3, 4) and is such that the portion of it intercepted between the axes is divided by the point in the ratio 2 : 3.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

Assuming:

The line meets the coordinate axes at A and B, So the coordinates A (a, 0) and B (0, b)

AP : BP =2 : 3

Here p = (3, 4)

$\therefore\ 3 = \frac{2 \times 0 + 3 \times a}{2 + 3}, 4 = \frac{2 \times b + 3 \times 0}{2 + 3}$

⇒ 3a = 15,2b = 20

⇒ a = 5, b = 10

Thus the equation of the line is

Formula used: $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{5} + \frac{y}{10} = 1$

⇒ 2x + y = 10

9. Question

Point R (h, k) divided line segments between the axes in the ratio 1 : 2. Find the equation of the line.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a}+\frac{y}{b}=1$

Given:

The line passes through R(h, k)

Explanation:

$\therefore\ \frac{h}{a}+\frac{k}{b}=1$ ……(i)

The line intersects the coordinate axes at A(a, 0) and B(0, b).

Here, AP : BP = 1 : 2

$\therefore\ h=\frac{1\times 0+2\times a}{1+2}, k=\frac{1\times b+2\times 0}{1+2}$

$\Rightarrow a=\frac{3h}{2}$, b = 3k

Substituting $a=\frac{3h}{2}$, b = 3k in $\frac{x}{a}+\frac{y}{b}=1$

$\frac{2x}{3h}+\frac{y}{3k}=1$

⇒2kx + hy – 3hk = 0

Hence, the equation of the line is 2kx + hy – 3hk = 0

10. Question

Find the equation of the straight line which passes through the point (-3, 8) and cuts off positive intercepts on the coordinate axes whose sum is 7

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a}+\frac{y}{b}=1$

Given:

Here a + b = 7, b = 7 – a

Explanation:

The line is passing through (-3, 8).

$\frac{-3}{a}+\frac{8}{b}=1$

Substituting b =7 – a, we get

$\frac{x}{a}+\frac{y}{7-a}=1$

$\Rightarrow -3(7-a)+8a=7a-a^2$

⇒ a^2 + 4a – 21 = 0

⇒ (a – 3)(a + 7)= 0

⇒ a = 3 (since, a can only be positive)

Substituting a = 3 in equation (i) we get,

b = 7 – 3 = 4

Hence, the equation of the line is $\frac{x}{3} + \frac{y}{4} = 1$

11. Question

Find the equation to the straight line which passes through the point (-4, 3) and is such that the portion of it between the axes is divided by the point in the ratio 5 : 3.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

Given:

The line $\frac{x}{a} + \frac{y}{b} = 1$ intersects the axes (a,0) and (0,b).

Explanation:

So, (-4,3) divides the line segment AB and the ratio 5:3

$-4 = \frac{5+3a}{5+3}, 3 = \frac{5b}{5+3}$

⇒ a = $-\frac{32}{3}$, b = $\frac{24}{5}$

So, the equation of the line is $\frac{x}{-\frac{32}{3}} + \frac{y}{\frac{24}{5}} = 1$

⇒ 9x – 20y = -96

Hence, the equation of line is 9x – 20y = -96

12. Question

Find the equation of a line which passes through the point (22, -6) and is such that the intercept on x-axis exceeds the intercept on the y-axis by 5.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

Given:

Here, a =b+5 ....(1)

Explanation:

The line passing through the point (22, -6)

$\frac{22}{a} + \frac{6}{b} = 1$.........(2)

Substituting a = b + 5 from equation (1) in equation (2)

$$\frac{22}{b-5} + \frac{6}{b} = 1$$

$22b - 6b - 30 = b^2 + 5b$

(b – 5)(b – 6)=0

b = 5, 6

From equation (1)

When b = 5 then a = 10

When b = 6 then a = 11

Thus the equation of the required line is

$\frac{x}{10}+\frac{y}{5}=1$ and $\frac{x}{11}+\frac{y}{6}=1$

Thus the equations are

x + 2y=10, 6x +11y=66

Hence, the equation of line is x + 2y=10, 6x +11y=66

13. Question

Find the equation of the line, which passes through P(1, -7) and meets the axes at A and B respectively so that 4AP – 3BP = 0.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a}+\frac{y}{b}=1$

Assuming:

The line meets the coordinate axes at A and B, So the coordinates A (a, 0) and B (0, b)

Given:

4AP – 3BP = 0

Explanation:

⇒ AP : BP =3 : 4

Here p= (1, -7)

$$\therefore\ 1=\frac{3\times 0+4\times a}{3+4},-7=\frac{3\times b+4\times 0}{3+4}$$

⇒4a = 7,3b = - 49

⇒a $=\frac{7}{4}$, b $=-\frac{49}{3}$

Thus the equation of the line is

$\frac{x}{\frac{7}{4}}+\frac{y}{-\frac{49}{3}}=1$

$\Rightarrow\frac{4x}{7}+\frac{-3y}{49}=1$

⇒ 28x – 3y = 49

14. Question

Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a}+\frac{y}{b}=1$

Given:

Here, a+b = 9

Explanation:

⇒ b = 9 – a ......(i)

The line is passing through (2, 2).

∴ $\frac{2}{a}+\frac{2}{b}=1$ ......(ii)

From equation (i) and (ii)

$$\frac{2}{a}+\frac{2}{9-a}=1$$

⇒18 – 2a + 2a = 9a – a^2

⇒a^2 – 9a +18 = 0

⇒ (a – 3)(a – 6) = 0

⇒a = 3, 6

For a = 3, b = 9 – 3 = 6

For a = 6, b = 9 – 6 = 3

Thus the equation of line is

$\frac{x}{3}+\frac{y}{6}=1$ or $\frac{x}{6}+\frac{y}{3}=1$

⇒2x + y = 6 or x + 2y = 6

Hence, the equation of line is 2x + y = 6 or x + 2y = 6

15. Question

Find the equation of the straight line which passes through the point P(2, 6) and cuts the Coordinates axes at the point A and B respectively so that $\frac{\mathrm{AP}}{\mathrm{BP}}=\frac{2}{3}$.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a}+\frac{y}{b}=1$

Assuming:

The line meets the coordinate axes at A and B, So the coordinates A (a, 0) and B (0, b)

Given:

AP : BP =2 : 3

Explanation:

Here p= (2, 6)

$$\therefore\ 2=\frac{2\times0+3\times a}{2+3}, 6=\frac{2\times b+3\times0}{2+3}$$

⇒3a = 10,2b = 30

⇒$a\ =\ \frac{10}{3}$, b = 15

Thus the equation of the line is

$\frac{x}{\frac{10}{3}} + \frac{y}{15} = 1$

$\Rightarrow \frac{3x}{10} + \frac{y}{15} = 1$

⇒ 9x + 2y = 30

16. Question

Find the equations of the straight lines each of which passes through the point (3, 2) and cuts off intercepts a and b respectively on x and y-axes such that a – b = 2.

Answer

Concept Used:

The equation of the line with intercepts a and b is $\frac{x}{a} + \frac{y}{b} = 1$

Given:

Here, a – b = 2

⇒ a = b + 2 ......(i)

Explanation:

The line is passing through (3,2).

$\therefore\ \frac{3}{a} + \frac{2}{b} = 1$ ......(ii)

From equation (i) and (ii)

$$\frac{3}{b+2} + \frac{2}{b} = 1$$

⇒3b + 2b +4 = b^2 + 2b

⇒b^2 – 3b – 4 = 0

⇒ (b – 4)(b + 1) = 0⇒b = 4, -1

Now, from equation (i)

For b = 4, a = 4 + 2 = 6

For b = -1, b = -1 + 2 = 1

Thus the equation of line is

$\frac{x}{1} + \frac{y}{-1} = 1$ or $\frac{x}{6} + \frac{y}{4} = 1$

⇒x – y = 1 or 2x + 3y = 12

17. Question

Find the equations of the straight lines which pass through the origin and trisect the portion of the straight line 2x + 3y = 6 which is intercepted between the axes.

Answer

To find: Equations of the straight lines which pass through the origin and trisect the portion of the line which is intercepted between the axes.

Assuming:

The line 2x + 3y = 6 intercept the x-axis and the y-axis at A and B, respectively.

Explanation:

At x = 0 we have,

$3y + 0 = 6$

$\Rightarrow 3y = 6$

$\Rightarrow y = 2$

At $y = 0$ we have,

$2x + 0 = 6$

$\Rightarrow x = 3$

A = (3, 0) and B = (0, 2)

Let $y = m_1x$ and $y = m_2x$ pass through origin trisecting the line $2x + 3y = 6$ at P and Q.

AP = PQ = QB

Let us find the coordinates of P and Q using the section formula

$P = \left(\frac{2\times3+1\times0}{2+1}, \frac{2\times0+1\times2}{2+1}\right) = (2, \frac{2}{3})$

$Q = \left(\frac{1\times3+2\times0}{2+1}, \frac{1\times0+2\times2}{2+1}\right) = (1, \frac{4}{3})$

Clearly, P and Q lie on $y = m_1x$ and $y = m_2x$, respectively

$\therefore \frac{2}{3} = m_1 \times 2 \text{ and } \frac{4}{3} = m_2$

$\Rightarrow m_1 = \frac{1}{3}$ and $m_2 = \frac{4}{3}$

Hence, the required lines are

$y = \frac{x}{3}$ and $y = \frac{4x}{3}$

$\Rightarrow x - 3y = 0$ and $4x - 3y = 0$

Hence, the equation of line is $x - 3y = 0$ and $4x - 3y = 0$

18. Question

Find the equation of the straight line passing through the point (2, 1) and bisecting the portion of the straight line $3x - 5y = 15$ lying between the axes.

Answer

Concept Used:

The equation of a line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$

Given:

The line passes through (2, 1)

$\therefore \frac{2}{a} + \frac{1}{b} = 1$ ......(i)

Assuming:

The line $3x - 5y = 15$ intercept the x-axis and the y-axis at A and B, respectively.

Explanation:

At $x = 0$ we have,

$0 - 5y = 15$

$\Rightarrow 5y = -15$

$\Rightarrow y = -3$

At y = 0 we have,

3x – 0 =15

⇒ x = 5

A= (0, -3) and B = (5, 0)

The midpoint of AB is $(\frac{5}{2}, -\frac{3}{2})$

Clearly, the point $(\frac{5}{2}, -\frac{3}{2})$ lies on the line $\frac{x}{a} + \frac{y}{b} = 1$

$\therefore\ \frac{5}{2a} - \frac{3}{2b} = 1$ ……(ii)

Using $\frac{3}{2} \times$ eq(i) + eq(ii) we get,

$$\frac{3}{a} + \frac{5}{2a} = \frac{3}{2} + 1$$

$\Rightarrow a = \frac{11}{5}$

For $a = \frac{11}{5}$ we have,

$$\frac{10}{11} + \frac{1}{b} = 1$$

⇒ b = 11

Therefore, the equation of the required line is:

$$\frac{x}{\frac{11}{5}} + \frac{y}{11} = 1$$

$$\frac{5x}{11} + \frac{y}{11} = 1$$

⇒ 5x + y = 11

Hence, the equation of line is 5x + y = 11

19. Question

Find the equation of the straight line passing through the origin and bisecting the portion of the line ax + by + c = 0 intercepted between the coordinate axes.

Answer

Concept Used:

The equation of the line passing through the origin is y = mx

To find:

Equation of the straight line passing through the origin and bisecting the portion of a line intercepted between the coordinate axes.

Assuming:

The line ax + by + c = 0 meets the coordinate axes at A and B.

Explanation:

So, the coordinate of A and B are $A(-\frac{c}{a}, 0)$ and B $(0, -\frac{c}{b})$

Now,

The midpoint of AB is $\left(-\frac{c}{2a}, -\frac{c}{2b}\right)$

$\therefore -\frac{c}{2b} = m \times -\frac{c}{2a}$

$\Rightarrow m = \frac{a}{b}$

Hence, the equation of the required line is

$y = \frac{a}{b}x$

⇒ ax – by = 0

Exercise 23.7

1 A. Question

Find the equation of a line for which

p = 5, α = 60°

Answer

Given: p = 5, α = 60°

Concept Used:

Equation of line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

x cos 60° + y sin 60° = 5

$\Rightarrow \frac{x}{2} + \frac{\sqrt{3}y}{2} = 5$

⇒ x + √3y = 10

Hence, the equation of line in normal form is x + √3y = 10.

1 B. Question

Find the equation of a line for which

p = 4, α = 150°

Answer

Given: p = 4, α = 150°

Concept Used:

Equation of line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

x cos 150° + y sin 150° = 4

cos (180° – θ) = – cos θ , sin (180° – θ) = sin θ

⇒ x cos(180° – 30°) + y sin(180° – 30°) = 4

⇒ – x cos 30° + y sin 30° = 4

$\Rightarrow -\frac{\sqrt{3}x}{2} + \frac{y}{2} = 4$

⇒ √3x – y + 8 = 0

Hence, the equation of line in normal form is √3 x – y + 8 = 0

1 C. Question

Find the equation of a line for which

p = 8, α = 225°

Answer

Given p = 8, α = 225°

Concept Used:

Equation of line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

x cos 225° + y sin 225° = 8

We know, cos (180° + θ) = – cos θ , sin (180° + θ) = – sin θ

⇒ – cos 45° – ysin 45° = 8

$\Rightarrow -\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 8$

⇒ x + y + 8√2 = 0

Hence, the equation of line in normal form is x + y + 8√2 = 0

1 D. Question

Find the equation of a line for which

p = 8, α = 300°

Answer

Given: p = 8, α = 300°

Concept Used:

Equation of line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

x cos 300° + y sin 300° = 8

⇒ x cos (360° – 60°) + y sin (360° – 60°) = 8

We know, cos (360° – θ) = cos θ, sin (360° – θ) = – sin θ

⇒ x cos60° – y sin60° = 8

$\Rightarrow \frac{x}{2} - \frac{\sqrt{3}y}{2} = 8$

⇒ x – √3y = 16

Hence, the equation of line in normal form is x – √3y = 16

2. Question

Find the equation of the line on which the length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x–axis is 30°.

Answer

Given: p = 4, α = 30∘

Concept Used:

Equation of line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

⇒ x cos30° + y sin30° = 4

⇒ $x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 4$

$\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$

⇒ √3 x + y = 8

Hence, the equation of line is √3 x + y = 8.

3. Question

Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with the positive direction of x–axis is 15°.

Answer

Given: p = 4, α = 15°

Concept Used:

Equation of line in normal form.

Explanation:

We know that, cos 15° = cos (45° – 30°) = cos45°cos30° + sin45°sin30°

cos(A – B) = cosAcosB + sinAsinB

$$\Rightarrow \cos 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

And sin 15 = sin (45° – 30°) = sin 45° cos 30° – cos 45° sin 30°

sin (A – B) = sinAcosB – cosAsinB

$$\Rightarrow \sin 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

$$\Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = 4$$

⇒ (√3 + 1)x + (√3 – 1)y = 8√2

Hence, the equation of line in normal form is (√3 + 1)x + (√3–1)y = 8√2

4. Question

Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle α given by $\tan\alpha = \frac{5}{12}$ with the positive direction of x–axis.

Answer

Given: p = 3, $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\therefore \tan\alpha = \frac{5}{12}$

$\Rightarrow \sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$

Concept Used:

The equation of a line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

$\Rightarrow \frac{12x}{13} + \frac{5y}{13} = 3$

⇒ 12x + 5y = 39

Hence, the equation of line in normal form is 12x + 5y = 39

5. Question

Find the equation of the straight line on which the length of the perpendicular from the origin is 2 and the perpendicular makes an angle α with x–axis such that $\sin\alpha = \frac{1}{3}$.

Answer

Given: p = 2, sinα = 1/3

We know that, cos $\alpha = \sqrt{1 - \sin^2\alpha}$

$\Rightarrow \cos\alpha = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

Concept Used:

The equation of a line in normal form.

Explanation:

So, the equation of the line in normal form is

Formula Used: x cos α + y sin α = p

$\Rightarrow \frac{2\sqrt{2}}{3}x + \frac{y}{3} = 2$

⇒ 2√2x + y = 6

Hence, the equation of line in normal form is 2√2x + y = 6

6. Question

Find the equation of the straight line upon which the length of the perpendicular from the origin is 2, and the slope of this perpendicular is $\frac{5}{12}$.

Answer

Assuming:

The perpendicular drawn from the origin make acute angle α with the positive x–axis. Then, we have, tanα = 5/12

We know that, tan(180∘ + α) = tanα

So, there are two possible lines, AB and CD, on which the perpendicular drawn from the origin has a slope equal to 5/12 .

Given:

Now tan α = 5/12

⇒ $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = 12/13$

Explanation:

So, the equations of the lines in normal form are

Formula Used: x cos α + y sin α = p

⇒ x cos α + y sin α = p and x cos(180° + α) + ysin(180° + α) = p

⇒ x cos α + y sin α = 2 and –x cos α – ysin α = 2

cos (180° + θ) = – cos θ , sin (180° + θ) = – sin θ

⇒ $\frac{12x}{13} + \frac{5y}{13} = 26$ and 12x + 5y = – 26

Hence, the equation of line in normal form is $\frac{12x}{13} + \frac{5y}{13} = 26$ and 12x + 5y = – 26

7. Question

The length of the perpendicular from the origin to a line is 7, and the line makes an angle of 150^0 with the positive direction of y–axis. Find the equation of the line.

Answer

Assuming:

AB be the given line which makes an angle of 150^0 with the positive direction of y–axis and OQ be the perpendicular drawn from the origin on the line.

Given:

p = 7 and α = 30°

Explanation:

So, the equation of the line AB is

Formula Used: x cos α + y sin α = p

⇒ x cos 30° + y sin 30° = 7

⇒ $\frac{\sqrt{3}x}{2} + \frac{y}{2} = 7$

⇒ √3 x + y = 14

Hence, the equation of line in normal form is √3 x + y = 14

8. Question

Find the value of θ and p if the equation x cos θ + y sin θ = p is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Answer

Given: the normal form of a line is x cos θ + y sin θ = p .... (1)

To find:

P and θ.

Explanation:

Let us try to write down the equation √3 + y + 2 = 0 in its normal form.

Now √3 + y + 2 = 0

⇒ √3 + y = – 2

Dividing both sides by 2,

⇒ – √3/2 – y/2 = 1

⇒ $\left(-\frac{\sqrt{3}}{2}\right)x + \left(-\frac{1}{2}\right)y = 1$ ...... (2)

Comparing equations (1) and (2) we get,

$\cos\theta = -\frac{\sqrt{3}}{2}$ and p = 1

⇒ θ = 210° = 7π/6 and p = 1

Hence, θ = 210° = 7π/6 and p = 1

9. Question

Find the equation of the straight line which makes a triangle of the area $96\sqrt{3}$ with the axes and perpendicular from the origin to it makes an angle of 30^0 with y–axis.

Answer

Assuming:

AB be the given line, and OL = p be the perpendicular drawn from the origin on the line.

Given:

α = 60°

Explanation:

So, the equation of the line AB is

Formula Used: x cos θ + y sin θ = p

⇒ x cos 60° + y sin 60° = p

⇒ $\frac{x}{2} + \frac{\sqrt{3}}{2}y = p$

⇒ x + √3y = 2p ...... (1)

Now, in triangles OLA and OLB

Cos 60° = $\frac{OL}{OA}$ cos30° = $\frac{OL}{OB}$

⇒ $\frac{1}{2} = \frac{o}{OA}$ and $\frac{\sqrt{3}}{2} = \frac{p}{OB}$

⇒ OA = 2p and OB = $\frac{2p}{\sqrt{3}}$

It is given that the area of triangle OAB is 96√3

$\therefore \frac{1}{2} \times OA \times OB = 96\sqrt{3}$

$\Rightarrow \frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = 96\sqrt{3}$

$\Rightarrow p^2 = 12^2$

⇒ p = 12

Substituting the value of p in (1)

x + √3 y = 24

Hence, the equation of the line AB is x + √3 y = 24

10. Question

Find the equation of a straight line on which the perpendicular from the origin makes an angle of 30° with x–axis and which forms a triangle of the area $50\sqrt{3}$ with the axes.

Answer

Assuming: AB be the given line, and OL = p be the perpendicular drawn from the origin on the line.

Given: α = 60°

Explanation:

So, the equation of the line AB is

x cos θ + y sin θ = p

⇒ x cos 30 + y sin 30 = p

$\Rightarrow \frac{\sqrt{3}}{2}x + \frac{y}{2} = p$

⇒ √3x + y = 2p ...... (1)

Now, in triangles OLA and OLB

Cos 30° $= \frac{OL}{OA}$, cos6° $= \frac{OL}{OB}$

$\Rightarrow \frac{1}{2} = \frac{o}{OB}, \frac{\sqrt{3}}{2} = \frac{p}{OA}$

⇒ OA = $\frac{2p}{\sqrt{3}}$ and OB = 2p

It is given that the area of triangle OAB is 50√3

$\therefore \frac{1}{2} \times OA \times OB = 50\sqrt{3}$

$\Rightarrow \frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = 50\sqrt{3}$

$\Rightarrow p^2 = 75$

⇒ p = √75

Substituting the value of p in (1)

√3 x + y = √75

Hence, the equation of the line AB is √3 x + y = √75

Exercise 23.8

1. Question

A line passes through a point A (1, 2) and makes an angle of 60^0 with the x–axis and intercepts the line x + y = 6 at the point P. Find AP.

Answer

Given: (x_1,y_1) = A(1, 2), θ = 60°

To find:

Distance AP.

Explanation:

So, the equation of the line is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$\Rightarrow \frac{x-1}{\cos 60°} = \frac{y-2}{\sin 60°} = r$

$\Rightarrow \frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$

Here, r represents the distance of any point on the line from point A (1, 2).

The coordinate of any point P on this line are $\left(1 + \frac{r}{2}, 2 + \frac{\sqrt{3}}{2}r\right)$

Clearly, P lies on the line x + y = 6

$\Rightarrow 1 + \frac{r}{2} + 2 + \frac{\sqrt{3}}{2}r = 6$

$\Rightarrow \frac{\sqrt{3}}{2}r + \frac{r}{2} = 3$

$\Rightarrow r(\sqrt{3} + 1) = 6$

$\Rightarrow r = \frac{6}{\sqrt{3}+1} = 3\left(\sqrt{3} - 1\right)$

Therefore, AP = $3(\sqrt{3} - 1)$

2. Question

If the straight line through the point P(3, 4) makes an angle $\pi / 6$ with the x–axis and meets the line 12x + 5y + 10 = 0 at Q, find the length PQ.

Answer

Given: (x_1,y_1) = A(3, 4), $\theta = \frac{\pi}{6} = 30°$

To find:

Length PQ.

Explanation:

So, the equation of the line is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$\Rightarrow \frac{x-3}{\cos 30°} = \frac{y-4}{\sin 30°} = r$

$\Rightarrow \frac{x-3}{\frac{\sqrt{3}}{2}} - \frac{y-4}{\frac{1}{2}} = r$

$\Rightarrow x - \sqrt{3}y + 4\sqrt{3} - 3 = 0$

Let PQ = r

Then, the coordinate of Q are given by

$$\frac{x-3}{\cos 30°} = \frac{y-4}{\sin 30°} = r$$

$\Rightarrow x = 3 + \frac{\sqrt{3}}{2}r,\ y = 4 + \frac{r}{2}$

The coordinate of point Q is $\left(3 + \frac{\sqrt{3}}{2}r, 4 + \frac{r}{2}\right)$

Clearly, Q lies on the line 12x + 5y + 10 = 0

$$\therefore\ 12\left(3 + \frac{\sqrt{3}}{2}r\right) + 5\left(4 + \frac{r}{2}\right) + 10 = 0$$

$\Rightarrow 66 + \frac{12\sqrt{3}+5}{2}r = 0$

$\Rightarrow r = -\frac{132}{5+12\sqrt{3}}$

$\therefore$ PQ = $|r| = \frac{132}{5+12\sqrt{3}}$

Hence, the length of PQ is $\frac{132}{5+12\sqrt{3}}$

3. Question

A straight line drawn through the point A (2, 1) making an angle $\pi/4$ with positive x–axis intersects another line x + 2y + 1 = 0 in the point B. Find length AB.

Answer

Given: (x_1, y_1) = A(2, 1), $\theta = \frac{\pi}{4} = 45°$

To find:

Length AB.

Explanation:

So, the equation of the line is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$\Rightarrow \frac{x-2}{\cos 45°} = \frac{y-1}{\sin 45°} = r$

$\Rightarrow \frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r$

$\Rightarrow$ x - y - 1 = 0

Let PQ = r

Then, the coordinate of Q is given by

$$\frac{x-2}{\cos 45°} = \frac{y-1}{\sin 45°} = r$$

$\Rightarrow x = 2 + \frac{1}{\sqrt{2}}r,\ y = 1 + \frac{r}{\sqrt{2}}$

The coordinate of point Q is $\left(2 + \frac{1}{\sqrt{2}}r, 1 + \frac{r}{\sqrt{2}}\right)$

Clearly, Q lies on the line x + 2y + 1 = 0

$\therefore\ 2 + \frac{1}{\sqrt{2}}r + 2\left(1 + \frac{r}{\sqrt{2}}\right) + 1 = 0$

$\Rightarrow 5 + \frac{3r}{\sqrt{2}}r = 0$

$\Rightarrow r = -\frac{5\sqrt{2}}{3}$

Hence, the length of AB is $-\frac{5\sqrt{2}}{3}$

4. Question

A line a drawn through A (4, – 1) parallel to the line 3x – 4y + 1 = 0. Find the coordinates of the two points on this line which are at a distance of 5 units from A.

Answer

Given: (x_1,y_1) = A(4, – 1)

To find:

Coordinates of the two points on this line which are at a distance of 5 units from A.

Explanation:

Line 3x – 4y + 1 = 0

⇒ 4y = 3x + 1

$\Rightarrow y = \frac{3}{4}x + \frac{1}{4}$

Slope $\tan\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$

So, the equation of the line passing through A (4, −1) and having slope $\frac{3}{4}$ is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$\Rightarrow \frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}}$

⇒ 3x – 4y = 16

Here, AP = r = ± 5Thus, the coordinates of P are given by

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$\Rightarrow \frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}} = r$

$\Rightarrow x = \frac{4r}{5} + 4$ and $y = \frac{3r}{5} - 1$

$\Rightarrow x = \frac{4(\pm 5)}{5} + 4$ and $y = \frac{3(\pm 5)}{5} - 1$

⇒ x = ±4 + 4 and y = ±3–1

So x = 8, 0 and y = 2, – 4

Hence, the coordinates of the two points at a distance of 5 units from A are (8, 2) and (0, −4).

5. Question

The straight line through $P(x_1, y_1)$ inclined at an angle θ with the x–axis meets the line ax + by + c = 0 in Q.

Find the length of PQ.

Answer

Given: the equation of the line that passes through $P(x_1, y_1)$ and makes an angle of θ with the x–axis.

To find:

Length of PQ.

Explanation:

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta}$$

Let PQ = rThen, the coordinates of Q are given by

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$\Rightarrow x = x_1 + r\cos\theta, y = y_1 + r\sin\theta$

Thus, the coordinates of Q are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

Clearly, Q lies on the line ax + by + c = 0.

$\therefore a(x_1 + r\cos\theta) + b(y_1 + r\sin\theta) + c = 0$

$\Rightarrow r = -\frac{ax_1 + by_1 + c}{\cos\theta + \sin\theta}$

$\therefore PQ = -\frac{ax_1 + by_1 + c}{\cos\theta + \sin\theta}$

Thus, length $PQ = -\frac{ax_1 + by_1 + c}{\cos\theta + \sin\theta}$

6. Question

Find the distance of the point (2, 3) from the line 2x – 3y + 9 = 0 measured along a line making an angle of 45° with the x–axis.

Answer

Given: $(x_1,y_1) = A(2, 3)$, $\theta = \frac{\pi}{4} = 45°$

To find:

Distance of point from line.

Explanation:

So, the equation of the line is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$\Rightarrow \frac{x-2}{\cos 45°} = \frac{y-3}{\sin 45°} = r$

$\Rightarrow \frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-3}{\frac{1}{\sqrt{2}}} = r$

$\Rightarrow x - y + 1 = 0$

Let PQ = r

Then, the coordinate of Q are given by

$$\frac{x-2}{\cos 45°} = \frac{y-3}{\sin 45°} = r$$

$\Rightarrow x = 2 + \frac{1}{\sqrt{2}}r,\ y = 3 + \frac{r}{\sqrt{2}}$

The coordinate of point Q is $\left(2 + \frac{1}{\sqrt{2}}r, 3 + \frac{r}{\sqrt{2}}\right)$

Clearly, Q lies on the line 2x – 3y + 9 = 0

$$\therefore\ 2\left(2 + \frac{1}{\sqrt{2}}r\right) - 3\left(3 + \frac{r}{\sqrt{2}}\right) + 9 = 0$$

$\Rightarrow 4 + \frac{2r}{\sqrt{2}} - 9 - \frac{3r}{\sqrt{2}} + 9 = 0$

$\Rightarrow r = 4\sqrt{2}$

Hence, the distance of the point from the given line is $4\sqrt{2}$.

7. Question

Find the distance of the point (3, 5) from the line 2x + 3y = 14 measured parallel to a line having slope 1/2.

Answer

Given: (x_1,y_1) = A(3, 5), $\tan\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{1^2+2^2}}$ and $\cos\theta = \frac{2}{\sqrt{1^2+2^2}}$

To find:

The distance of a point from the line parallel to another line.

Explanation:

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$\Rightarrow \frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{\sqrt{5}}}$

$\Rightarrow$ x – 2y + 7 = 0

Let x – 2y + 7 = 0 intersect the line 2x + 3y = 14 at point P.

Let AP = r

Then, the coordinate of P is given by

$$\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{\sqrt{5}}} = r$$

$\Rightarrow x = 3 + \frac{2r}{\sqrt{5}}$ and $y = 5 + \frac{r}{\sqrt{5}}$

Thus, the coordinate of P is $\left(3 + \frac{2r}{\sqrt{5}}, 5 + \frac{r}{\sqrt{5}}\right)$

Clearly, P lies on the line 2x + 3y = 14

$$\therefore 2\left(3 + \frac{2r}{\sqrt{5}}\right) + 3\left(5 + \frac{r}{\sqrt{5}}\right) = 14$$

$\Rightarrow 6 + \frac{4r}{\sqrt{5}} + 15 + \frac{3r}{\sqrt{5}} = 14$

$\Rightarrow \frac{7r}{\sqrt{5}} = -7$

$\Rightarrow r = -\sqrt{5}$

Hence, the distance of the point (3, 5) from the line 2x + 3y = 14 is $\sqrt{5}$

8. Question

Find the distance of the point (2, 5) from the line 3x + y + 4 = 0 measured parallel to a line having slope 3/4.

Answer

Given: (x_1, y_1) = A(2, 5), $\tan\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{3}{\sqrt{3^2+4^2}}$ and $\cos\theta = \frac{4}{\sqrt{3^2+4^2}}$

To find:

The distance of a point from the line parallel to another line.

Explanation:

So, the equation of the line passing through (2, 5) and having a slope $\frac{3}{4}$ is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$\Rightarrow \frac{x-3}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}}$

⇒ 3x – 4y + 14 = 0

Let 3x – 4y + 7 = 0 intersect the line 3x + y + 4 = 0 at point P.

Let AP = r

Then, the coordinate of P are given by

$$\frac{x-3}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}} = r$$

$\Rightarrow x = 3 + \frac{4r}{5}$ and $y = 5 + \frac{3r}{5}$

Thus, the coordinate of P is $\left(3 + \frac{4r}{5}, 5 + \frac{3r}{5}\right)$

Clearly, P lies on the line 3x + y + 4 = 0

$\therefore 2\left(3 + \frac{4r}{5}\right) + 3\left(5 + \frac{3r}{5}\right) + 4 = 0$

$\Rightarrow 6 + \frac{12r}{5} + 5 + \frac{3r}{5} + 4 = 0$

⇒ 3r = – 15

⇒ r = – 5

Hence, the distance of the point (2, 5) from the line 3x + y + 4 = 0 is 5

9. Question

Find the distance of the point (3, 5) from the line 2x + 3y = 14 measured parallel to the line x – 2y = 1.

Answer

Given: (x_1, y_1) = A(3, 5)

To find:

The distance of a point from the line parallel to another line.

Explanation:

It is given that the required line is parallel to x – 2y = 1

⇒ 2y = x – 1

⇒ $y = \frac{1}{2}x - \frac{1}{2}$

$\therefore \tan\theta = \frac{1}{2}$

⇒ $\sin\theta = \frac{1}{\sqrt{1^2+2^2}}$ and $\cos\theta = \frac{2}{\sqrt{1^2+2^2}}$

So, the equation of the line is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

⇒ $\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{\sqrt{5}}}$

⇒ x – 2y + 7 = 0

Let x – 2y + 7 = 0 intersect the line 2x + 3y = 14 at point P.

Let AP = r

Then, the coordinate of P is given by

$$\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{\sqrt{5}}} = r$$

⇒ $x = 3 + \frac{2r}{\sqrt{5}}$ and $y = 5 + \frac{r}{\sqrt{5}}$

Thus, the coordinate of P is $\left(3 + \frac{2r}{\sqrt{5}}, 5 + \frac{r}{\sqrt{5}}\right)$

Clearly, P lies on the line 2x + 3y = 14

$\therefore 2\left(3 + \frac{2r}{\sqrt{5}}\right) + 3\left(5 + \frac{r}{\sqrt{5}}\right) = 14$

⇒ $6 + \frac{4r}{\sqrt{5}} + 15 + \frac{3r}{\sqrt{5}} = 14$

⇒ $\frac{7r}{\sqrt{5}} = -7$

⇒ $r = -\sqrt{5}$

Hence, the distance of the point (3, 5) from the line 2x + 3y = 14 is $\sqrt{5}$

10. Question

Find the distance of the point (2, 5) from the line 3x + y + 4 = 0 measured parallel to the line 3x – 4y + 8 = 0.

Answer

Given: (x_1,y_1) = A(2,5)

To find:

The distance of a point from the line parallel to another line.

Explanation:

It is given that the required line is parallel to $3x - 4y + 8 = 0$

$\Rightarrow 4y = 3x + 8$

$\Rightarrow y = \frac{3}{4}x + 2$

$\therefore \tan\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$

So, the equation of the line is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta}$$

$$\Rightarrow \frac{x - 1}{\frac{4}{5}} = \frac{y - 5}{\frac{3}{5}}$$

$\Rightarrow 3x - 6 = 4y - 20$

$\Rightarrow 3x - 4y + 14 = 0$

Let the line $3x - 4y + 14 = 0$ cut the line $3x + y + 4 = 0$ at P.

Let AP = r Then, the coordinates of P are given by

$$\frac{x - 1}{\frac{4}{5}} = \frac{y - 5}{\frac{3}{5}} = r$$

$\Rightarrow x = 2 + \frac{4r}{5}, y = 5 + \frac{3r}{5}$

Thus, the coordinates of P are $\left(2 + \frac{4r}{5}, 5 + \frac{3r}{5}\right)$

Clearly, P lies on the line $3x + y + 4 = 0$.

$\therefore 3\left(2 + \frac{4r}{5}\right) + 5 + \frac{3r}{5} + 4 = 0$

$\Rightarrow 6 + \frac{12r}{5}5 + \frac{3r}{5} + 4 = 0$

$\Rightarrow 15 + \frac{15r}{5} = 0$

$\Rightarrow r = -5$

$\therefore AP = |r| = 5$

11. Question

Find the distance of the line $2x + y = 3$ from the point $(-1, -3)$ in the direction of the line whose slope is 1.

Answer

Given: $(x_1, y_1) = A(-1, -3)$

And $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

To find:

The distance of a point from the line in the direction of the line.

Explanation:

So, the equation of the line passing through (– 1, – 3) and having slope 1 is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$\Rightarrow \frac{x+1}{\frac{1}{\sqrt{2}}} = \frac{y+3}{\frac{1}{\sqrt{2}}}$

⇒ x – y = 2

Let x – y = 2 intersect the line 2x + y = 3 at point P.

Let AP = r

Then, the coordinate of P is given by

$$\frac{x+1}{\frac{1}{\sqrt{2}}} = \frac{y+3}{\frac{1}{\sqrt{2}}} = r$$

$\Rightarrow x = \frac{r}{\sqrt{2}} - 1$ and $y = \frac{r}{\sqrt{2}} - 3$

Thus, the coordinate of P is $\left(\frac{r}{\sqrt{2}} - 1, \frac{r}{\sqrt{2}} - 3\right)$

Clearly, P lies on the line 2x + y = 3

$\therefore 2\left(\frac{r}{\sqrt{2}} - 1\right) + \left(\frac{r}{\sqrt{2}} - 3\right) = 3$

$\Rightarrow \frac{3r}{\sqrt{2}} - 5 = 3$

$\Rightarrow 3r = 8\sqrt{2}$

$\Rightarrow r = \frac{8\sqrt{2}}{3}$

Hence, the distance of the point (– 1, – 3) from the line 2x + y = 3 is $\frac{8\sqrt{2}}{3}$

12. Question

A line is such that its segment between the straight line 5x – y – 4 = 0 and 3x + 4y – 4 = 0 is bisected at the point (1, 5). Obtain its equation.

Answer

Assuming:

P_1 P_2 be the intercept between the lines 5x − y − 4 = 0 and 3x + 4y − 4 = 0.

P_1 P_2 makes an angle θ with the positive x–axis.

Given: (x_1, y_1) = A (1, 5)

Explanation:

So, the equation of the line passing through A (1, 5) is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$$\Rightarrow \frac{x-1}{\cos\theta} = \frac{y-5}{\sin\theta}$$

$$\Rightarrow \frac{y-5}{x-1} = \tan\theta$$

Let $AP_1 = AP_2 = r$

Then, the coordinates of P1 and P2 are given by

$\frac{x-1}{\cos\theta} = \frac{y-5}{\sin\theta} = r$ and $\frac{x-1}{\cos\theta} = \frac{y-5}{\sin\theta} = -r$

So, the coordinates of P_1 and P_2 are 1 + rcosθ, 5 + rsinθ and 1 – rcosθ, 5 – rsinθ, respectively.

Clearly, P_1 and P_2 lie on $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$, respectively.

∴5(1 + rcos θ) – 5 – rsin θ – 4 = 0 and 3(1 – r cos θ) + 4(5 – rsin θ) + 4(5 – r sin θ) – 4 = 0

$\Rightarrow r = \frac{4}{5\cos\theta - \sin\theta}$ and $r = \frac{19}{3\cos\theta + 4\sin\theta}$

$$\Rightarrow \frac{4}{5\cos\theta - \sin\theta} = \frac{19}{3\cos\theta + 4\sin\theta}$$

⇒ 95 cos θ – 19 sinθ = 12 cos + 16 sinθ

⇒ 83 cosθ = 35 sinθ

⇒ tan θ = 83/35

Thus, the equation of the required line is

$$\frac{y-5}{x-1} = \tan\theta$$

$$\Rightarrow \frac{y-5}{x-1} = \frac{83}{35}$$

⇒ 83x – 35y + 92 = 0

Hence, the equation of line is 83x – 35 y + 92 = 0

13. Question

Find the equation of straight line passing through (– 2, – 7) and having an intercept of length 3 between the straight lines 4x + 3y = 12 and 4x + 3y = 3.

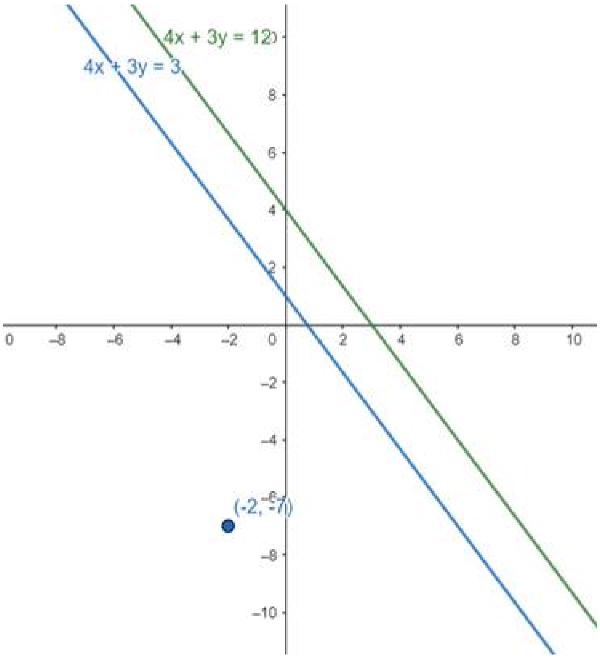
Answer

Given: (x_1, y_1) = A (– 2, – 7)

To find:

Equation of required line.

Explanation:



So, the equation of the line is

Formula Used: $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$\Rightarrow \dfrac{x + 2}{\cos\theta} = \dfrac{y + 7}{\sin\theta}$

Let the required line intersect the lines 4x + 3y = 3 and 4x + 3y = 12 at P_1 and P_2.

Let $AP_1 = r_1$ and $AP_2 = r_2$

Then, the coordinates of P_1 and P_2 are given by $\frac{x+2}{\cos\theta} = \frac{y+7}{\sin\theta} = r_1$ and $\frac{x+2}{\cos\theta} = \frac{y+7}{\sin\theta} = r_2$ respectively. Thus, the coordinates of P_1 and P_2 are $(-2 + r_1\cos\theta, -7 + r_1\sin\theta)$ and $(-2 + r_2\cos\theta, -7 + r_2\sin\theta)$, respectively.

Clearly, the points P_1 and P_2 lie on the lines 4x + 3y = 3 and 4x + 3y = 12

$4(-2 + r_1\cos\theta) + 3(-7 + r_1\sin\theta) = 3$ and $4(-2 + r_2\cos\theta) + 3(-7 + r_2\sin\theta) = 12$,

$\Rightarrow r_1 = \frac{32}{4\cos\theta + 3\sin\theta}$ and $r_2 = \frac{41}{4\cos\theta + 3\sin\theta}$

Here $AP_2 - AP_1 = 3$

$\Rightarrow r_2 - r_1 = 3$

$\Rightarrow \dfrac{41}{4\cos\theta + 3\sin\theta} - \dfrac{32}{4\cos\theta + 3\sin\theta} = 3$

$\Rightarrow 3 = 4\cos\theta + 3\sin\theta$

$\Rightarrow 3(1 - \sin\theta) = 4\cos\theta$

$\Rightarrow 9(1 + \sin^2\theta - 2\sin\theta) = 16\cos^2\theta = 16(1 - \sin^2\theta)$

$\Rightarrow 25\sin^2\theta - 18\sin\theta - 7 = 0$

$\Rightarrow (\sin\theta - 1)(25\sin\theta + 7) = 0$

$\Rightarrow$ Sin θ = 1, sin θ = – 7/25

$\Rightarrow$ Cos θ = 0, cos θ = 24/25

Thus, the equation of the required line is

x + 2 = 0 or $\frac{x+2}{\frac{24}{25}} = \frac{y+7}{-\frac{7}{25}}$

⇒ x + 2 = 0 or 7x + 24 y + 182 = 0

Hence, the equation of line is x + 2 = 0 or 7x + 24 y + 182 = 0

Exercise 23.9

1. Question

Reduce the equation $\sqrt{3}x + y + 2 = 0$ to:

(i) slope - intercept form and find slope and y - intercept;

(ii) Intercept form and find intercept on the axes

(iii) The normal form and find p and α.

Answer

(i) Given: $\sqrt{3}x + y + 2 = 0$

Explanation:

$\Rightarrow y = -\sqrt{3}x - 2$

This is the slope intercept form of the given line.

Hence, slope = - √3 and y - intercept = - 2

(ii) Given: $\sqrt{3}x + y + 2 = 0$

Explanation:

$\Rightarrow \sqrt{3}\,x + y = -2$

Dividing both sides by - 2

$$\Rightarrow \frac{\sqrt{3}}{-2}x + \frac{y}{-2} = 1$$

Hence, the intercept form of the given line. Here, x - intercept = - 23 and y - intercept = - 2

(iii) Given: $\sqrt{3}x + y + 2 = 0$

Explanation:

$\Rightarrow -\sqrt{3}\,x - y = 2$

$$\Rightarrow -\frac{\sqrt{3}x}{\sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}} - \frac{y}{\sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}} = \frac{2}{\sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$\Rightarrow -\frac{\sqrt{3}x}{2} - \frac{y}{2} = 1$$

This is the normal form of the given line.

Hence, p = 1 cos $\alpha = -\frac{\sqrt{3}}{2}$ and sinα = - 1/2

Hence, α = 210

2 A. Question

Reduce the following equations to the normal form and find p and α in each case :

$x + \sqrt{3}y - 4 = 0$

Answer

Given: $x + \sqrt{3}y - 4 = 0$

Explanation:

$\Rightarrow x + \sqrt{3}y = 4$

$$\Rightarrow \frac{x}{\sqrt{1^2 + (\sqrt{3})^2}} + \frac{\sqrt{3}y}{\sqrt{1^2 + (\sqrt{3})^2}} = \frac{4}{\sqrt{1^2 + (\sqrt{3})^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$\Rightarrow \frac{x}{2} + \frac{\sqrt{3}y}{2} = 2$$

Hence, the normal form of the given line, where p = 2, cosα = 1/2 and sin α $=\frac{\sqrt{3}}{2}$

⇒ α = π/3

2 B. Question

Reduce the following equations to the normal form and find p and α in each case :

$x + y + \sqrt{2} = 0$

Answer

Given: $x + y + \sqrt{2} = 0$

Explanation:

$\Rightarrow -x - y = \sqrt{2}$

$$\Rightarrow \frac{-x}{\sqrt{(-1)^2 + (-1)^2}} + \frac{y}{\sqrt{(-1)^2 + (-1)^2}} = \frac{\sqrt{2}}{\sqrt{(-1)^2 + (-1)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$\Rightarrow -\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 1$$

Hence, the normal form of the given line, where p = 1, cosα $= -\frac{1}{\sqrt{2}}$ and sin α $= -\frac{1}{\sqrt{2}}$

⇒ α = 225 [The coefficient of xand y are negative So lies in third quadrant]

2 C. Question

Reduce the following equations to the normal form and find p and α in each case :

$x - y + 2\sqrt{2} = 0$

Answer

Given: $x - y + 2\sqrt{2} = 0$

Explanation:

$\Rightarrow -x + y = 2\sqrt{2}$

$$\Rightarrow \frac{-x}{\sqrt{(-1)^2 + (1)^2}} + \frac{y}{\sqrt{(-1)^2 + (1)^2}} = \frac{2\sqrt{2}}{\sqrt{(-1)^2 + (1)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$\Rightarrow -\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 2$$

Hence, the normal form of the given line, where p = 2, cosα $= \frac{-1}{\sqrt{2}}$ and sin α = $\frac{1}{\sqrt{2}}$

⇒ α = 135

The coefficient of x and y are negative and positive respectively. So, α lies in the second quadrant

2 D. Question

Reduce the following equations to the normal form and find p and α in each case :

x – 3 = 0

Answer

Given: x – 3 = 0

Explanation:

⇒ x = 3

⇒ x + 0 × y = 3

$$\Rightarrow \frac{x}{\sqrt{(1)^2 + (0)^2}} + 0 \times \frac{y}{\sqrt{(1)^2 + (0)^2}} = \frac{3}{\sqrt{(1)^2 + (0)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

⇒ x + 0 × y = 3

Hence, the normal form of the given line, where p = 3, cosα = 1 and sin α = 0

⇒ α = 0

2 E. Question

Reduce the following equations to the normal form and find p and α in each case :

y – 2 = 0

Answer

Given: y – 2 = 0

Explanation:

⇒ y = 2

⇒ 0*x + y = 2

$$\Rightarrow 0 \times \frac{x}{\sqrt{(1)^2 + (0)^2}} + \frac{y}{\sqrt{(1)^2 + (0)^2}} = \frac{2}{\sqrt{(1)^2 + (0)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } y)^2 + (\text{coefficient of } x)^2}$

⇒ 0 * x + y = 2

Hence, the normal form of the given line, where p = 2, cosα = 0 and sin α = 1

⇒ α = 90

3. Question

Put the equation $\frac{x}{a}+\frac{y}{b}=1$ the slope intercept form and find its slope and y - intercept.

Answer

Given: the equation is $\frac{x}{a} + \frac{y}{b} = 1$

Concept Used:

General equation of line y = mx + c.

Explanation:

bx + ay = ab

⇒ ay = - bx + ab

⇒ $y = -\frac{b}{a}x + b$

Hence, the slope intercept form of the given line.

∴ Slope = - b/a and y - intercept = b

4. Question

Reduce the lines 3x – 4y + 4 = 0 and 2x + 4y – 5 = 0 to the normal form and hence find which line is nearer to the origin.

Answer

Given:

The normal forms of the lines 3x − 4y + 4 = 0 and 2x + 4y − 5 = 0.

To find:

In given normal form of a line, Which is nearer to the origin.

Explanation:

$\Rightarrow\ -3x + 4y = 4$

$$\Rightarrow -\frac{3x}{\sqrt{(-3)^2 + (4)^2}} + 4\frac{y}{\sqrt{(-3)^2 + (4)^2}} = \frac{4}{\sqrt{(-3)^2 + (4)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$\Rightarrow\ -\frac{3}{5}x + \frac{4}{5}y = \frac{4}{5}$ …... (1)

Now 2x + 4y = - 5

⇒ - 2x – 4y = 5

$$\Rightarrow -\frac{2x}{\sqrt{(-2)^2 + (-4)^2}} - 4\frac{y}{\sqrt{(-2)^2 + (-4)^2}} = \frac{5}{\sqrt{(-2)^2 + (-4)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$\Rightarrow\ -\frac{2}{2\sqrt{5}}x - \frac{4}{2\sqrt{5}}y = \frac{5}{2\sqrt{5}}$ …... (2)

From equations (1) and (2):

45<525

Hence, the line 3x − 4y + 4 = 0 is nearer to the origin.

5. Question

Show that the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.

Answer

Given: The lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.

To prove:

The origin is equidistant from the lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.

Explanation:

Let us write down the normal forms of the given lines.

First line: 4x + 3y + 10 = 0

⇒ - 4x - 3y = 10

$$\Rightarrow -\frac{4x}{\sqrt{(-4)^2 + (-3)^2}} - 3\frac{y}{\sqrt{(-4)^2 + (-3)^2}} = \frac{10}{\sqrt{(-4)^2 + (-3)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$\Rightarrow -\frac{4}{5}x - \frac{3}{5}y = 2$$

∴ p = 2

Second line: 5x − 12y + 26 = 0

⇒ - 5x + 12y = 26

$$\Rightarrow -\frac{5\,x}{\sqrt{(-5)^2 + (12)^2}} + 12\frac{y}{\sqrt{(-5)^2 + (12)^2}} = \frac{26}{\sqrt{(-5)^2 + (12)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$\Rightarrow -\frac{5}{13}x + \frac{12}{13}y = 2$$

∴ p = 2

Third line: 7x + 24y = 50

$$\Rightarrow \frac{7\,x}{\sqrt{(7)^2 + (24)^2}} + 24\frac{y}{\sqrt{(7)^2 + (24)^2}} = \frac{50}{\sqrt{(7)^2 + (24)^2}}$$

Dividing both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$\Rightarrow \frac{7}{25}x + \frac{24}{25}y = 2$$

∴ p = 2

Hence, the origin is equidistant from the given lines.

6. Question

Find the values of θ and p, if the equation x cos θ + y sin θ = p is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Answer

Given: The normal form of the line $\sqrt{3}x + y + 2 = 0$.

To find:

Value of θ and p.

Explanation:

$\sqrt{3}x + y + 2 = 0$

Divide Both side by 2

$$\frac{\sqrt{3}}{2}x + \frac{y}{2} + 1 = 0$$

$$-\frac{\sqrt{3}}{2}x - \frac{y}{2} = 1$$

Comparing the equations xcos θ + ysin θ = p we get,

cos θ $= -\frac{\sqrt{3}}{2}$, sin θ $= -\frac{1}{2}$ and p = 1

∴ θ = 210∘ and p = 1

Hence, θ = 210∘ and p = 1

7. Question

Reduce the equation 3x – 2y + 6 = 0 to the intercept form and find the x and y - intercepts.

Answer

Given: equation is 3x − 2y + 6 = 0

Concept Used:

Line in intercepts form is $\frac{x}{a} + \frac{y}{b} = 1$(a and b are x and y intercepts resp.)

Explanation:

3x − 2y = - 6

⇒ $\frac{3}{-6}x + \frac{2y}{6} = 1$ [Dividing both sides by - 6]

⇒ $\frac{x}{-2} + \frac{y}{3} = 1$

Thus, the intercept form of the given line

∴ x - intercept = −2 and y - intercept = 3

8. Question

The perpendicular distance of a line from the origin is 5 units, and its slope is - 1. Find the equation of the line.

Answer

Given: slope = - 1 and p = 5

Assuming: c be the intercept on the y - axis.

Explanation:

Then, the equation of the line is

$y = - x + c$ [$\because m = - 1$]

$\Rightarrow x + y = c$

$\Rightarrow \frac{x}{\sqrt{1^2+1^2}} + \frac{y}{\sqrt{1^2+1^2}} = \frac{c}{\sqrt{1^2+1^2}}$

Dividing both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{c}{\sqrt{2}}$

This is the normal form of the given line.

Therefore, $\frac{c}{\sqrt{2}}$ denotes the length of the perpendicular from the origin.

But, the length of the perpendicular is 5 units.

$\therefore \left|\frac{c}{\sqrt{2}}\right| = 5$

$\Rightarrow c = \pm 5\sqrt{2}$

Thus, substituting $c = \pm 5\sqrt{2}$ in $y = - x + c$, we get the equation of line to be $y = -x + 5\sqrt{2}$ or $x + y - 5\sqrt{2} = 0$

Exercise 23.10

1 A. Question

Find the point of intersection of the following pairs of lines:

$2x - y + 3 = 0$ and $x + y - 5 = 0$

Answer

Given:

The equations of the lines are as follows:

$2x - y + 3 = 0$ ... (1)

$x + y - 5 = 0$ ... (2)

Concept Used:

Point of intersection of two lines.

To find:

Point of intersection of pair of lines.

Explanation:

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{5-3} = \frac{y}{3+10} = \frac{1}{2+1}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{13} = \frac{1}{3}$$

$\Rightarrow x = 2/3$ and $y = 13/3$

Hence, the point of intersection is $\left(\frac{2}{3}, \frac{13}{3}\right)$

1 B. Question

Find the point of intersection of the following pairs of lines:

bx + ay = ab and ax + by = ab

Answer

Given:

The equations of the lines are as follows:

bx + ay = ab

To find:

Point of intersection of pair of lines.

Concept Used:

Point of intersection of two lines.

Explanation:

⇒ bx + ay − ab = 0 … (1)

ax + by = ab ⇒ ax + by − ab = 0 … (2)

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{-a^2b + ab^2} = \frac{y}{-a^2b + ab^2} = \frac{1}{b^2 - a^2}$$

$$\Rightarrow \frac{x}{ab(b-a)} = \frac{y}{ab(b-a)} = \frac{1}{(a + b)(b-a)}$$

$\Rightarrow x = \frac{ab}{a+b}$ and $y = \frac{ab}{a+b}$

Hence, the point of intersection is $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$

1 C. Question

Find the point of intersection of the following pairs of lines:

$$y = m_1x + \frac{a}{m_1} \text{ and } y = m_2x + \frac{a}{m_2}$$

Answer

Given:

The equations of the lines are

$y = m_1x + \frac{a}{m_1}$ and $y = m_2x + \frac{a}{m_2}$

To find:

Point of intersection of pair of lines.

Concept Used:

Point of intersection of two lines.

Explanation:

Thus, we have:

$m_1x - y + \frac{a}{m_1} = 0$.........(1)

$m_2x - y + \frac{a}{m_2} = 0$..........(2)

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{-\frac{a}{m_2}+\frac{a}{m_1}}=\frac{y}{\frac{am_2}{m_1}-\frac{am_1}{m_2}}=\frac{1}{-m_1+m_2}$$

$$\Rightarrow x=\frac{-\frac{a}{m_2}+\frac{a}{m_1}}{-m_1+m_2}, y=\frac{\frac{am_2}{m_1}-\frac{am_1}{m_2}}{-m_1+m_2}$$

$\Rightarrow x=\frac{a}{m_1m_2}$ and $y=\frac{a(m_1+m_2)}{m_1m_2}$

Hence, the point of intersection is $\left(\frac{a}{m_1m_2},\frac{a(m_1+m_2)}{m_1m_2}\right)$ or $\left(\frac{a}{m_1m_2},a\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\right)$

2 A. Question

Find the coordinates of the vertices of a triangle, the equations of whose sides are :

x + y – 4 = 0, 2x – y + 3 0 and x – 3y + 2 = 0

Answer

Given:

x + y − 4 = 0, 2x − y + 3 = 0 and x − 3y + 2 = 0

To find:

Point of intersection of pair of lines.

Concept Used:

Point of intersection of two lines.

Explanation:

x + y − 4 = 0 … (1)

2x − y + 3 = 0 … (2)

x − 3y + 2 = 0 … (3)

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{3-4}=\frac{y}{-8-3}=\frac{1}{-1-2}$$

⇒ x = 1/3, y = 11/3

Solving (1) and (3) using cross - multiplication method:

$$\frac{x}{2-12}=\frac{y}{-4-2}=\frac{1}{-3-1}$$

⇒ x = 5/2, y = 3/2

Similarly, solving (2) and (3) using cross - multiplication method:

$$\frac{x}{-2+9}=\frac{y}{3-4}=\frac{1}{-6+1}$$

⇒ x = - 7/5, y = 1/5

Hence, the coordinates of the vertices of the triangle are $\left(\frac{1}{3},\frac{11}{3}\right),\left(\frac{5}{2},\frac{3}{2}\right)$ and $\left(-\frac{7}{5},\frac{1}{5}\right)$

2 B. Question

Find the coordinates of the vertices of a triangle, the equations of whose sides are :

$y(t_1 + t_2) = 2x + 2at_1t_2$. $y(t_2 + t_3) = 2x + 2at_2t_3$ and, $y(t_3 + t_1) = 2x + 2at_1t_3$.

Answer

Given:

$y\ (t_1 + t_2) = 2x + 2a\ t_1t_2$, $y\ (t_2 + t_3) = 2x + 2a\ t_2t_3$ and $y\ (t_3 + t_1) = 2x + 2a\ t_1t_3$

To find:

Point of intersection of pair of lines.

Concept Used:

Point of intersection of two lines.

Explanation:

$2x - y\ (t_1 + t_2) + 2a\ t_1t_2 = 0$ … (1)

$2x - y\ (t_2 + t_3) + 2a\ t_2t_3 = 0$ … (2)

$2x - y\ (t_3 + t_1) + 2a\ t_1t_3 = 0$ … (3)

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{-(t_1 + t_2) \times 2at_2t_3 + (t_2 + t_3)2at_1t_2} = \frac{-y}{4at_2t_3 - 4at_1t_2} = \frac{1}{-2(t_2 + t_3) + 2(t_1 + t_2)}$$

$$x = \frac{-(t_1 + t_2) \times 2at_2t_3 + (t_2 + t_3)2at_1t_2}{-2(t_2 + t_3) + 2(t_1 + t_2)} = at_2^2$$

$$y = -\frac{4at_2t_3 - 4at_1t_2}{-2(t_2 + t_3) + 2(t_1 + t_2)} = 2at_2$$

Solving (1) and (3) using cross - multiplication method:

$$\frac{x}{-(t_1 + t_2) \times 2at_1t_3 + (t_3 + t_1)2at_1t_2} = \frac{-y}{4at_1t_3 - 4at_1t_2} = \frac{1}{-2(t_3 + t_1) + 2(t_1 + t_2)}$$

$$x = \frac{-(t_1 + t_2) \times 2at_1t_3 + (t_3 + t_1)2at_1t_2}{-2(t_3 + t_1) + 2(t_1 + t_2)} = at_1^2$$

$$y = -\frac{4at_1t_3 - 4at_1t_2}{-2(t_3 + t_1) + 2(t_1 + t_2)} = 2at_1$$

Solving (2) and (3) using cross - multiplication method:

$$\frac{x}{-(t_2 + t_3) \times 2at_1t_3 + (t_3 + t_1)2at_2t_3} = \frac{-y}{4at_1t_3 - 4at_2t_3} = \frac{1}{-2(t_3 + t_1) + 2(t_2 + t_3)}$$

$$x = \frac{-(t_2 + t_3) \times 2at_1t_3 + (t_3 + t_1)2at_2t_3}{-2(t_3 + t_1) + 2(t_2 + t_3)} = at_3^2$$

$$y = -\frac{4at_1t_3 - 4at_2t_3}{-2(t_3 + t_1) + 2(t_2 + t_3)} = 2at_3$$

Hence, the coordinates of the vertices of the triangle are $(at_2^2, 2at_2), (at_1^2, 2at_1)$ and $(at_3^2, 2at_3)$.

3 A. Question

Find the area of the triangle formed by the lines

$y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$

Answer

Given:

$y = m_1x + c_1$ ... (1)

$y = m_2x + c_2$ ... (2)

$x = 0$ ... (3)

Explanation:

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively. Solving (1) and (2):

$$x = \frac{c_2 - c_1}{m_1 - m_2}, y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

Thus, AB and BC intersect at B $\left(\frac{c_2-c_1}{m_1-m_2}, \frac{m_1c_2-m_2c_1}{m_1-m_2}\right)$

Solving (1) and (3):

$x = 0$, $y = c_1$

Thus, AB and CA intersect at A $0,c_1$.

Similarly, solving (2) and (3):

$x = 0$, $y = c_2$

Thus, BC and CA intersect at C $0,c_2$.

$$\therefore \text{Area of triangle ABC} = \frac{1}{2}\begin{vmatrix} 0 & c_1 & 1 \\ 0 & c_2 & 1 \\ \frac{c_2-c_1}{m_1-m_2} & \frac{m_1c_2-m_2c_1}{m_1-m_2} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left(\frac{c_2 - c_1}{m_1 - m_2}\right)(c_1 - c_2) = \frac{\frac{1}{2}(c_1 - c_2)^2}{m_2 - m_1}$$

3 B. Question

Find the area of the triangle formed by the lines

$y = 0$, $x = 2$ and $x + 2y = 3$

Answer

Given:

$y = 0$ ... (1)

$x = 2$ ... (2)

$x + 2y = 3$ ... (3)

Assuming:

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (2):

x = 2, y = 0

Thus, AB and BC intersect at B (2, 0).

Solving (1) and (3):

x = 3, y = 0

Thus, AB and CA intersect at A (3, 0).

Similarly, solving (2) and (3):

x = 2, y = 12

Thus, BC and CA intersect at C(2, 12).

$\therefore$ Area of triangle ABC $= \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 3 & 0 & 1 \\ 2 & \frac{1}{2} & 1 \end{vmatrix} = \frac{1}{4}$

Hence, area of triangle ABC is $\frac{1}{4}$.

3 C. Question

Find the area of the triangle formed by the lines

x + y – 6 = 0, x – 3y – 2 = 0 and 5x – 3y + 2 = 0

Answer

Given:

x + y − 6 = 0 … (1)

x − 3y − 2 = 0 … (2)

5x − 3y + 2 = 0 … (3)

Assuming:

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (2):

x = 5, y = 1

Thus, AB and BC intersect at B (5, 1).

Solving (1) and (3):

x = 2, y = 4

Thus, AB and CA intersect at A (2, 4).

Similarly, solving (2) and (3):

x = −1, y = −1

Thus, BC and CA intersect at C (−1, −1).

$\therefore$ Area of triangle ABC $= \frac{1}{2}\begin{vmatrix} 5 & 1 & 1 \\ 2 & 4 & 1 \\ -1 & -1 & 1 \end{vmatrix} = 12$

Hence, area of triangle ABC is $\frac{1}{4}$.

4. Question

Find the equations of the medians of a triangle, the equations of whose sides are :

3x + 2y + 6 = 0, 2x – 5y + 4 = 0 and x – 3y – 6 = 0

Answer

Given: equations are as follows:

3x + 2y + 6 = 0 … (1)

2x − 5y + 4 = 0 … (2)

x − 3y − 6 = 0 … (3)

Assuming:

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (2):

x = −2, y = 0

Thus, AB and BC intersect at B (−2, 0).

Solving (1) and (3):

x = - 6/11, y = - 24/11

Thus, AB and CA intersect at $A\left(-\frac{6}{11}, -\frac{24}{11}\right)$

Similarly, solving (2) and (3):

x = −42, y = −16

Thus, BC and CA intersect at C (−42, −16).

Let D, E and F be the midpoints the sides BC, CA and AB, respectively. Then,

Then, we have:

$$D = \left(\frac{-2-42}{2}, \frac{0-16}{2}\right) = (-22, -8)$$

$$E = \left(\frac{-\frac{6}{11}-42}{2}, \frac{-\frac{24}{11}-16}{2}\right) = \left(-\frac{234}{11}, -\frac{100}{11}\right)$$

$$F = \left(\frac{-\frac{6}{11}-2}{2}, \frac{-\frac{24}{11}+0}{2}\right) = \left(-\frac{14}{11}, -\frac{12}{11}\right)$$

Now, the equation of the median AD is

$$y + \frac{24}{11} = \frac{-8 + \frac{24}{11}}{-22 + \frac{6}{11}}\left(x + \frac{6}{11}\right)$$

⇒ 16x – 59y – 120 = 0

The equation of the median BE is

$$y - 0 = \frac{-\frac{100}{11} - 0}{-\frac{234}{11} + 2}(x + 2)$$

⇒ 25x – 53y + 50 = 0

And, the equation of median CF is

$$y + 16 = \frac{-\frac{12}{11} + 16}{-\frac{14}{11} + 42}(x + 42)$$

⇒ 41 x – 112 y – 70 = 0

5. Question

Prove that the lines $y = \sqrt{3}x + 1$, y = 4 and $y = -\sqrt{3}x + 2$ form an equilateral triangle.

Answer

Given: equations are as follows:

$y = \sqrt{3}x + 1$......(1)

$y = 4$ ....(2)

$y = -\sqrt{3}x + 2$.......(3)

Assuming:

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

To prove:

Lines $y = \sqrt{3}x + 1$, $y = 4$ and $y = -\sqrt{3}x + 2$ form an equilateral triangle.

Explanation:

Solving (1) and (2):

$x = \sqrt{3}$, $y = 4$

Thus, AB and BC intersect at B($\sqrt{3}$,4)

Solving (1) and (3):

$$x = \frac{1}{2\sqrt{3}}, y = \frac{3}{2}$$

Thus, AB and CA intersect at A$\left(\frac{1}{2\sqrt{3}}, \frac{3}{2}\right)$

Similarly, solving (2) and (3):

$x = -\frac{2}{\sqrt{3}}$, y = 4

Thus, BC and AC intersect at C$\left(-\frac{2}{\sqrt{3}},4\right)$

Now, we have:

$$AB = \sqrt{\left(\frac{1}{2\sqrt{3}} - \sqrt{3}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

$$BC = \sqrt{\left(\frac{1}{2\sqrt{3}} + \frac{2}{\sqrt{3}}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

$$AC = \sqrt{\left(\frac{1}{2\sqrt{3}} + \frac{2}{\sqrt{3}}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

Hence Proved, the given lines form an equilateral triangle

6. Question

Classify the following pairs of lines as coincident, parallel or intersecting:

(i) $2x + y - 1 = 0$ and $3x + 2y + 5 = 0$

(ii) $x - y = 0$ and $3x - 3y + 5 = 0$

(iii) $3x + 2y - 4 = 0$ and $6x + 4y - 8 = 0$

Answer

Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be the two lines.

(a) The lines intersect if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ is true.

(b) The lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is true.

(c) The lines are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is true.

(i) Given: $2x + y - 1 = 0$ and $3x + 2y + 5 = 0$

Explanation:

Here, $\frac{2}{3} \neq \frac{1}{2}$

Therefore, the lines $2x + y - 1 = 0$ and $3x + 2y + 5 = 0$ intersect.

(ii) Given: $x - y = 0$ and $3x - 3y + 5 = 0$

Explanation:

Here, $\frac{1}{3} = -\frac{1}{-3} \neq \frac{0}{5}$

Therefore, the lines $x - y = 0$ and $3x - 3y + 5 = 0$ are parallel.

(iii) Given: $3x + 2y - 4 = 0$ and $6x + 4y - 8 = 0$

Explanation:

Here, $\frac{3}{6} = \frac{2}{4} = -\frac{4}{-8}$

Therefore, the lines $3x + 2y - 4 = 0$ and $6x + 4y - 8 = 0$ are coincident.

7. Question

Find the equation of the line joining the point (3, 5) to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$.

Answer

Given:

$4x + y - 1 = 0$ … (1)

$7x - 3y - 35 = 0$ … (2)

Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{-35-3} = \frac{y}{-7+140} = \frac{1}{-12-7}$$

⇒ x = 2 ,y = - 7

Thus, the point of intersection of the given lines is (2, - 7).

So, the equation of the line joining the points (3, 5) and (2, - 7) is

$$y-5 = \frac{-7-5}{2-3}(x-3)$$

⇒ y - 5 = 12x - 36

⇒ 12x – y – 31 = 0

Hence, the required equation of line is 12x – y – 31 = 0

8. Question

Find the equation of the line passing through the point of intersection of the lines 4x – 7y – 3 = 0 and 2x – 3y + 1 = 0 that has equal intercepts on the axes.

Answer

Given:

4x − 7y − 3 = 0 ... (1)

2x − 3y + 1 = 0 ... (2)

To find:

Equation of line passing through the point of intersection of lines.

Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{-7-9} = \frac{y}{-6-4} = \frac{1}{-12+14}$$

⇒ x = - 8 , y = - 5

Thus, the point of intersection of the given lines is (- 8, - 5).

Now, the equation of a line having equal intercept as a is $\frac{x}{a} + \frac{y}{a} = 1$

This line passes through (- 8, - 5)

$$\therefore -\frac{8}{a}-\frac{5}{a} = 1$$

⇒ - 8 - 5 = a

⇒ a = - 13

Hence, the equation of the required line is $\frac{x}{-13} + \frac{y}{-13} = 1$ or x + y + 13 = 0

9. Question

Show that the area of the triangle formed by the lines y = m_1x, y = m_2x and y = c is equal to $\frac{c^2}{4}(\sqrt{33}+\sqrt{11})$, where m_1, m_2 are the roots of the equation $x^2+(\sqrt{3}+2)x+\sqrt{3}-1=0$.

Answer

Given: lines are as follows:

y = m1 x ... (1)

y = m2 x ... (2)

y = c ... (3)

To prove:

The area of the triangle formed by the lines y = m_1x, y = m_2x and y = c is equal to $\frac{c^2}{4}\left(\sqrt{33}+\sqrt{11}\right)$.

Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (2), we get (0, 0) as their point of intersection.

Solving (1) and (3), we get $\left(\frac{c}{m_1}, c\right)$ as their point of intersection.

Similarly, solving (2) and (3), we get $\left(\frac{c}{m_2}, c\right)$ as their point of intersection.

∴ Area of the triangle formed by these lines $= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{c}{m_1} & c & 1 \\ \frac{c}{m_2} & c & 1 \end{vmatrix} = \frac{1}{2}\left(\frac{c^2}{m_1}-\frac{c^2}{m_2}\right) = \frac{c^2}{2}\left|\frac{m_2-m_1}{m_2 m_1}\right|$

It is given that m1 and m2 are the roots of the

$$x^2 + (\sqrt{3}+2)x + \sqrt{3}-1 = 0$$

$$\therefore\ m_1 + m_2 = -(\sqrt{3}+2),\ m_1 m_2 = \sqrt{3}-1$$

$$\Rightarrow m_2 - m_1 = \sqrt{(m_1+m_2)^2 - 4m_1 m_2}$$

$$\Rightarrow m_2 - m_1 = \sqrt{\{-(\sqrt{3}+2)\}^2 - 4\sqrt{3} + 4}$$

$$\Rightarrow m_2 - m_1 = \sqrt{7 + 4\sqrt{3} - 4\sqrt{3} + 4} = \sqrt{11}$$

∴ AREA $= \frac{c^2}{2}\left|\frac{\sqrt{11}}{\sqrt{3}-1}\right| = \frac{c^2}{2}\left|\frac{(\sqrt{3}+1)\sqrt{11}}{(\sqrt{3}+1)(\sqrt{3}-1)}\right|$

$$= \frac{c^2}{2}\left|\frac{\sqrt{33}+\sqrt{11}}{2}\right| = \frac{c^2}{4}\left(\sqrt{33}+\sqrt{11}\right)$$

Hence Proved.

10. Question

If the straight line $\frac{x}{a}+\frac{y}{b}=1$ passes through the point of intersection of the lines x + y = 3 and 2x – 3y = 1 and is parallel to x – y – 6 = 0, find a and b.

Answer

Given: lines are x + y = 3 and 2x − 3y = 1.

To find:

a and b.

Concept Used:

Point of intersection of two lines.

Explanation:

x + y − 3 = 0 … (1)

2x − 3y − 1 = 0 … (2)

Solving (1) and (2) using cross - multiplication method:

$$\frac{x}{-1-9} = \frac{y}{-6+1} = \frac{1}{-3-2}$$

⇒ x = 2 , y = 1

Thus, the point of intersection of the given lines is (2, 1).

It is given that the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through (2, 1).

$\therefore \frac{2}{a} + \frac{1}{b} = 1$......(3)

It is also given that the line $\frac{x}{a} + \frac{y}{b} = 1$ is parallel to the line x − y − 6 = 0.

Hence, Slope of $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow y = -\frac{b}{a}x + b$ is equal to the slope of x − y − 6 = 0 or, y = x – 6

∴ - b/a = 1

⇒ b = - a … (4)

From (3) and (4):

$\therefore \frac{2}{a} - \frac{1}{a} = 1$

⇒ a = 1

From (4):

b = −1

∴ a = 1,

b = −1

Hence, a = 1, b = - 1

11. Question

Find the orthocenter of the triangle the equations of whose sides are x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0.

Answer

Given: Sides of triangle are are x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0.

Assuming: AB, BC and AC be the sides of triangle whose equation is are x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0.

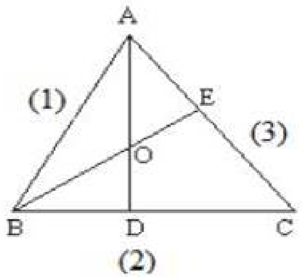
To find:

Orthocenter of triangle.

Concept Used:

Point of intersection of two lines.

Explanation:



x + y – 1 = 0 ...... (i)

2x + 3y – 6 = 0 ...... (ii)

4x – y + 4 = 0. ...... (iii)

By solving equation (i) and (ii) By cross multiplication

$$\frac{x}{-6 + 3} = \frac{-y}{-6 + 2} = \frac{1}{3 - 2}$$

⇒ x = - 3 ,y = 4

∴B(- 3, 4)

By Solving equation (i) and (iii) By cross multiplication

$$\frac{x}{4 - 1} = \frac{-y}{4 + 4} = \frac{1}{-3 - 2}$$

⇒ $x = -\frac{3}{5}$, $y = \frac{8}{5}$

∴ $A(-\frac{3}{5}, \frac{8}{5})$

Equation of BC is 2x + 3y = 6

Altitude AD is perpendicular to BC,

Therefore, equation of AD is x + y + k = 0

AD is passing through $A(-\frac{3}{5}, \frac{8}{5})$

$$\left(-\frac{3}{5}\right) + \left(\frac{8}{5}\right) + k = 0$$

⇒ k = - 1

∴ Equation of AD is x + y – 1 = 0 ...... (iv)

Altitude BE is perpendicular to AC.

⇒ Let the equation of DE be x – 2y = k

BE is passing through D(- 3, 4)

⇒ - 3 – 8 = k

⇒ k = - 11

Equation of BE is x – 2y = - 11 ...... (v)

By solving equation (iv) and (v),

We get, x = - 3 and y = 4

Hence, the orthocenter of triangle is (- 3, 4).

12. Question

Three sides AB, BC and CA of a triangle ABC are 5x – 3y + 2 = 0, x – 3y – 2 = 0 and x + y – 6 = 0 respectively. Find the equation of the altitude through the vertex A.

Answer

Given:

The sides AB, BC and CA of a triangle ABC are as follows:

5x − 3y + 2 = 0 … (1)

x − 3y − 2 = 0 … (2)

x + y − 6 = 0 … (3)

To find:

The equation of the Altitude through the vertex A.

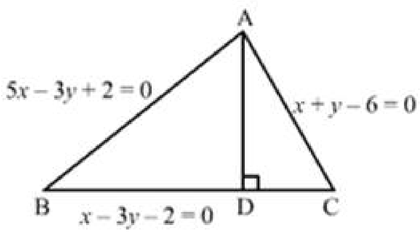
Concept Used:

Point of intersection of two lines.

Explanation:

Solving (1) and (3):

x = 2 , y = 4



Thus, AB and CA intersect at A (2, 4).

Let AD be the altitude.

AD⊥BC

∴ Slope of AD × Slope of BC = −1

Here, slope of BC = slope of the line (2) = 1/3

∴ Slope of AD×1/3 = - 1 ⇒ Slope of AD = - 3

Hence, the equation of the altitude AD passing through A (2, 4) and having slope −3 is y - 4 = - 3x - 2⇒ 3x + y = 10

13. Question

Find the coordinates of the orthocenter of the triangle whose vertices are (- 1, 3), (2, - 1) and (0, 0).

Answer

Given: coordinates of the orthocenter of the triangle whose vertices are (- 1, 3), (2, - 1) and (0, 0).

Assuming:

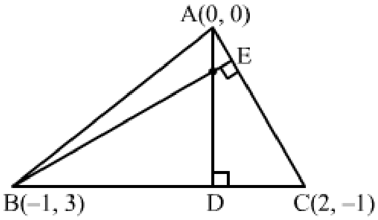
A (0, 0), B (−1, 3) and C (2, −1) be the vertices of the triangle ABC.

Let AD and BE be the altitudes.

To find:

Orthocenter of the triangle.

Explanation:



AD⊥BC and BE⊥AC

∴ The slope of AD × Slope of BC = −1

The slope of BE × Slope of AC = −1

Here, the slope of BC = $\frac{-1-3}{2+1} = -\frac{4}{3}$

and slope of AC = $\frac{-1-0}{2-0} = -\frac{1}{2}$

∴ slope of AD × (- 4/3) = - 1 and slope of BE × (- 1/2) = - 1

⇒ slope of AD = $\frac{3}{4}$ and slope of BE = 2

The equation of the altitude AD passing through A (0, 0) and having slope $\frac{3}{4}$ is

y - 0 = $\frac{3}{4}$ (x - 0)

⇒ y = $\frac{3}{4}$ x .....(1)

The equation of the altitude BE passing through B (−1, 3) and having slope 2 is

y - 3 = 2(x + 1)

⇒ 2x – y + 5 = 0 ....(2)

Solving (1) and (2):

x = − 4, y = − 3

Hence, the coordinates of the orthocentre is (−4, −3).

14. Question

Find the coordinates of the incentre and centroid of the triangle whose sides have the equations 3x – 4y = 0, 12y + 5x = 0 and y – 15 = 0.

Answer

Given: lines are as follows:

3x − 4y = 0 … (1)

12y + 5x = 0 … (2)

y − 15 = 0 … (3)

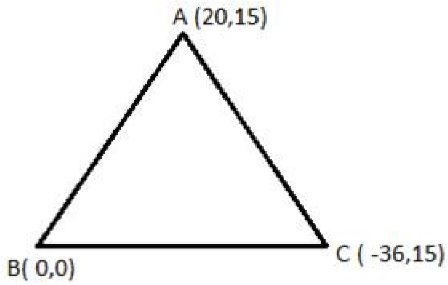
Assuming:

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

Concept Used:

Point of intersection of two lines.

Explanation:



Solving (1) and (2):

x = 0, y = 0

Thus, AB and BC intersect at B (0, 0).

Solving (1) and (3):

x = 20 , y = 15

Thus, AB and CA intersect at A (20, 15).

Solving (2) and (3): x = −36 , y = 15

Thus, BC and CA intersect at C (−36, 15).

Let us find the lengths of sides AB, BC and CA.

$$AB = \sqrt{(20-0)^2 + (15-0)^2} = 25$$

$$BC = \sqrt{(0+36)^2 + (0-15)^2} = 39$$

$$AC = \sqrt{(20+36)^2 + (15-15)^2} = 56$$

Here, a = BC = 39, b = CA = 56 and c = AB = 25

Also, x_1, y_1 = A (20, 15), x_2, y_2 = B (0, 0) and x_3, y_3 = C (−36, 15)

$$\therefore \text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{20 + 0 - 36}{3}, \frac{15 + 0 + 15}{3}\right) = \left(-\frac{16}{3}, 10\right)$$

AND incentre $= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

$$= \left(\frac{39 \times 20 + 56 \times 0 - 25 \times 36}{39 + 56 + 25}, \frac{39 \times 15 + 56 \times 0 - 25 \times 15}{39 + 56 + 25}\right)$$

$$= \left(-\frac{120}{120}, 120 \times \frac{8}{120}\right)$$

$$= (-1, 8)$$

Hence, coordinate of incenter and centroid are $\left(-\frac{16}{3}, 10\right)$ and (- 1, 8)

15. Question

Prove that the lines $\sqrt{3}x + y = 0$, $\sqrt{3}y + x = 0$, $\sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$ form a rhombus.

Answer

Given: lines are as follows:

$\sqrt{3}x + y = 0, \sqrt{3}y + x = 0, \sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$

To prove:

$\sqrt{3}x + y = 0, \sqrt{3}y + x = 0, \sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$ lines form a rhombus.

Assuming:

In quadrilateral ABCD, let equations (1), (2), (3) and (4) represent the sides AB, BC, CD and DA, respectively.

Explanation:

Lines (1) and (3) are parallel and lines (2) and (4) are parallel.

Solving (1) and (2):

x = 0, y = 0.

Thus, AB and BC intersect at B (0, 0).

Solving (1) and (4):

$x = -\frac{1}{2}, y = \frac{\sqrt{3}}{2}$

Thus, AB and DA intersect $A\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Solving (3) and (2):

$x = \frac{\sqrt{3}}{2}, y = -\frac{1}{2}$

Thus, BC and CD intersect at $C\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Solving (3) and (4):

$x = \frac{(\sqrt{3}-1)}{2}, y = \frac{(\sqrt{3}-1)}{2}$

Thus, DA and CD intersect at $D\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}-1}{2}\right)$

Let us find the lengths of sides AB, BC and CD and DA.

$$AB = \sqrt{\left(0-\frac{1}{2}\right)^2 + \left(0-\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$BC = \sqrt{\left(\frac{\sqrt{3}}{2}-0\right)^2 + \left(-\frac{1}{2}-0\right)^2} = 1$$

$$CB = \sqrt{\left(\frac{\sqrt{3}-1}{2}-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}+\frac{1}{2}\right)^2} = 1$$

$$DA = \sqrt{\left(\frac{\sqrt{3}-1}{2}+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}-\frac{\sqrt{3}}{2}\right)^2} = 1$$

Hence Proved, the given lines form a rhombus.

16. Question

Find the equation of the line passing through the intersection of the lines 2x + y = 5 and x + 3y + 8 = 0 and parallel to the line 3x + 4y = 7.

Answer

Given: Line passing through point of intersection of lines 2x + y = 5 and x + 3y + 8 = 0 and parallel to the line 3x + 4y = 7.

To find:

The equation of the required line.

Concept Used:

Point of intersection of two lines.

Explanation:

2x + y – 5 = 0 ...... (i)

x + 3y + 8 = 0 ...... (ii)

By solving equation (i) and (ii) ,By cross multiplication,

$$\frac{x}{8 + 15} = \frac{-y}{16 + 5} = \frac{1}{6 - 1}$$

$\Rightarrow x = \frac{23}{5}, y = -\frac{21}{5}$

Point of intersection $\left(\frac{23}{5}, -\frac{21}{5}\right)$

Equation of line parallel to 3x + 4y – 7 = 0 is

3x + 4y + k = 0 ...... (iii)

Equation (iii) passing through point of intersection of line.

$\Rightarrow 3\left(\frac{23}{5}\right) + 4\left(-\frac{21}{5}\right) + k = 0$

$\Rightarrow k = 3$

Hence, required equation of line is 3x + 4y + 3 = 0.

17. Question

Find the equation of the straight line passing through the point of intersection of the lines 5x – 6y – 1 = 0 and 3x + 2y + 5 = 0 and perpendicular to the line 3x – 5y + 11 = 0.

Answer

Given: the point of intersection of the lines 5x – 6y – 1 = 0 and 3x + 2y + 5 = 0 and perpendicular to the line 3x – 5y + 11 = 0.

To find:

The equation of the required line.

Concept Used:

Point of intersection of two lines.

Explanation:

5x – 6y – 1 = 0 ...... (i)

3x + 2y + 5 = 0 ...... (ii)

By solving equation (i) and (ii) ,By cross multiplication,

$$\frac{x}{-30 + 2} = \frac{-y}{25 + 3} = \frac{1}{10 + 18}$$

$\Rightarrow x = -1, y = -1$

Point of intersection (- 1, - 1)

Now, the slope of the line 3x – 5y + 11 = 0 or y= $\frac{3}{5}x + \frac{11}{5}$ is $\frac{3}{5}$

Now, we know that rhe product of the slope of two perpendicular lines is - 1.

Assuming: the slope of required line is m

$$m \times \frac{3}{5} = -1$$

$$\Rightarrow m = -\frac{5}{3}$$

Now, the equation of the required line passing through (- 1, - 1) and having slope $-\frac{5}{3}$ is given by,

$$Y + 1 = -\frac{5}{3}(x + 1)$$

⇒ 3y + 3 = - 5x – 5

⇒ 5x + 3y + 8 = 0

Hence, equation of required line is 5x + 3y + 8 = 0.

Exercise 23.11

1 A. Question

Prove that the following sets of three lines are concurrent:

15x – 18y + 1 = 0, 12x + 10y – 3 = 0 and 6x + 66y – 11 = 0

Answer

Given:

15x – 18y + 1 = 0 ...... (i)

12x + 10y – 3 = 0 ...... (ii)

6x + 66y – 11 = 0 ...... (iii)

To prove:

Sets of given three lines are concurrent.

Explanation:

Now, consider the following determinant:

$$\begin{vmatrix} 15 & -18 & 1 \\ 12 & 19 & -3 \\ 6 & 66 & -11 \end{vmatrix} = 15(-110 + 198) + 18(-132 + 18) + 1(792 - 60)$$

⇒ 1320 – 2052 + 732 = 0

Hence proved, the given lines are concurrent.

1 B. Question

Prove that the following sets of three lines are concurrent:

3x – 5y – 11 = 0, 5x + 3y – 7 = 0 and x + 2y = 0

Answer

Given:3x − 5y − 11 = 0 ...... (i)

5x + 3y − 7 = 0 ...... (ii)

x + 2y = 0 ...... (iii)

To prove:

Sets of given three lines are concurrent.

Explanation:

Now, consider the following determinant:

$$\begin{vmatrix} 3 & -5 & -11 \\ 5 & 3 & -7 \\ 1 & 2 & 0 \end{vmatrix} = 3 \times 14 + 5 \times 7 - 11 \times 7 = 0$$

Hence, the given lines are concurrent.

1 C. Question

Prove that the following sets of three lines are concurrent:

$\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1$ and y = x.

Answer

Given:bx + ay – ab = 0 … (1)

ax + by – ab = 0 … (2)

x − y = 0 … (3)

To prove:

Sets of given three lines are concurrent.

Explanation:

Now, consider the following determinant:

$$\begin{vmatrix} b & a & -ab \\ a & b & -ab \\ 1 & -1 & 0 \end{vmatrix} = -b \times ab - a \times ab - ab \times (-a - b) = 0$$

Hence proved, the given lines are concurrent.

2. Question

For what value of λ are the three lines 2x – 5y + 3 = 0, 5x – 9y + λ = 0 and x – 2y + 1 = 0 concurrent?

Answer

Given:2x − 5y + 3 = 0 … (1)

5x − 9y + λ = 0 … (2)

x − 2y + 1 = 0 … (3)

To find:

Value of λ.

Concept Used:

Determinant of equation is zero.

Explanation:

It is given that the three lines are concurrent.

$$\therefore \begin{vmatrix} 2 & -5 & 3 \\ 5 & -9 & \lambda \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-9 + 2\lambda) + 5(5 - \lambda) + 3(-10 + 9) = 0$

$\Rightarrow -18 + 4\lambda + 25 - 5\lambda - 3 = 0$

$\Rightarrow \lambda = 4$

Hence, $\lambda = 4$.

3. Question

Find the conditions that the straight lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ may meet in a point.

Answer

Given:

The given lines can be written as follows:

$m_1x - y + c_1 = 0$ ... (1)

$m_2x - y + c_2 = 0$ ... (2)

$m_3x - y + c_3 = 0$ ... (3)

To find:

Conditions that the straight lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ may meet in a point.

Concept Used:

Determinant of equation is zero.

Explanation:

It is given that the three lines are concurrent.

$$\therefore \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$$

$\Rightarrow m_1(-c_3 + c_2) + 1(m_2c_3 - m_3c_2) + c_1(-m_2 + m_3) = 0$

$\Rightarrow m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

Hence, the required condition is $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

4. Question

If the lines $p_1x + q_1y = 1$, $p_2x + q_2y = 1$ and $p_3x + q_3y = 1$ be concurrent, show that the points (p_1, q_1), (p_2, q_2) and (p_3, q_3) are collinear.

Answer

Given:

$p_1x + q_1y = 1$

$p_2x + q_2y = 1$

$p_3x + q_3y = 1$

To prove:

The points (p_1, q_1), (p_2, q_2) and (p_3, q_3) are collinear.

Concept Used:

If three lines are concurrent then determinant of equation is zero.

Explanation:

The given lines can be written as follows:

$p_1 x + q_1 y - 1 = 0$ … (1)

$p_2 x + q_2 y - 1 = 0$ … (2)

$p_3 x + q_3 y - 1 = 0$ … (3)

It is given that the three lines are concurrent.

$$\therefore \begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

Hence proved, This is the condition for the collinearity of the three points, (p_1, q_1), (p_2, q_2) and (p_3, q_3).

5. Question

Show that the straight lines $L_1 = (b + c)x + ay + 1 = 0$, $L_2 = (c + a)x + by + 1 = 0$ and $L_3 = (a + b)x + cy + 1 = 0$ are concurrent.

Answer

Given:

$L_1 = (b + c)x + ay + 1 = 0$

$L_2 = (c + a)x + by + 1 = 0$

$L_3 = (a + b)x + cy + 1 = 0$

To prove:

The straight lines $L_1 = (b + c)x + ay + 1 = 0$, $L_2 = (c + a)x + by + 1 = 0$ and $L_3 = (a + b)x + cy + 1 = 0$ are concurrent.

Concept Used:

If three lines are concurrent then determinant of equation is zero.

Explanation:

The given lines can be written as follows:

$(b + c)\, x + ay + 1 = 0$ … (1)

$(c + a)\, x + by + 1 = 0$ … (2)

$(a + b)\, x + cy + 1 = 0$ … (3)

Consider the following determinant.

$$\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix}$$

Applying the transformation $C_1 \to C_1 + C_2$,

$$\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = \begin{vmatrix} a+b+c & a & 1 \\ c+a+b & b & 1 \\ a+b+c & c & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$\Rightarrow \Rightarrow \begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = 0$$

Hence proved, the given lines are concurrent.

6. Question

If the three lines $ax + a^2y + 1 = 0$, $bx + b^2y + 1 = 0$ and $cx + c^2y + 1 = 0$ are concurrent, show that at least two of three constant a, b, c are equal.

Answer

Given:

$ax + a^2y + 1 = 0$

$bx + b^2y + 1 = 0$

$cx + c^2y + 1 = 0$

To prove:

At least two of three constant a, b, c are equal.

Concept Used:

If three lines are concurrent then determinant of equation is zero.

Explanation:

The given lines can be written as follows:

$ax + a^2y + 1 = 0$ ... (1)

$bx + b^2y + 1 = 0$ ... (2)

$cx + c^2y + 1 = 0$ ... (3)

The given lines are concurrent.

$$\therefore \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

Applying the transformation $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$:

$$\begin{vmatrix} a-b & a^2-b^2 & 0 \\ b-c & b^2-c^2 & 0 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (a-b)(b-c)\begin{vmatrix} 1 & a+b & 1 \\ 1 & b+c & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

⇒ (a – b)(b – c)(c – a) = 0

⇒ a – b = 0 or b – c = 0 or c – a = 0

⇒ a = b or b = c or c = a

Hence proved, atleast two of the constants a,b,c are equal .

7. Question

If a, b, c are in A. P., prove that the straight lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent.

Answer

Given:

ax + 2y + 1 = 0

bx + 3y + 1 = 0

cx + 4y + 1 = 0

To prove:

The straight lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent.

Concept Used:

If three lines are concurrent then determinant of equation is zero.

Explanation:

The given lines can be written as follows:

ax + 2y + 1 = 0 … (1)

bx + 3y + 1 = 0 … (2)

cx + 4y + 1 = 0 … (3)

Consider the following determinant. $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix}$

Applying the transformation $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$,

$$\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = \begin{vmatrix} a-b & -1 & 0 \\ b-c & -1 & 0 \\ c & 4 & 1 \end{vmatrix}$$

$\Rightarrow \begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} =$ (-a + b + b – c) = 2b – a – c

Given:2b = a + c

$\Rightarrow \begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} =$ a + c –a – c = 0

Hence proved, the given lines are concurrent, provided 2b = a + c.

8. Question

Show that the perpendicular bisectors of the sides of a triangle are concurrent.

Answer

To prove:

Perpendicular bisectors of the sides of a triangle are concurrent.

Assuming:

ABC be a triangle with vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3).

Let D, E and F be the midpoints of the sides BC, CA and AB, respectively.

Explanation:

Thus, the coordinates of D, E and F are $D\left(\frac{x_2+x_3}{2}+\frac{y_2+y_3}{2}\right), E\left(\frac{x_1+x_3}{2}+\frac{y_1+y_3}{2}\right)$ and $F\left(\frac{x_1+x_2}{2}+\frac{y_1+y_2}{2}\right)$

Let mD, mE and mF be the slopes of AD, BE and CF respectively.

∴ Slope of BC × mD = -1

$\Rightarrow \frac{y_3-y_2}{x_3-x_2} \times mD = -1$

$\Rightarrow mD = -\frac{x_3-x_2}{y_3-y_2}$

Thus, the equation of AD $y-\frac{y_1+y_3}{2} = -\frac{x_3-x_2}{y_3-y_2}\left(x-\frac{x_2+x_3}{2}\right)$

$\Rightarrow y-\frac{y_1+y_3}{2} = -\frac{x_3-x_2}{y_3-y_2}\left(x-\frac{x_2+x_3}{2}\right)$

$\Rightarrow 2y(y_3-y_2)-(y_3^2-y_2^2) = -2x(x_3-x_2)+x_3^2-x_2^2$

$\Rightarrow 2x(x_3-x_2)+2y(y_3-y_2)-(x_3^2-x_2^2)-(y_3^2-y_2^2) = 0$

Similarly, the respective equations of BE and CF are

$2x(x_1-x_3)+2y(y_1-y_3)-(x_1^2-x_3^2)-(y_1^2-y_3^2) = 0$

$2x(x_2-x_1)+2y(y_2-y_1)-(x_2^2-x_1^2)-(y_2^2-y_1^2) = 0$

Let L_1, L_2 and L_3 represent the lines (1), (2) and (3), respectively.Adding all the three lines,

We observe:$1 \cdot L_1 + 1 \cdot L_2 + 1 \cdot L_3 = 0$

Hence proved, the perpendicular bisectors of the sides of a triangle are concurrent.

Exercise 23.12

1. Question

Find the equation of a line passing through the point (2, 3) and parallel to the line 3x– 4y + 5 = 0.

Answer

<u>Given:</u> equation is parallel to 3x − 4y + 5 = 0 and pass through (2, 3)

<u>To find:</u>

Equation of required line.

Explanation:

The equation of the line parallel to $3x - 4y + 5 = 0$ is

$3x - 4y + \lambda = 0$,

Where, λ is a constant.

It passes through (2, 3).

$\therefore 6 - 12 + \lambda = 0$

$\Rightarrow \lambda = 6$

Hence, the required line is $3x - 4y + 6 = 0$.

2. Question

Find the equation of a line passing through (3, -2) and perpendicular to the line $x - 3y + 5 = 0$.

Answer

Given: equation is perpendicular to $x - 3y + 5 = 0$ and passes through (3,-2)

To find:

Equation of required line.

Explanation:

The equation of the line perpendicular to $x - 3y + 5 = 0$ is

$3x + y + \lambda = 0$,

Where λ is a constant.

It passes through (3, − 2).

$9 - 2 + \lambda = 0$

$\Rightarrow \lambda = -7$

Substituting $\lambda = -7$ in $3x + y + \lambda = 0$,

Hence, we get $3x + y - 7 = 0$, which is the required line.

3. Question

Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1).

Answer

Given: A (1, 3) and B (3, 1) be the points joining the perpendicular bisector

To find:

The equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1).

Explanation:

Let C be the midpoint of AB.

$\therefore$ coordinates of c $= \left(\frac{1+3}{2}, \frac{3+1}{2}\right) = (2,2)$

Slope of AB $= \frac{1-3}{3-1} = -1$

$\therefore$ Slope of the perpendicular bisector of AB = 1

Thus, the equation of the perpendicular bisector of AB is

$y - 2 = 1(x - 2)$

$\Rightarrow x - y = 0$

Or, y = x

Hence, the equation is y = x.

4. Question

Find the equations of the altitudes of a ΔABC whose vertices are A (1, 4), B(-3, 2) and C(-5, -3).

Answer

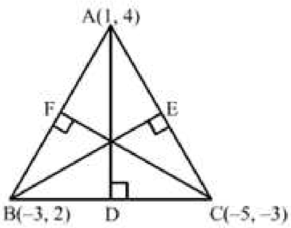
Given: The vertices of ΔABC are A (1, 4), B (– 3, 2) and C (– 5, – 3).

To find:

The equations of the altitudes of a ΔABC whose vertices are A (1, 4), B (-3, 2) and C (-5, -3).

Explanation:

Diagram:



Slope of AB $= \frac{2-4}{-3-1} = \frac{1}{2}$

Slope of BC $= \frac{-3-2}{-5+3} = \frac{5}{2}$

Slope of CA $= \frac{4+3}{1+5} = \frac{7}{6}$

Thus, we have:

Slope of CF = -2

Slope of AD $= -\frac{2}{5}$

Slope of BE $= -\frac{6}{7}$

Hence,

Equation of CF is: y + 3 = -2(x + 5)

⇒ 2x + y + 13 = 0

Equation of AD is: y – 4 $= -\frac{2}{5}$(x – 1)

⇒ 2x + 5y – 22 = 0

Equation of BE is: y – 2 $= -\frac{6}{7}$(x + 3)

⇒ 6x + 7y + 4 = 0

5. Question

Find the equation of a line which is perpendicular to the line $\sqrt{3}x - y + 5 = 0$ and which cuts off an intercept of 4 units with the negative direction of y-axis.

Answer

Given: equation is perpendicular to $\sqrt{3}x - y + 5 = 0$ equation and cuts off an intercept of 4 units with the negative direction of y-axis

To find:

The equation of a line which is perpendicular to the line $\sqrt{3}x - y + 5 = 0$ and which cuts off an intercept of 4 units with the negative direction of y-axis.

Explanation:

The line perpendicular to $\sqrt{3}x - y + 5 = 0$ is $x + \sqrt{3}y + \lambda = 0$

It is Given that the line $x + \sqrt{3}y + \lambda = 0$ cuts off an intercept of 4 units with the negative direction of the y-axis.

This means that the line passes through (0,-4).

$\therefore 0 - \sqrt{3} \times 4 + \lambda = 0$

$\Rightarrow \lambda = 4\sqrt{3}$

Substituting the value of λ, we get $+ \sqrt{3}y + 4\sqrt{3} = 0$, which is the equation of the required line.

6. Question

If the image of the point (2, 1) with respect to a line mirror is (5, 2), find the equation of the mirror.

Answer

Given: image of (2 1) is (5,2)

To find:

The equation of the mirror.

Explanation:

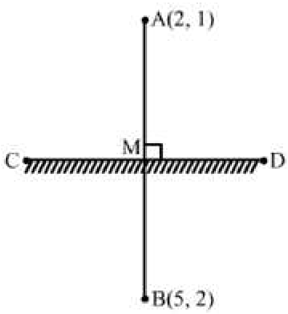
Let the image of A (2, 1) be B (5, 2).

Also, let M be the midpoint of AB.

$\therefore$ Coordinates of M $= \left(\frac{2+5}{2}, \frac{1+2}{2}\right)$

$= \left(\frac{7}{2}, \frac{3}{2}\right)$

Diagram:



Let CD be the mirror.

Line AB is perpendicular to the mirror CD.

$\therefore$ Slope of AB × Slope of CD $= -1$

$\Rightarrow \frac{2-1}{5-2} \times$ Slope of CD = -1

⇒ Slope of CD = -3

Equation of the mirror CD

$$y - \frac{3}{2} = -3\left(x - \frac{7}{2}\right)$$

$\Rightarrow 2y - 3 = -6x + 21$

$\Rightarrow 6x + 2y - 24 = 0$

$\Rightarrow 3x + y - 12 = 0$

Hence, the equation of mirror is $3x + y - 12 = 0$

7. Question

Find the equation of the straight line through the point (α, β) and perpendicular to the line $lx + my + n = 0$.

Answer

Given: equation is perpendicular to $lx + my + n = 0$ and passing through (α, β)

To find:

The equation of the straight line through the point (α, β) and perpendicular to the line $lx + my + n = 0$.

Explanation:

The line perpendicular to $lx + my + n = 0$ is

$mx - ly + \lambda = 0$

This line passes through (α, β).

$\therefore m\alpha - l\beta + \lambda = 0 \Rightarrow \lambda = l\beta - m\alpha$

Substituting the value of λ:

$mx - ly + l\beta - m\alpha = 0$

$\Rightarrow mx - \alpha = ly - \beta$

Hence, equation of the required line is $mx - \alpha = ly - \beta$

8. Question

Find the equation of the straight line perpendicular to $2x - 3y = 5$ and cutting off an intercept 1 on the positive direction of the x-axis.

Answer

Given: equation is perpendicular to $2x - 3y = 5$ and cutting off an intercept 1 on the positive direction of the x-axis.

Explanation:

The line perpendicular to $2x - 3y = 5$ is

$3x + 2y + \lambda = 0$

It is given that the line $3x + 2y + \lambda = 0$ cuts off an intercept of 1 on the positive direction of the x axis.

This means that the line $3x + 2y + \lambda = 0$ passes through the point (1, 0).

$\therefore 3 + 0 + \lambda = 0$

$\Rightarrow \lambda = -3$

Substituting the value of λ, we get $3x + 2y - 3 = 0$,

Hence, equation of the required line.

9. Question

Find the equation of the straight line perpendicular to $5x - 2y = 8$ and which passes through the mid-point of the line segment joining (2, 3) and (4, 5).

Answer

Given: equation is perpendicular to $5x - 2y = 8$ and pass through mid-point of the line segment joining (2, 3) and (4, 5).

To find:

The equation of the straight line perpendicular to 5x – 2y = 8 and which passes through the mid-point of the line segment joining (2, 3) and (4, 5).

Explanation:

The line perpendicular to 5x − 2y = 8 is 2x + 5y + λ = 0

Coordinates of the mid points of (2,3) and (4,5) = $\left(\frac{2+4}{2}, \frac{3+5}{2}\right) = (3,4)$

∴ 6 + 20 + λ = 0

⇒ λ = -26

Substituting the value of λ,

We get 2x + 5y-26 = 0,

Hence, the required equation of line is 2x + 5y-26 = 0.

10. Question

Find the equation of the straight line which has y-intercept equal to 4/3 and is perpendicular to 3x – 4y + 11 = 0.

Answer

Given: equation is perpendicular to 3x – 4y + 11 = 0 and has y-intercept equal to $\frac{4}{3}$

To find:

The equation of the straight line which has y-intercept equal to $\frac{4}{3}$ and is perpendicular to 3x – 4y + 11 = 0.

Explanation:

The line perpendicular to 3x − 4y + 11 = 0 is 4x + 3y + λ = 0

It is given that the line 4x + 3y + λ = 0 has y – intercept equal to $\frac{4}{3}$

This means that the line passes through $\left(0, \frac{4}{3}\right)$

∴ 0 + 4 + λ = 0

⇒ λ = -4

Substituting the value of λ,

We get 4x + 3y – 4 = 0,

Hence, equation of the required line is 4x + 3y – 4 = 0

11. Question

Find the equation of the right bisector of the line segment joining the points (a, b) and (a_1, b_1).

Answer

Given: A (a, b) and B (a_1, b_1) be the given points

To find:

Equation of the right bisector of the line segment joining the points (a, b) and (a_1, b_1).

Explanation:

Let C be the midpoint of AB.

$\therefore$ coordinates of C = $\left(\frac{a+a_1}{2}, \frac{b+b_1}{2}\right)$

And, slope of AB = $\frac{b_1 - b}{a_1 - a}$

So, the slope of the right bisector of AB is $-\frac{a_1 - a}{b_1 - b}$

Thus, the equation of the right bisector of the line segment joining the points (a, b) and (a_1, b_1) is

$$y - \frac{b + b_1}{2} = -\frac{a_1 - a}{b_1 - b}\left(x - \frac{a + a_1}{2}\right)$$

$\Rightarrow 2 (a_1-a)x + 2y(b_1-b) + (a^2 + b^2) - (a_1^2 + b_1^2) = 0$

Hence, equation of the required line $2 (a_1 - a)x + 2y(b_1- b) + (a^2 + b^2) - (a_1^2 + b_1^2) = 0$

12. Question

Find the image of the point (2, 1) with respect to the line mirror x + y – 5 = 0.

Answer

Given: (2,1) is given point and line mirror is x + y – 5 = 0

To find:

Image of the point with respect to mirror line.

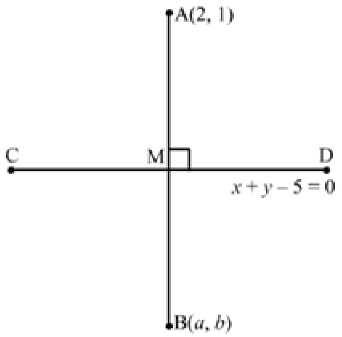
Explanation:

Let the image of A (2, 1) be B (a, b).

Let M be the midpoint of AB.

$\therefore$ Coordinates of M are = $\left(\frac{2+a}{2}, \frac{1+b}{2}\right)$

Diagram:



The point M lies on the line x + y − 5 = 0

$$\therefore \frac{2 + a}{2} + \frac{1 + b}{2} - 5 = 0$$

$\Rightarrow$ a + b = 7 … (1)

Now, the lines x + y − 5 = 0 and AB are perpendicular.

$\therefore$ Slope of AB × Slope of CD = − 1

$$\Rightarrow \frac{b - 1}{a - 2} \times (-1) = -1$$

$\Rightarrow$ a – 2 = b – 1 ……(2)

Adding eq (1) and eq (2):

2a = 8

⇒ a = 4

Now, from equation (1):

4 + b = 7

⇒ b = 3

Hence, the image of the point (2, 1) with respect to the line mirror x + y − 5 = 0 is (4, 3).

13. Question

If the image of the point (2, 1) with respect to the line mirror be (5, 2), find the equation of the mirror.

Answer

Given: image of (2,1) is (5,2)

To find:

The equation of the mirror.

Explanation:

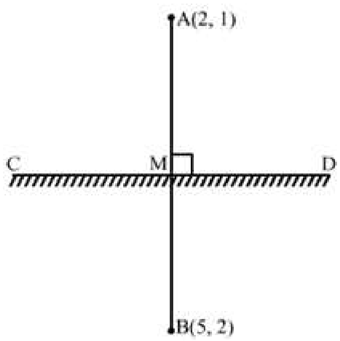
Let the image of A (2, 1) be B (5, 2).

Let M be the midpoint of AB.

Coordinates of M $= \left(\frac{2+5}{2}, \frac{1+2}{2}\right)$

$= \left(\frac{7}{2}, \frac{3}{2}\right)$

Diagram:



Let CD be the mirror.

The line AB is perpendicular to the mirror CD.

∴ Slope of AB × Slope of CD = − 1

⇒ Slope of CD = -3

Thus, the equation of the mirror CD is

$$y - \frac{3}{2} = -3\left(x - \frac{7}{2}\right)$$

⇒ 2y – 3 = -6x + 21

⇒ 6x + 2y -24 = 0

⇒ 3x + y – 12 = 0

Hence, the equation of mirror is 3x + y – 12 = 0.

14. Question

Find the equation to the straight line parallel to 3x – 4y + 6 = 0 and passing through the middle point of the join of points (2, 3) and (4, -1).

Answer

Given: equation parallel to 3x – 4y + 6 = 0 and passing through the middle point of the join of points (2, 3) and (4, -1).

To find:

The equation to the straight line parallel to 3x – 4y + 6 = 0 and passing through the middle point of the join of points (2, 3) and (4, -1).

Explanation:

Let the Given points be A (2, 3) and B $(4, -1)$. Let M be the midpoint of AB.

∴ Coordinates of M $= \left(\frac{2+4}{2}, \frac{3-1}{2}\right) = (3,1)$

The equation of the line parallel to $3x - 4y + 6 = 0$ is $3x - 4y + \lambda = 0$

This line passes through M (3, 1).

∴ $9 - 4 + \lambda = 0$

⇒ $\lambda = -5$

Substituting the value of λ in $3x - 4y + \lambda = 0$, we get $3x - 4y - 5 = 0$

Hence, the equation of the required line is $3x - 4y - 5 = 0$.

15. Question

Prove that the lines 2x – 3y + 1 = 0, x + y = 3, 2x – 3y = 2 and x + y = 4 form a parallelogram.

Answer

Given: 2x – 3y + 1 = 0,

x + y = 3,

2x – 3y = 2

x + y = 4 are given equation

To prove:

The lines 2x – 3y + 1 = 0, x + y = 3, 2x – 3y = 2 and x + y = 4 form a parallelogram.

Explanation:

The given lines can be written as

$y = \frac{2}{3}x + \frac{1}{3}$ ... (1)

$y = -x + 3$ ... (2)

$y = \frac{2}{3}x - \frac{2}{3}$ ... (3)

$y = -x + 4$ ... (4)

The slope of lines (1) and (3) is $\frac{2}{3}$ and that of lines (2) and (4) is -1.

Thus, lines (1) and (3), and (2) and (4) are two pair of parallel lines.

If both pair of opposite sides are parallel then, we can say that it is a parallelogram.

Hence proved, the given lines form a parallelogram.

16. Question

Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point where it meets the y-axis.

Answer

Given: equation is perpendicular to $\frac{x}{4} + \frac{y}{6} = 1$ and it meets the y-axis.

To find:

The equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point where it meets the y-axis.

Explanation:

Let us find the intersection of the line $\frac{x}{4} + \frac{y}{6} = 1$ with y-axis.

At x = 0,

$$0 + \frac{y}{6} = 1$$

$\Rightarrow$ y = 6

Thus, the given line intersects y-axis at (0, 6).

The line perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ is

$$\frac{x}{6} - \frac{y}{4} + \lambda = 0$$

This line passes through (0, 6).

$$0 - \frac{6}{4} + \lambda = 0$$

$\Rightarrow \lambda = \frac{3}{2}$

Now, substituting the value of λ, we get:

$$\frac{x}{6} - \frac{y}{4} + \frac{3}{2} = 0$$

$\Rightarrow$ 2x -3y + 18 = 0

Hence, the equation of the required line is 2x -3y + 18 = 0

17. Question

The perpendicular from the origin to the line y = mx + c meets it at the point (-1, 2). Find the values of m and c.

Answer

Given: perpendicular from the origin and meets at the point (-1, 2)

Explanation:

The given line is y = mx + c which can be written as mx –y + c = 0 … (1)

The slope of the line perpendicular to y = mx + c is $-\frac{1}{m}$

So, the equation of the line with slope $-\frac{1}{m}$ and passing through the origin is

$$y = -\frac{1}{m}x$$

x + my = 0 … (2)

Solving eq(1) and eq(2) by cross multiplication, we get

$$\frac{x}{mc - 0} = \frac{y}{0 - c} = \frac{1}{-1 - m^2}$$

$$\Rightarrow x = -\frac{mc}{m^2 + 1}, y = \frac{c}{m^2 + 1}$$

Thus, the point of intersection of the perpendicular from the origin to the line y = mx + c is $\left(-\frac{mc}{m^2+1}, \frac{c}{m^2+1}\right)$

It is given that the perpendicular from the origin to the line y = mx + c meets it at the point (-1, 2)

$-\frac{mc}{m^2+1} = -1$ and $\frac{c}{m^2+1} = 2$

$\Rightarrow m^2 + 1 = mc$ and $m^2 + 1 = \frac{c}{2}$

$\Rightarrow mc = \frac{c}{2}$

$\Rightarrow m = \frac{1}{2}$

Now, substituting the value of m in $m^2 + 1 = mc$, we get

$$\frac{1}{4} + 1 = \frac{1}{2}c$$

$\Rightarrow c = \frac{5}{2}$

Hence, $m = \frac{1}{2}$ and $c = \frac{5}{2}$

18. Question

Find the equation of the right bisector of the line segment joining the points (3, 4) and (-1, 2).

Answer

Given: A (3, 4) and B (– 1, 2) be the given points

To find:

Equation of the right bisector of the line segment joining the points (3, 4) and (-1, 2).

Explanation:

Let C be the midpoint of AB.

$\therefore C = \left(\frac{3-1}{2}, \frac{4+2}{2}\right) = (1,3)$

$\because$ Slope of AB $= \frac{2-4}{-1-3} = \frac{1}{2}$

$\therefore$ Slope of the perpendicular bisector of AB = -2

Thus, the equation of the perpendicular bisector of AB is

y-3 = -2(x-1)

$\Rightarrow$ 2x + y – 5 = 0

Hence, the required line is 2x + y – 5 = 0.

19. Question

The line through (h, 3) and (4, 1) intersects the line 7x – 9y – 19 = 0 at right angle. Find the value of h.

Answer

Given: A (h,3) and B (4,1) be the points intersect At right angle at the line 7x – 9y – 19 = 0

To find:

The value of h.

Explanation:

The line 7x − 9y − 19 = 0 can be written as

$$y = \frac{7}{9}x - \frac{19}{9}$$

So, the slope of this line is $\frac{7}{9}$

It is given that the line joining the points A (h,3) and B (4,1) is perpendicular to the line 7x − 9y − 19 = 0.

$$\frac{7}{9} \times \frac{1-3}{4-h} = -1$$

⇒ 9h – 36 = -14

⇒ 9h = 22

⇒ h = $\frac{22}{9}$

Hence, the value of h is $\frac{22}{9}$

20. Question

Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

Answer

Given: (3, 8) is given point and line mirror is x + 3y – 7 = 0.

To find:

Image of point with respect to mirror line.

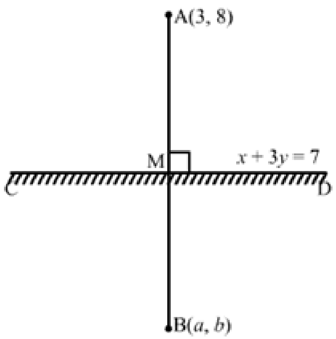
Explanation:

Let the image of A (3,8) be B (a,b).

Also, let M be the midpoint of AB.

∴ Coordinates of M = $\left(\frac{3+a}{2}, \frac{8+b}{2}\right)$

Diagram:



Point M lies on the line x + 3y = 7

$$\therefore \frac{3+a}{2} + 3 \times \left(\frac{8+b}{2}\right) = 7$$

⇒ a + 3b + 13 = 0 … (1)

Lines CD and AB are perpendicular

∴ Slope of AB × Slope of CD = − 1

$$\Rightarrow \frac{b-8}{a-3} \times \left(-\frac{1}{3}\right) = -1$$

⇒ b – 8 = 3a – 9

⇒ 3a-b-1 = 0 … (2)

Solving (1) and (2) by cross multiplication, we get:

$$\frac{a}{-3+13} = \frac{b}{39+1} = \frac{1}{-1-9}$$

⇒ a = -1, b = -4

Hence, the image of the point (3, 8) with respect to the line mirror x + 3y = 7 is

(− 1, − 4).

21. Question

Find the coordinates of the foot of the perpendicular from the point (-1, 3) to the line 3x – 4y – 16 = 0.

Answer

Given: equation is perpendicular from the point (-1, 3) to the line 3x – 4y – 16 = 0.

To find:

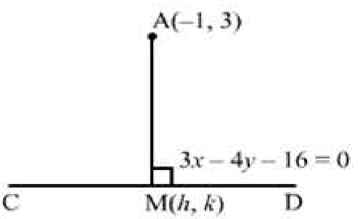
The coordinates of the foot of the perpendicular from the point (-1, 3) to the line 3x – 4y – 16 = 0.

Explanation:

Let A (− 1, 3) be the given point.

Also, let M (h, k) be the foot of the perpendicular drawn from A (− 1, 3) to the line 3x − 4y − 16 = 0

Diagram:



Point M (h, k) lies on the line 3x − 4y − 16 = 0

3h − 4k − 16 = 0 … (1)

Lines 3x − 4y − 16 = 0 and AM are perpendicular.

$$\therefore \frac{k-3}{h+1} \times \frac{3}{4} = -1$$

⇒ 4h + 3k – 5 = 0 … (2)

Solving eq (1) and eq (2) by cross multiplication, we get:

$$\frac{h}{20+48} = \frac{k}{-64+15} = \frac{1}{9+16}$$

$$\Rightarrow a = \frac{68}{25}, b = -\frac{49}{25}$$

Hence, the coordinates of the foot of perpendicular are $\left(\frac{68}{25}, -\frac{49}{25}\right)$

22. Question

Find the projection of the point (1, 0) on the line joining the points (-1, 2) and (5, 4).

Answer

Given:

The points (-1, 2) and (5, 4).

To find:

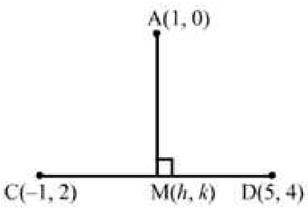
The projection of the point (1, 0) on the line joining the points (-1, 2) and (5, 4).

Explanation:

Let A (– 1, 2) be the given point whose projection is to be evaluated and C (– 1, 2) and D (5, 4) be the other two points.

Also, let M (h, k) be the foot of the perpendicular drawn from A (– 1, 2) to the line joining the points C (– 1, 2) and D (5, 4).

Diagram:



Clearly, the slope of CD and MD are equal.

$$\therefore \frac{4-k}{5-h} = \frac{4-2}{5+1}$$

⇒ h – 3k + 7 = 0…….(1)

The lines segments AM and CD are perpendicular.

$$\frac{k-0}{h-1} \times \frac{4-2}{5+1} = -1$$

⇒ 3h + k – 3 = 0 …….(2)

Solving (1) and (2) by cross multiplication, we get:

$$\frac{h}{9-7} = \frac{k}{21+3} = \frac{1}{1+9}$$

$$\Rightarrow h = \frac{1}{5}, k = \frac{12}{5}$$

Hence, the projection of the point (1, 0) on the line joining the points (-1, 2) and (5, 4) is $\left(\frac{1}{5}, \frac{12}{5}\right)$

23. Question

Find the equation of a line perpendicular to the line $\sqrt{3}x - y + 5 = 0$ and at a distance of 3 units from the origin.

Answer

Given:

Line $\sqrt{3}x - y + 5 = 0$ and distance of 3 units from the origin.

To find:

The equation of a line perpendicular to the line $\sqrt{3}x - y + 5 = 0$ and at a distance of 3 units from the origin.

Explanation:

The line perpendicular to $\sqrt{3}x - y + 5 = 0$ is $x + \sqrt{3}y + \lambda = 0$

It is given that the line $x + \sqrt{3}y + \lambda = 0$ is at a distance of 3 units from the origin.

$$\therefore \left|\frac{\lambda}{\sqrt{1 + 3}}\right| = 3$$

$\Rightarrow \lambda = \pm 6$

Substituting the value of λ,

We get $x + \sqrt{3}y \pm 6 = 0$,

Hence, equation of the required line is $x + \sqrt{3}y \pm 6 = 0$

24. Question

The line 2x + 3y = 12 meets the x-axis at A and y-axis at B. The line through (5, 5) perpendicular to AB meets the x-axis and the line AB at C and E respectively. If O is the origin of coordinates, find the area of figure OCEB.

Answer

Given:

Line 2x + 3y = 12 meets the x-axis at A and y-axis at B

To find:

The area of figure OCEB.

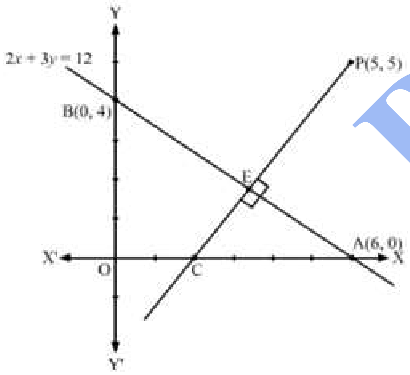
Explanation:

The given line is 2x + 3y = 12, which can be written as

$\frac{x}{6} + \frac{y}{4} = 1$......(1)

So, the coordinates of the points A and B are (6, 0) and (0, 4), respectively.

Diagram:



The equation of the line perpendicular to line (1) is

$$\frac{x}{4} - \frac{y}{6} + \lambda = 0$$

This line passes through the point (5, 5).

$$\therefore \frac{5}{4} - \frac{5}{6} + \lambda = 0$$

$$\Rightarrow \lambda = -\frac{5}{12}$$

Now, substituting the value of λ in $\frac{x}{4} - \frac{y}{6} + \lambda = 0$, we get:

$$\frac{x}{4} - \frac{y}{6} - \frac{5}{12} = 0$$

$$\Rightarrow \frac{x}{\frac{5}{3}} - \frac{y}{\frac{5}{2}} = 1 \text{......(2)}$$

Thus, the coordinates of intersection of line (1) with the x-axis is $C\left(\frac{5}{3}, 0\right)$

To find the coordinates of E, let us write down equations (1) and (2) in the following manner:

2x + 3y – 12 = 0 ... (3)

3x – 2y – 5 = 0 ... (4)

Solving (3) and (4) by cross multiplication, we get:

$$\frac{x}{-15 - 24} = \frac{y}{-36 + 10} = \frac{1}{-4 - 9}$$

⇒ x = 3, y = 2

Thus, the coordinates of E are (3, 2).

From the figure,

$$EC = \sqrt{\left(\frac{5}{3} - 3\right)^2 + (0 - 2)^2} = \frac{2\sqrt{13}}{3}$$

$$EA = \sqrt{(6 - 3)^2 + (0 - 2)^2} = \sqrt{13}$$

Now,

Area(OCEB) = Area(ΔOAB) –Area(ΔCAE)

$$\Rightarrow \text{Area(OCEB)} = \frac{1}{2} \times 6 \times 4 - \frac{1}{2} \times \frac{2\sqrt{13}}{3} \times \sqrt{13}$$

$= \frac{23}{3}$ sq units

Hence, area of figure OCEB is $= \frac{23}{3}$ sq units

25. Question

Find the equation of the straight line which cuts off intercepts on x-axis twice that on y-axis and is at a unit distance from the origin.

Answer

To find:

The equation of the straight line which cuts off intercepts on x-axis twice that on y-axis and is at a unit distance from the origin.

Assuming:

Intercepts on x-axis and y-axis be 2a and a, respectively.

Explanation:

So, the equation of the line with intercepts 2a on x-axis and a on y-axis be

$$\frac{x}{2a} + \frac{y}{a} = 1$$

⇒ x + 2y = 2a ... (1)

Let us change equation (1) into normal form.

$$\frac{x}{\sqrt{1 + 2^2}} + \frac{2y}{\sqrt{1 + 2^2}} = \frac{2a}{\sqrt{1 + 2^2}}$$

$$\frac{x}{\sqrt{5}} + \frac{2y}{\sqrt{5}} = \frac{2a}{\sqrt{5}}$$

Thus, the length of the perpendicular from the origin to the line (1) is

$p = \left|\frac{2a}{\sqrt{5}}\right|$ Given:

P = 1

$$\therefore \left|\frac{2a}{\sqrt{5}}\right| = 1$$

$$\Rightarrow a = \pm\frac{\sqrt{5}}{2}$$

Required equation of the line:

$x + 2y = \pm\frac{2\sqrt{5}}{2}$

$\Rightarrow x + 2y + \sqrt{5} = 0$

Hence, equation of required line is $x + 2y + \sqrt{5} = 0$.

26. Question

The equations of perpendicular bisectors of the sides AB and AC of a triangle ABC are x – y + 5 = 0 and x + 2y = 0 respectively. If the point A is (1, -2), find the equation of the line BC.

Answer

Given:

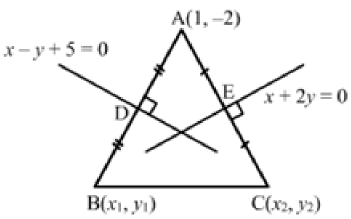
Sides AB and AC of a triangle ABC are x – y + 5 = 0 and x + 2y = 0.

To find:

The equation of the line BC.

Explanation:

Diagram:



Let the perpendicular bisectors $x - y + 5 = 0$ and $x + 2y = 0$ of the sides AB and AC intersect at D and E, respectively.

Let (x_1,y_1) and (x_2,y_2) be the coordinates of points B and C.

Coordinates of D $= \left(\frac{x_1 + 1}{2}, \frac{y_1 - 2}{2}\right)$

And coordinates of E $= \left(\frac{x_2 + 1}{2}, \frac{y_2 - 2}{2}\right)$

Point D lies on the line $x - y + 5 = 0$

$$\therefore \frac{x_1 + 1}{2} - \frac{y_1 - 2}{2} + 5 = 0$$

$\Rightarrow x_1 - y_1 + 13 = 0 \ldots (1)$

Point E lies on the line x + 2y = 0

$\therefore \frac{x_2 + 1}{2} + 2 \times \frac{y_2 - 2}{2} = 0$

$\Rightarrow x_2 + 2y_2 - 3 = 0 \ldots (2)$

Side AB is perpendicular to the line x − y + 5 = 0

$\therefore 1 \times \frac{y_1 - 2}{x_1 - 1} = -1$

$\Rightarrow x_1 + y_1 + 1 \ldots (3)$

Similarly, side AC is perpendicular to the line x + 2y = 0

$\therefore -\frac{1}{2} \times \frac{y_2 + 2}{x_2 - 1} = -1$

$\Rightarrow 2x_2 - y_2 - 4 = 0 \ldots (4)$

Now, solving eq (1) and eq (3) by cross multiplication, we get:

$$\frac{x_1}{-1 - 13} = \frac{y_1}{13 - 1} = \frac{1}{1 + 1}$$

$\Rightarrow x_1 = -7, y_1 = 6$

Thus, the coordinates of B are (-7, 6)

Similarly, solving (2) and (4) by cross multiplication, we get:

$$\frac{x_2}{-8 - 3} = \frac{y_2}{-6 + 4} = \frac{1}{-1 - 4}$$

$\Rightarrow x_2 = \frac{11}{5}, y_2 = \frac{2}{5}$

Thus, coordinates of C are $\left(\frac{11}{5}, \frac{2}{5}\right)$

Therefore, equation of line BC is

$$y - 6 = \frac{\frac{2}{5} - 6}{\frac{11}{5} + 7}(x + 7)$$

$\Rightarrow y - 6 = -\frac{28}{46}(x + 7)$

$\Rightarrow$ 14x + 23 y – 40 = 0

Hence, the equation of line BC is 14x + 23 y – 40 = 0

Exercise 23.13

1 A. Question

Find the angles between each of the following pairs of straight lines :

3x + y + 12 = 0 and x + 2y – 1 = 0

Answer

Given:

The equations of the lines are

3x + y + 12 = 0 … (1)

x + 2y − 1 = 0 … (2)

To find:

Angles between two lines.

Explanation:

Let m_1 and m_2 be the slopes of these lines.

$m_1 = -3,\ m_2 = -\frac{1}{2}$

Let θ be the angle between the lines.Then,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}} = 1$$

$$\Rightarrow \theta = \frac{\pi}{4} \text{ or } 45°$$

Hence, the acute angle between the lines is 45°

1 B. Question

Find the angles between each of the following pairs of straight lines :

3x – y + 5 = 0 and x – 3y + 1 = 0

Answer

Given:

The equations of the lines are

3x − y + 5 = 0 … (1)

x − 3y + 1 = 0 … (2)

To find:

Angles between two lines.

Explanation:

Let m_1 and m2 be the slopes of these lines.

$m_1 = 3,\ m_2 = \frac{1}{3}$

Let θ be the angle between the lines.Then,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{3 + \frac{1}{3}}{1 + 1} = \frac{4}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Hence, the acute angle between the lines is $\tan^{-1}\left(\frac{4}{3}\right)$

1 C. Question

Find the angles between each of the following pairs of straight lines :

3x + 4y – 7 = 0 and 4x – 3y + 5 = 0

Answer

Given:

The equations of the lines are

$3x + 4y - 7 = 0$ … (1)

$4x - 3y + 5 = 0$ … (2)

To find:

Angles between two lines.

Explanation:

Let m_1 and m_2 be the slopes of these lines.

$m_1 = -\frac{3}{4}, m_2 = \frac{4}{3}$

$\because m_1 m_2 = -\frac{3}{4} \times \frac{4}{3} = -1$

Hence, the given lines are perpendicular.Therefore, the angle between them is 90°.

1 D. Question

Find the angles between each of the following pairs of straight lines :

x – 4y = 3 and 6x – y = 11

Answer

Given:

The equations of the lines are

$x - 4y = 3$ … (1)

$6x - y = 11$ … (2)

To find:

Angles between two lines.

Explanation:

Let m_1 and m_2 be the slopes of these lines.

$m_1 = \frac{1}{4}, m_2 = 6$

Let θ be the angle between the lines.Then,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{4} + 6}{1 + \frac{3}{2}} = \frac{23}{10}$$

$\Rightarrow \theta = \tan^{-1}(\frac{23}{10})$

Hence, the acute angle between the lines is $\tan^{-1}(\frac{23}{10})$

1 E. Question

Find the angles between each of the following pairs of straight lines :

$(m^2 - mn)\, y = (mn + n^2)x + n^3$ and $(mn + m^2)y = (mn - n^2)x + m^3$.

Answer

Given:

The equations of the lines are

$(m^2 - mn)\, y = (mn + n^2)\, x + n^3$ … (1)

$(mn + m^2)\, y = (mn - n^2)\, x + m^3$ … (2)

To find:

Angles between two lines.

Explanation:

Let m_1 and m_2 be the slopes of these lines.

$$\therefore m_1 = \frac{mn + n^2}{m^2 - mn}, m_2 = \frac{mn - n^2}{m^2 + mn}$$

Let θ be the angle between the lines.

Then,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{mn + n^2}{m^2 - mn} - \frac{mn - n^2}{m^2 + mn}}{1 + \left(\frac{mn + n^2}{m^2 - mn}\right)\left(\frac{mn - n^2}{m^2 + mn}\right)}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{(mn + n^2)(m^2 + mn) - (mn - n^2)(m^2 - mn)}{(m^2 - mn)(m^2 + mn) + (mn + n^2)(mn - n^2)}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{(mn + n^2)(m^2 + mn) - (mn - n^2)(m^2 - mn)}{(m^2 - mn)(m^2 + mn) + (mn + n^2)(mn - n^2)}\right|$$

Then,

$$\Rightarrow \tan\theta = \left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4m^2n^2}{m^4 - n^4}\right)$$

Hence, the acute angle between the lines is $\tan^{-1}\left(\frac{4m^2n^2}{m^4 - n^4}\right)$

2. Question

Find the acute angle between the lines $2x - y + 3 = 0$ and $x + y + 2 = 0$.

Answer

Given:

The equations of the lines are

$2x - y + 3 = 0$ … (1)

$x + y + 2 = 0$ … (2)

To find:

Angles between two lines.

Explanation:

Let m_1 and m_2 be the slopes of these lines.

$m_1 = 2$, $m_2 = -1$

Let θ be the angle between the lines.Then,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{2 + 1}{1 + 2} = 3$$

$$\Rightarrow \theta = \tan^{-1}(3)$$

Hence, the acute angle between the lines is $\tan^{-1}(3)$.

3. Question

Prove that the points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram and find the angle between its diagonals.

Answer

To prove:

The points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram

To find:

The angle between diagonals of parallelogram.

Assuming:

A(2, − 1), B(0, 2), C(2, 3) and D(4, 0) be the vertices.

Explanation:

Slope of AB $= \frac{2 + 1}{0 - 2} = -\frac{3}{2}$

Slope of BC $= \frac{3 - 2}{2 - 0} = \frac{1}{2}$

Slope of CD $= \frac{0 - 3}{4 - 2} = -\frac{3}{2}$

Slope of DA $= \frac{-1 - 0}{2 - 4} = \frac{1}{2}$

Thus, AB is parallel to CD and BC is parallel to DA.

Therefore, the given points are the vertices of a parallelogram.

Now, let us find the angle between the diagonals AC and BD.

Let m_1 and m_2 be the slopes of AC and BD, respectively.

$$\therefore m_1 = \frac{3 + 1}{2 - 2} = \infty$$

$\therefore m_2 = \frac{0-2}{4-0} = -\frac{1}{2}$

Thus, the diagonal AC is parallel to the y-axis.

$\therefore \angle ODB = \tan^{-1}\left(\frac{1}{2}\right)$

In triangle MND,

$\angle DMN = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

Hence proved, the acute angle between the diagonal is $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$.

4. Question

Find the angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1.

Answer

Given:

Points (2, 0), (0, 3) and the line x + y = 1.

Assuming:

Let A (2, 0), B (0, 3) be the given points.

To find:

Angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1.

Explanation:

Slope of AB = $m_1 = \frac{3-0}{0-2} = -\frac{3}{2}$

Slope of the line x + y = 1 is -1

$\therefore m_2 = -1$

Let θ be the angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{-\frac{3}{2}+1}{1+\frac{3}{2}} = \frac{1}{5}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{5}\right)$

Hence, the acute angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1 is $\tan^{-1}\left(\frac{1}{5}\right)$.

5. Question

If θ is the angle which the straight line joining the points (x_1, y_1) and (x_2, y_2) subtends at the origin, prove that $\tan\theta = \frac{x_2 y_1 - x_1 y_2}{x_1 x_2 + y_1 y_2}$ and $\cos\theta = \frac{x_1 y_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}$.

Answer

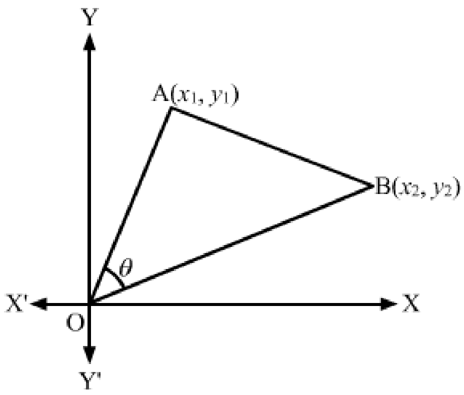
To prove:

$\tan\theta = \frac{x_2y_1 - x_1y_2}{x_1x_2 + y_1y_2}$ and $\cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}}$.

Assuming:

A (x_1, y_1) and B (x_2, y_2) be the given points and O be the origin.

Explanation:



Slope of OA = m_1 = y_{1x1}

Slope of OB = m_2 = y_{2x2}

It is given that θ is the angle between lines OA and OB.

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$= \frac{\frac{y_1}{x_1} - \frac{y_2}{x_2}}{1 + \frac{y_1}{x_1} \times \frac{y_2}{x_2}}$

$\Rightarrow \tan\theta = \frac{x_2y_1 - x_1y_2}{x_1x_2 + y_1y_2}$

Now,

As we know that $\cos\theta = \sqrt{\frac{1}{1+\tan^2\theta}}$

$\therefore \cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{(x_2y_1 - x_1y_2)^2 + (x_1x_2 + y_1y_2)^2}}$

$\Rightarrow \cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2y_1^2 + x_1^2y_2^2 + x_1^2x_2^2 + y_1^2y_2^2}}$

$\Rightarrow \cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}}$.

Hence proved.

6. Question

Prove that the straight lines (a + b)x + (a – b)y = 2ab, (a – b)x + (a + b)y = 2ab and x + y = 0 form an isosceles triangle whose vertical angle is $2\tan^{-1}\left(\frac{a}{b}\right)$.

Answer

Given:

The given lines are

(a + b) x + (a − b) y = 2ab … (1)

(a − b) x + (a + b) y = 2ab … (2)

x + y = 0 … (3)

To prove:

The straight lines (a + b)x + (a – b)y = 2ab, (a – b)x + (a + b)y = 2ab and x + y = 0 form an isosceles triangle whose vertical angle is $2\tan^{-1}\left(\frac{a}{b}\right)$.

Assuming:

Let m_1, m_2 and m_3 be the slopes of the lines (1), (2) and (3), respectively.

Explanation:

Now,

Slope of the first line = $m_1 = -\left(\frac{a+b}{a-b}\right)$

Slope of the second line = $m_2 = -\left(\frac{a-b}{a+b}\right)$

Slope of the third line = m_3 = -1

Let θ_1 be the angle between lines (1) and (2), θ_2 be the angle between lines (2) and (3) and θ_3 be the angle between lines (1) and (3).

$$\therefore \tan\theta_1 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\left(\frac{a+b}{a-b}\right) + \left(\frac{a-b}{a+b}\right)}{1 + \left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right)}\right|$$

$$\Rightarrow \tan\theta_1 = \left|\frac{2ab}{a^2 - b^2}\right|$$

$$\Rightarrow \theta_1 = \tan^{-1}\left(\frac{2ab}{a^2 - b^2}\right)$$

$$\therefore \tan\theta_2 = \left|\frac{m_2 - m_3}{1 + m_2 m_3}\right|$$

$$= \left|\frac{-\left(\frac{a-b}{a+b}\right) + 1}{1 + \left(\frac{a-b}{a+b}\right)}\right|$$

$$\Rightarrow \tan\theta_2 = \left|\frac{b}{a}\right|$$

$$\Rightarrow \theta_2 = \tan^{-1}\left(\frac{b}{a}\right)$$

$\therefore \tan\theta_3 = \left|\frac{m_1 - m_3}{1 + m_1 m_3}\right|$

$= \left|\frac{-\left(\frac{a + b}{a - b}\right) + 1}{1 + \left(\frac{a + b}{a - b}\right)}\right|$

$\Rightarrow \tan\theta_3 = \left|\frac{b}{a}\right|$

$\Rightarrow \theta_3 = \tan^{-1}\left(\left|\frac{b}{a}\right|\right)$

Here,$\theta_2 = \theta_3$ and $\theta = 2\tan^{-1}\left(\frac{a}{b}\right)$

Hence proved, the given lines form an isosceles triangle whose vertical angle is $2\tan^{-1}\left(\frac{a}{b}\right)$

7. Question

Find the angle between the lines x = a and by + c = 0.

Answer

Given:

x = a

by + c = 0

To find:

Angle between the lines x = a and by + c = 0.

Concept Used:

Angle between two lines.

Explanation:

The given lines can be written as

x = a … (1)

y = -cb … (2)

Hence, Lines (1) and (2) are parallel to the y-axis and x-axis, respectively. Thus, they intersect at right angle, i.e. at 90°.

8. Question

Find the tangent of the angle between the lines which have intercepts 3, 4 and 1, 8 on the axes respectively.

Answer

Given:

Intercepts 3, 4 and 1, 8 on the axes

To find:

Tangent of the angle between the lines

Concept Used:

Angle between two lines.

Explanation:

The respective equations of the lines having intercepts 3, 4 and 1, 8 on the axes are

$\frac{x}{3} + \frac{y}{4} = 1$ ... (1)

$\frac{x}{1} + \frac{y}{8} = 1$ ... (2)

Assuming:

m_1 and m_2 be the slope of the lines (1) and (2), respectively.

$\therefore m_1 = -\frac{4}{3}$, $m_2 = -8$

Let θ be the angle between the lines (1) and (2).

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{-\frac{4}{3} + 8}{1 + \frac{32}{3}} = \frac{4}{7}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{7}\right)$

Hence, the tangent of the angles between the lines is $\frac{4}{7}$.

9. Question

Show that the line $a^2x + ay + 1 = 0$ is perpendicular to the line $x - ay = 1$ for all non-zero real values of a.

Answer

Given:

Line $a^2x + ay + 1 = 0$ is perpendicular to the line $x - ay = 1$

To prove:

The line $a^2x + ay + 1 = 0$ is perpendicular to the line $x - ay = 1$ for all non-zero real values of a.

Concept Used:

Product of slope of perpendicular line is -1.

Explanation:

The given lines are

$a^2x + ay + 1 = 0$ ... (1)

$x - ay = 1$ ... (2)

Let m_1 and m_2 be the slopes of the lines (1) and (2).

$m_1m_2 = -\frac{a^2}{a} \times \frac{1}{a} = -1$

Hence proved, line $a^2x + ay + 1 = 0$ is perpendicular to the line $x - ay = 1$ for all non-zero real values of a.

10. Question

Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.

Answer

Given:

$\frac{x}{a} + \frac{y}{b} = 1$ ......(i)

$\frac{x}{a} - \frac{y}{b} = 1$ ......(ii)

To prove:

The tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$

Concept Used:

Angle between two lines.

Assuming:

m_1 and m_2 be the slope of the lines (1) and (2), respectively.

Explanation:

$\therefore m_1 = -\frac{b}{a}$, $m_2 = \frac{b}{a}$

Let θ be the angle between the lines (1) and (2).

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(\frac{-b}{a}\right)\left(\frac{b}{a}\right)}\right|$$

$$= \left|\frac{-\frac{2b}{a}}{1 - \frac{b^2}{a^2}}\right|$$

$$= \left|-\frac{2ab}{a^2 - b^2}\right|$$

$$= \frac{2ab}{a^2 - b^2}$$

Hence proved, the tangent of the angles between the lines is $\frac{2ab}{a^2 - b^2}$.

Exercise 23.14

1. Question

Find the values of α so that the point P(α ^{2}, α) lies inside or on the triangle formed by the lines x – 5y + 6 = 0, x – 3y + 2 = 0 and x – 2y – 3 = 0.

Answer

Given: x – 5y + 6 = 0, x – 3y + 2 = 0 and x – 2y – 3 = 0 forming a triangle and point P(α^2, α) lies inside or on the triangle

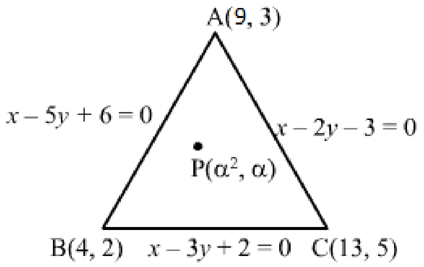
To find: value of α

Explanation:

Let ABC be the triangle of sides AB, BC and CA whose equations are $x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$, respectively.On solving the equations,

We get A (9, 3), B (4, 2) and C (13, 5) as the coordinates of the vertices.

Diagram:



It is given that point P (α^2, α) lies either inside or on the triangle. The three conditions are given below.

(i) A and P must lie on the same side of BC.

(ii) B and P must lie on the same side of AC.

(iii) C and P must lie on the same side of AB.

If A and P lie on the same side of BC, then

$(9 - 9 + 2)(\alpha^2 - 3\alpha + 2) \geq 0$

$\Rightarrow (\alpha - 2)(\alpha - 1) \geq 0$

$\Rightarrow \alpha \in (-\infty, 1] \cup [2, \infty)$ ... (1)

If B and P lie on the same side of AC, then

$(4 - 4 - 3)(\alpha^2 - 2\alpha - 3) \geq 0$

$\Rightarrow (\alpha - 3)(\alpha + 1) \leq 0$

$\Rightarrow \alpha \in [-1, 3]$ ... (2)

If C and P lie on the same side of AB, then

$(13 - 25 + 6)(\alpha^2 - 5\alpha + 6) \geq 0$

$\Rightarrow (\alpha - 3)(\alpha - 2) \leq 0$

$\Rightarrow \alpha \in [2, 3]$ ... (3)

From (1), (2) and (3), we get:

$\alpha \in [2, 3]$

Hence, $\alpha \in [2, 3]$

2. Question

Find the values of the parameter a so that the point (a, 2) is an interior point of the triangle formed by the lines $x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$.

Answer

Given:

$x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$ forming a triangle and point (a, 2)is an interior point of the triangle

To find:

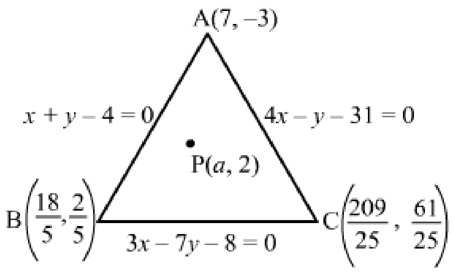
Value of a

Explanation:

Let ABC be the triangle of sides AB, BC and CA whose equations are $x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$, respectively.

On solving them, we get A (7, - 3), B $\left(\frac{18}{5}, \frac{2}{5}\right)$ and C $\left(\frac{209}{25}, \frac{61}{25}\right)$ as the coordinates of the vertices.Let P (a, 2) be the given point.

Diagram:



It is given that point P (a, 2) lies inside the triangle. So, we have the following:

(i) A and P must lie on the same side of BC.

(ii) B and P must lie on the same side of AC.

(iii) C and P must lie on the same side of AB.

Thus, if A and P lie on the same side of BC, then

$21 + 21 - 8 - 3a - 14 - 8 > 0$

$\Rightarrow a > \frac{22}{3}$ ... (1)

If B and P lie on the same side of AC, then

$4 \times \frac{18}{5} - \frac{2}{5} - 31 - 4a - 2 - 31 > 0$

$\Rightarrow a < \frac{33}{4}$ ... (2)

If C and P lie on the same side of AB, then

$\frac{209}{25} + \frac{61}{25} - 4 - a + 2 - 4 > 0$

$\Rightarrow \frac{34}{5} - 4 - a + 2 - 4 > 0$

$\Rightarrow a > 2$ ... (3)

From (1), (2) and (3), we get:

$A \in \left(\frac{22}{3}, \frac{33}{4}\right)$

Hence, $A \in \left(\frac{22}{3}, \frac{33}{4}\right)$

3. Question

Determine whether the point (-3, 2) lies inside or outside the triangle whose sides are given by the equations x + y – 4 = 0, 3x – 7y + 8 = 0, 4x – y – 31 = 0.

Answer

Given:

x + y – 4 = 0, 3x – 7y + 8 = 0, 4x – y – 31 = 0 forming a triangle and point (-3, 2)

To find:

Point (-3, 2) lies inside or outside the triangle

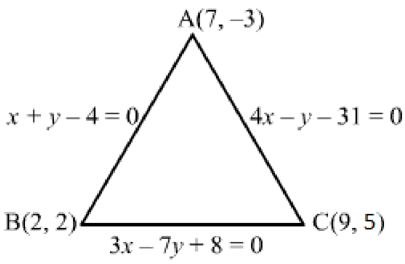
Explanation:

Let ABC be the triangle of sides AB, BC and CA, whose equations x + y− 4 = 0, 3x − 7y + 8 = 0 and 4x − y − 31 = 0, respectively.

On solving them, we get A (7, - 3), B (2, 2) and C (9, 5) as the coordinates of the vertices.

Let P (− 3, 2) be the given point.

Diagram:



The given point P (− 3, 2) will lie inside the triangle ABC, if

(i) A and P lies on the same side of BC

(ii) B and P lies on the same side of AC

(iii) C and P lies on the same side of AB

Thus, if A and P lie on the same side of BC, then

21 + 21 + 8 – 9 – 14 + 8 > 0

⇒ 50 × - 15 > 0

⇒ - 750 > 0,

Which is false

Hence, the point (− 3, 2) lies outside triangle ABC.

Exercise 23.15

1. Question

Find the distance of the point (4, 5) from the straight line 3x – 5y + 7 = 0.

Answer

Given:

Line 3x – 5y + 7 = 0

Concept Used:

Distance of a point from a line.

To find:

The distance of the point (4, 5) from the straight line 3x – 5y + 7 = 0.

Explanation:

Comparing ax + by + c = 0 and 3x − 5y + 7 = 0, we get:

a = 3, b = − 5 and c = 7

So, the distance of the point (4, 5) from the straight line 3x − 5y + 7 = 0 is

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$\Rightarrow d = \left|\frac{3 \times 4 - 5 \times 5 + 7}{\sqrt{3^2 + (-5^2)}}\right| = \frac{6}{\sqrt{34}}$$

Hence, the required distance is $\frac{6}{\sqrt{34}}$

2. Question

Find the perpendicular distance of the line joining the points (cosθ, sinθ) and (cosϕ, sinϕ) from the origin.

Answer

Given: points (cosθ, sinθ) and (cosϕ, sinϕ) from the origin.

To find:

The perpendicular distance of the line joining the points (cosθ, sinθ) and (cosϕ, sinϕ) from the origin

Concept Used:

Distance of a point from a line.

Explanation:

The equation of the line joining the points (cos θ, sin θ) and (cos ϕ, sin ϕ) is given below:

$$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$$

$$\Rightarrow (\cos\phi - \cos\theta)y - \sin\theta(\cos\phi - \cos\theta) = (\sin\phi - \sin\theta)x - (\sin\phi - \sin\theta)\cos\theta$$

$$\Rightarrow (\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$$

Let d be the perpendicular distance from the origin to the line $(\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$

-sinθ2 + cosϕ-cosθ2

$$\Rightarrow d = \left|\frac{\sin\theta - \phi}{\sqrt{(\sin\phi - \sin\theta)^2 + (\cos\phi - \cos\theta)^2}}\right|$$

$$\Rightarrow d = \left|\frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \sin^2\theta - 2\sin\phi\sin\theta + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}}\right|$$

$$\Rightarrow d = \left|\frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \cos^2\phi + \sin^2\theta + \cos^2\phi + \cos^2\theta - 2\cos(\theta - \phi)}}\right|$$

$$\Rightarrow d = \left|\frac{\frac{1}{\sqrt{2}}(\sin(\theta-\phi))}{\sqrt{1-\cos(\theta-\phi)}}\right|$$

$$\Rightarrow d = \frac{1}{\sqrt{2}}\left|\frac{\sin(\theta-\phi)}{\sqrt{2\sin^2\left(\frac{\theta-\phi}{2}\right)}}\right|$$

$$\Rightarrow d = \frac{1}{2}\left|\frac{2\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)}{\sin\left(\frac{\theta-\phi}{2}\right)}\right|$$

$$\Rightarrow d = \cos\left(\frac{\theta-\phi}{2}\right)$$

Hence, the required distance is $\cos\left(\frac{\theta-\phi}{2}\right)$

3. Question

Find the length of the perpendicular from the origin to the straight line joining the two points whose coordinates are (a cos α, a sin α) and (a cos β, a sin β).

Answer

Given:

Coordinates are (a cos α, a sin α) and (a cos β, a sin β).

To find:

The length of the perpendicular from the origin to the straight line joining the two points whose coordinates are (a cos α, a sin α) and (a cos β, a sin β).

Concept Used:

Distance of a point from a line.

Explanation:

Equation of the line passing through (acosα, asinα) and (acosβ, asinβ) is

$$y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$\Rightarrow y - a\sin\alpha = \frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}(x - a\cos\alpha)$$

$$\Rightarrow y - a\sin\alpha = \frac{2\cos\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}{2\sin\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow y - a\sin\alpha = -\cot\left(\frac{\beta+\alpha}{2}\right)(x - a\cos\alpha)$$

$$\Rightarrow y - a\sin\alpha = -\cot\left(\frac{\alpha+\beta}{2}\right)(x - a\cos\alpha)$$

$$\Rightarrow x\cot\left(\frac{\alpha+\beta}{2}\right) + y - a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right) = 0$$

The distance of the line from the origin is

$$d = \left|\frac{-a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\cot^2\left(\frac{(\alpha+\beta)}{2}\right) + 1}}\right|$$

$$d = \left|\frac{-a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\operatorname{cosec}^2\left(\frac{(\alpha+\beta)}{2}\right)}}\right|$$

$\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$$\Rightarrow d = a\left|\sin\left(\frac{\alpha+\beta}{2}\right)\sin\alpha + \cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)\right|$$

$$\Rightarrow d = a\left|\sin\alpha\sin\left(\frac{\alpha+\beta}{2}\right) + \cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)\right|$$

$$\Rightarrow d = a\left|\cos\left(\frac{\alpha+\beta}{2} - \alpha\right)\right| = a\cos\left(\frac{\beta-\alpha}{2}\right)$$

Hence, the required distance is $a\cos\left(\frac{\beta-\alpha}{2}\right)$

4. Question

Show that the perpendicular let fall from any point on the straight line 2x + 11y – 5 = 0 upon the two straight lines 24x + 7y = 20 and 4x – 3y – 2 = 0 are equal to each other.

Answer

<u>Given:</u>

Lines 24x + 7y = 20 and 4x – 3y – 2 = 0

To prove:

The perpendicular let fall from any point on the straight line 2x + 11y – 5 = 0 upon the two straight lines 24x + 7y = 20 and 4x – 3y – 2 = 0 are equal to each other.

<u>Concept Used:</u>

Distance of a point from a line.

Assuming:

P(a, b) be any point on 2x + 11y − 5 = 0

Explanation:

∴ 2a + 11b − 5 = 0

$\Rightarrow b = \frac{5-2a}{11}$ ....(i)

Let d_1 and d_2 be the perpendicular distances from point Pon the lines 24x + 7y = 20 and 4x − 3y − 2 = 0, respectively.

$d_1 = \left|\frac{24a + 7b - 20}{\sqrt{24^2 + 7^2}}\right| = \left|\frac{24a + 7b - 20}{25}\right|$

$\Rightarrow d_1 = \left|\frac{24a + 7 \times \frac{5 - 2a}{11} - 20}{25}\right|$

From (1)

$\Rightarrow d_1 = \left|\frac{50a - 37}{55}\right|$

Similarly,

$d_2 = \left|\frac{4a - 3b - 2}{\sqrt{3^2 + (-4)^2}}\right| = \left|\frac{4a - 3 \times \frac{5 - 2a}{11} - 2}{5}\right|$

$\Rightarrow d_2 = \left|\frac{44a - 15 + 6a - 22}{11 \times 5}\right|$

From (1)

$\Rightarrow d_2 = \left|\frac{50a - 37}{55}\right|$

$\therefore d_1 = d_2$

Hence proved.

5. Question

Find the distance of the point of intersection of the lines 2x + 3y = 21 and 3x – 4y + 11 = 0 from the line 8x + 6y + 5 = 0.

Answer

Given:

Lines 2x + 3y = 21 and 3x – 4y + 11 = 0

To find:

The distance of the point of intersection of the lines 2x + 3y = 21 and 3x – 4y + 11 = 0 from the line 8x + 6y + 5 = 0.

Concept Used:

Distance of a point from a line.

Explanation:

Solving the lines 2x + 3y = 21 and 3x − 4y + 11 = 0 we get:

$$\frac{x}{33 - 84} = \frac{y}{-63 - 22} = \frac{1}{-8 - 9}$$

⇒ x = 3, y = 5

So, the point of intersection of 2x + 3y = 21 and 3x − 4y + 11 = 0 is (3, 5).

Now, the perpendicular distance d of the line 8x + 6y + 5 = 0 from the point (3, 5) is $d = \left|\frac{24 + 30 + 5}{\sqrt{8^2 + 6^2}}\right| = \frac{59}{10}$

Hence, distance is $\frac{59}{10}$.

6. Question

Find the length of the perpendicular from the point (4, -7) to the line joining the origin and the point of intersection of the lines 2x – 3y + 14 = 0 and 5x + 4y – 7 = 0.

Answer

Given:

Lines 2x – 3y + 14 = 0 and 5x + 4y – 7 = 0.

To find:

The length of the perpendicular from the point (4, -7) to the line joining the origin and the point of intersection of the lines 2x – 3y + 14 = 0 and 5x + 4y – 7 = 0.

Concept Used:

Distance of a point from a line.

Explanation:

Solving the lines 2x − 3y + 14 = 0 and 5x + 4y − 7 = 0 we get:

$$\frac{x}{21 - 56} = \frac{y}{70 + 14} = \frac{1}{8 + 15}$$

$\Rightarrow x = -\frac{35}{23}, y = \frac{84}{23}$

So, the point of intersection of 2x − 3y + 14 = 0 and 5x + 4y − 7 = 0 is $-\frac{35}{23}, \frac{84}{23}$.

The equation of the line passing through the origin and the point $\left(-\frac{35}{23}, \frac{84}{23}\right)$ is

$$y - 0 = \frac{\frac{84}{23} - 0}{-\frac{35}{23} - 0}(x - 0)$$

$\Rightarrow y = \frac{84}{-35}x$

$\Rightarrow y = -\frac{12}{5}x$

$\Rightarrow 12x + 5y = 0$

Let d be the perpendicular distance of the line 12x + 5y = 0 from the point (4, − 7)

$\therefore d = \left|\frac{48 - 35}{\sqrt{12^2 + 5^2}}\right| = \frac{13}{13} = 1$

Hence, Length of perpendicular is 1.

7. Question

What are the points on X-axis whose perpendicular distance from the straight line $\frac{x}{a} + \frac{y}{b} = 1$ is a ?

Answer

Given:

The points on x-axis whose perpendicular distance from the straight line $\frac{x}{a} + \frac{y}{b} = 1$ is a

To find:

Points on x-axis

Concept Used:

Distance of a point from a line.

Explanation:

Let (t, 0) be a point on the x-axis.

It is given that the perpendicular distance of the line $\frac{x}{a} + \frac{y}{b} = 1$ from a point is a.

$$\therefore \frac{\frac{t}{a} + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = a$$

$$\Rightarrow a^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) = \frac{t^2}{a^2} + 1 - \frac{2t}{a}$$

$$\Rightarrow 1 + \frac{a^2}{b^2} = \frac{t^2}{a^2} + 1 - \frac{2t}{a}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{t^2}{a^2} - \frac{2t}{a}$$

$$\Rightarrow b^2t^2 - 2ab^2t - a^4 = 0$$

$$\Rightarrow t = \frac{2ab^2 \pm 2\sqrt{a^2b^4 + b^2a^4}}{2b^2}$$

$$\Rightarrow t = \frac{a}{b}\left(b \pm \sqrt{a^2 + b^2}\right)$$

Hence, the required points on the x-axis are $\left(\frac{a}{b}\left(b - \sqrt{a^2 + b^2}\right), 0\right)$ and $\left(\frac{a}{b}\left(b + \sqrt{a^2 + b^2}\right), 0\right)$

8. Question

Show that the product of perpendicular on the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ from the points $(\pm\sqrt{a^2 - b^2}, 0)$ is b^2.

Answer

Given:

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and points $(\pm\sqrt{a^2 - b^2}, 0)$

To prove:

The product of perpendicular on the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ from the points $(\pm\sqrt{a^2 - b^2}, 0)$ is b^2.

Concept Used:

Distance of a point from a line.

Explanation:

Let d_1 and d_2 be the perpendicular distances of line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ from points $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ respectively.

$$\therefore d_1 = \left|\frac{\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right| = b\left|\frac{\sqrt{a^2-b^2}\cos\theta - a}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}\right|$$

Similarly,

$$d_1 = -\left|\frac{-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right| = b\left|\frac{-\sqrt{a^2-b^2}\cos\theta - a}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}\right| = b\left|\frac{\sqrt{a^2-b^2}\cos\theta - a}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}\right|$$

Now,

$$d_1 d_2 = b\left|\frac{\sqrt{a^2-b^2}\cos\theta - a}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}\right| \times b\left|\frac{\sqrt{a^2-b^2}\cos\theta - a}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}\right|$$

$$\Rightarrow d_1 d_2 = b^2\left|\frac{(a^2-b^2)\cos^2\theta - a^2}{a^2\sin^2\theta + b^2\cos^2\theta}\right|$$

$$\Rightarrow d_1 d_2 = b^2\left|\frac{a^2(\cos^2\theta - 1) - b^2\cos^2\theta}{a^2\sin^2\theta + b^2\cos^2\theta}\right|$$

$$\Rightarrow d_1 d_2 = b^2\left|\frac{-a^2\sin^2\theta - b^2\cos^2\theta}{a^2\sin^2\theta + b^2\cos^2\theta}\right| = b^2\left|\frac{a^2\sin^2\theta + b^2\cos^2\theta}{a^2\sin^2\theta + b^2\cos^2\theta}\right| = b^2$$

Hence proved.

9. Question

Find the perpendicular distance from the origin of the perpendicular from the point (1, 2) upon the straight line $x - \sqrt{3}y + 4 = 0$.

Answer

<u>**Given:**</u>

Line $x - \sqrt{3}y + 4 = 0$

<u>**To find:**</u>

The perpendicular distance from the origin of the perpendicular from the point (1, 2) upon the straight line $x - \sqrt{3}y + 4 = 0$.

<u>Concept Used:</u>

Distance of a point from a line.

Explanation:

The equation of the line perpendicular to $x - \sqrt{3}y + 4 = 0$ is $\sqrt{3}x + y + \lambda = 0$. This line passes through (1, 2).

$\therefore \sqrt{3} + 2 + \lambda = 0$

$\Rightarrow \lambda = \sqrt{-3} - 2$

Substituting the value of λ, we get $\sqrt{3x} + y - \sqrt{3} - 2 = 0$

Let d be the perpendicular distance from the origin to the line $\sqrt{3x} + y - \sqrt{3} - 2 = 0$

$$d = \frac{0 - 0 - \sqrt{3} - 2}{\sqrt{1+3}} = \frac{\sqrt{3}+2}{2}$$

Hence, the required perpendicular distance is $\frac{\sqrt{3}+2}{2}$

10. Question

Find the distance of the point (1, 2) from the straight line with slope 5 and passing through the point of intersection of x + 2y = 5 and x – 3y = 7.

Answer

Given:

Lines x + 2y = 5 and x – 3y = 7, slope = 5.

To find:

The distance of the point (1, 2) from the straight line with slope 5 and passing through the point of intersection of x + 2y = 5 and x – 3y = 7.

Concept Used:

Distance of a point from a line.

Explanation:

To find the point intersection of the lines x + 2y = 5 and x − 3y = 7, let us solve them.

$$\frac{x}{-14-15} = \frac{y}{-5+7} = \frac{1}{-3-2}$$

$$\Rightarrow x = \frac{29}{5}, y = -\frac{2}{5}$$

So, the equation of the line passing through $\left(\frac{29}{5}, -\frac{2}{5}\right)$ with slope 5 is

$$y + \frac{2}{5} = 5x - \frac{29}{5}$$

$\Rightarrow 5y + 2 = 25x - 145$

$\Rightarrow 25x - 5y - 147 = 0$

Let d be the perpendicular distance from the point (1, 2) to the line 25x – 5y – 147 = 0

$$\therefore d = \left|\frac{25 - 10 - 147}{\sqrt{25^2 + 5^2}}\right| = \frac{132}{5\sqrt{26}}$$

Hence, the required perpendicular distance is $\frac{132}{5\sqrt{26}}$

11. Question

What are the points on y-axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units?

Answer

Given:

Distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4.

To find:

Points on y-axis

Concept Used:

Distance of a point from a line.

Explanation:

Let (0, t) be a point on the y-axis.

It is given that the perpendicular distance of the line $\frac{x}{3} + \frac{y}{4} = 1$ from the point (0, t) is 4 units.

$$\therefore \left|\frac{0 + \frac{t}{4} - 1}{\sqrt{\frac{1}{3^2} + \frac{1}{4^2}}}\right| = 4$$

$$\Rightarrow |t - 4| = 4 \times 4 \times \frac{5}{3 \times 4}$$

$$\Rightarrow |t - 4| = \frac{20}{3}$$

$$\Rightarrow t - 4 = \pm\frac{20}{3}$$

$$\Rightarrow t = 4 \pm \frac{20}{3}$$

$$\Rightarrow t = 4 + \frac{20}{3}, 4 - \frac{20}{3}$$

$$\Rightarrow t = \frac{32}{3}, -\frac{8}{3}$$

Hence, the required points on the y-axis are $\left(0, \frac{32}{3}\right)$ and $\left(0, -\frac{8}{3}\right)$.

12. Question

In the triangle ABC with vertices A(2, 3), B(4, -1) and C(1, 2) find the equation and the length of the altitude from the vertex A.

Answer

Given:

A(2, 3), B(4, -1) and C(1, 2).

To find:

The equation and the length of the altitude from the vertex A.

Concept Used:

Distance of a point from a line.

Explanation:

Equation of side BC:

$$y + 1 = \frac{2 + 1}{1 - 4}(x - 4)$$

$\Rightarrow x + y - 3 = 0$

The equation of the altitude that is perpendicular to $x + y - 3 = 0$ is $x - y + \lambda = 0$.

Line $x - y + \lambda = 0$ passes through (2, 3).

$\therefore 2 - 3 + \lambda = 0$

$\Rightarrow \lambda = 1$

Thus, the equation of the altitude from the vertex A (2, 3) is $x - y + 1 = 0$.

Let d be the length of the altitude from A (2, 3).

$$d = \left|\frac{2 + 3 - 3}{\sqrt{1^2 + 1^2}}\right|$$

$\Rightarrow d = \sqrt{2}$

Hence, the required distance is $\sqrt{2}$.

13. Question

Show that the path of a moving point such that its distances from two lines $3x - 2y = 5$ and $3x + 2y = 5$ are equal is a straight line.

Answer

Given:

Two lines $3x - 2y = 5$ and $3x + 2y = 5$

To prove:

The path of a moving point such that its distances from two lines $3x - 2y = 5$ and $3x + 2y = 5$ are equal is a straight line

Concept Used:

Distance of a point from a line.

Explanation:

Let P(h, k) be the moving point such that it is equidistant from the lines $3x - 2y = 5$ and $3x + 2y = 5$

$$\left|\frac{3h - 2k - 5}{\sqrt{3^2 + 2^2}}\right| = \left|\frac{3h + 2k - 5}{\sqrt{3^2 + 2^2}}\right|$$

$\Rightarrow |3h - 2k - 5| = |3h + 2k - 5|$

$\Rightarrow 3h - 2k - 5 = \pm(3h + 2k - 5)$

$\Rightarrow 3h - 2k - 5 = 3h + 2k - 5$ and $3h - 2k - 5 = -3h + 2k - 5$

⇒ k = 0 and 3h = 5

Hence proved, the path of the moving points are 3x = 5 or y = 0. These are straight lines.

14. Question

If sum of perpendicular distances of a variable point P(x, y) from the lines x + y – 5 = 0 and 3x – 2y + 7 = 0 is always 10. Show that P must move on a line.

Answer

Given:

Sum of perpendicular distances of a variable point P(x, y) from the lines x + y – 5 = 0 and 3x – 2y + 7 = 0 is always 10

To prove:

P must move on a line.

Concept Used:

Distance of a point from a line.

Explanation:

It is given that the sum of perpendicular distances of a variable point P (x, y) from the lines x + y − 5 = 0 and 3x − 2y + 7 = 0 is always 10

$$\therefore \left|\frac{x+y-5}{\sqrt{1^2+1^2}}\right| + \left|\frac{3x-2y+7}{\sqrt{3^2+2^2}}\right| = 10$$

$$\Rightarrow \left|\frac{x+y-5}{\sqrt{2}}\right| + \left|\frac{3x-2y+7}{\sqrt{13}}\right| = 10$$

$$(3\sqrt{2} + \sqrt{13})x + (\sqrt{13} - 2\sqrt{2})y + 7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26} = 0$$

It is a straight line.

Hence proved.

15. Question

If the length of the perpendicular from the point (1, 1) to the line ax – by + c = 0 be unity, Show that $\frac{1}{c}+\frac{1}{a}-\frac{1}{b}=\frac{c}{2ab}$.

Answer

Given:

Line ax – by + c = 0 and point (1, 1)

To prove:

$$\frac{1}{c}+\frac{1}{a}-\frac{1}{b}=\frac{c}{2ab}$$

Concept Used:

Distance of a point from a line.

Explanation:

The distance of the point (1, 1) from the straight line ax − by + c = 0 is 1

$\therefore 1 = \left|\frac{a - b + c}{\sqrt{a^2 + b^2}}\right|$

$\Rightarrow a^2 + b^2 + c^2 - 2ab + 2ac - 2bc = a^2 + b^2$

$\Rightarrow ab + bc - ac = \frac{c^2}{2}$

Dividing both the sides by abc, we get:

$$\frac{1}{c} + \frac{1}{a} - \frac{1}{b} = \frac{c}{2ab}$$

Hence proved.

Exercise 23.16

1 A. Question

Determine the distance between the following pair of parallel lines:

$4x - 3y - 9 = 0$ and $4x - 3y - 24 = 0$

Answer

Given: The parallel lines are

$4x - 3y - 9 = 0$ … (1)

$4x - 3y - 24 = 0$ … (2)

To find:

Distance between the givens parallel lines

Explanation:

Let d be the distance between the given lines.

$$\Rightarrow d = \left|\frac{-9 + 24}{\sqrt{4^2 + (-3)^2}}\right| = \frac{15}{5} = 3 \text{ units}$$

Hence, distance between givens parallel line is 3 units

1 B. Question

Determine the distance between the following pair of parallel lines:

$8x + 15y - 34 = 0$ and $8x + 15y + 31 = 0$

Answer

Given: The parallel lines are

$8x + 15y - 34 = 0$ … (1)

$8x + 15y + 31 = 0$ … (2)

To find:

Distance between the givens parallel lines

Explanation:

Let d be the distance between the given lines.

$$\Rightarrow d = \left|\frac{-34 - 31}{\sqrt{8^2 + 15^2}}\right| = \frac{65}{17}\text{units}$$

Hence, distance between givens parallel line is $\frac{65}{17}$units

1 C. Question

Determine the distance between the following pair of parallel lines:

y = mx + c and y = mx + d

Answer

Given: The parallel lines are

y = mx + c and y = mx + d

To find:

Distance between the givens parallel lines

Explanation:

The parallel lines can be written as

mx − y + c = 0 … (1)

mx − y + d = 0 … (2)

Let d be the distance between the given lines.

$$\Rightarrow d = \left|\frac{c - d}{\sqrt{m^2 + 1}}\right|$$

Hence, distance between givens parallel line is $\left|\frac{c - d}{\sqrt{m^2 + 1}}\right|$

1 D. Question

Determine the distance between the following pair of parallel lines:

4x + 3y – 11 = 0 and 8x + 6y = 15

Answer

Given: The parallel lines are

4x + 3y – 11 = 0 and 8x + 6y = 15

To find:

Distance between the givens parallel lines

Explanation:

The given parallel lines can be written as

4x + 3y − 11 = 0 … (1)

$4x + 3y - \frac{15}{2} = 0$ … (2)

Let d be the distance between the given lines.

$$\Rightarrow d = \left|\frac{-11 + \frac{15}{2}}{\sqrt{4^2 + 3^2}}\right| = \frac{7}{2 \times 5} = \frac{7}{10} \text{ units}$$

Hence, distance between givens parallel line is $\frac{7}{10}$ units

2. Question

The equations of two sides of a square are 5x – 12y – 65 = 0 and 5x – 12y + 26 = 0. Find the area of the square.

Answer

Given: Two side of square are 5x – 12y – 65 = 0 and 5x – 12y + 26 = 0

To find: area of the square

Explanation:

The sides of a square are

5x − 12y − 65 = 0 … (1)

5x − 12y + 26 = 0 … (2)

We observe that lines (1) and (2) are parallel.

So, the distance between them will give the length of the side of the square.

Let d be the distance between the given lines.

$$\Rightarrow d = \left|\frac{-65 - 26}{\sqrt{5^2 + (-12)^2}}\right| = \frac{91}{13}$$

∴ Area of the square = 7^2= 49 square units

3. Question

Find the equation of two straight lines which are parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1).

Answer

Given: equation is parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1)

To find: equation of two straight lines

Explanation:

The equation of given line is

x + 7y + 2 = 0 … (1)

The equation of a line parallel to line x + 7y + 2 = 0 is given below:

X + 7y + λ = 0 … (2)

The line x + 7y + λ = 0 is at a unit distance from the point (1, − 1).

∴ 1 = 1 – 7 + λ1 + 49

$$\Rightarrow \lambda - 6 = \pm 5\sqrt{2}$$

$$\Rightarrow \lambda = 6 + 5\sqrt{2}, 6 - 5\sqrt{2}$$

Hence, the required lines:

$x + 7y + 6 + 5\sqrt{2} = 0$ and $x + 7y + 6 - 5\sqrt{2} = 0$

4. Question

Prove that the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6.

Answer

Given: lines A 2x + 3y = 19 and B 2x + 3y + 7 = 0 also a line C 2x + 3y = 6.

To prove:

Line A and B are equidistant from the line C

Proof:

Let d1 be the distance between lines 2x + 3y = 19 and 2x + 3y = 6,

While d2 is the distance between lines 2x + 3y + 7 = 0 and 2x + 3y = 6

$$\therefore d1 = \left|\frac{-19 - (-6)}{\sqrt{2^2 + 3^2}}\right| \text{and } d2 = \left|\frac{7 - (-6)}{\sqrt{2^2 + 3^2}}\right|$$

$$\Rightarrow d1 = \left|-\frac{13}{\sqrt{13}}\right| = \sqrt{13} \text{ and } d2 = \left|\frac{13}{\sqrt{13}}\right| = \sqrt{13}$$

Hence proved, the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6

5. Question

Find the equation of the line mid-way between the parallel lines 9x + 6y – 7 = 0 and 3x + 2y + 6 = 0.

Answer

Given:

9x + 6y – 7 = 0 and 3x + 2y + 6 = 0 are parallel lines

To find:

The equation of the line mid-way between the givens line

Explanation:

The given equations of the lines can be written as:

$3x + 2y - \frac{7}{3} = 0$ … (1)

3x + 2y + 6 = 0 … (2)

Let the equation of the line midway between the parallel lines (1) and (2) be

3x + 2y + λ = 0 … (3)

The distance between (1) and (3) and the distance between (2) and (3) are equal.

$$\therefore \left|\frac{-\frac{7}{3} - \lambda}{\sqrt{3^2 + 2^2}}\right| = \left|\frac{6 - \lambda}{\sqrt{3^2 + 2^2}}\right|$$

$$\Rightarrow \left|-\lambda + \frac{7}{3}\right| = |6 - \lambda|$$

$$\Rightarrow 6 - \lambda = \lambda + \frac{7}{3}$$

$$\Rightarrow \lambda = \frac{11}{6}$$

Equation of the required line:

$$3x + 2y + \frac{11}{6} = 0$$

$\Rightarrow 18x + 12y + 11 = 0$

Hence, equation of required line is $18x + 12y + 11 = 0$.

6. Question

Find the ratio in which the line $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$

Answer

Given:

Lines A: $3x + 4y + 5 = 0$ and B: $3x + 4y - 5 = 0$

And also C: $3x + 4y + 2 = 0$

To find:

Ratio in which line C divides the distance between the lines A and B

Explanation:

The distance between two parallel line $ax + by + c1 = 0$ and $ax + by + c2 = 0$ is

$$\frac{|(c1 - c2)|}{\sqrt{a^2 + b^2}}$$

Therefore the distance between two parallel line $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ is

$$\frac{|5 - 2|}{\sqrt{3^2 + 4^2}} = \frac{3}{\sqrt{25}} = \frac{3}{5}$$

The distance between the line $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ is

$$\frac{|2 - (-5)|}{\sqrt{3^2 + 4^2}} = \frac{7}{5}$$

Hence, the required ratio $= \frac{\frac{3}{5}}{\frac{7}{5}} = \frac{3}{7}$

Exercise 23.17

1. Question

Prove that the area of the parallelogram formed by the lines

$a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1}\right|$ sq. units.

Deduce the condition for these lines to form a rhombus.

Answer

Given:

The given lines are

$a_1x + b_1y + c_1 = 0$ … (1)

$a_1x + b_1y + d_1 = 0$ … (2)

$a_2x + b_2y + c_2 = 0$ … (3)

$a_2x + b_2y + d_2 = 0$ … (4)

To prove:

The area of the parallelogram formed by the lines

$a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$ sq. units.

Explanation:

The area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_2x + b_2y + d_2 = 0$ is given below:

$$\text{Area} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}\right|$$

$$\because \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$\therefore \text{Area} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1b_2 - a_2b_1)}\right| = \left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$$

If the given parallelogram is a rhombus, then the distance between the pair of parallel lines are equal.

$$\therefore \left|\frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

Hence proved.

2. Question

Prove that the area of the parallelogram formed by the lines 3x – 4y + a = 0, 3x – 4y + 3a = 0, 4x – 3y – a = 0 and 4x – 3y – 2a = 0 is $\frac{2a^2}{7}$ sq. units.

Answer

Given:

The given lines are

$3x - 4y + a = 0$ … (1)

$3x - 4y + 3a = 0$ … (2)

$4x - 3y - a = 0$ … (3)

$4x - 3y - 2a = 0$ … (4)

To prove:

The area of the parallelogram formed by the lines 3x – 4y + a = 0, 3x – 4y + 3a = 0, 4x – 3y – a = 0 and 4x – 3y – 2a = 0 is $\frac{2a^2}{7}$ sq. units.

Explanation:

From Above solution, We know that

Area of the parallelogram $= \left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1b_2 - a_2b_1)}\right|$

⇒ Area of the parallelogram $= \left|\frac{(a - 3a)(2a - a)}{(-9 + 16)}\right| = \frac{2a^2}{7}$ square units

Hence proved.

3. Question

Show that the diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my + n' = 0, mx + ly + n = 0 and mx + ly + n' = 0 include an angle $\frac{\pi}{2}$.

Answer

Given:

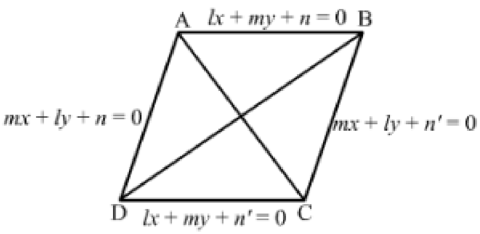
The given lines are

lx + my + n = 0 … (1)

mx + ly + n' = 0 … (2)

lx + my + n' = 0 … (3)

mx + ly + n = 0 … (4)



To prove:

The diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my + n' = 0, mx + ly + n = 0 and mx + ly + n' = 0 include an angle $\frac{\pi}{2}$.

Explanation:

Solving (1) and (2), we get,

B $= \left(\frac{mn' - ln}{l^2 - m^2}, \frac{mn - ln'}{l^2 - m^2}\right)$

Solving (2) and (3), we get,

C $= \left(-\frac{n'}{m + l}, -\frac{n'}{m + l}\right)$

Solving (3) and (4), we get,

D $= \left(\frac{mn - ln'}{l^2 - m^2}, \frac{mn' - ln}{l^2 - m^2}\right)$

Solving (1) and (4), we get,

A $= \left(\frac{-n}{m + l}, \frac{-n}{m + l}\right)$

Let m_1 and m_2 be the slope of AC and BD.

$$m_1 = \frac{\frac{-n'}{m+l} + \frac{n}{m+l}}{\frac{-n'}{m+l} + \frac{n}{m+l}} = 1$$

$$m_2 = \frac{\frac{mn' - ln}{l^2 - m^2} - \frac{mn - ln'}{l^2 - m^2}}{\frac{mn - ln'}{l^2 - m^2} - \frac{mn' - ln}{l^2 - m^2}} = -1$$

$\therefore m_1 m_2 = -1$

Hence proved, diagonals of the parallelogram intersect at an angle $\frac{\pi}{2}$.

Exercise 23.18

1. Question

Find the equation of the straight lines passing through the origin and making an angle of 45^0 with the straight line $\sqrt{3}x + y = 11$.

Answer

Given:

Equation passes through (0, 0) and make an angle of 45° with the line $\sqrt{3}x + y = 11$

To find:

Equation of given line

Explanation:

We know that, the equations of two lines passing through a point x1,y1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, $x_1 = 0$, $y_1 = 0$, $\alpha = 45°$ and $m = -\sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{-\sqrt{3} + \tan 45°}{1 + \sqrt{3}\tan 45°}(x - 0) \text{ and } y - 0 = \frac{-\sqrt{3} - \tan 45°}{1 - \sqrt{3}\tan 45°}(x - 0)$$

$$\Rightarrow y = \frac{-\sqrt{3} + 1}{1 + \sqrt{3}}x \text{ and } y = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}x$$

$$\Rightarrow y = -\frac{3 + 1 - 2\sqrt{3}}{3 - 1}x \text{ and } y = \frac{3 + 1 + 2\sqrt{3}}{3 - 1}x$$

$$\Rightarrow y = (\sqrt{3} - 2)x \text{ and } y = (\sqrt{3} + 2)x$$

Hence, Equation of given line is $y = (\sqrt{3} - 2)x$ and $y = (\sqrt{3} + 2)x$

2. Question

Find the equations to the straight lines which pass through the origin and are inclined at an angle of 75^0 to the straight line $x + y + \sqrt{3}(y - x) = a$.

Answer

Given:

Equation passes through (0,0) and make an angle of 75° with the line $x + y + \sqrt{3}(y - x) = a$

To find:

Equation of given line

Explanation:

We know that the equations of two lines passing through a point x1,y1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,Equation of the given line is,

$$x + y + \sqrt{3}(y - x) = a$$

$$\Rightarrow \left(\sqrt{3} + 1\right)y = \left(\sqrt{3} - 1\right)x + a$$

$$\Rightarrow y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}x + \frac{a}{\sqrt{3} + 1}$$

Comparing this equation with y = mx + c

We get,

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$ ∴ $x_1 = 0$, $y_1 = 0$, $\alpha = 75°$, $m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$

and tan75∘ = 2 + $\sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{2 - \sqrt{3} + \tan 75°}{1 - \left(2 - \sqrt{3}\right)\tan 75°}(x - 0) \text{ and } y - 0$$
$$= \frac{2 - \sqrt{3} - \tan 75°}{1 + (2 - \sqrt{3})\tan 75°}(x - 0)$$

$$\Rightarrow y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - \left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)}x \text{ and } y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + \left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)}x$$

$$\Rightarrow y = \frac{4}{1 - 1}x \text{ and } y = -\sqrt{3}x$$

$$\Rightarrow x = 0 \text{ and } \sqrt{3}x + y = 0$$

Hence, Equation of given line is $x = 0 \text{ and } \sqrt{3}x + y = 0$

3. Question

Find the equations of straight lines passing through (2, -1) and making an angle of 45^0 with the line 6x + 5y – 8 = 0.

Answer

Given: equation passes through (2,-1) and make an angle of 45° with the line 6x + 5y – 8 = 0

To find: equation of given line

Explanation:

We know that the equations of two lines passing through a point x1,y1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,Equation of the given line is,

6x + 5y – 8 = 0

⇒ 5y = - 6x + 8

$$\Rightarrow y = -\frac{6}{5}x + \frac{8}{5}$$

Comparing this equation with y = mx + c

we get, m $= -\frac{6}{5}$

$x_1 = 2$, $y_1 = -1$, α = 45°, m $= -\frac{6}{5}$

So, the equations of the required lines are

$$y + 1 = \frac{\left(-\frac{6}{5} + \tan 45°\right)}{\left(1 + \frac{6}{5}\tan 45°\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - \tan 45°\right)}{\left(1 - \frac{6}{5}\tan 45°\right)}(x - 2)$$

$$\Rightarrow y + 1 = \frac{\left(-\frac{6}{5} + 1\right)}{\left(1 + \frac{6}{5}\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - 1\right)}{\left(1 - \frac{6}{5}\right)}(x - 2)$$

$$\Rightarrow y + 1 = -\frac{1}{11}(x - 2) \text{ and } y + 1 = -\frac{11}{-1}(x - 2)$$

⇒ x + 11y + 9 = 0 and 11x – y – 23 = 0

Hence, Equation of given line is x + 11y + 9 = 0 and 11x – y – 23 = 0

4. Question

Find the equations to the straight lines which pass through the point (h, k) and are inclined at angle $\tan^{-1}$ m to the straight line y = mx + c.

Answer

Given: equation passes through (h, k) and make an angle of $\tan^{-1}$ m with the line y = mx + c

To find: equation of given line

Explanation:

We know that the equations of two lines passing through a point x1,y1 and making an angleα with the given line y = m’x + c are

m’ = m so,

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,x_1 = h, y_1 = k, α $= \tan^{-1} m,$ m’ = m.

So, the equations of the required lines are

$$y - k = \frac{m + m}{1 - m^2}(x - h) \text{ and } y - k = \frac{m - m}{1 + m^2}(x - h)$$

$$\Rightarrow y - k = \frac{2m}{1 - m^2}(x - h) \text{ and } y - k = 0$$

$\Rightarrow (y - k)(1 - m^2) = 2m(x - h)$ and y = k

Hence, Equation of given line is $(y - k)(1 - m^2) = 2m(x - h)$ and y = k

5. Question

Find the equations to the straight lines passing through the point (2, 3) and inclined at an angle of 45^0 to the lines 3x + y – 5 = 0.

Answer

<u>Given:</u> equation passes through (2,3) and make an angle of $45^0$with the line 3x + y – 5 = 0.

<u>To find:</u> equation of given line

Explanation:

We know that the equations of two lines passing through a point x1,y1 and making an angleα with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,Equation of the given line is,

3x + y – 5 = 0

⇒ y = - 3x + 5

Comparing this equation with y = mx + c we get, m = - 3x_1 = 2, y_1 = 3, α = 45∘, m = - 3.

So, the equations of the required lines are

$$y - 3 = \frac{-3 + \tan 45°}{1 + 3\tan 45°}(x - 2) \text{ and } y - 3 = \frac{-3 - \tan 45°}{1 - 3\tan 45°}(x - 2)$$

$$\Rightarrow y - 3 = \frac{-3 + 1}{1 + 3}(x - 2) \text{ and } y - 3 = \frac{-3 - 1}{1 - 3}(x - 2)$$

$$\Rightarrow y - 3 = \frac{-1}{2}(x - 2) \text{ and } y - 3 = 2(x - 2)$$

⇒ x + 2y – 8 = 0 and 2x – y – 1 = 0

Hence, Equation of given line is x + 2y – 8 = 0 and 2x – y – 1 = 0

6. Question

Find the equations to the sides of an isosceles right angled triangle the equation of whose hypotenuse is 3x + 4y = 4 and the opposite vertex is the point (2, 2).

Answer

Given: hypotenuse is 3x + 4y = 4 of isosceles right angled triangle the opposite vertex is the point (2, 2).

To find: equation of side of isosceles right angle triangle

Explanation:

Here,

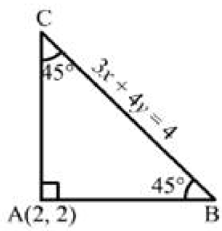
we are given △ABC is an isosceles right angled triangle .

∠A + ∠B + ∠C = 180°

⇒ 90° + ∠B + ∠B = 180°

⇒ ∠B = 45°, ∠C = 45°

Diagram:



Now, we have to find the equations of the sides AB and AC, where 3x + 4y = 4 is the equation of the hypotenuse BC.

We know that the equations of two lines passing through a point x1,y1 and making an angleα with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,Equation of the given line is,

3x + 4y = 4

⇒ 4y = - 3x + 4

$$\Rightarrow y = -\frac{3}{4}x + 1$$

Comparing this equation with y = mx + c

we get, $m = -\frac{3}{4}$

x1 = 2, y1 = 2, α = 45∘, m $= -\frac{3}{4}$

So, the equations of the required lines are

$$y - 2 = \frac{-\frac{3}{4} + \tan 45°}{1 + \frac{3}{4}\tan 45°}(x - 2) \text{ and } y - 2 = \frac{-\frac{3}{4} - \tan 45°}{1 - \frac{3}{4}\tan 45°}(x - 2)$$

$$\Rightarrow y - 2 = \frac{-\frac{3}{4} + 1}{1 + \frac{3}{4}}(x - 2) \text{ and } y - 2 = \frac{-\frac{3}{4} - 1}{1 - \frac{3}{4}}(x - 2)$$

$$\Rightarrow y - 2 = \frac{1}{7}(x - 2) \text{ and } y - 2 = -\frac{7}{1}(x - 2)$$

⇒ x – 7y + 12 = 0 and 7x + y – 16 = 0

Hence, Equation of given line is x – 7y + 12 = 0 and 7x + y – 16 = 0

7. Question

The equation of one side of an equilateral triangle is x – y = 0 and one vertex is $(2+\sqrt{3}, 5)$. Prove that a second side is $y+(2-\sqrt{3})x = 6$ and find the equation of the third side.

Answer

Given: equation of one side of an equilateral triangle is x – y = 0 and one vertex is $(2 + \sqrt{3}, 5)$

To prove: second side is $y + (2 - \sqrt{3})x = 6$

To find: the equation of the third side.

Explanation:

Let A$(2 + \sqrt{3}, 5)$ be the vertex of the equilateral triangle ABC and x − y = 0 be the equation of BC.Here, we have to find the equations of sides AB and AC, each of which makes an angle of 60∘ with the line x − y = 0

We know the equations of two lines passing through a point x1,y1 and making an angle α with the line whose slope is m.

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,$x_1 = 2 + \sqrt{3}$, y_1 = 5, α = 60∘, m = 1

So, the equations of the required sides are

$y - 5 = \frac{1 + \tan 60°}{1 - \tan 60°}(x - 2 - \sqrt{3})$ and $y - 5 = \frac{1 - \tan 60°}{1 + \tan 60°}(x - 2 - \sqrt{3})$

$\Rightarrow y - 5 = -(2 + \sqrt{3})(x - 2 - \sqrt{3})$ and $y - 5 = -(2 - \sqrt{3})(x - 2 - \sqrt{3})$

$\Rightarrow y - 5 = -(2 + \sqrt{3})x + (2 + \sqrt{3})^2$ and $y - 5 = -(2 - \sqrt{3})x + (2 + \sqrt{3})(2 - \sqrt{3})$

$\Rightarrow (2 + \sqrt{3})x + y = 2 + 4\sqrt{3}$ and $(2 - \sqrt{3})x + y - 6 = 0$

Hence, the second side is $y + (2 - \sqrt{3})x = 6$ and the equation of the third side is

$\Rightarrow (2 + \sqrt{3})x + y = 2 + 4\sqrt{3}$

8. Question

Find the equations of the two straight lines through (1, 2) forming two sides of a square of which 4x + 7y = 12 is one diagonal.

Answer

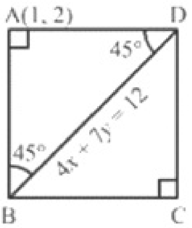
Given: 4x + 7y = 12 is one diagonal and opposite vertex is (1,2)

To find: equation of straight line

Explanation:

Let A (1, 2) be the vertex of square ABCD and BD be the diagonal that is along the line4x + 7y = 12

Diagram:



Here, we have to find the equations of sides AB and AD, each of which makes an angle of 45∘ with line 4x + 7y = 12

We know that the equations of two lines passing through a point x1,y1 and making an angleα with the line whose slope is m.

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Equation of given line is

4x + 7y = 9

$$\Rightarrow y = -\frac{4}{7}x + \frac{9}{4}$$

$m = -\frac{4}{7}$, $x_1 = 1$, $y_1 = 2$, $\alpha = 45^\circ$

So, the equations of the required sides are

$$y - 2 = \frac{-\frac{4}{7} + \tan 45^\circ}{1 + \frac{4}{7}\tan 45^\circ}(x - 1) \text{ and } y - 2 = \frac{-\frac{4}{7} - \tan 45^\circ}{1 - \frac{4}{7}\tan 45^\circ}(x - 1)$$

$$\Rightarrow y - 2 = \frac{-\frac{4}{7} + 1}{1 + \frac{4}{7}}(x - 1) \text{ and } y - 2 = \frac{-\frac{4}{7} - 1}{1 - \frac{4}{7}}(x - 1)$$

⇒ 3x – 11y + 19 = 0 and 11x + 3y – 17 = 0

Hence, equation of straight line 3x – 11y + 19 = 0 and 11x + 3y – 17 = 0

9. Question

Find the equations of two straight lines passing through (1, 2) and making an angle of 60^0 with the lines x + y = 0. Find also the area of the triangle formed by the three lines.

Answer

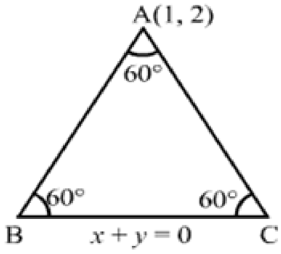
Given: equation passes through (1,2) and make an angle of 60° with the line x + y = 0

To find: equation of given line and the area of the triangle formed by the three lines

Explanation:

Let A(1, 2) be the vertex of the triangle ABC and x + y = 0 be the equation of BC.

Diagram:



Here, we have to find the equations of sides AB and AC, each of which makes an angle of 60∘ with the line x + y = 0.

We know the equations of two lines passing through a point x1,y1 and making an angle α with the line whose slope is m.

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, $x_1 = 1$, $y_1 = 2$, $\alpha = 60^\circ$, $m = -1$

So, the equations of the required sides are

$y - 2 = \frac{-1 + \tan 60^\circ}{1 + \tan 60^\circ}(x - 1)$ and $y - 2 = \frac{-1 - \tan 60^\circ}{1 - \tan 60^\circ}(x - 1)$

$$\Rightarrow y - 2 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}(x - 1) \text{ and } y - 2 = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}(x - 1)$$

$$\Rightarrow y - 2 = (2 - \sqrt{3})(x - 1) \text{ and } y - 2 = (2 + \sqrt{3})(x - 1)$$

Solving x + y = 0 and $y - 2 = (2 - \sqrt{3})(x - 1)$, we get:

$$x = -\frac{\sqrt{3} + 1}{2}, y = \frac{\sqrt{3} + 1}{2}$$

$$\therefore B = \left(-\frac{\sqrt{3} + 1}{2}, \frac{\sqrt{3} + 1}{2}\right) \text{ or } C = \left(\frac{\sqrt{3} - 1}{2}, -\frac{\sqrt{3} - 1}{2}\right)$$

AB = BC = AD = $\sqrt{6}$ units

∴ Area of the required triangle = $\sqrt{3} \times \frac{(\sqrt{6})^2}{4} = \frac{3\sqrt{3}}{2}$ square units

Hence, area of the required triangle = $\frac{3\sqrt{3}}{2}$ square units

10. Question

Two sides of an isosceles triangle are given by the equations 7x – y + 3 = 0 and x + y – 3 = 0 and its third side passes through the point (1, -10). Determine the equation of the third side.

Answer

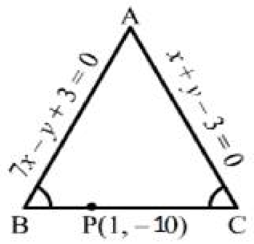
Given: two side of an isosceles triangle are 7x – y + 3 = 0 and x + y – 3 = 0 and its third side passes through the point (1, -10)

To find: third side of isosceles triangle

Explanation:

Let ABC be the isosceles triangle, where $7x - y + 3 = 0$ and $x + y - 3 = 0$ represent the sides AB and AC, respectively.Let AB = BC

Diagram:



∵ AB = BC

∴ tan B = tan C

Here,

Slope of AB = 7

Slope of AC = − 1

Let m be the slope of BC.

Then, $\left|\frac{m-7}{1+7m}\right| = \left|\frac{m+1}{1-m}\right| = \left|\frac{m+1}{m-1}\right|$

$\Rightarrow \frac{m - 7}{1 + 7m} = \pm\frac{m + 1}{m - 1}$

Taking the positive sign, we get:

$m^2 - 8m + 7 = 7m^2 + 8m + 1$

$\Rightarrow (m + 3)\left(m - \frac{1}{3}\right) = 0$

$\Rightarrow m = -3, \frac{1}{3}$

Now, taking the negative sign, we get:

(m – 7) (m – 1) = - (7m + 1)(m + 1)

$\Rightarrow m^2 - 8m + 7 = -7m^2 - 8m - 1$

$\Rightarrow m^2 = -1$ (not possible)

Equations of the third side is

y + 10 = - 3(x – 1) and $y + 10 = \frac{1}{3}(x - 1)$

⇒ 3x + y + 7 = 0 and x – 3y – 31 = 0

Hence, third side of isosceles triangle is 3x + y + 7 = 0 and x – 3y – 31 = 0

11. Question

Show that the point (3, -5) lies between the parallel lines 2x + 3y – 7 = 0 and 2x + 3y + 12 = 0 and find the equation of lines through (3, -5) cutting the above lines at an angle of 45°.

Answer

Given:

Parallel lines 2x + 3y – 7 = 0 and 2x + 3y + 12 = 0 and

To prove:

The point (3, -5) lies between the parallel lines 2x + 3y – 7 = 0 and 2x + 3y + 12 = 0

To find:

Lines through (3, -5) cutting the above lines at an angle of 45^0.

Explanation:

We observed that (0, -4) lies on the line 2x + 3y + 12 = 0

If (3, 5) lies between the lines 2x + 3y – 7 = 0 and 2x + 3y + 12 = 0, then we have,

$(ax_1 + by_1 + c_1)(ax_2 + by_2 + c_1) > 0$

Here, $x_1 = 0$, $y_1 = -4$, $x_2 = 3$, $y_2 = -5$, a = 2, b = 3, c1 = -7

Now,

$(ax_1 + by_1 + c_1)(ax_2 + by_2 + c_1) = (2 \times 0 - 3 \times 4 - 7)\,(2 \times 3 - 3 \times 5 - 7)$

$(ax_1 + by_1 + c_1)(ax_2 + by_2 + c_1) = -19 \times (-16) > 0$

Thus, point (3, -5) lies between the given parallel lines.

The equation of the lines passing through (3, -5) and making angle of 45 with the given parallel lines is given below:

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,$x_1 = 3$, $y_1 = -5$, $\alpha = 45°$, $m = -\frac{2}{3}$

So, the equations of the required sides are

$$y + 5 = \frac{-\frac{2}{3} \pm \tan 45°}{1 \mp (-\frac{2}{3})\tan 45°}(x - 3)$$

$y + 5 = \frac{-\frac{2}{3} + \tan 45°}{1 - \left(-\frac{2}{3}\right)\tan 45°}(x - 3)$ and $y + 5 = \frac{-\frac{2}{3} - \tan 45°}{1 + \left(-\frac{2}{3}\right)\tan 45°}(x - 3)$

$\Rightarrow y + 5 = \frac{1}{5}(x - 3)$ and $y + 5 = -5(x - 3)$

$\Rightarrow$ x – 5y – 28 = 0 and 5x + y – 10 = 0

Hence, equation of required line is x – 5y – 28 = 0 and 5x + y – 10 = 0

Hence proved.

12. Question

The equation of the base of an equilateral triangle is x + y = 2 and its vertex is (2, -1). Find the length and equations of its sides.

Answer

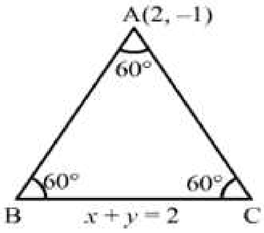
Given: equation of the base of an equilateral triangle is x + y = 2 and its vertex is (2, -1)

To find: length and equations of its sides

Explanation:

Let A (2, − 1) be the vertex of the equilateral triangle ABC and x + y = 2 be the equation of BC.

Diagram:



Here, we have to find the equations of the sides AB and AC, each of which makes an angle of 60° with the line x + y = 2

The equations of two lines passing through point x1,y1 and making an angle α with the line whose slope is m is given below:

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, $x_1 = 2$, $y_1 = -1$, α = 60°, m = -1

So, the equations of the required sides are

$y + 1 = \frac{-1 + \tan 60^\circ}{1 + \tan 60^\circ}(x - 2)$ and $y + 1 = \frac{-1 - \tan 60^\circ}{1 - \tan 60^\circ}(x - 2)$

$\Rightarrow y + 1 = \frac{-1 + \sqrt{3}}{1 + \sqrt{3}}(x - 2)$ and $y + 1 = \frac{-1 - \sqrt{3}}{1 - \sqrt{3}}(x - 2)$

$\Rightarrow y + 1 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}(x - 2)$ and $y + 1 = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}(x - 2)$

$\Rightarrow y + 1 = (2 - \sqrt{3})(x - 2)$ and $y + 1 = (2 + \sqrt{3})(x - 2)$

Solving x + y = 2 and $y + 1 = (2 - \sqrt{3})(x - 2)$, we get:

$$x = \frac{15 + \sqrt{3}}{6}, y = \frac{-3 + \sqrt{3}}{6}$$

$$\therefore B = \left(\frac{15 + \sqrt{3}}{6}, \frac{-3 + \sqrt{3}}{6}\right) \text{ or } C = \left(\frac{15 - \sqrt{3}}{6}, \frac{-3 - \sqrt{3}}{6}\right)$$

$\therefore AB = BC = AC = \sqrt{\frac{2}{3}}$Hence, equations of its sides are given below: $(2 - \sqrt{3})x - y + \sqrt{\frac{2}{3}} = 0$,

$(2 - \sqrt{3})x - y - \sqrt{\frac{2}{3}} = 0$

13. Question

If two opposites vertices of a square are (1, 2) and (5, 8), find the coordinates of its other two vertices and the equations of its sides.

Answer

Given: two opposites vertices of square are (1,2) and (5,8)

To find: opposite's vertices of a square and equation of sides.

Explanation:

Let A (1, 2) be the vertex of square ABCD and BD be the diagonal that is along the line 8x− 15y = 0

Equation of the given line is, 8x – 15y = 0

⇒ - 15y = - 8x

$$\Rightarrow y = \frac{8}{15}x$$

Comparing this equation with y = mx + c

We get, m $= \frac{8}{15}$

So, the slope of BD will be $\frac{8}{15}$.Here, we have to find the equations of sides AB and AD.

We know that the equations of two lines passing through a point x1,y1 and making an angle α with the line whose slope is m.

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

m $= \frac{8}{15}$, $x_1 = 1$, $y_1 = 2$, α = 45∘

So, the equations of the required sides are

$$y - 2 = \frac{\frac{8}{15} + \tan 45°}{1 - \frac{8}{15}\tan 45°}(x - 1) \text{ and } y - 2 = \frac{\frac{8}{15} - \tan 45°}{1 + \frac{8}{15}\tan 45°}(x - 1)$$

$$\Rightarrow y - 2 = \frac{\frac{8}{15} + 1}{1 - \frac{8}{15}}(x - 1) \text{ and } y - 2 = \frac{\frac{8}{15} - 1}{1 + \frac{8}{15}}(x - 1)$$

⇒ 23x – 7y – 9 = 0 and 7x + 23y – 53 = 0

Hence, equation of sides is 23x – 7y – 9 = 0 and 7x + 23y – 53 = 0

Exercise 23.19

1. Question

Find the equation of a straight line through the point of intersection of the lines 4x – 3y = 0 and 2x – 5y + 3 = 0 and parallel to 4x + 5 y + 6 = 0.

Answer

Given:

Lines 4x – 3y = 0 and 2x – 5y + 3 = 0 and parallel to 4x + 5 y + 6 = 0

To find:

The equation of a straight line through the point of intersection of the lines

Explanation:

The equation of the straight line passing through the points of intersection of $4x - 3y = 0$ and $2x - 5y + 3 = 0$ is given below:

$4x - 3y + \lambda (2x - 5y + 3) = 0$

$\Rightarrow (4 + 2\lambda)x + (-3 - 5\lambda)y + 3\lambda = 0$

$\Rightarrow y = \left(\frac{4 + 2\lambda}{3 + 5\lambda}\right)x + \frac{3\lambda}{(3 + 5\lambda)}$

The required line is parallel to $4x + 5y + 6 = 0$ or, $y = -\frac{4}{5}x - \frac{6}{5} \therefore \frac{4 + 2\lambda}{3 + 5\lambda} = -\frac{4}{5}$

$\Rightarrow \lambda = -\frac{16}{15}$

Hence, the required equation is $\left(4 - \frac{32}{15}\right)x - \left(3 - \frac{80}{15}\right)y - \frac{48}{15} = 0$

$\Rightarrow 28x + 35y - 48 = 0$

2. Question

Find the equation of a straight line passing through the point of intersection of $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ and perpendicular to the straight line $x - y + 9 = 0$.

Answer

Given:

$x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$

To find:

The equation of a straight line passing through the point of intersection of $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ and perpendicular to the straight line $x - y + 9 = 0$.

Explanation:

The equation of the straight line passing through the points of intersection of $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ is

$x + 2y + 3 + \lambda(3x + 4y + 7) = 0$

$\Rightarrow (1 + 3\lambda)x + (2 + 4\lambda)y + 3 + 7\lambda = 0$

$\Rightarrow y = -\left(\frac{1 + 3\lambda}{2 + 4\lambda}\right)x - \left(\frac{3 + 7\lambda}{2 + 4\lambda}\right)$

The required line is perpendicular to $x - y + 9 = 0$ or, $y = x + 9$

$\therefore \left(\frac{-1 + 3\lambda}{2 + 4\lambda}\right) \times 1 = -1$

$\Rightarrow \lambda = -1$

Required equation is given below:

$(1 - 3)x + (2 - 4)y + 3 - 7 = 0$

$\Rightarrow x + y + 2 = 0$

Hence, required equation is $x + y + 2 = 0$

3. Question

Find the equation of the line passing through the point of intersection of $2x - 7y + 11 = 0$ and $x + 3y - 8 = 0$ and is parallel to (i) x = axis (ii) y-axis.

Answer

Given:

$2x - 7y + 11 = 0$ and $x + 3y - 8 = 0$

To find:

The equation of the line passing through the point of intersection of $2x - 7y + 11 = 0$ and $x + 3y - 8 = 0$ and is parallel to (i) x = axis (ii) y-axis.

Explanation:

The equation of the straight line passing through the points of intersection of $2x - 7y + 11 = 0$ and $x + 3y - 8 = 0$ is given below:

$2x - 7y + 11 + \lambda(x + 3y - 8) = 0$

$\Rightarrow (2 + \lambda)x + (-7 + 3\lambda)y + 11 - 8\lambda = 0$(i) The required line is parallel to the x-axis. So, the coefficient of x should be zero.

$\therefore 2 + \lambda = 0$

$\Rightarrow \lambda = -2$

Hence, the equation of the required line is

$0 + (-7 - 6)y + 11 + 16 = 0$

$\Rightarrow 13y - 27 = 0$

(ii) The required line is parallel to the y-axis. So, the coefficient of y should be zero.

$\therefore -7 + 3\lambda = 0$

$\Rightarrow \lambda = \frac{7}{3}$

Hence, the equation of the required line is

$$\left(2 + \frac{7}{3}\right)x + 0 + 11 - 8 \times \frac{7}{3} = 0$$

$\Rightarrow 13x - 23 = 0$

4. Question

Find the equation of the straight line passing through the point of intersection of $2x + 3y + 1 = 0$ and $3x - 5y - 5 = 0$ and equally inclined to the axes.

Answer

Given:

$2x + 3y + 1 = 0$ and $3x - 5y - 5 = 0$

To find:

The equation of the straight line passing through the point of intersection of $2x + 3y + 1 = 0$ and $3x - 5y - 5 = 0$ and equally inclined to the axes.

Explanation:

The equation of the straight line passing through the points of intersection of $2x + 3y + 1 = 0$ and $3x - 5y - 5 = 0$ is

$2x + 3y + 1 + \lambda(3x - 5y - 5) = 0$

$\Rightarrow (2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0 \Rightarrow y = -\left(\frac{2 + 3\lambda}{3 - 5\lambda}\right) - \left(\frac{1 - 5\lambda}{3 - 5\lambda}\right)$

The required line is equally inclined to the axes. So, the slope of the required line is either 1 or − 1.

$\therefore -\left(\frac{2 + 3\lambda}{3 - 5\lambda}\right) = 1$ and $-\left(\frac{2 + 3\lambda}{3 - 5\lambda}\right) = -1$

$\Rightarrow -2 - 3\lambda = 3 - 5\lambda$ and $2 + 3\lambda = 3 - 5\lambda$

$\Rightarrow \lambda = \frac{5}{2}$ and $\frac{1}{8}$

Substituting the values of λ in $(2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0$, we get the equations of the required lines.

$$\left(2 + \frac{15}{2}\right)x + \left(3 - \frac{25}{2}\right)y + 1 - \frac{25}{2} = 0 \text{ and } \left(2 + \frac{3}{8}\right)x + \left(3 - \frac{5}{8}\right)y + 1 - \frac{5}{8} = 0$$

$\Rightarrow 19x - 19y - 23 = 0$ and $19x + 19y + 3 = 0$

Hence, required equation is $19x - 19y - 23 = 0$ and $19x + 19y + 3 = 0$

5. Question

Find the equation of the straight line drawn through the point of intersection of the lines x + y = 4 and 2x – 3y = 1 and perpendicular to the line cutting off intercepts 5, 6 on the axes.

Answer

Given:

lines x + y = 4 and 2x – 3y = 1

To find:

The equation of the straight line drawn through the point of intersection of the lines x + y = 4 and 2x – 3y = 1 and perpendicular to the line cutting off intercepts 5, 6 on the axes.

Explanation:

The equation of the straight line passing through the point of intersection of x + y = 4 and 2x− 3y = 1 is

$x + y - 4 + \lambda(2x - 3y - 1) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 - 3\lambda)y - 4 - \lambda = 0$ … (1)

$\Rightarrow y = -\left(\frac{1 + 2\lambda}{1 - 3\lambda}\right)x + \left(\frac{4 + \lambda}{1 - 3\lambda}\right)$

The equation of the line with intercepts 5 and 6 on the axis is

$\frac{x}{5} + \frac{y}{6} = 1$ … (2)

The slope of this line is $-\frac{6}{5}$

The lines (1) and (2) are perpendicular.

$\therefore -\frac{6}{5} \times \left(\frac{-1 + 2\lambda}{1 - 3\lambda}\right) = -1$

$\Rightarrow \lambda = \frac{11}{3}$

Substituting the values of λ in (1), we get the equation of the required line.

$\Rightarrow \left(1 + \frac{22}{3}\right)x + (1 - 11)y - 4 - \frac{11}{3} = 0$

⇒ 25x – 30y – 23 = 0

Hence, required equation is 25x – 30y – 23 = 0

6. Question

Prove that the family of lines represented by x(1 + λ) + y(2 – λ) + 5 = 0, λ being arbitrary, pass through a fixed point. Also, find the fixed point.

Answer

Given:

Lines represented by x(1 + λ) + y(2 – λ) + 5 = 0, λ being arbitrary

To prove:

The family of lines represented by x(1 + λ) + y(2 – λ) + 5 = 0, λ being arbitrary, pass through a fixed point. Also, find the fixed point.

Explanation:

The given family of lines can be written as

x + 2y + 5 + λ (x − y) = 0

This line is of the form $L_1 + \lambda L_2 = 0$, which passes through the intersection of $L_1 = 0$ and $L_2 = 0$. ⇒ x + 2y + 5 = 0 ⇒ x − y = 0

Now, solving the lines: $\left(-\frac{5}{3}, -\frac{5}{3}\right)$ This is a fixed point.

Hence proved.

7. Question

Show that the straight lines given by (2 + k)x + (1 + k)y = 5 + 7k for different values of k pass through a fixed point. Also, find that point.

Answer

Given:

lines given by (2 + k)x + (1 + k)y = 5 + 7k

To prove:

The straight lines given by (2 + k)x + (1 + k)y = 5 + 7k for different values of k pass through a fixed point

Explanation:

The given straight line (2 + k)x + (1 + k)y = 5 + 7k can be written in the following way:

2x + y − 5 + k (x + y − 7) = 0

This line is of the form $L_1 + kL_2 = 0$, which passes through the intersection of the lines $L_1 = 0$ and $L_2 = 0$, i.e. 2x + y − 5 = 0 and x + y − 7 = 0.

Solving 2x + y − 5 = 0 and x + y − 7 = 0, we get (− 2, 9), which is the fixed point.

Hence proved.

8. Question

Find the equation of the straight line passing through the point of intersection of $2x + y - 1 = 0$ and $x + 3y - 2 = 0$ and making with the coordinate axes a triangle of area 3/8 sq. units.

Answer

Given:

$2x + y - 1 = 0$ and $x + 3y - 2 = 0$

To find:

The equation of the straight line passing through the point of intersection of $2x + y - 1 = 0$ and $x + 3y - 2 = 0$ and making with the coordinate axes a triangle of area 3/8 sq. units.

Explanation:

The equation of the straight line passing through the point of intersection of $2x + y - 1 = 0$ and $x + 3y - 2 = 0$ is given below:

$2x + y - 1 + \lambda (x + 3y - 2) = 0$

$\Rightarrow (2 + \lambda)x + (1 + 3\lambda)y - 1 - 2\lambda = 0$

$\Rightarrow \left(\frac{x}{\frac{1+2\lambda}{2+\lambda}}\right) + \left(\frac{y}{\frac{1+2\lambda}{1+3\lambda}}\right) = 1$

So, the points of intersection of this line with the coordinate axes are $\left(\frac{1+2\lambda}{2+\lambda}, 0\right)$ and $\left(0, \frac{1+2\lambda}{1+3\lambda}\right)$

It is given that the required line makes an area of $\frac{3}{8}$ square units with the coordinate axes.

$$\left|\left(\frac{1 + 2\lambda}{2 + \lambda}\right) \times \left(\frac{1 + 2\lambda}{1 + 3\lambda}\right)\right| = \frac{3}{8}$$

$\Rightarrow 3\,|3\lambda^2 + 7\lambda + 2| = 4\,|4\lambda^2 + 4\lambda + 1|$

$\Rightarrow 9\lambda^2 + 21\lambda + 6 = 16\lambda^2 + 16\lambda + 4$

$\Rightarrow 7\lambda^2 - 5\lambda - 2 = 0$

$\Rightarrow \lambda = 1, -\frac{2}{7}$

Hence, the equations of the required lines are

$3x + 4y - 1 - 2 = 0$ and $\left(2 - \frac{2}{7}\right)x + \left(1 - \frac{6}{7}\right)y - 1 + \frac{4}{7} = 0$

$\Rightarrow 3x + 4y - 3 = 0$ and $12x + y - 3 = 0$

9. Question

Find the equation of the straight line which passes through the point of intersection of the lines $3x - y = 5$ and $x + 3y = 1$ and makes equal and positive intercepts on the axes.

Answer

Given:

Lines $3x - y = 5$ and $x + 3y = 1$

To find:

The equation of the straight line which passes through the point of intersection of the lines $3x - y = 5$ and $x + 3y = 1$ and makes equal and positive intercepts on the axes.

Explanation:

The equation of the straight line passing through the point of intersection of $3x - y = 5$ and $x + 3y = 1$ is

$3x - y - 5 + \lambda(x + 3y - 1) = 0$

$\Rightarrow (3 + \lambda)x + (-1 + 3\lambda)y - 5 - \lambda = 0$ ... (1) $\Rightarrow y = -\left(\frac{3+\lambda}{-1+\lambda}\right)x + \left(\frac{5+\lambda}{-1+\lambda}\right)$

The slope of the line that makes equal and positive intercepts on the axis is -1.

From equation (1), we have:

$$-\left(\frac{3 + \lambda}{-1 + 3\lambda}\right) = -1$$

$\Rightarrow \lambda = 2$

Substituting the value of λ in (1), we get the equation of the required line.

$\Rightarrow 3 + 2x + -1 + 6y - 5 - 2 = 0$

$\Rightarrow 5x + 5y - 7 = 0$

Hence, equation of required line is $5x + 5y - 7 = 0$

10. Question

Find the equations of the lines through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$.

Answer

Given:

Lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$

To find:

The equations of the lines through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$.

Explanation:

The equation of the straight line passing through the point of intersection of $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ is given below:

$x - 3y + 1 + \lambda(2x + 5y - 9) = 0$

$\Rightarrow (1 + 2\lambda)x + (-3 + 5\lambda)y + 1 - 9\lambda = 0$ ... (1)

The distance of this line from the origin is $\sqrt{5}$

$$\left|\frac{1 - 9\lambda}{\sqrt{(1 + 2\lambda)^2 + (5\lambda - 3)^2}}\right| = \sqrt{5}$$

$\Rightarrow 1 + 81\lambda^2 - 18\lambda = 145\lambda^2 - 130\lambda + 50$

$\Rightarrow 64\lambda^2 - 112\lambda + 49 = 0$

$\Rightarrow (8\lambda - 7)^2 = 0$

$\Rightarrow \lambda = \frac{7}{8}$

Substituting the value of λ in (1), we get the equation of the required line.

$$\left(1 + \frac{14}{8}\right)x + \left(-3 + \frac{35}{8}\right)y + 1 - \frac{63}{8} = 0$$

⇒ $22x + 11y - 55 = 0$

⇒ $2x + y - 5 = 0$

Hence, equation of required line is $2x + y - 5 = 0$.

11. Question

Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ whose distance from the point (3, 2) is $\frac{7}{5}$.

Answer

Given:

Lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$

To find:

The equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ whose distance from the point (3, 2) is 7/5.

Explanation:

The equation of the straight line passing through the point of intersection of $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ is given below:

$x - y + 1 + \lambda(2x - 3y + 5) = 0$

⇒ $(1 + 2\lambda)x + (-3\lambda - 1)y + 5\lambda + 1 = 0$ ... (1)

The distance of this line from the point is given by

$$\left|\frac{3(1 + 2\lambda) + 2(-3\lambda - 1) + 5\lambda + 1}{\sqrt{(1 + 2\lambda)^2 + (-3\lambda - 1)^2}}\right| = \frac{7}{5}$$

$$\left|\frac{|5\lambda + 2|}{\sqrt{13\lambda^2 + 10\lambda + 2}}\right| = \frac{7}{5}$$

⇒ $25(5\lambda + 2)^2 = 49(13\lambda^2 + 10\lambda + 2)$

⇒ $6\lambda^2 - 5\lambda - 1 = 0$

⇒ $\lambda = 1, \frac{-1}{6}$

Substituting the value of λ in (1), we get the equation of the required line.

⇒ $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$

Hence, equation of required line is $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$.

Very Short Answer

1. Question

Write an equation representing a pair of lines through the point (a, b) and parallel to the coordinates axes.

Answer

Given:

Point (a, b)

To find:

Equation representing a pair of lines through the point (a, b) and parallel to the coordinates axes.

Explanation:

The lines passing through (a, b) and parallel to the x-axis and y-axis are y = b and x = a, respectively.

Therefore, their combined equation is given below:

$(x - a)(y - b) = 0$

2. Question

Write the coordinates of the orthocenter of the triangle formed by the lines $x^2 - y^2 = 0$ and $x + 6y = 18$.

Answer

Given:

Lines $x^2 - y^2 = 0$ and $x + 6y = 18$.

To find:

The coordinates of the orthocenter of the triangle formed by the lines $x^2 - y^2 = 0$ and $x + 6y = 18$.

Explanation:

The equation $x^2 - y^2 = 0$ represents a pair of straight line, which can be written in the following way:

$(x + y)(x - y) = 0$

So, the lines can be written separately in the following manner:

$x + y = 0$ ... (1)

$x - y = 0$ ... (2)

The third line is

$x + 6y = 18$ ... (3)

Lines (1) and (2) are perpendicular to each other as their slopes are -1 and 1, respectively $\Rightarrow -1 \times 1 = -1$

Therefore, the triangle formed by the lines (1), (2) and (3) is a right-angled triangle.

Thus, the orthocentre of the triangle formed by the given lines is the intersection of $x + y = 0$ and $x - y = 0$, which is (0, 0).

3. Question

If the centroid of a triangle formed by the points (0, 0), (cos θ, sin θ) and (sinθ, - cosθ) lies on the line y = 2x, then write the value of tanθ.

Answer

Given:

The points (0, 0), (cos θ, sin θ) and (sinθ, - cosθ) lies on the line y = 2x

To find:

The value of tanθ.

Explanation:

The centroid of a triangle with vertices (x_1,y_1), (x_2, y_2) and $(x_3, y_3)$is given below:

$$\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

Therefore, the centre of the triangle having vertices (0, 0,), (cos θ, sin θ) and (sin θ, − cos θ) is

$\left(\frac{0 + \cos\theta + \sin\theta}{3}\right), \left(\frac{0 + \sin\theta - \cos\theta}{3}\right) = \left(\frac{\cos\theta + \sin\theta}{3}\right), \left(\frac{\sin\theta - \cos\theta}{3}\right)$

This point lies on the line y = 2x.

$\frac{\sin\theta - \cos\theta}{3} = 2 \times \frac{\cos\theta + \sin\theta}{3}$

⇒ sinθ – cosθ = 2cosθ + 2sinθ

⇒ tanθ = -3

∴ tanθ = − 3

Hence, tanθ = − 3

4. Question

Write the value of $\theta \in \left(0, \frac{\pi}{2}\right)$ for which area of the triangle formed by points O(0, 0), A(a cos θ, b sin θ) and (a cos θ, - b sin θ) is maximum.

Answer

Given:

Points O(0, 0), A(a cos θ, b sin θ) and (a cos θ, - b sin θ)

To find:

The value of $\theta \in \left(0, \frac{\pi}{2}\right)$ for which area of the triangle formed by points O(0, 0), A(a cos θ, b sin θ) and (a cos θ, - b sin θ) is maximum.

Explanation:

Let A be the area of the triangle formed by the points O (0,0), A (acosθ, bsinθ) and B (acosθ, − bsinθ)

$A = \begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ a\cos\theta & -b\sin\theta & 1 \end{vmatrix}$

⇒ $A = \frac{1}{2}|(-ab\sin\theta\cos\theta - ab\sin\theta\cos\theta)|$

⇒ A = ab sinθcosθ $= \frac{1}{2}\sin 2\theta$

Now,∴Amax $= \frac{1}{2}$, when sin2θ = 1

⇒ ∴Amax $= \frac{1}{2}$, when 2θ = π/2

⇒ $\theta = \frac{\pi}{4}$

Hence, the area of the triangle formed by the given points is maximum when $\theta = \frac{\pi}{4}$.

5. Question

Write the distance the lines 4x + 3y – 11 = 0 and 8x + 6y – 15 = 0.

Answer

Given:

Lines 4x + 3y – 11 = 0 and 8x + 6y – 15 = 0

To find:

Distance **between lines.**

Explanation:

The distance between the two parallel lines ax + by + c_1 = 0 and ax + by + c_2 = 0 is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$the given lines can be written as

4x + 3y − 11 = 0 … (1)

8x + 6y – 15 = 0

⇒ $4x + 3y - \frac{15}{2} = 0$ … (2)

Let d be the distance between the lines (1) and (2).

$$d = \left|\frac{-11 - \left(-\frac{15}{2}\right)}{\sqrt{4^2 + 3^2}}\right| = \frac{7}{10} \text{units}$$

Hence, $d = \frac{7}{10}$units

6. Question

Write the coordinates of the orthocenter of the triangle formed by the lines xy = 0 and x + y = 1

Answer

Given:

Lines xy = 0 and x + y = 1

To find:

The coordinates of the orthocenter of the triangle formed by the lines xy = 0 and x + y = 1

Explanation:

The equation xy = 0 represents a pair of straight lines.

The lines can be written separately in the following way:

x = 0 … (1)

y = 0 … (2)

The third line is

x + y = 1 … (3)

Lines (1) and (2) are perpendicular to each other as they are coordinate axes.

Therefore, the triangle formed by the lines (1), (2) and (3) is a right-angled triangle.

Thus, the orthocentre of the triangle formed by the given lines is the intersection of x = 0 and y = 0, which is (0, 0).

7. Question

If the lines x + ay + a = 0, bx + y + b = 0 and cx + cy + 1 = 0 are concurrent then write the value of 2abc – ab – bc – ca.

Answer

Given:

Lines x + ay + a = 0, bx + y + b = 0 and cx + cy + 1 = 0

To find:

The value of 2abc – ab – bc – ca.

Explanation:

The given lines are

x + ay + a = 0 … (1)

bx + y + b = 0 … (2)

cx + cy + 1 = 0 … (3)

It is given that the lines (1), (2) and (3) are concurrent.

$$\therefore \begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$$

⇒ (1 – bc) – a(b – bc) + a(bc – c) = 0

⇒ 1 – bc – ab + abc + abc – ac = 0

⇒ 2abc – ab – bc – ca = -1

Hence, the value of 2abc − ab − bc − ca is − 1

8. Question

Write the area of the triangle formed by the coordinate axes and the line (sec θ – tan θ)x + (sec θ + tan θ) y = 2.

Answer

Given:

Line (sec θ – tan θ)x + (sec θ + tan θ) y = 2.

To find:

The area of the triangle formed by the coordinate axes and the line (sec θ – tan θ)x + (sec θ + tan θ) y = 2.

Explanation:

The point of intersection of the coordinate axes is (0, 0).Let us find the intersection of the line (secθ − tan θ) x + (sec θ + tan θ) y = 2 and the coordinate axis.

For x-axis:

y = 0, x $= \frac{2}{\sec\theta - \tan\theta}$

For y-axis:

x = 0, y $= \frac{2}{\sec\theta + \tan\theta}$

Thus, the coordinates of the triangle formed by the coordinate axis and the line (sec θ − tan θ) x + (sec θ + tan θ) y = 2 are (0, 0), $\left(\frac{2}{\sec\theta - \tan\theta}, 0\right)$and $\left(0, \frac{2}{\sec\theta + \tan\theta}\right)$.

Let A be the area of the required triangle.

$$\therefore A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{2}{\sec\theta - \tan\theta} & 0 & 1 \\ 0 & \frac{2}{\sec\theta + \tan\theta} & 1 \end{vmatrix}$$

$$\Rightarrow A = \frac{1}{2} \times \frac{2}{\sec\theta - \tan\theta} \times \frac{2}{\sec\theta + \tan\theta}$$

$$\Rightarrow A = \frac{2}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)} = 2$$

Hence, the area of the triangle is 2 square units.

9. Question

If the diagonals of the quadrilateral formed by the lines $l_1x + m_1y + n_1 = 0$, $l_2x + m_2y + n_2 = 0$, $l_1x + m_1y + n'_1 = 0$ and $l_2x + m_2y + n_2' = 0$ are perpendicular, then write the value of $l^{12} - l_2^2 + m^1{}_2 - m^2{}_2$.

Answer

Given:

$l_1x + m_1y + n_1 = 0$ … (1)

$l_2x + m_2y + n_2 = 0$ … (2)

$l_1x + m_1y + n'_1 = 0$ … (3)

$l_2x + m_2y + n_2' = 0$ … (4)

To find:

The value of $l^{12} - l_2^2 + m^1{}_2 - m^2{}_2$.

Explanation:

Assuming:

(1), (2), (3) and (4) represent the sides AB, BC, CD and DA, respectively.

The equation of diagonal AC passing through the intersection of (2) and (3) is given by$l_1x + m_1y + n'_1 + \lambda(l_2x + m_2y + n_2) = 0$

$\Rightarrow (l_1 + \lambda l_2)x + (m_1 + \lambda m_2)y + (n_1' + \lambda n_2) = 0$

$\Rightarrow$ Slope of diagonal AC $= \left(\frac{l_1 + \lambda l_1}{m_1 + \lambda m_2}\right)$

Also, the equation of diagonal BD, passing through the intersection of (1) and (2), is given by$l_1x + m_1y + n_1 + \mu(l_2x + m_2y + n_2) = 0$

$\Rightarrow l_1 + \mu l_2x + m_1 + \mu m_2y + n_1 + \mu n_2 = 0$

$\Rightarrow$ Slope of diagonal BD $= \frac{l_1 + \mu l_1}{m_1 + \mu m_2}$

The diagonals are perpendicular to each other.

$$\therefore \left(\frac{l_1 + \lambda l_1}{m_1 + \lambda m_2}\right)\left(\frac{l_1 + \mu l_1}{m_1 + \mu m_2}\right) = -1$$

$\Rightarrow (l_1 + \lambda l_2)(l_1 + \lambda l_2) = (-m_1 + \lambda m_2)(m_1 + \mu m_2)$

Let λ = -1, μ = 1

⇒ $(l_1 - l_2)(l_1 + l_2) = (-m_1-m_2)(m_1 + m_2)$

⇒ $(l_1^2-l_2^2) = (-m_1^2-m_2^2)$

⇒ $(l_1^2-l_2^2) + (m_1^2-m_2^2) = 0$

Hence, $(l_1^2-l_2^2) + (m_1^2-m_2^2) = 0$

10. Question

Write the coordinates of the image of the point (3, 8) in the line x + 3y – 7 = 0.

Answer

Given:

Line x + 3y – 7 = 0, point (3, 8).

To find:

The coordinates of the image.

Explanation:

Let the given point be A(3,8) and its image in the line x + 3y − 7 = 0 is B(h,k).

The midpoint of AB is $\frac{3+h}{2}, \frac{8+k}{2}$ that lies on the line x + 3y − 7 = 0.

$$\therefore \frac{3+h}{2} + 3 \times \frac{8+k}{2} - 7 = 0$$

h + 3k + 13 = 0 … (1)

AB and the line x + 3y − 7 = 0 are perpendicular.

∴Slope of AB × Slope of the line = - 1

$$\Rightarrow \left(\frac{k-8}{h-3}\right) \times -\frac{1}{3} = -1$$

⇒ 3h – k – 1 = 0 … (2)

Solving (1) and (2), we get:(h, k) = (− 1, − 4)

Hence, the image of the point (3,8) in the line x + 3y − 7 = 0 is (− 1, − 4).

11. Question

Write the integral values of m for which the x-coordinate of the point of intersection of the lines y = mx + 1 and 3x + 4y = 9 is an integer.

Answer

Given:

Lines y = mx + 1 and 3x + 4y = 9

To find:

The integral values of m

Explanation:

The given lines can be written as

mx – y + 1 = 0 … (1)

3x + 4y – 9 = 0 … (2)

Solving (1) and (2) by cross multiplication, we get:

$$\frac{x}{9-4} = \frac{y}{3+9m} = \frac{1}{4m+3}$$

$\Rightarrow x = 5(4m+3),$

$y = \frac{9m+3}{4m+3}$For x to be integer we have, 4m + 3 = 1, -1, 5 and -5

$\Rightarrow m = -\frac{1}{2}, -1, \frac{1}{2}$ and -2Hence, the integral values of m are -1 and -2.

12. Question

If a ≠ b ≠ c, write the condition for which the equation (b – c)x + (c – a) y + (a – b) = 0 and $(b^3 - c^3)x + (c^3 - a^3)y + (a^3 - b^3) = 0$ represent the same line

Answer

Given:

The equation (b – c)x + (c – a) y + (a – b) = 0 and $(b^3 - c^3)x + (c^3 - a^3)y + (a^3 - b^3) = 0$

To find:

The condition for which the equation (b – c)x + (c – a) y + (a – b) = 0 and $(b^3 - c^3)x + (c^3 - a^3)y + (a^3 - b^3) = 0$ represent the same line

Explanation:

The given lines are

(b − c)x + (c − a)y + (a − b) = 0 … (1)

$(b^3 - c^3)x + (c^3 - a^3)y + (a^3 - b^3) = 0$ … (2)

The lines (1) and (2) will represent the same lines if

$$\frac{b-c}{b^3-c^3} = \frac{c-a}{c^3-a^3} = \frac{a-b}{a^3-b^3}$$

$$\Rightarrow \frac{b-c}{(b-c)(b^2+bc+c^2)} = \frac{c-a}{(c-a)(c^2+ac+a^2)} = \frac{a-b}{(a-b)(a^2+ab+b^2)}$$

$$\Rightarrow \frac{1}{b^2+bc+c^2} = \frac{1}{c^2+ac+a^2} = \frac{1}{a^2+ab+b^2}$$

∵ (a ≠ b ≠ c)

$\Rightarrow b^2 + bc + c^2 = c^2 + ac + a^2$ and $c^2 + ac + a^2 = a^2 + ab + b^2$

⇒ (a – b) (a + b + c) = 0 and (b – c) (b + c + a) = 0

⇒ a + b + c = 0 ∵(a ≠ b ≠ c)

Hence, the given lines will represent the same lines if a + b + c = 0.

13. Question

If a, b, c are in G.P. write the area of the triangle formed by the line ax + by + c = 0 with the coordinates axes.

Answer

Given:

a, b, c are in G.P.

To find:

Area of the triangle formed by the line ax + by + c = 0 with the coordinates axes.

Explanation:

The point of intersection of the line ax + by + c = 0 with the coordinate axis are (–c/a,0) and (0,-c/b).

So, the vertices of the triangle are (0, 0), $\left(-\frac{c}{a}, 0\right)$ and $\left(0, -\frac{c}{b}\right)$.

Let A be the area of the required triangle.

$$A = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{c}{a} & 0 & 1 \\ 0 & -\frac{c}{b} & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left|-\frac{c}{a} \times -\frac{c}{b}\right| = \frac{1}{2}\left|\frac{c^2}{ab}\right|$$

It is given that a, b and c are in GP.

$\therefore b^2 = ac$

$$\Rightarrow A = \frac{1}{2}\left|-\frac{c}{a} \times -\frac{c}{b}\right| = \frac{1}{2}\left|\frac{c^2}{ab}\right|$$

Hence, area $A = \frac{1}{2}\left|\frac{c^2}{ab}\right|$

14. Question

Write the area of the figure formed by the lines a|x| + b|y| + c = 0

Answer

Given:

a x + b y + c = 0; x, y ≥ 0 … (1)

-a x + b y + c = 0; x < 0 y ≥ 0 … (2)

-a x – b y + c = 0; x < 0 y < 0 … (3)

a x – b y + c = 0; x ≥ 0 y < 0 … (4)

To find:

The area of the figure formed by the lines a|x| + b|y| + c = 0

Explanation:

The given lines can be written separately in the following way:

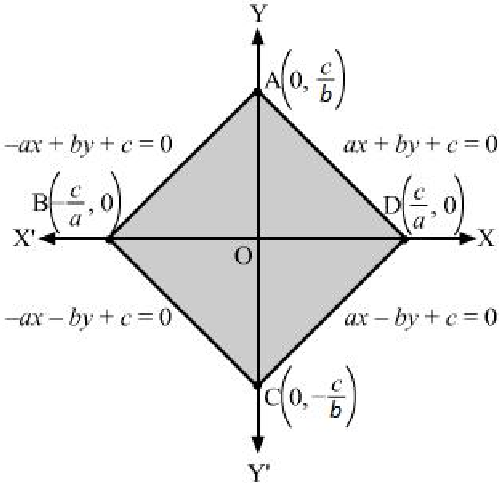
a x + b y + c = 0; x, y ≥ 0 … (1)

-a x + b y + c = 0; x < 0 y ≥ 0 … (2)

-a x – b y + c = 0; x < 0 y < 0 … (3)

a x – b y + c = 0; x ≥ 0 y < 0 … (4)

The lines and the region enclosed between them is shown below.



So, the area of the figures formed by the lines a |x| + b |y| + c = 0 is

$4 \times \frac{1}{2}\left|\frac{c}{a} \times \frac{c}{b}\right| = \frac{2c^2}{|ab|}$ Square units

15. Question

Write the locus of a point the sum of whose distances from the coordinate's axes is unity.

Answer

Given:

Distances from the coordinate's axes is unity.

To find:

The locus of a point the sum of whose distances from the coordinate's axes is unity.

Assuming:

(h, k) be the locus.

Explanation:

It is given that the sum of distances of (h, k) from the coordinate axis is unity.

∴ |h| + |k| = 1

Taking locus of (h, k), we get:

|x| + |y| = 1

Hence, this represents a square.

16. Question

If a, b, c are in A.P., then the line ax + by + c = 0 passes through a fixed point. Write the coordinates of that point.

Answer

Given:

a, b, c are in A.P.

To find:

The coordinates of that point.

Explanation:

If a, b, c are in A.P., then

a + c = 2b

⇒ a – 2b + c = 0

Comparing the coefficient of ax + by + c = 0 and a – 2b + c = 0, we get x = 1 and y = -2

Hence, the coordinate of that point is (1, -2).

17. Question

Write the equation of the line passing through the point (1, -2) and cutting off equal intercepts from the axes.

Answer

Given:

Line passing through the point (1, -2) and cutting off equal intercepts from the axes.

To find:

The equation of the line

Explanation:

Let the required equation of the line is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Now, passes through (1, -2)

$$\frac{1}{a} - \frac{2}{a} = 1$$

⇒ a = -1

Hence, the required equation is:

$$\frac{x}{-1} + \frac{y}{-1} = 1$$

⇒ x + y + 1 = 0

Hence, equation of required line is x + y + 1 = 0.

18. Question

Find the locus of the mid-points of the portion of the line x sinθ + y cos θ = p intercepted between the axes.

Answer

Given:

Line x sin θ + y cos θ = p

To find:

The locus of the mid-points of the portion of the line x sinθ + y cos θ = p intercepted between the axes.

Explanation:

If the equation of the given line is

x sin θ + y cos θ = p, then the solution is shown below:

The line

x sin θ + y cos θ = p intercepts the axes.

Thus, the coordinate of the poin where the line intercepts x – axis is

$$\left(\frac{p}{\cos\theta}, 0\right)$$

Thus, the coordinate of the poin where the line intercepts y – axis is

$(0, \frac{p}{\sin\theta})$

The midpoint R of the line is given by

R(h, k) $= \left(\frac{\frac{p}{\cos\theta}+0}{2}, \frac{0+\frac{p}{\sin\theta}}{2}\right) = \left(\frac{p}{2\cos\theta}, \frac{p}{2\sin\theta}\right)$

$\Rightarrow h = \frac{p}{2\cos\theta}, k = \frac{p}{2\sin\theta}$

Eliminating the sine and cosine terms, we get

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \frac{p^2}{4h^2} + \frac{p^2}{4k^2} = 1$

$\Rightarrow p^2(h^2 + k^2) = 4h^2k^2$

Thus, the locus is given by

$p^2(x^2 + y^2) = 4x^2y^2$

MCQ

1. Question

L is variable line such that the algebraic sum of the distances of the points (1, 1), (2, 0) and (0, 2) from the line is equal to zero. The line L will always pass through

A. (1, 1)

B. (2, 1)

C. (1, 2)

D. none of these

Answer

Let ax + by + c = 0 be the variable line. It is given that the algebraic sum of the distancesof the points (1, 1), (2, 0) and (0, 2) from the line is equal to zero.

$\therefore \frac{a+b+c}{\sqrt{a^2+b^2}} + \frac{2a+0+c}{\sqrt{a^2+b^2}} + \frac{0+2b+c}{\sqrt{a^2+b^2}} = 0$

$\Rightarrow 3a + 3b + 3c = 0$

$\Rightarrow a + b + c = 0$

Substituting c = – a – b in ax + by + c = 0, we get:

ax + by – a – b = 0

$\Rightarrow a(x - 1) + b(y - 1) = 0$

$\Rightarrow x - 1 + \frac{b}{a}(y - 1) = 0$

This line is of the form $L_1 + \lambda L_2 = 0$, which passes through the intersection of $L_1 = 0$ and $L_2 = 0$, i.e. x – 1 = 0 and y – 1 = 0.

$\Rightarrow x = 1, y = 1$

2. Question

The acute angle between the medians drawn from the acute of a right angled isosceles triangle is

A. $\cos^{-1}\left(\frac{2}{3}\right)$

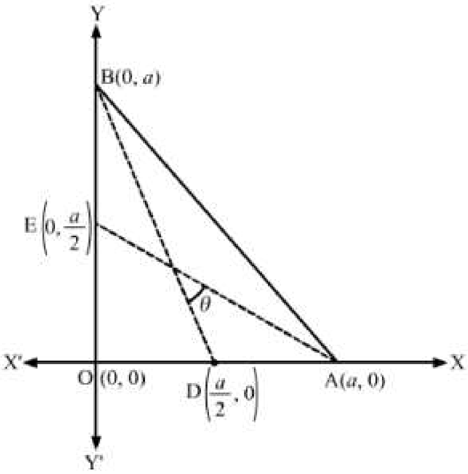
B. $\cos^{-1}\left(\frac{3}{4}\right)$

C. $\cos^{-1}\left(\frac{4}{5}\right)$

D. $\cos^{-1}\left(\frac{5}{6}\right)$

Answer

Let the coordinates of the right-angled isosceles triangle be O(0, 0), A(a, 0) and B(0, a).



Here, BD and AE are the medians drawn from the acute angles B and A, respectively.

∴ Slope of BD = $m_1 = \frac{0 - a}{\frac{a}{2} - 0} = -2$

Slope of AE = $m_2 = \frac{\frac{a}{2} - 0}{0 - a} = -\frac{1}{2}$

Let θ be the angle between BD and AE.

$\tan\theta = \left|\frac{-2 + \frac{1}{2}}{1 + 1}\right| = \frac{3}{4}$

$\Rightarrow \cos\theta = \frac{4}{\sqrt{3^2 + 4^2}}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right)$

Hence, the acute angle between the medians is $\cos^{-1}\left(\frac{4}{5}\right)$

3. Question

The distance between the orthocenter and circumcentre of the triangle with vertices (1, 2) (2, 1) and $\left(\frac{3+\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$ is

A. 0

B. $\sqrt{2}$

C. $3+\sqrt{3}$

D. none of these

Answer

Let A(1, 2), B(2, 1) and $C\left(\left(\frac{3+\sqrt{3}}{2}\right), \left(\frac{3+\sqrt{3}}{2}\right)\right)$ be the given points.

$\therefore AB = \sqrt{(2-1)^2 + (1-2)^2} = \sqrt{2}$

$BC = \sqrt{\left(\frac{3+\sqrt{3}}{2} - 2\right)^2 + \left(\frac{3+\sqrt{3}}{2} - 1\right)^2} = \sqrt{2}$

$AC = \sqrt{\left(\frac{3+\sqrt{3}}{2} - 1\right)^2 + \left(\frac{3+\sqrt{3}}{2} - 2\right)^2} = \sqrt{2}$

Thus, ABC is an equilateral triangle.

We know that the orthocentre and the circumcentre of an equilateral triangle are same.

So, the distance between the orthocentre and the circumcentre of the trianglewith vertices (1, 2), (2, 1) and $\left(\left(\frac{3+\sqrt{3}}{2}\right), \left(\frac{3+\sqrt{3}}{2}\right)\right)$ is 0.

4. Question

The equation of the straight line which passes through the point (-4, 3) such that the portion of the line between the axes is divided internally by the point in the ratio 5 : 3 is

A. 9x – 20y + 96 = 0

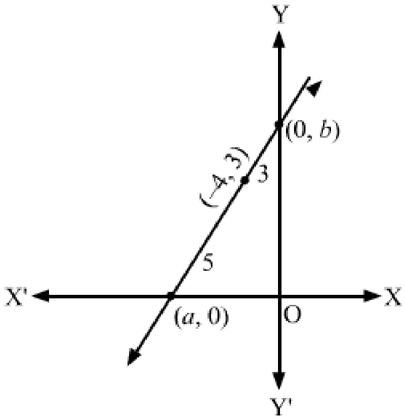
B. 9x + 20y = 24

C. 20x + 9y + 53 = 0

D. none of these

Answer

Let the required line intersects the coordinate axis at (a, 0) and (0, b).



The point (− 4, 3) divides the required line in the ratio 5 : 3

$\therefore$ -4 $= \frac{5 \times 0 + 3 \times a}{5 + 3}$ and $3 = \frac{5 \times b + 3 \times 0}{5 + 3}$

$\Rightarrow$ a $= -\frac{32}{3}$ and b $= \frac{24}{5}$

Hence, the equation of the required line is given below:

$$\frac{x}{-\frac{32}{3}} + \frac{y}{\frac{24}{5}} = 1$$

$\Rightarrow -\frac{3x}{32} + \frac{5y}{24} = 1$

⇒ -9x + 20y = 96

⇒ 9x – 20y + 96 = 0

5. Question

Which point which divides the join of (1, 2) and (3, 4) externally in the ratio of 1 : 1.

A. lies in the III quadrant

B. lies in the II quadrant

C. lies in the I quadrant

D. cannot be found

Answer

The point which divides the join of (1, 2) and (3, 4) externally in the ratio 1 :1 is

$\left(\frac{1 \times 3 - 1 \times 1}{1 - 1}, \frac{1 \times 4 - 1 \times 2}{1 - 1}\right)$ which is not defined .

Therefore, it is not possible to externally divide the line joining two points in the ratio 1:1

6. Question

A line passes through the point (2, 2) and is perpendicular to the line 3x + y = 3. Its y-intercepts is

A. 1/3

B. 2/3

C. 1

D. 4/3

Answer

The equation of the line perpendicular to $3x + y = 3$ is given below:

$x - 3y + \lambda = 0$

This line passes through (2, 2).

$2 - 6 + \lambda = 0$

$\Rightarrow \lambda = 4$

So, the equation of the line will be

$x - 3y + 4 = 0$

$\Rightarrow y = 13x + 43$

Hence, the y-intercept is $\frac{4}{3}$.

7. Question

If the lines $ax + 12y + 1 = 0$, $bx + 13y + 1 = 0$ and $cx + 14y - 1 = 0$ are concurrent, then a, b, c are in

A. H.P.

B. G.P.

C. A.P.

D. none of these

Answer

The given lines are

$ax + 12y + 1 = 0$ … (1)

$bx + 13y + 1 = 0$ … (2)

$cx + 14y + 1 = 0$ … (3)

It is given that (1), (2) and (3) are concurrent.

$$\begin{vmatrix} a & 12 & 1 \\ b & 13 & 1 \\ c & 14 & 1 \end{vmatrix} = 0$$

$\Rightarrow a(13 - 14) - 12(b - c) + 14b - 13c = 0$

$\Rightarrow -a - 12b + 12c + 14b - 13c = 0$

$\Rightarrow -a + 2b - c = 0$

$\Rightarrow 2b = a + c$

Hence, a, b and c are in AP.

8. Question

The number of real values of λ for which the lines $x - 2y + 3 = 0$, $\lambda x + 3y + 1 = 0$ and $4x - \lambda y + 2 = 0$ are concurrent is

A. 0

B. 1

C. 2

D. infinite

Answer

The given lines are

$x - 2y + 3 = 0$ ... (1)

$\lambda x + 3y + 1 = 0$ ... (2)

$4x - \lambda y + 2 = 0$ ... (3)

It is given that (1), (2) and (3) are concurrent.

$$\therefore \begin{vmatrix} 1 & -2 & 3 \\ \lambda & 3 & 1 \\ 4 & -\lambda & 2 \end{vmatrix} = 0$$

$\Rightarrow (6 + \lambda) + 2(2\lambda - 4) + 3(-\lambda^2 - 12) = 0$

$\Rightarrow 6 + \lambda + 4\lambda - 8 - 3\lambda^2 - 36 = 0$

$\Rightarrow 5\lambda - 3\lambda^2 - 38 = 0$

$\Rightarrow 3\lambda^2 - 5\lambda + 38 = 0$

The discriminant of this equation is $25-4 \times 3 \times 38 = -431$

Hence, there is no real value of λ for which the lines $x - 2y + 3 = 0$, $\lambda x + 3y + 1 = 0$ and $4x - \lambda y + 2 = 0$ are concurrent.

9. Question

The equations of the sides AB, BC and CA of ΔABC are $y - x = 2$, $x + 2y = 1$ and $3x + y + 5 = 0$ respectively. The equation of the altitude through B is

A. $x - 3y + 1$ 0

B. $x - 3y + 4 = 0$

C. $3x - y + 2 = 0$

D. none of these

Answer

The equation of the sides AB, BC and CA of ΔABC are $y - x = 2$, $x + 2y = 1$ and $3x + y + 5 = 0$, respectively.

Solving the equations of AB and BC, i.e. $y - x = 2$ and $x + 2y = 1$, we get:

$x = -1, y = 1$

So, the coordinates of B are $(-1, 1)$.

The altitude through B is perpendicular to AC.

$\therefore$ Slope of AC = -3

Thus, slope of the altitude through B is 13.

Equation of the required altitude is given below:

$y - 1 = 13x + 1$

$\Rightarrow x - 3y + 4 = 0$

10. Question

If p_1 and p_2 are the lengths of the perpendiculars form the origin upon the lines $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$ respectively, then

A. $4p_1^2 + p_2^2 = a^2$

B. $p_1^2 + 4p_2^2 = a^2$

C. $p_1^2 + p_2^2 = a^2$

D. none of these

Answer

The given lines are

$x \sec\theta + y \operatorname{cosec}\theta = a$ … (1)

$x \cos\theta - y \sin\theta = a \cos 2\theta$ … (2)

p_1 and p_2 are the perpendiculars from the origin upon the lines (1) and (2), respectively.

$$p_1 = \left| -\frac{a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} \right| \text{ and } p_2 = \left| -\frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} \right|$$

$$\Rightarrow p_1 = \frac{1}{2}|-a \times 2\sin\theta\cos\theta| \text{ and } p_2 = |-a\cos 2\theta|$$

$$\Rightarrow p_1 = \frac{1}{2}|-a\sin 2\theta| \text{ and } p_2 = |-a\cos 2\theta|$$

$\Rightarrow 4p_1^2 + p_2^2 = a^2 (\sin^2 2\theta + \cos^2 2\theta) = a^2$

11. Question

Area of the triangle formed by the points $((a + 3)(a + 4), a + 3)$, $((a + 2)(a + 3), (a + 2))$ and $((a + 1)(a + 2), (a + 1))$ is

A. $25a^2$

B. $5a^2$

C. $24a^2$

D. none of these

Answer

The given points are $(a + 3)(a + 4), a + 3$, $(a + 2)(a + 3), (a + 2)$ and $(a + 1)(a + 2), (a + 1)$.

Let A be the area of the triangle formed by these points.

Then, $A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\Rightarrow A = \frac{1}{2}[(a + 3)(a + 4)(a + 2 - a - 1) - (a + 2)(a + 3)(a + 1 - a - 3) + (a + 1)(a + 2)(a + 3 - a - 2)]$$

$$\Rightarrow A = \frac{1}{2}[(a + 3)(a + 4) - 2(a + 2)(a + 3) + (a + 1)(a + 2)]$$

$$\Rightarrow A = \frac{1}{2}[a^2 + 7a + 12 - 2a^2 - 10a - 12 + a^2 + 3a + 2]$$

$$\Rightarrow A = 1$$

12. Question

If $a + b + c = 0$, then the family of lines $3ax + by + 2c = 0$ pass through fixed point

A. (2. 2/3)

B. (2/3, 2)

C. (-2, 2/3)

D. none of these

Answer

Given:

a + b + c = 0

Substituting c = − a − b in 3ax + by + 2c = 0, we get:

3ax + by – 2a – 2b = 0

⇒ a (3x – 2) + b (y – 2) = 0

$$\Rightarrow (3x - 2) + \frac{b}{a}(y - 2) = 0$$

This line is of the form $L_1 + \lambda L_2 = 0$,

which passes through the intersection of the lines L_1 and L_2, i.e. 3x – 2 = 0 and y – 2 = 0 .

Solving 3 x – 2 = 0 and y – 2 = 0, we get:

$$x = \frac{2}{3}, y = 2$$

Hence, the required fixed point is $\left(\frac{2}{3}, 2\right)$

13. Question

The line segment joining the points (-3, -4) and (1, -2) is divided by y-axis in the ratio

A. 1 : 3

B. 2 : 3

C. 3 : 1

D. 3 : 2

Answer

Let the points (− 3, − 4) and (1, − 2) be divided by y-axis at (0, t) in the ratio m:n.

$$\therefore \left(\frac{m - 3n}{m + n}, \frac{-2m - 4n}{m + n}\right) = (0, t)$$

$$\Rightarrow 0 = \frac{m - 3n}{m + n}$$

$$\Rightarrow m:n = 3:1$$

14. Question

The area of a triangle with vertices at (-4, -1), (1, 2) and (4, -3) is

A. 17

B. 16

C. 15

D. none of these

Answer

Let A be the area of the triangle formed by the points (− 4, − 1), (1, 2) and (4, − 3).

$$\therefore A = \frac{1}{2}|\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}|$$

$\Rightarrow A = \frac{1}{2}|\{-4(2 + 3) + 1(-3 + 1) + 4(-1 - 2)\}|$

⇒ A = 17

15. Question

The line segment joining the points (1, 2) and (-2, 1) is divided by the line 3x + 4y = 7 in the ratio

A. 3 : 4

B. 4 : 3

C. 9 : 4

D. 4 : 9

Answer

Let the line segment joining the points (1, 2) and (− 2, 1) be divided by the line 3x + 4y = 7 in the ratio m:n.

Then, the coordinates of this point will be $\left(\frac{-2m+n}{m+n}, \frac{m+2n}{m+n}\right)$ that lie on the line 3x + 4y = 7

$$3 \times \frac{-2m + n}{m + n} + 4 \times \frac{m + 2n}{m. + n} = 7$$

⇒ - 2 + 11 n = 7 m + 7 n

⇒ - 9 m = - 4 n

⇒ m: n = 4 : 9

16. Question

If the point (5, 2) bisects the intercept of a line between the axes, then its equation is

A. 5x + 2y = 20

B. 2x + 5y = 20

C. 5x – 2y = 20

D. 2x – 5y = 20

Answer

Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$

The coordinates of the intersection of this line with the coordinate axes are (a, 0) and (0, b).

The midpoint of (a, 0) and (0, b) is, $\left(\frac{a}{2}, \frac{b}{2}\right)$

According to the question:

$$\left(\frac{a}{2}, \frac{b}{2}\right) = (5,2)$$

$$\Rightarrow \frac{a}{2} = 5, \frac{b}{2} = 2$$

⇒ a = 10, b = 4

The equation of the required line is given below:

$$\frac{x}{10} + \frac{y}{4} = 1$$

⇒ 2x + 5y = 20

17. Question

A(6, 3), B(-3, 5), C(4, -2) and (x, 3x) are four points. If $\Delta DBC : \Delta ABC = 1 : 2$, then x is equal to

A. 11/8

B. 8/11

C. 3

D. none of these

Answer

The area of a triangle with vertices D (x, 3x), B (– 3, 5) and C (4, – 2) is given below:

Area of $\Delta DBC = \frac{1}{2}\{x(5 + 2) - 3(-2 - 3x) + 4(3x - 5)\}$

⇒ Area of ΔDBC = 14x – 7 sq units

Similarly, the area of a triangle with vertices A (6, 3), B (– 3, 5) and C (4, – 2) is given below:

$$\Delta ABC = \frac{1}{2}\{6(5 + 2) - 3(-2 - 3) + 4(3 - 5)\}$$

$$\Rightarrow \Delta ABC = \frac{49}{2}\text{sq units}$$

Given:

ΔDBC: ΔABC = 1: 2

$$\frac{2(14x - 7)}{49} = \frac{1}{2}$$

⇒ 8x – 4 = 7

$$\Rightarrow x = \frac{11}{8}$$

18. Question

If p be the length of the perpendicular from the origin on the line x/a + y/b = 1, then

A. $p^2 = a^2 + b^2$

B. $p^2 = \frac{1}{a^2} + \frac{1}{b^2}$

C. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

D. none of these

Answer

It is given that p is the length of the perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{1}{a}x + \frac{1}{b}y - 1 = 0$$

$$\therefore p = \left|\frac{0 + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right|$$

Squaring both sides

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

19. Question

If equation of the line passing through (1, 5) and perpendicular to the line 3x – 5y + 7 = 0 is

A. 5x + 3y – 20 = 0

B. 3x – 5y + 7 = 0

C. 3x – 5y + 6 = 0

D. none of these

Answer

A line perpendicular to 3x − 5y + 7 = 0 is given by

5x + 3y + λ = 0

This line passes through (1, 5).

5 + 15 + λ = 0

⇒ λ = – 20

Therefore, the equation of the required line is 5x + 3y – 20 = 0

20. Question

The figure formed by the lines ax ± by ± c = 0 is

A. a rectangle

B. a square

C. a rhombus

D. none of these

Answer

The given lines can be written separately in the following manner:

ax + by + c = 0 ... (1)

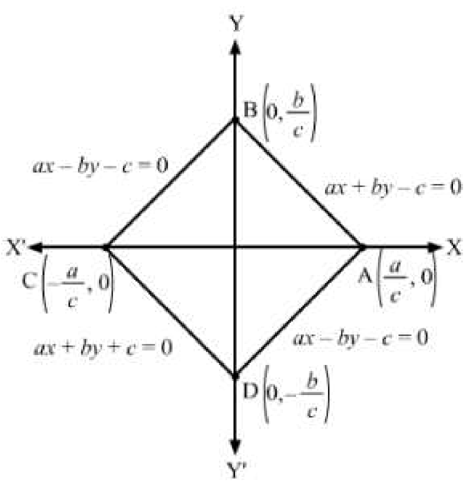
ax + by − c = 0 ... (2)

ax − by − c = 0 ... (3)

ax − by − c = 0 ... (4)

Graph of the given lines is given below:

Diagram:



Clearly, AB = BC = CD = DA $= \sqrt{\frac{a^2}{c^2} + \frac{b^2}{c^2}} = \frac{\sqrt{a^2 + b^2}}{|c|}$

Thus, the region formed by the given lines is ABCD, which is a rhombus

21. Question

Two vertices of a triangle are (-2, -1) and (3, 2) and third vertex lies on the line x + y = 5. If the area of the triangle is 4 square units, then the third vertex is

A. (0, 5) or, (4, 1)

B. (5, 0) or, (1, 4)

C. (5, 0) or, (4, 1)

D. (0, 5) or, (1, 4)

Answer

Let (h, k) be the third vertex of the triangle.

It is given that the area of the triangle with vertices (h, k), (− 2, − 1) and (3, 2) is 4 square units.

$$\frac{1}{2}|h(-1 - 2) - 3(-1 - k) - 2(2 - k)| = 4$$

⇒ 3h – 5k + 1 = ± 8

Taking positive sign, we get,

3h – 5k + 1 = 8

3h – 5k – 7 = 0 … (1)

Taking negative sign, we get,

3h – 5k + 9 = 0 … (2)

The vertex (h, k) lies on the line x + y = 5.

H + k – 5 = 0 … (3)

On solving (1) and (3), we find (4, 1) to be the coordinates of the third vertex.

Similarly, on solving (2) and (3), we find (2, 3) to be the coordinates of the third vertex.

22. Question

The inclination of the straight line passing through the point (-3, 6) and the mid-point of the line joining the point (4, -5) and (-2, 9) is

A. $\pi / 4$

B. $\pi / 6$

C. $\pi / 3$

D. $3\pi / 4$

Answer

The midpoint of the line joining the points (4, − 5) and (− 2, 9) is (1, 2).

Let θ be the inclination of the straight line passing through the points (− 3, 6) and (1, 2).

Then $\tan\theta = \frac{2-6}{1+3} = -1$

$$\Rightarrow \theta = \frac{3\pi}{4}$$

23. Question

Distance between the lines 5x + 3y – 7 = 0 and 15x + 9y + 14 = 0 is

A. $\frac{35}{\sqrt{34}}$

B. $\frac{1}{3\sqrt{34}}$

C. $\frac{35}{3\sqrt{34}}$

D. $\frac{35}{2\sqrt{34}}$

Answer

The given lines can be written as

5x + 3y − 7 = 0 ... (1)

$5x + 3y + \frac{14}{3} = 0$......(2)

Let d be the distance between the lines 5x + 3y − 7 = 0 and 15x + 9y + 14 = 0

Then, $d = \left|\frac{\left(-7-\frac{14}{3}\right)}{\sqrt{5^2+3^2}}\right|$

$\Rightarrow d = \frac{35}{3\sqrt{34}}$

24. Question

The angle between the lines 2x – y + 3 = 0 and x + 2y + 3 = 0 is

A. 90°

B. 60°

C. 45°

D. 30°

Answer

Let m1 and m2 be the slope of the lines 2x − y + 3 = 0 and x + 2y + 3 = 0, respectively.

Let θ be the angle between them.

Here, m1 = 2 and m2 $= -\frac{1}{2}$

∵ m1m2 = − 1

Therefore, the angle between the given lines is 90°.

25. Question

The value of λ for which the lines 3x + 4y = 5, 5x + 4y = 4 and λx + 4y = 6 meet at a point is

A. 2

B. 1

C. 4

D. 3

Answer

It is given that the lines 3x + 4y = 5, 5x + 4y = 4 and λx + 4y = 6 meet at a point.

In other words, the given lines are concurrent.

$$\begin{vmatrix} 3 & 4 & -5 \\ 5 & 4 & -4 \\ \lambda & 4 & -6 \end{vmatrix} = 0$$

⇒ 3(- 24 + 16) – 4 (- 30 + 4λ) – 5 (20 – 4 λ) = 0

⇒ - 24 + 120 – 16 λ – 100 + 20 λ = 0

⇒ 4 λ = 4

⇒ λ = 1

26. Question

Three vertices of a parallelogram taken in order are (-1, -6), (2, -5) and (7, 2). The fourth vertex is

A. (1, 4)

B. (4, 1)

C. (1, 1)

D. (4, 4)

Answer

Let A (− 1, − 6), B(2, − 5) and C(7, 2) be the given vertex.

Let D(h, k) be the fourth vertex.

The midpoints of AC and BD are $(3, -2)$and $\left(\frac{2+h}{2}, \frac{-5+k}{2}\right)$ respectively.

We know that the diagonals of a parallelogram bisect each other .

$$\therefore 3 = \frac{2 + h}{2} \text{ and } -2 = \frac{-5 + k}{2}$$

⇒ h = 4 and k = 1

27. Question

The centroid of a triangle is (2, 7) and two of its vertices are (4, 80 and (-2, 6). The third vertex is

A. (0, 0)

B. (4, 7)

C. (7, 4)

D. (7, 7)

Answer

Let A(4, 8) and B(− 2, 6) be the given vertex. Let C (h, k) be the third vertex.

The centroid of △ABC is $\left(\frac{4-2+h}{3}, \frac{8+6+k}{3}\right)$

It is given that the centroid of triangle ABC is (2, 7).

$$\therefore \frac{4 - 2 + h}{3} = 2, \frac{8 + 6 + k}{3} = 7$$

⇒ h = 4, k = 7

Thus, the third vertex is (4, 7).

28. Question

If the lines x + q = 0, y – 2 = 0 and 3x + 2y + 5 + 0 are concurrent, then the value of q will be

A. 1

B. 2

C. 3

D. 5

Answer

The lines x + q = 0, y − 2 = 0 and 3x + 2y + 5 = 0 are concurrent.

$\therefore \begin{vmatrix} 1 & 0 & q \\ 0 & 1 & -2 \\ 3 & 2 & 5 \end{vmatrix} = 0$

$\Rightarrow 1(5 + 4) - 0 + q(0 - 3) = 0$

⇒ 3q = 9

⇒ q = 3

29. Question

The medians AD and BE of a triangle with vertices A(0, b), B(0, 0) and C(a, 0) are perpendicular to each other, if

A. $a = \frac{b}{2}$

B. $b = \frac{a}{2}$

C. ab = 1

D. $a = \pm\sqrt{2}b$

Answer

The midpoints of BC and AC are $D\left(\frac{a}{2}, 0\right)$ and $E\left(\frac{a}{2}, \frac{b}{2}\right)$

Slope of AD = $(0 - b)/(\frac{a}{2} - 0)$

Slope of BE = $\frac{-\frac{b}{2}}{-\frac{a}{2}}$

It is given that the medians are perpendicular to each other.

$\frac{0 - b}{\frac{a}{2} - 0} \times \frac{-\frac{b}{2}}{-\frac{a}{2}} = 1$

$\Rightarrow a = \pm\sqrt{2}b$

30. Question

The equation of the line with slope -3/2 and which is concurrent with the lines 4x + 3y – 7 = 0 and 8x + 5y – 1 = 0 is

A. 3x + 2y – 63 = 0

B. 3x + 2y – 2 = 0

C. 2y – 3x – 2 = 0

D. none of these

Answer

Given:

4x + 3y − 7 = 0 … (1)

8x + 5y − 1 = 0 … (2)

The equation of the line with slope – 3/2 is given below:

y = - 32x + c

⇒ 32x + y - c = 0 … (3)

The lines (1), (2) and (3) are concurrent.

$$\therefore \begin{vmatrix} 4 & 3 & -7 \\ 8 & 5 & -1 \\ \frac{3}{2} & 1 & -c \end{vmatrix} = 0$$

$$\Rightarrow 4(-5c + 1) - 3\left(-8c + \frac{3}{2}\right) - 7\left(8 - \frac{15}{2}\right) = 0$$

$$\Rightarrow -20c + 4 + 24c - \frac{9}{2} - 56 + \frac{105}{2} = 0$$

$$\Rightarrow \frac{-40c + 8 + 48c - 9 - 112 + 105}{2} = 0$$

⇒ 8c = 8

⇒ c = 1

On substituting c = 1 in $= -\frac{3}{2}x + c$, we get:

$$y = -\frac{3}{2}x + 1$$

⇒ 3x + 2y – 2 = 0

31. Question

The vertices of a triangle are (6, 0), (0, 6) and (6, 6). The distance between its circumcentre and centroid is

A. $2\sqrt{2}$

B. 2

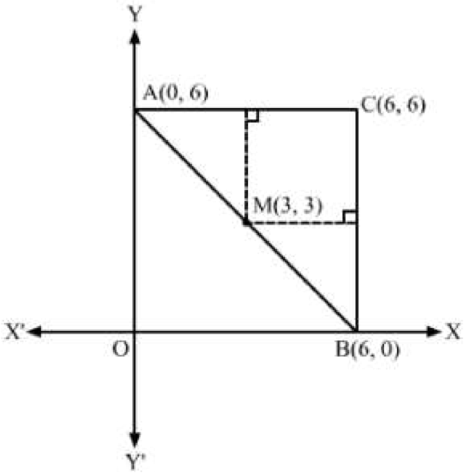
C. $\sqrt{2}$

D. 1

Answer

Let A(0, 6), B(6, 0) and C(6, 6) be the vertices of the given triangle.

Diagram:



Coordinates of N = $\left(\frac{6+6}{2}, \frac{6+0}{2}\right)$

= (6, 3)

Coordinates of P = $\left(\frac{0+6}{2}, \frac{6+6}{2}\right)$ = (3, 6)

Equation of MN is y = 3

Equation of MP is x = 3

As we know that circumcentre of a triangle is the intersection of the perpendicular bisectors of any two sides .Therefore, coordinates of circumcentre is (3,3)

Thus, the coordinates of the circumcentre are (3, 3) and the centroid of the triangle is (4,4).

Let d be the distance between the circumcentre and the centroid.

$\therefore\ d = \sqrt{(4-3)^2 + (4-3)^2} = \sqrt{2}$

32. Question

A point equidistant from the line 4x + 3y + 10 = 0, 5x – 12y + 26 = 0 and 7x + 24y – 50 = 0 is

A. (1, -1)

B. (1, 1)

C. (0, 0)

D. (0, 1)

Answer

Given equations are AB 4x + 3y + 10 = 0

Normalizing AB, we get

= > 4x + 3y + 10 = 0

Dividing by 5, we get

$\frac{4x}{5} + \frac{3y}{5} + 2 = 0$ ......(1)

Consider BC 5x - 12y + 26 = 0

Normalizing BC we get,

= > $\frac{5x}{13} - \frac{12y}{13} + 2 = 0$ ......(2)

Consider AC 7x + 24y - 50 = 0

Normalizing AC we get

$= > \frac{7x}{25} + \frac{24y}{25} - 2 = 0$ ......(3)

Adding (1) + (3), we get Angular bisector of A: $\frac{27x}{25} + \frac{39y}{25} = 0$ ......(4)

Adding (2) + (3), we get Angular bisector of C: $\frac{216x}{325} + \frac{12y}{325} = 0$ ......(5)

Finding point of intersection of lines (4) and (5), we get I(0, 0) which is the

Incenter of the given triangle which is the point equidistant from its sides of a triangle.

33. Question

The ratio in which the line 3x + 4y + 2 = 0 divides the distance between the lines 3x + 4y + 5 = 0 and 3x + 4y – 5 = 0 is

A. 1 : 2

B. 3 : 7

C. 2 : 3

D. 2 : 5

Answer

The distance between two parallel line 3x + 4y + 5 = 0 and 3x + 4y + 2 = 0 is

$$\frac{|5 - 2|}{\sqrt{3^2 + 4^2}} = \frac{3}{\sqrt{25}} = \frac{3}{5}$$

The distance between two parallel line 3x + 4y + 2 = 0 and 3x + 4y - 5 = 0 is

$$\frac{|2 - (-5)|}{\sqrt{3^2 + 4^2}} = \frac{7}{\sqrt{25}} = \frac{7}{5}$$

Thus required ratio is $\frac{\frac{3}{5}}{\frac{7}{5}} = \frac{3}{7}$

34. Question

The coordinates of the foot of the perpendicular from the point (2, 3) on the line x + y – 11 = 0 are

A. (-6, 5)

B. (3, 4)

C. (0, 0)

D. (6, 5)

Answer

Let the coordinate of the foot of perpendicular from the point (2, 3) on the line x + y – 11 = 0 be (x, y)

Now, the slope of the perpendicular = -1

The equation of perpendicular is given by

y – 3 = 1(x – 2)

= > x – y + 1 = 0

Solving x + y – 11 = 0 and x – y + 1 = 0, we get

ð x = 5 and y = 6

35. Question

The reflection of the point (4, -13) about the line 5x + y + 6 = 0 is

A. (-1, -14)

B. (3, 4)

C. (0, 0)

D. (1, 2)

Answer

Given point (4, -13)

Line is 5x + y + 6 = 0 ......(i)

Let (h, k) be the image of A w.r.t (i)

We know that P(h, k) be the image of A(x_1, y_1) w.r.t ax + by + c = 0

Then,

$$\frac{h-4}{5} = \frac{k+13}{1} = -\frac{2(20-13+6)}{25+1}$$

$$\frac{h-4}{5} = \frac{k+13}{1} = -\frac{26}{26} = -1$$

$h - 4 = -5$, $k + 13 = -1$

h = -1, k = -14

Image of(4, -13) is (-1, -14).