23 Measures of Central Tendency

23.1 MEASURES OF CENTRAL TENDENCY

Central tendency. Often, we observe that every statistical data has a tendency to cluster around a certain value, possibly at the centre of observations. This tendency of the statistical data is known as its *central tendency*.

Measure of central tendency. A numerical value which represents (approximately) the entire statistical data is called a *measure of central tendency* of that data.

The numerical value which represents the entire statistical data is neither the lowest nor the highest value in the data, rather it lies in between the two extreme values of the data.

The different ways of measuring central tendency of a statistical data are :

(*i*) *Mean* (*ii*) *Median* (*iii*) *Mode*.

23.2 MEAN (ARITHMETIC MEAN)

23.2.1 Mean of ungrouped data

The *mean (arithmetic mean)* of *n* variates (observations) $x_1, x_2, x_3, \ldots, x_n$ is given by the formula :

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\Sigma x_i}{n}$$

where $\Sigma x_i = x_1 + x_2 + x_3 + \ldots + x_n$.

Note

The Greek letter Σ (read as sigma) represents the sum.

ILLUSTRATIVE EXAMPLES

Example 1. *The marks obtained by 12 students in a monthly test are 11, 19, 07, 13, 18, 21, 09, 05, 20, 17, 16, 21. Find :*

(i) the mean of their marks.

(ii) the mean of their marks when the marks of each student are increased by 3.

Solution.

(*i*) The sum of the marks of all students

$= 11 + 19 + 07 + 13 + 18 + 21 + 09 + 05 + 20 + 17 + 16 + 21 = 177.$

$\therefore$ The mean of marks $= \frac{\Sigma x_i}{n} = \frac{\text{sum of marks of all students}}{\text{number of students}}$

$= \frac{177}{12} = \frac{59}{4} = 14{\cdot}75.$

(*ii*) When the marks of each student are increased by 3, then the sum of their marks increases by 12×3 *i.e.* by 36.

$\therefore$ The new sum of marks of all students $= 177 + 36 = 213.$

$\therefore$ The new mean of marks $= \frac{\text{new sum of marks}}{\text{number of students}}$

$= \frac{213}{12} = \frac{71}{4} = 17{\cdot}75.$

Note that the new mean of marks also increases by 3.

Example 2. *The mean of 7 variates is 12. If six of them are 5, 13, 9, 17, 14 and 10, find the seventh variate.*

Solution. Let the seventh variate be x. By def.,

$$\text{mean} = \frac{\text{sum of variates}}{\text{number of variates}}$$

$$\Rightarrow \quad 12 = \frac{5 + 13 + 9 + 17 + 14 + 10 + x}{7}$$

$$\Rightarrow \quad 12 = \frac{68 + x}{7} \Rightarrow 68 + x = 12 \times 7$$

$$\Rightarrow \quad x = 84 - 68 = 16.$$

Hence, the 7th variate is 16.

Example 3. *The mean height of 8 students is 152 cm. Two more students of heights 143 cm and 156 cm join the group. What is the new mean height?*

Solution. Mean height of 8 students $= \frac{\text{sum of heights of 8 students}}{8}$,

$\therefore \quad 152 \text{ cm} = \frac{\text{sum of heights of 8 students}}{8}$

$\Rightarrow$ sum of heights of 8 students $= (152 \times 8)$ cm $= 1216$ cm.

Now two more students of heights 143 cm and 156 cm join the group.

$\therefore$ Sum of heights of 10 students $= (1216 + 143 + 156)$ cm $= 1515$ cm.

$\therefore$ New mean height $= \frac{\text{sum of heights of 10 students}}{10}$

$= \frac{1515}{10}$ cm $= 151{\cdot}5$ cm.

Example 4. *In an examination, the mean of marks scored by a class of 30 students was calculated as 58·5. Later on, it was detected that the marks of one student was wrongly copied as 57 instead of 75. Find the correct mean.*

Solution. Mean of marks = $\frac{\text{Incorrect sum of marks of 30 students}}{30}$,

$\therefore \quad 58 \cdot 5 = \frac{\text{Incorrect sum of marks}}{30}$

$\Rightarrow$ Incorrect sum of marks = $58 \cdot 5 \times 30 = 1755$.

As marks of one student was wrongly copied as 57 instead of 75, correct sum of marks $= 1755 - 57 + 75 = 1773$.

$\therefore \quad$ Correct mean $= \frac{1773}{30} = 59 \cdot 1$.

23.2.2 Mean of (discrete) grouped data

If the variates $x_1, x_2, x_3, \ldots, x_n$ have frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then the mean is given by the formula :

$$Mean = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

This method of finding the mean is called the **direct method.**
In some problems, where the number of variates (observations) is large or the values of x_i or f_i are larger, then the calculations become tedious. In order to overcome this difficulty, we use another method usually called **short cut** (or **deviation**) **method.** In this method, an approximate mean (called *assumed mean* or *provisional mean*) is taken (preferably near the middle), say A, and we calculate the deviation $d_i = x_i - A$ for each variate x_i. Then the mean is given by the formula :

$$Mean = A + \frac{\Sigma f_id_i}{\Sigma f_i}$$

ILLUSTRATIVE EXAMPLES

Example 1. *The following table shows the gain in weight by 25 children in a year :*

Gain in weight (in kg)	1·5	2	2·4	3	3·2	3·4
No. of children	4	5	8	5	2	1

Find the mean of gain in weight.

Solution. To calculate the mean of gain in weight, we construct the following table :

x_i	f_i	f_ix_i
1·5	4	6
2	5	10
2·4	8	19·2
3	5	15
3·2	2	6·4
3·4	1	3·4
Total	25	60

$\therefore \quad$ Mean $= \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{60}{25} = \frac{12}{5} = 2 \cdot 4$.

$\therefore \quad$ The mean of gain in weight = 2·4 kg.

Example 2. *The distribution of heights of 50 children (measured to the nearest cm) was as under :*

Height	*110*	*115*	*118*	*120*	*121*	*125*
No. of students	6	8	14	15	4	3

Calculate the mean height for this distribution correct to one place of decimal.

Solution. We shall use *short cut method.* Construct the table as under by taking assumed mean A = 118.

x_i	$d_i = x_i - A$	f_i	$f_i d_i$
110	− 8	6	− 48
115	− 3	8	− 24
Assumed mean → 118	0	14	0
120	2	15	30
121	3	4	12
125	7	3	21
Total		50	−9

$\therefore$ Mean $= A + \dfrac{\Sigma f_i d_i}{\Sigma f_i} = 118 + \dfrac{-9}{50}$

$= 118 - \cdot 18 = 117 \cdot 82.$

$\therefore$ Mean height $= 117 \cdot 8$ cm.

Example 3. *Find the value of p for the following distribution whose mean is 10 :*

Variate (x_i)	5	7	9	11	13	15	20
Frequency (f_i)	4	4	p	7	3	2	1

Solution. We construct the following table :

x_i	f_i	$f_i x_i$
5	4	20
7	4	28
9	p	$9p$
11	7	77
13	3	39
15	2	30
20	1	20
Total	$21 + p$	$214 + 9p$

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{214 + 9p}{21 + p}.$

But mean = 10 (given) $\Rightarrow \dfrac{214 + 9p}{21 + p} = 10$

$\Rightarrow$ $210 + 10p = 214 + 9p \Rightarrow p = 4.$

23.2.3 Mean of grouped data when the frequency distribution is given in the form of classes (continuous or discontinuous)

In this case, we assume that the frequency in each class is centred at its class mark. If there are n classes and f_i, y_i denote the frequency and the class mark respectively of the ith class, then the mean is given by the formula :

$$Mean = \frac{f_1y_1 + f_2y_2 + f_3y_3 ... + f_ny_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\Sigma f_iy_i}{\Sigma f_i}$$

This method of finding the mean is called the **direct method.**

In some problems, where the number of classes is large or the values of f_i or y_i are large, then the calculations become tedious. In order to overcome this difficulty, we shall use **short cut** (or **deviation**) **method.** In this method, an approximate mean (called *assumed mean*) is taken (preferably near the middle), say A, and we calculate the deviation $d_i = y_i - A$ for each class mark y_i. Then the mean is given by the formula :

$$Mean = A + \frac{\Sigma f_id_i}{\Sigma f_i}$$

In most of the problems, width of all classes is the same, so we can further simplify the calculations of the mean by computing the **coded mean** *i.e.* the mean of $u_1, u_2, u_3, ..., u_n$ where

$u_i = \frac{y_i - A}{c}$ and c is the width of each class.

Then the mean is given by the formula :

$$Mean = A + c \times \frac{\Sigma f_iu_i}{\Sigma f_i}$$

This method of finding the mean is called **step-deviation method.**

Remark

If the class sizes are unequal, and y_i are large, we can still apply the step-deviation method by taking c to be a suitable divisor of all the d_i's. In fact, the formula

$$\bar{x} = A + c \times \frac{\Sigma f_iu_i}{\Sigma f_i}$$

holds if A and c are not as above, but are any non-zero numbers such that $u_i = \frac{y_i - A}{c}$.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the mean for the following frequency distribution by*
(i) direct method *(ii) short cut method* *(iii) step-deviation method.*

Class-intervals	84 – 90	90 – 96	96 – 102	102 – 108	108 – 114
Frequency	8	12	15	10	5

Solution. (*i*) ***Direct method***

Construct the table as under :

Classes	Class mark y_i	Frequency f_i	f_iy_i
84 – 90	87	8	696
90 – 96	93	12	1116
96 – 102	99	15	1485
102 – 108	105	10	1050
108 – 114	111	5	555
Total		50	4902

$\therefore$ Mean $= \frac{\Sigma f_iy_i}{\Sigma f_i} = \frac{4902}{50} = 98{\cdot}04.$

(ii) ***Short cut method***

Construct the table as under, taking assumed mean A = 99.

Classes	Class mark y_i	deviation $d_i = y_i - A$	frequency f_i	$f_i d_i$
84 – 90	87	−12	8	−96
90 – 96	93	−6	12	−72
96 – 102	99	0	15	0
102 – 108	105	6	10	60
108 – 114	111	12	5	60
Total			50	−48

$$\therefore \quad \text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 99 + \frac{-48}{50}$$
$$= 99 - \cdot96 = 98\cdot04.$$

(iii) ***Step-deviation method***

Construct the table as under, taking assumed mean A = 99.

Here, c (width of each class) = 6.

Classes	Class mark y_i	$u_i = \frac{y_i - A}{c}$	frequency f_i	$f_i u_i$
84 – 90	87	−2	8	−16
90 – 96	93	−1	12	−12
96 – 102	99	0	15	0
102 – 108	105	1	10	10
108 – 114	111	2	5	10
Total			50	−8

$$\therefore \quad \text{Mean} = A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 99 + 6 \times \frac{-8}{50}$$
$$= 99 - \cdot96 = 98\cdot04.$$

Example 2. *Find the mean of the following distribution by step deviation method :*

Class-intervals	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	10	6	8	12	5	9

(2013, 06)

Solution. Construct the table as under, taking assumed mean A = 45. Here, c (width of each class) = 10.

Classes	Class mark y_i	$u_i = \frac{y_i - A}{c}$	frequency f_i	$f_i u_i$
20 – 30	25	−2	10	−20
30 – 40	35	−1	6	−6
40 – 50	45	0	8	0
50 – 60	55	1	12	12
60 – 70	65	2	5	10
70 – 80	75	3	9	27
Total			50	23

$$\therefore \quad \text{Mean} = A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i} \quad \text{(by step deviation method)}$$
$$= 45 + 10 \times \frac{23}{50} = 45 + \frac{23}{5} = 45 + 4\cdot6 = 49\cdot6.$$

Example 3. *The weights of 50 apples were recorded as given below :*

Weight in grams	*80 – 85*	*85 – 90*	*90 – 95*	*95 – 100*	*100 – 105*	*105 – 110*	*110 – 115*
Number of apples	*5*	*8*	*10*	*12*	*8*	*4*	*3*

Calculate the mean weight, to the nearest gram, by the ***Step Deviation Method.*** (2008)

Solution. Construct the table as under, taking assumed mean A = 97·5.
Here, c(width of each class) = 5.

Classes	*Class mark* y_i	$u_i = \frac{y_i - A}{c}$	*frequency* f_i	$f_i\ u_i$
80 – 85	82·5	–3	5	–15
85 – 90	87·5	–2	8	–16
90 – 95	92·5	–1	10	–10
95 – 100	97·5	0	12	0
100 – 105	102·5	1	8	8
105 – 110	107·5	2	4	8
110 – 115	112·5	3	3	9
Total			50	–16

$\therefore$ Mean = $A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 97{\cdot}5 + 5 \times \frac{-16}{50} = 97{\cdot}5 - 1{\cdot}6 = 95{\cdot}9$

$\therefore$ Mean = 96 gram (to the nearest gram).

Example 4. *Following table gives marks scored by students in an examination :*

Marks	*0 – 5*	*5 – 10*	*10 – 15*	*15 – 20*	*20 – 25*	*25 – 30*	*30 – 35*	*35 – 40*
No. of students	*3*	*7*	*15*	*24*	*16*	*8*	*5*	*2*

Calculate the mean mark correct to 2 decimal places.

Solution. We shall use step-deviation method. Construct the table as under, taking assumed mean A = 17·5. Here, c (width of each class) = 5.

Classes	*Class mark* y_i	$u_i = \frac{y_i - A}{c}$	*frequency* f_i	$f_i\ u_i$
0 – 5	2·5	– 3	3	– 9
5 – 10	7·5	– 2	7	–14
10 – 15	12·5	–1	15	–15
15 – 20	17·5	0	24	0
20 – 25	22·5	1	16	16
25 – 30	27·5	2	8	16
30 – 35	32·5	3	5	15
35 – 40	37·5	4	2	8
Total			80	17

$\therefore$ Mean = $A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 17{\cdot}5 + 5 \times \frac{17}{80}$

$= 17{\cdot}5 + \frac{17}{16} = 17{\cdot}5 + 1{\cdot}06 = 18{\cdot}56.$

Example 5. *A study of the yield of 150 tomato plants resulted in the following record :*

Tomatoes per plant	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
Number of plants	20	50	46	22	12

Calculate the mean of the number of tomatoes per plant.

Solution. We shall use step-deviation method.

Construct the table as under, taking assumed mean A = 13.

Here, c (width of each class) = 5.

Classes	Class mark y_i	$u_i = \frac{y_i - A}{c}$	frequency f_i	$f_i u_i$
1 – 5	3	–2	20	–40
6 – 10	8	–1	50	–50
11 – 15	13	0	46	0
16 – 20	18	1	22	22
21 – 25	23	2	12	24
Total			150	–44

$$\therefore \quad \text{Mean} = A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 13 + 5 \times \frac{-44}{150} = 13 - \frac{44}{30}$$

$$= 13 - 1{\cdot}466 = 11{\cdot}534.$$

$\therefore$ The mean of the number of tomatoes per plant = 11·53.

Note that the frequency distribution is discontinuous.

Example 6. *Find the mean for the following data :*

Marks obtained	*No. of students*
less than 20	*16*
less than 30	*21*
less than 40	*35*
less than 50	*52*
less than 60	*58*
less than 70	*78*
less than 80	*94*
less than 100	*100*

Solution. We shall use short cut method.

Construct the table as under, taking assumed mean A = 45.

Classes	Class mark y_i	deviation $d_i = y_i - A$	Cumulative frequency	Frequency f_i	$f_i d_i$
0 – 20	10	–35	16	16	–560
20 – 30	25	–20	21	5	–100
30 – 40	35	–10	35	14	–140
40 – 50	45	0	52	17	0
50 – 60	55	10	58	6	60
60 – 70	65	20	78	20	400
70 – 80	75	30	94	16	480
80 – 100	90	45	100	6	270
Total				100	410

$$\therefore \quad \text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 45 + \frac{410}{100} = 45 + 4{\cdot}1 = 49{\cdot}1.$$

Note that the sizes of classes are not equal.

Example 7. *The mean of the following distribution is 52 and the frequency of class interval 30 – 40 is f. Find f.*

Class interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	5	3	f	7	2	6	13

(2010)

Solution. We shall use step-deviation method. Construct the table as under, taking assumed mean A = 45. Here, c (width of each class) = 10.

Classes	Class mark y_i	$u_i = \frac{y_i - A}{c}$	frequency f_i	$f_i u_i$
10 – 20	15	– 3	5	– 15
20 – 30	25	– 2	3	– 6
30 – 40	35	– 1	f	$-f$
40 – 50	45	0	7	0
50 – 60	55	1	2	2
60 – 70	65	2	6	12
70 – 80	75	3	13	39
Total			$36 + f$	$32 - f$

$\therefore$ Mean $= A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i}$

$\Rightarrow$ $52 = 45 + 10 \times \frac{32 - f}{36 + f}$

$\Rightarrow$ $7 = 10 \times \frac{32 - f}{36 + f}$

$\Rightarrow$ $252 + 7f = 320 - 10f$

$\Rightarrow$ $17f = 68 \Rightarrow f = 4.$

Hence, $f = 4$.

Example 8. *The mean of the following frequency distribution is 62·8 and the sum of all the frequencies is 50. Find the values of p and q :*

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	p	10	q	7	8

Solution. We shall use step-deviation method. Construct the table as under, taking assumed mean A = 70. Here, c (width of each class) = 20.

Classes	Class mark y_i	$u_i = \frac{y_i - A}{c}$	frequency f_i	$f_i u_i$
0 – 20	10	–3	5	–15
20 – 40	30	–2	p	$-2p$
40 – 60	50	–1	10	–10
60 – 80	70	0	q	0
80 – 100	90	1	7	7
100 – 120	110	2	8	16
Total			$30 + p + q$	$-2p - 2$

Since sum of frequencies is 50,

$30 + p + q = 50$

$\Rightarrow p + q = 20$ ...(*i*)

Mean $= A + c \times \frac{\Sigma f_i u_i}{\Sigma f_i}$

$\Rightarrow 62 \cdot 8 = 70 + 20 \times \frac{-2p-2}{50}$

$\Rightarrow \frac{2}{5}(2p + 2) = 70 - 62 \cdot 8$

$\Rightarrow 4(p + 1) = 5 \times 7 \cdot 2$

$\Rightarrow 4(p + 1) = 36 \Rightarrow p + 1 = 9$

$\Rightarrow p = 8.$

$\therefore$ From (*i*), $8 + q = 20 \Rightarrow q = 12.$

Hence, $p = 8, q = 12.$

Exercise 23.1

1. (*a*) Calculate the arithmetic mean of 5·7, 6·6, 7·2, 9·3, 6·2.
   (*b*) The weights (in kg) of 8 new born babies are 3, 3·2, 3·4, 3·5, 4, 3·6, 4·1, 3·2. Find the mean weight of the babies.
2. The marks obtained by 15 students in a class test are 12, 14, 07, 09, 23, 11, 08, 13, 11, 19, 16, 24, 17, 03, 20. Find :
   (*i*) the mean of their marks.
   (*ii*) the mean of their marks when the marks of each student are increased by 4.
   (*iii*) the mean of their marks when 2 marks are deducted from the marks of each student.
   (*iv*) the mean of their marks when the marks of each student are doubled.
3. (*a*) The mean of the numbers 6, y, 7, x, 14 is 8. Express y in terms of x.
   (*b*) The mean of 9 variates is 11. If eight of them are 7, 12, 9, 14, 21, 3, 8 and 15, find the 9th variate.
4. (*a*) The mean age of 33 students of a class is 13 years. If one girl leaves the class, the mean becomes $12\frac{15}{16}$ years. What is the age of the girl?
   (*b*) In a class test, the mean of marks scored by a class of 40 students was calculated as 18·2. Later on, it was detected that the marks of one student was wrongly copied as 21 instead of 29. Find the correct mean.
5. Find the mean of 25 given numbers when the mean of 10 of them is 13 and the mean of the remaining numbers is 18.
6. Find the mean of the following distribution :

Number	5	10	15	20	25	30	35
Frequency	1	2	5	6	3	2	1

7. The contents of 100 matchboxes were checked to determine the number of matches they contained.

No. of matches	35	36	37	38	39	40	41
No. of boxes	6	10	18	25	21	12	8

(*i*) Calculate, correct to one decimal place, the mean number of matches per box.

(*ii*) Determine how many extra matches would have to be added to the total contents of the 100 boxes to bring the mean upto exactly 39 matches.

8. Calculate the mean for the following distribution :

Pocket money (in ₹)	60	70	80	90	100	110	120
No. of students	2	6	13	22	24	10	3

9. Six coins were tossed 1000 times, and at each toss the number of heads were counted and the results were recorded as under :

No. of heads	6	5	4	3	2	1	0
No. of tosses	20	25	160	283	338	140	34

Calculate the mean for this distribution.

10. Find the mean for the following distribution :

Numbers	60	61	62	63	64	65	66
Cumulative frequency	8	18	33	40	49	55	60

11.

Category	A	B	C	D	E	F	G
Wages in ₹ per day	50	60	70	80	90	100	110
No. of workers	2	4	8	12	10	6	8

(*i*) Calculate the mean wage, correct to the nearest rupee.

(*ii*) If the number of workers in each category is doubled, what would be the new mean wage ?

12. If the mean of the following distribution is 7·5, find the missing frequency f :

Variate	5	6	7	8	9	10	11	12
Frequency	20	17	f	10	8	6	7	6

(2005)

13. Find the value of the missing variate for the following distribution whose mean is 10 :

Variate (x_i)	5	7	9	11	...	15	20
Frequency (f_i)	4	4	4	7	3	2	1

14. Marks obtained by 40 students in a short assessment is given below, where a and b are two missing data :

Marks	5	6	7	8	9
No. of students	6	a	16	13	b

If the mean of the distribution is 7·2, find a and b. (2012)

15. In an examination taken by 50 candidates, the marks obtained are given in the table below. Calculate the mean mark.

Mark	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of candidates	8	14	13	10	5

16. Find the mean of the following distribution :

Class-intervals	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	10	6	8	12	5

(2007)

17. The frequency distribution of marks obtained by students of a class is as under. Calculate the Arithmetic mean.

Marks	0 – 8	8 – 16	16 – 24	24 – 32	32 – 40	40 – 48
Students	5	3	10	16	4	2

18. Find the mean of the following frequency distribution :

Class-intervals	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250	250 – 300
Frequency	4	8	16	13	6	3

(2003)

19. The following table gives the wages of workers in a factory :

Wages in ₹	45–50	50–55	55–60	60–65	65–70	70–75	75–80
No. of Workers	5	8	30	25	14	12	6

Calculate their mean by short cut method. (2009)

20. Weights of 50 eggs were recorded as given below :

Weight in gms	80–84	85–89	90–94	95–99	100–104	105–109	110–114
No. of eggs	5	10	12	12	8	2	1

Calculate their mean weight to the nearest gram.

21. The following table gives the daily wages of 50 workers of a factory :

Wages (in ₹)	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
No. of workers	2	1	5	9	21	10	2

Calculate the mean daily wage of a worker of the factory. If the daily wages of all the workers are increased by ₹ 8, what will be the new mean daily wage of a worker ?

22. The mean of the following distribution is 23·4. Find the value of p :

Class-intervals	0 – 8	8 – 16	16 – 24	24 – 32	32 – 40	40 – 48
Frequency	5	3	10	p	4	2

23. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the value of f :

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
No. of children	3	6	9	13	f	5	4

24. The mean of the following distribution is 50 and the sum of all the frequencies is 120. Find the values of p and q.

Class-intervals	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	17	p	32	q	19

25. The mean of the following frequency distribution is 57·6 and the sum of all the frequencies is 50. Find the values of p and q :

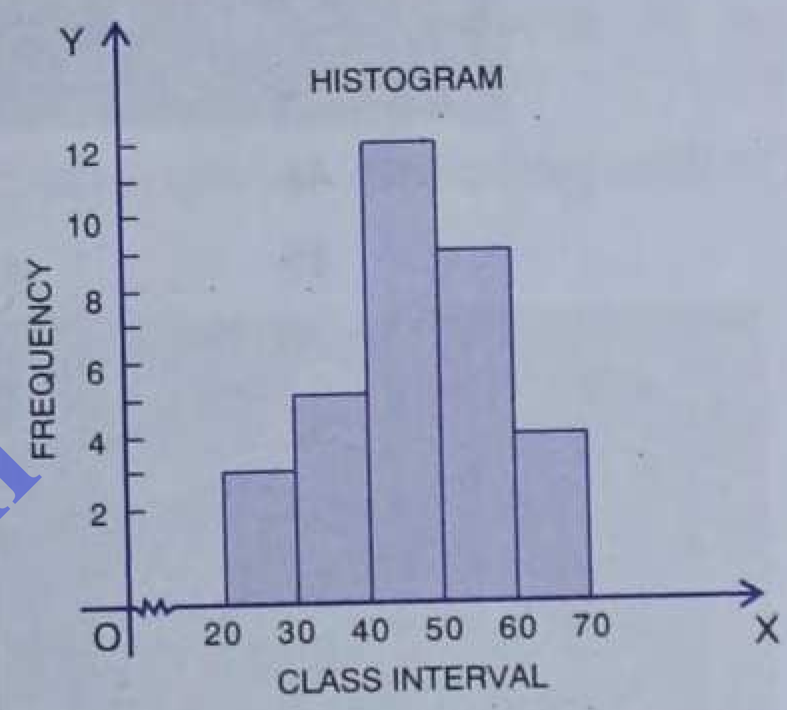
Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	7	p	12	q	8	5

26. The following table gives the life time in days of 100 electricity tubes of a certain make :

Life time in days	No. of tubes
less than 50	8
less than 100	23
less than 150	55
less than 200	81
less than 250	93
less than 300	100

Find the mean life time of electricity tubes.

27. Using the information given in the adjoining histogram, calculate the mean correct to one decimal place.



23.3 MEDIAN

Median *is the central value (or middle observation) of a statistical data if it is arranged in ascending or descending order.*

Thus, if there are n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ arranged in ascending or descending order, then

$$\text{median} = \begin{cases} \dfrac{n+1}{2}\text{ th observation, if } n \text{ is odd} \\ \dfrac{\dfrac{n}{2}\text{ th observation} + \left(\dfrac{n}{2}+1\right)\text{ th observation}}{2}, \text{ if } n \text{ is even.} \end{cases}$$

ILLUSTRATIVE EXAMPLES

Example 1. *Find the median of the following numbers :*

(i) 3, 5, 0, 4, 9, 7, 6, 2, 8 *(ii)* 3, 5, 1, 2, 4, 6, 0, 2, 2, 3

Solution.

(*i*) On arranging the given numbers in ascending order, we get

0, 2, 3, 4, 5, 6, 7, 8, 9.

Here, n (no. of observations) = 9, which is odd.

$\therefore$ Median = $\frac{n+1}{2}$ th observation = 5th observation = 5.

(*ii*) On arranging the given numbers in ascending order, we get

0, 1, 2, 2, 2, 3, 3, 4, 5, 6

Here, n (no. of observations) = 10, which is even.

$$\therefore \quad \text{Median} = \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\text{5th observation} + \text{6th observation}}{2} = \frac{2+3}{2} = \frac{5}{2} = 2{\cdot}5.$$

Example 2. *Calculate the median of the following distribution :*

Weight (in nearest kg)	46	48	50	52	53	54	55
No. of students	7	5	8	12	10	2	1

Solution. The given variates (weights of students) are already in ascending order. We construct the cumulative frequency table as under :

Variate (weight)	*Frequency (No. of students)*	*Cumulative frequency*
46	7	7
48	5	12
50	8	20
52	12	32
53	10	42
54	2	44
55	1	45

Here, n (total no. of students) = 45, which is odd.

$\therefore$ Median = $\frac{n+1}{2}$th observation = 23rd observation = 52.

($\because$ all observations from 21st to 32nd are equal, each = 52).

Example 3. *Calculate the median of the following distribution :*

No. of goals	0	1	2	3	4	5
No. of matches	2	4	7	6	8	3

Solution. The given variates (no. of goals) are already in ascending order. We construct the cumulative frequency table as under :

Variate (No. of goals)	*Frequency (No. of matches)*	*Cumulative frequency*
0	2	2
1	4	6
2	7	13
3	6	19
4	8	27
5	3	30

Here, n (total no. of matches) = 30, which is even.

$$\therefore \quad \text{Median} = \frac{\frac{n}{2}\text{ th observation} + \left(\frac{n}{2}+1\right)\text{ th observation}}{2}$$

$$= \frac{\text{15th observation + 16th observation}}{2} = \frac{3+3}{2} = 3.$$

($\because$ all observations from 14th to 19th are equal, each = 3).

23.3.1 Quartiles

Quartiles are the values of a statistical data which divide the whole set of observations (variates) into four equal parts.

Lower (or first) quartile

If the variates are arranged in ascending order, then the observation lying mid-way between the lower extreme and the median is called the ***lower*** (or ***first***) ***quartile***. It is denoted by Q_1.

Upper (or third) quartile

If the variates are arranged in ascending order, then the observation lying mid-way between the median and upper extreme is called the ***upper*** (or ***third***) ***quartile***. It is denoted by Q_3.

Thus, if there are n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ arranged in ascending order, then

$$\textit{lower quartile } (Q_1) = \begin{cases} \frac{n+1}{4} \text{ th observation, if } n \text{ is odd} \\ \frac{n}{4} \text{ th observation, if } n \text{ is even} \end{cases}$$

$$\textit{and upper quartile } (Q_3) = \begin{cases} \frac{3(n+1)}{4} \text{ th observation, if } n \text{ is odd} \\ \frac{3n}{4} \text{ th observation, if } n \text{ is even.} \end{cases}$$

Remark

- The *middle quartile* (Q_2) is the median.

23.3.2 Inter quartile range

*The difference between the upper quartile (Q_3) and the lower quartile (Q_1) is called the **interquartile range**.*

Thus, interquartile range = $Q_3 - Q_1$ *and semi interquartile range* = $\frac{Q_3 - Q_1}{2}$.

ILLUSTRATIVE EXAMPLES

Example 1. *In a class test, the marks scored by 11 students are*

13, 17, 20, 5, 3, 19, 7, 6, 11, 15, 17. Find :

(i) median (ii) lower quartile (iii) upper quartile (iv) inter quartile range.

Solution. On arranging the given variates (marks) in ascending order, we get

3, 5, 6, 7, 11, 13, 15, 17, 17, 19, 20.

(*i*) Here, n (no. of observations) = 11, which is odd.

$\therefore$ Median = $\frac{n+1}{2}$ th observation = 6th observation = 13.

(*ii*) Lower quartile (Q_1) = $\frac{n+1}{4}$ th observation = 3rd observation = 6.

(*iii*) Upper quartile (Q_3) = $\frac{3(n+1)}{4}$ th observation = 9th observation = 17.

(*iv*) Inter quartile range = $Q_3 - Q_1$ = 17 – 6 = 11.

Example 2. *The heights (in nearest cm) of 60 students of a certain school are given in the following frequency distribution table :*

Heights (in cm)	*151*	*152*	*153*	*154*	*155*	*156*	*157*
No. of students	*6*	*4*	*11*	*9*	*16*	*12*	*2*

Find : (i) median (ii) lower quartile (iii) upper quartile (iv) inter quartile range.

Solution. The given variates (heights of students) are already in ascending order. We construct the cumulative frequency table as under :

Variate (height)	*Frequency (No. of students)*	*Cumulative frequency*
151	6	6
152	4	10
153	11	21
154	9	30
155	16	46
156	12	58
157	2	60

Here, n (total no. of students) = 60, which is even.

(*i*) Median $= \dfrac{\frac{n}{2}\text{ th observation} + \left(\frac{n}{2}+1\right)\text{ th observation}}{2}$

$= \dfrac{\text{30th observation + 31st observation}}{2} = \dfrac{154+155}{2} = \dfrac{309}{2} = 154{\cdot}5.$

(*ii*) Lower quartile (Q_1) = $\frac{n}{4}$ th observation = 15th observation = 153.

(*iii*) Upper quartile (Q_3) = $\frac{3n}{4}$ th observation = 45th observation = 155.

(*iv*) Inter quartile range = $Q_3 - Q_1$ = 155 – 153 = 2.

23.3.3 Estimation of median and quartiles from ogives

In a continuous frequency distribution, the median and the quartiles (lower and upper) can be estimated from the *ogive* of the given (continuous) frequency distribution.

Procedure :

1. Construct cumulative frequency table. Let n be the sum of frequencies.
2. Draw ogive for the given distribution.
3. ***To find the median*** :

   Locate a point along y-axis representing frequency equal to

   (*i*) $\frac{n+1}{2}$ if n is odd and (*ii*) $\frac{n}{2}$ if n is even.

   Through this point, draw a horizontal line to meet the ogive, and through this point of the ogive, draw a vertical line to meet the x-axis at the point M (say). The variate at the point M is the required median :

   *The class in which the median lies is called the **median class**.*

4. ***To find the lower quartile :***

Locate a point along y-axis representing frequency equal to

(*i*) $\frac{n+1}{4}$ if n is odd and (*ii*) $\frac{n}{4}$ if n is even.

Through this point, draw a horizontal line to meet the ogive, and through this point of the ogive, draw a vertical line to meet the x-axis at the point N (say).

The variate at the point N is the required lower quartile.

5. ***To find the upper quartile :***

Locate a point along y-axis representing frequency equal to

(*i*) $\frac{3(n+1)}{4}$ if n is odd and (*ii*) $\frac{3n}{4}$ if n is even.

Now proceed as in 4 to find the required upper quartile.

Remark

If, in a problem, frequency distribution is discontinuous, first convert it into continuous distribution and then find median and quartiles as explained above.

ILLUSTRATIVE EXAMPLES

Example 1. *The marks obtained by 200 students in an examination are given below :*

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	05	10	11	20	27	38

continued

60 – 70	70 – 80	80 – 90	90 – 100
40	29	14	06

Using a graph paper, draw an ogive for the above distribution. Use your ogive to estimate :

(i) the median

(ii) the lower quartile

(iii) the number of students who obtained more than 80% marks in the examination and

(iv) the number of students who did not pass, if the pass percentage was 35. (2004)

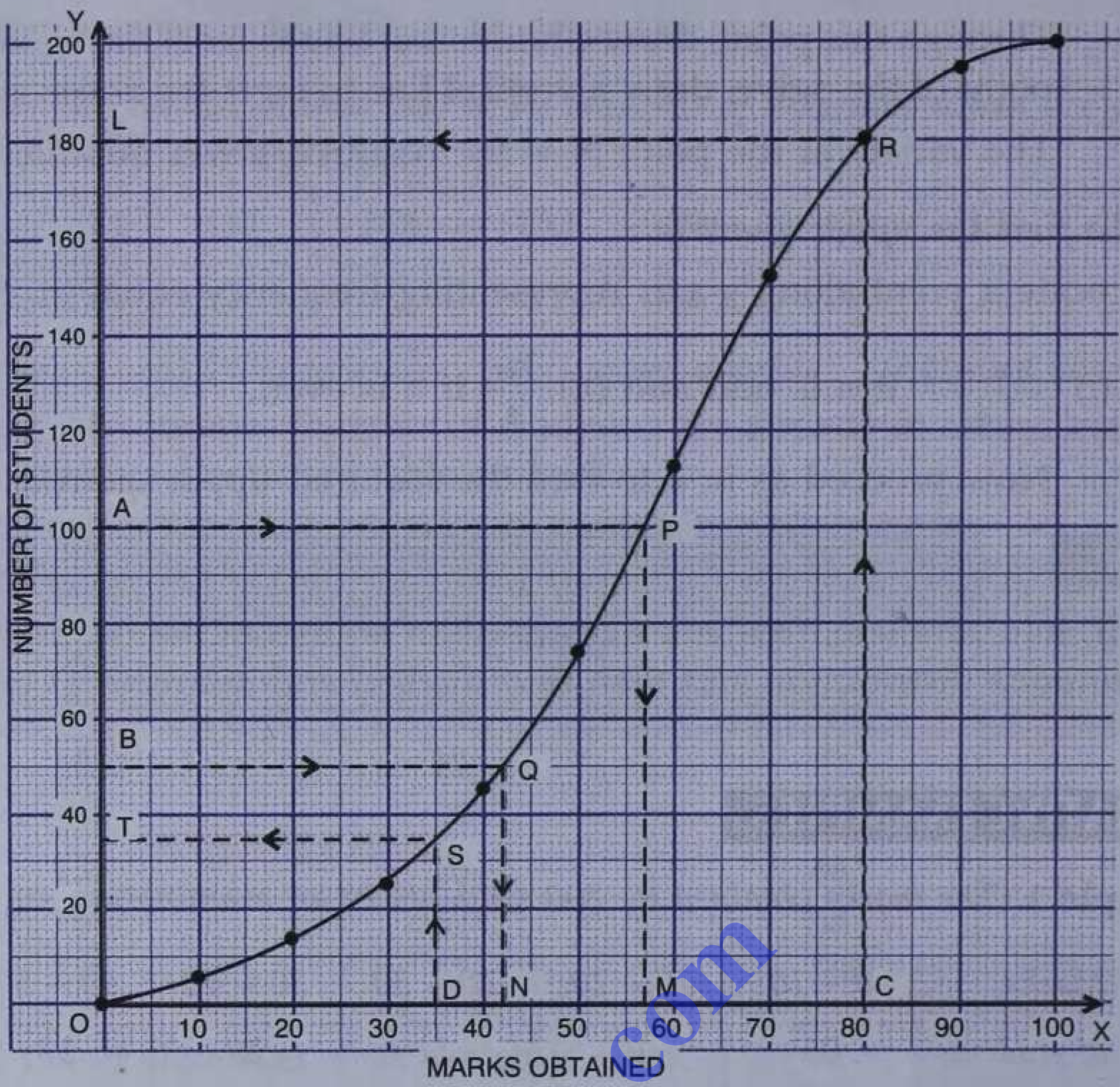
Solution. The cumulative frequency table for the given continuous distribution is :

Marks	*No. of students*	*Cumulative frequency*
0 – 10	05	05
10 – 20	10	15
20 – 30	11	26
30 – 40	20	46
40 – 50	27	73
50 – 60	38	111
60 – 70	40	151
70 – 80	29	180
80 – 90	14	194
90 – 100	06	200

Take 1 cm = 10 marks along x-axis

and 1 cm = 20 students along y-axis.

Plot the points (10, 5), (20, 15), (30, 26), (40, 46), (50, 73), (60, 111), (70, 151), (80, 180), (90, 194), (100, 200) and (0, 0). Join these points by a freehand drawing. The required ogive is shown on the graph paper given below.



Here, n (no. of students) = 200, which is even.

(*i*) *To find the median* :

Let A be the point on y-axis representing frequency

$$= \frac{n}{2} = \frac{200}{2} = 100.$$

Through A, draw a horizontal line to meet the ogive at P. Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents 57 marks.

∴ The required median = 57 marks.

(*ii*) *To find the lower quartile* :

Let B be the point on y-axis representing frequency $= \frac{n}{4} = \frac{200}{4} = 50.$

Through B, draw a horizontal line to meet the ogive at Q. Through Q, draw a vertical line to meet the x-axis at N. The abscissa of the point N represents 42 marks.

∴ The lower quartile = 42 marks.

(*iii*) 80% marks = 80% of 100 marks = 80 marks.

Let the point C on x-axis represent 80 marks. Through C, draw a vertical line to meet the ogive at the point R. Through R, draw a horizontal line to meet the y-axis at L. The ordinate of point L represents 180 students on y-axis.

∴ The number of students who obtained more than 80 marks

= total number of students – number of students who obtained ≤ 80 marks

= 200 – 180 = 20.

(*iv*) 35% marks = 35% of 100 marks = 35 marks.

Let the point D on x-axis represent 35 marks. Through D, draw a vertical line to meet the ogive at the point S. Through S, draw a horizontal line to meet the y-axis at T. The ordinate of the point T represents 34 students on y-axis.

∴ The number of students who did not pass

= the number of students who obtained < 35 marks

= 33.

Example 2. *Draw ogive for the following distribution :*

Monthly income (in ₹)	600 – 700	700 – 800	800 – 900	900 – 1000	1000 – 1100	1100 – 1200	1200 – 1300
No. of employees	40	68	86	120	90	40	26

Hence determine :

(*i*) *the median income.*

(*ii*) *the percentage of employees whose income exceeds ₹ 1180.*

(*iii*) *the lower and upper quartiles*

(*iv*) *the inter quartile range.*

Solution. The cumulative frequency table for the given continuous distribution is :

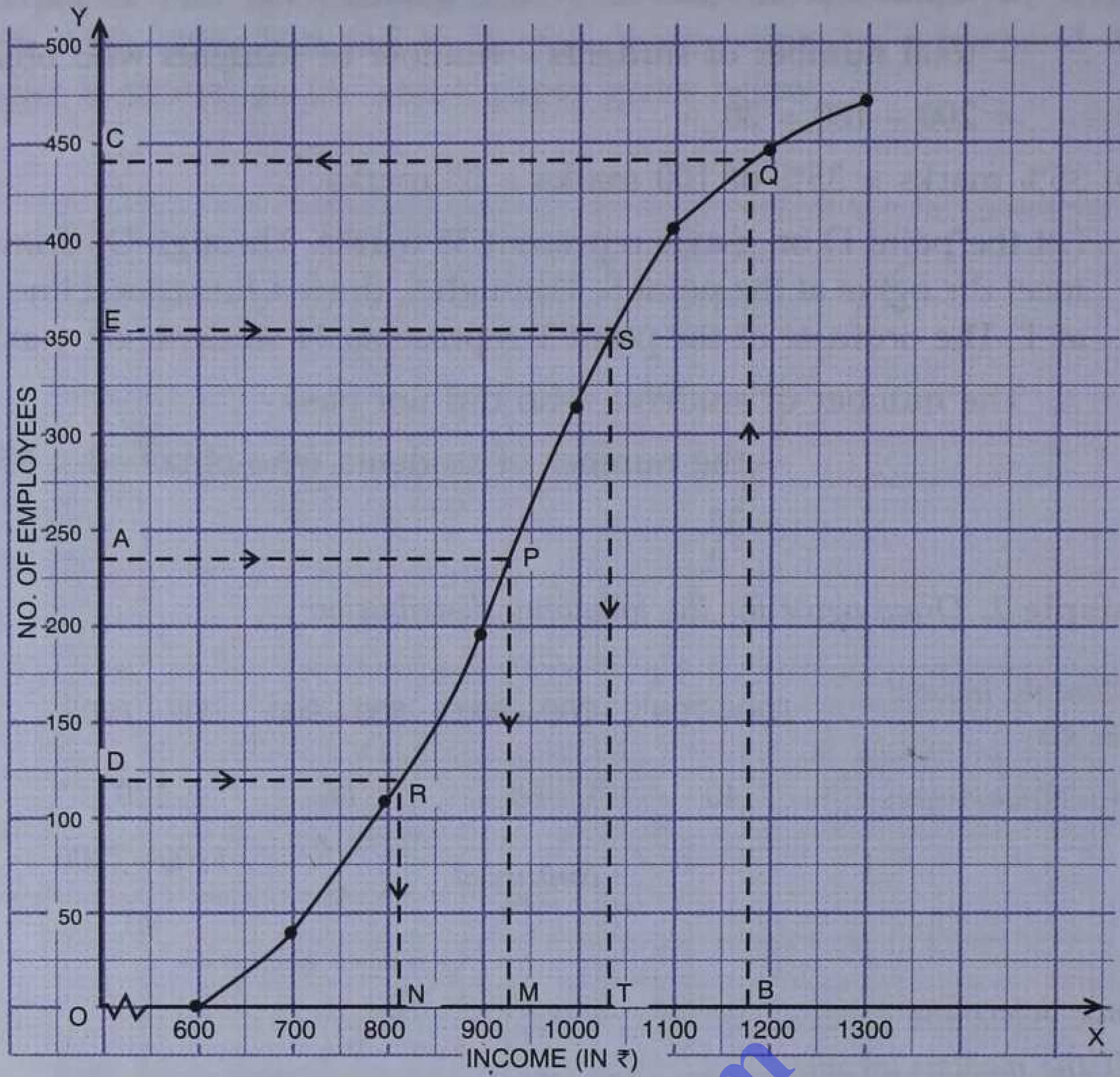
Monthly income in ₹ (class-intervals)	*No. of employees (frequency)*	*Cumulative frequency*
600 – 700	40	40
700 – 800	68	108 (40 + 68)
800 – 900	86	194 (108 + 86)
900 – 1000	120	314 (194 + 120)
1000 – 1100	90	404 (314 + 90)
1100 – 1200	40	444 (404 + 40)
1200 – 1300	26	470 (444 + 26)

Take 1 cm along x-axis = ₹ 100 and

1 cm along y-axis = 50 employees.

Since the scale on x-axis starts at 600, a kink is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 600.

Plot the points (700, 40), (800, 108), (900, 194), (1000, 314), (1100, 404), (1200, 444), (1300, 470) and (600, 0). Join these points by a free hand drawing. The required ogive is drawn on the graph paper given below.



Here, n (no. of employees) = 470, which is even.

(*i*) *To find the median* :

Let A be a point on y-axis representing frequency

$$= \frac{n}{2} = \frac{470}{2} = 235$$

Through A, draw a horizontal line to meet the ogive at P. Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents ₹ 930.

∴ The required median = ₹ 930.

(*ii*) Let the point B on x-axis represent ₹ 1180. Through B, draw a vertical line to meet the ogive at Q. Through Q, draw a horizontal line to meet the y-axis at C. The ordinate of the point C represents 436 employees on y-axis.

∴ The number of employees whose income exceeds ₹ 1180

= total no. of employees – no. of employees whose income ≤ ₹ 1180

= 470 – 436 = 34.

∴ The percentage of employees whose income exceeds ₹ 1180

$$= \left(\frac{34}{470} \times 100\right)\% = 7{\cdot}23\% \text{ (approximately)}$$

(*iii*) *To find the lower quartile* :

Let D be the point on y-axis representing frequency $= \frac{n}{4} = \frac{470}{4} = 117{\cdot}5$.

Through D, draw a horizontal line to meet the ogive at R. Through R, draw a vertical line to meet the x-axis at N. The abscissa of the point N represents ₹ 815.

∴ The lower quartile = ₹ 815.

To find the upper quartile :

Let E be the point on y-axis representing frequency = $\frac{3n}{4} = \frac{3 \times 470}{4}$ = 352·5.

Through E, draw a horizontal line to meet the ogive at S. Through S, draw a vertical line to meet the x-axis at T. The abscissa of the point T represents ₹ 1035.

∴ The upper quartile = ₹ 1035.

(*iv*) Inter quartile range = upper quartile – lower quartile = ₹ 1035 – ₹ 815

= ₹ 220.

Example 3. *The table below shows the distribution of the scores obtained by 120 shooters in a shooting competition. Using a graph sheet, draw an ogive for the distribution.*

Score obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of shooters	5	9	16	22	26	18

continued

60 – 70	70 – 80	80 – 90	90 – 100
11	6	4	3

Use your ogive to estimate :

(*i*) *the median*

(*ii*) *the interquartile range.*

(*iii*) *the number of shooters who obtained more than 75% scores.* (2007)

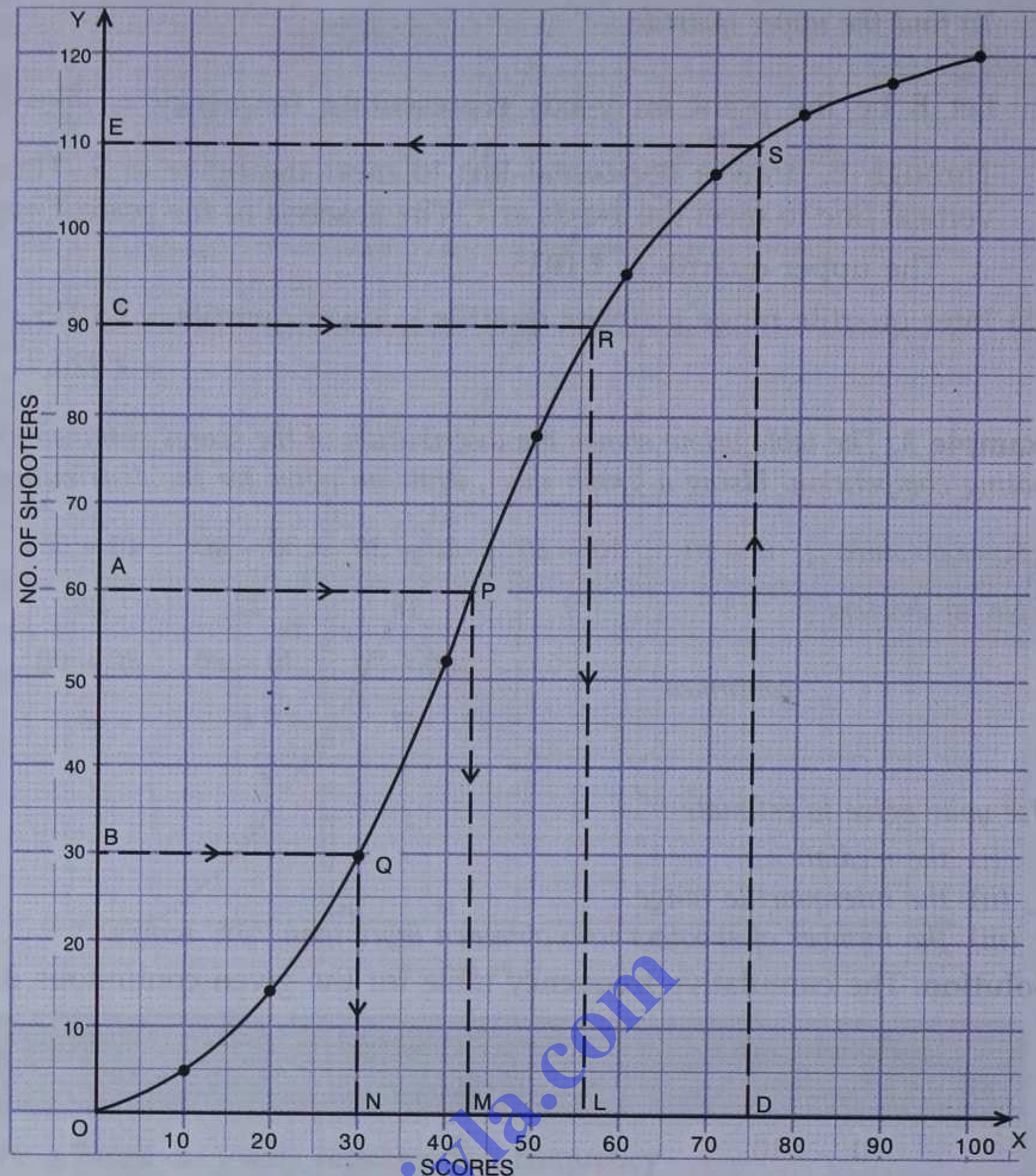
Solution. The cumulative frequency table for the given continuous distribution is :

Scores obtained (class-intervals)	*Number of shooters (frequency)*	*Cumulative frequencey*
0 – 10	5	5
10 – 20	9	14 (5 + 9)
20 – 30	16	30 (14 + 16)
30 – 40	22	52 (30 + 22)
40 – 50	26	78 (52 + 26)
50 – 60	18	96 (78 + 18)
60 – 70	11	107 (96 +11)
70 – 80	6	113 (107 + 6)
80 – 90	4	117 (113 + 4)
90 – 100	3	120 (117 + 3)

Take 1 cm along x-axis = 10 scores and

1 cm along y-axis = 10 shooters.

Plot the points (10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120) and (0, 0). Join these points by a free hand drawing. The required ogive is drawn on the graph sheet given below.



Here, n(number of shooters) = 120, which is even.

(i) ***To find the median :***

Let A be the point on y-axis representing frequency

$= \frac{n}{2} = \frac{120}{2} = 60.$

Through A, draw a horizontal line to meet the ogive at P. Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents 43.

∴ The required median = 43 score.

(ii) ***To find the interquartile range :***

To find the lower quartile :

Let B be the point on y-axis representing frequency $\frac{n}{4} = \frac{120}{4} = 30.$

Through B, draw a horizontal line to meet the ogive at Q. Through Q, draw a vertical line to meet the x-axis at the point N. The abscissa of the point N represents 30.

∴ The lower quartile (Q_1) = 30 score.

To find the upper quartile :

Let C be the point on y-axis representing frequency $= \frac{3n}{4} = \frac{3 \times 120}{4} = 90.$

Through C, draw a horizontal line to meet the ogive at R. Through R, draw a vertical line to meet the x-axis at L. The abscissa of the point L represents 56.

∴ The upper quartile (Q_3) = 56 score.

∴ The required interquartile range = $Q_3 - Q_1$ = (56 – 30) score = 26 score.

(*iii*) Let D be the point on x-axis representing 75% score *i.e.* 75 score. Through D, draw a vertical line to meet the ogive at the point S. Through S, draw a horizontal line to meet the y-axis at the point E. The ordinate of the point E represents 110 shooters on the y-axis.

∴ The number of shooters who obtained more than 75% scores

= total number of shooters – nnumber of shooters who obtained ≤ 75% score

= 120 – 110 = 10.

Example 4. *Attempt this question on graph paper :*

Marks obtained by 200 students in examination are given below :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	5	10	14	21	25	34	36	27	16	12

continued

Draw an ogive for the given distribution. From the graph, find :

(*i*) *the median*

(*ii*) *the upper quartile*

(*iii*) *the number of students scoring above 65 marks.*

(*iv*) *If 10 students qualify for merit scholarship, find the minimum marks required to qualify.* (2009)

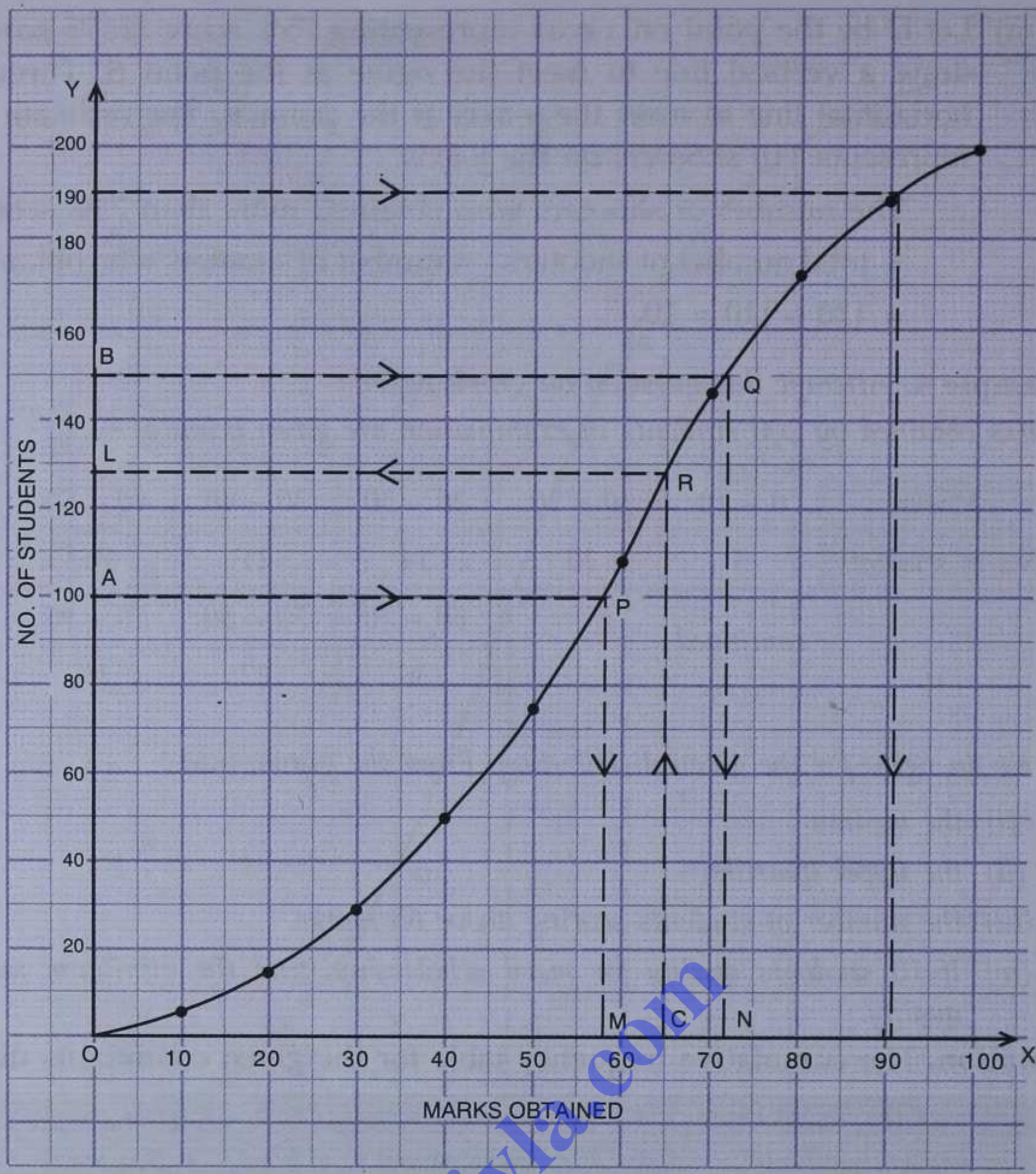
Solution. The cumulative frequency table for the given continuous distribution is :

Marks obtained (class-intervals)	*Number of students (frequency)*	*Cumulative frequency*
0 – 10	5	5
10 – 20	10	15
20 – 30	14	29
30 – 40	21	50
40 – 50	25	75
50 – 60	34	109
60 – 70	36	145
70 – 80	27	172
80 – 90	16	188
90 – 100	12	200

Take 1 cm along x-axis = 10 marks and

1 cm along y-axis = 20 students.

Plot the points (10, 5), (20, 15), (30, 29), (40, 50), (50, 75), (60, 109), (70, 145), (80, 172), (90, 188), (100, 200) and (0, 0). Join these points by a freehand drawing. The required ogive is drawn on the graph sheet given below.



Here, n(number of students) = 200, which is even.

(*i*) ***To find the median :***

Let A be the point on y-axis representing frequency

$$= \frac{n}{2} = \frac{200}{2} = 100.$$

Through A, draw a horizontal line to meet the ogive at P.

Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents 58 marks.

∴ The required median = 58 marks.

(*ii*) ***To find the upper quartile :***

Let B be the point on y-axis representing frequency

$$= \frac{3n}{4} = \frac{3 \times 200}{4} = 150.$$

Through B, draw a horizontal line to meet the ogive at Q. Through Q, draw a vertical line to meet the x-axis at N. The abscissa of the point N represents 72 marks.

∴ The upper quartile = 72 marks.

(*iii*) ***To find number of students scoring above 65 marks :***

Let the point C on x-axis represent 65 marks. Through C, draw a vertical line to meet the ogive at R. Through R, draw a horizontal line to meet the y-axis at L. The ordinate of the point L represents 128 students on y-axis.

∴ The number of students scoring above 65 marks

= total no. of students – no. of students scoring ≤ 65 marks

= 200 – 128 = 72.

(*iv*) ***To find minimum marks required to qualify for merit scholarship :***

As 10 students qualify for merit sholarship and the marks are already arranged in ascending order, so, the students from 191 to 200 qualify for merit scholarship. Note that 190th student does not qualify for merit scholarship but the next one does so.

From ogive, we find that 190th student scores 91 marks and he does not qualify for merit scholarship. Therefore, the next student who qualifies for merit scholarship scores more marks than 91.

$\therefore$ The minimum marks required to qualify for merit scholarship = 92.

Example 5. *The marks of 200 students in a test were recorded as follows :*

Marks %	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of students	7	11	20	46	57

continued

60 – 69	70 – 79	80 – 89
37	15	7

Draw the cumulative frequency table.
Draw the ogive and use it to find

(*i*) *the median*

(*ii*) *the number of students who scored more than 35% marks.*

Solution. The given frequency distribution is discontinuous, to convert it into continuous distribution,

$$\text{adjustment factor} = \frac{20 - 19}{2} = \cdot 5.$$

Cumulative (continuous) frequency table for the given data is :

Marks % *(Classes before adjustment)*	*Marks %* *(Classes after adjustment)*	*Frequency*	*Cumulative frequency*
10 – 19	9·5 – 19·5	7	7
20 – 29	19·5 – 29·5	11	18
30 – 39	29·5 – 39·5	20	38
40 – 49	39·5 – 49·5	46	84
50 – 59	49·5 – 59·5	57	141
60 – 69	59·5 – 69·5	37	178
70 – 79	69·5 – 79·5	15	193
80 – 89	79·5 – 89·5	7	200

Take 1 cm along x-axis = 10% marks and

1 cm along y-axis = 20 students.

Plot the points (19·5, 7), (29·5, 18), (39·5, 38), (49·5, 84), (59·5, 141), (69·5, 178), (79·5, 193), (89·5, 200) and (9·5, 0). Join these points by a free hand drawing. The required ogive is drawn on the next page.

Here, n (no. of students) = 200, which is even.

(*i*) *To find the median* :

Let A be a point on y-axis representing frequency

$$= \frac{n}{2} = \frac{200}{2} = 100.$$

Through A, draw a horizontal line to meet the ogive at P. Through P, draw a vertical line to meet x-axis at M. The abscissa of point M represents 52%.

$\therefore$ The required median = 52% marks.

(*ii*) Let the point B on x-axis represent 35% marks. Through B, draw a vertical line to meet the ogive at Q. Through Q, draw a horizontal line to meet y-axis at C. The ordinate of the point C represents 28 students on y-axis.

$\therefore$ The number of students who scored more than 35% marks

= total no. of students – no. of students who scored $\leq$ 35%

= 200 – 28 = 172.

Exercise 23.2

1. A student scored the following marks in 11 questions of a question paper :
   3, 4, 7, 2, 5, 6, 1, 8, 2, 5, 7.
   Find the median marks.
2. (*a*) Find the median of the following set of numbers :
   9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7.
   (*b*) For the following set of numbers, find the median :
   10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9, 15. *(2004)*
3. If 3, 8, 10, x, 14, 16, 18, 20 are in ascending order and their median is 13, calculate the numerical value of x.
4. Calculate the mean and the median of the numbers : 2, 1, 0, 3, 1, 2, 3, 4, 3, 5.
5. The median of the observations 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. *(2013)*
6. The mean of the numbers 1, 7, 5, 3, 4, 4 is m. The numbers 3, 2, 4, 2, 3, 3, p have mean $m - 1$ and median q. Find (*i*) p (*ii*) q (*iii*) the mean of p and q.

7. Find the median for the following distribution :

Wages per day (in ₹)	38	45	48	55	62	65
No. of workers	14	8	7	10	6	2

8. Find the median for the following distribution :

Marks	35	45	50	64	70	72
No. of students	3	5	8	10	5	5

9. Marks obtained by 70 students are given below :

Marks	20	70	50	60	75	90	40
No. of students	8	12	18	6	9	5	12

Calculate the median marks.

Hint

Arrange the variates in ascending order.

10. Calculate the mean and the median for the following distribution :

Number	5	10	15	20	25	30	35
Frequency	1	2	5	6	3	2	1

11. The daily wages (in rupees) of 19 workers are :

41, 21, 38, 27, 31, 45, 23, 26, 29, 30, 28, 25, 35, 42, 47, 53, 29, 31, 35.

Find :

(*i*) the median (*ii*) lower quartile

(*iii*) upper quartile (*iv*) inter quartile range.

12. From the following frequency distribution, find :

(*i*) the median (*ii*) lower quartile

(*iii*) upper quartile (*iv*) inter quartile range.

Variate	15	18	20	22	25	27	30
Frequency	4	6	8	9	7	8	6

13. For the following frequency distribution, find :

(*i*) the median (*ii*) lower quartile (*iii*) upper quartile.

Variate	25	31	34	40	45	48	50	60
Frequency	3	8	10	15	10	9	6	2

14. Use graph paper for this question.

The table given below shows the monthly wages of some factory workers.

(*i*) Using the table, calculate the cumulative frequencies of workers.

(*ii*) Draw the cumulative frequency curve.

Use 2 cm = ₹ 500, starting the origin at ₹ 6500 on x-axis, and 2 cm = 10 workers on y-axis.

(*iii*) Use your graph to write down the median wage in ₹.

Wages (in ₹)	6500 – 7000	7000 – 7500	7500 – 8000	8000 – 8500
Frequency	10	18	22	25
	continued	8500 – 9000	9000 – 9500	9500 – 10000
		17	10	8

15. The following table shows the distribution of the heights of a group of factory workers.

Height (cm)	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180	180–185
No. of workers	6	12	18	20	13	8	6

(*i*) Determine the cumulative frequencies.

(*ii*) Draw the cumulative frequency curve on a graph paper. Use 2 cm = 5 cm height on one axis and 2 cm = 10 workers on the other.

(*iii*) From your graph, write down the median height in cm. (*2000*)

16. Using the data given below construct the cumulative frequency table and draw the ogive. From the ogive, determine the median.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	3	8	12	14	10	6	5	2

(*2001*)

17. Use graph paper for this question.

The following table shows the weights in gm of a sample of 100 potatoes taken from a large consignment :

Weight (gm)	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110	110 – 120	120 – 130
Frequency	8	10	12	16	18	14	12	10

(*i*) Calculate the cumulative frequencies.

(*ii*) Draw the cumulative frequency curve and from it determine the median weight of the potatoes.

18. Attempt this question on graph paper.

Age (yrs)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
No. of casualties due to accidents	6	10	15	13	24	8	7

(*i*) Construct the 'less than' cumulative frequency curve for the above data, using 2 cm = 10 years, on one axis and 2 cm = 10 casualties on the other.

(*ii*) From your graph determine

(*a*) the median and (*b*) the upper quartile.

19. The daily wages of 160 workers in a building project are given below :

Wages (in ₹)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of workers	12	20	30	38	24	16	12	8

Using a graph paper, draw an ogive for the above distribution. Use your ogive to estimate :

(*i*) the median wage of the workers.

(*ii*) the upper quartile wage of the workers.

(*iii*) the lower quartile wage of the workers.

(*iv*) the percentage of workers who earn more than ₹ 45 a day. (*2006*)

20. Marks obtained by 200 students in an examination are given below :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	5	11	10	20	28	37
continued			60 – 70	70 – 80	80 – 90	90 – 100
			40	29	14	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine:

(*i*) The median marks.

(*ii*) The number of students who failed if minimum marks required to pass is 40.

(*iii*) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination. (*2011*)

21. The monthly income of a group of 320 employees in a company is given below

Monthly Income (in ₹)	6000 – 7000	7000 – 8000	8000 – 9000	9000 – 10000
No. of Employees	20	45	65	95
continued		10000 – 11000	11000 – 12000	12000 – 130000
		60	30	5

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹ 1000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine

(*i*) the median wage.

(*ii*) the number of employees whose income is below ₹ 8500.

(*iii*) If the salary of a senior employee is above ₹ 11500, find the number of senior employees in the company.

(*iv*) the upper quartile. (*2010*)

22. Using a graph paper, draw an ogive for the following distribution which shows a record of weight in kilograms of 200 students :

Weight (kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
Frequency	5	17	22	45	51	31	20	9

Use your ogive to estimate the following :

(*i*) The percentage of students weighing 55 kg or more.

(*ii*) The weight above which the heaviest 30% of the students fall.

(*iii*) The number of students who are (1) under-weight and (2) over-weight, if 55·70 kg is considered as standard weight. (*2005*)

23. Marks scored by 400 students in an examination are as follows :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	10	20	22	40	55	75
continued			60 – 70	70 – 80	80 – 90	90 – 100
			80	58	28	12

Draw the ogive and from it determine :

(*i*) the median mark, and

(*ii*) the pass marks if 80% of the students pass examination.

24. The marks obtained by 120 students in a Mathematics test are given below :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	5	9	16	22	26	18
continued			60 – 70	70 – 80	80 – 90	90 – 100
			11	6	4	3

Draw an ogive for the given distribution on a graph sheet. Use a suitable scale for ogive to estimate the following :

(*i*) the median.

(*ii*) the number of students who obtained more than 75% marks in the test.

(*iii*) the number of students who did not pass in the test if the pass percentage was 40. *(2013, 02)*

25. The following distribution represents the height of 160 students of a school.

Height (in cm)	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165
No. of students	12	20	30	38	24
continued			165 – 170	170 – 175	175 – 180
			16	12	8

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine :

(*i*) The median height.

(*ii*) The inter quartile range.

(*iii*) The number of students whose height is above 172 cm. *(2012)*

26. 100 pupils in a school have heights as tabulated below :

Height (in cm)	121 – 130	131 – 140	141 – 150	151 – 160	161 – 170	171 – 180
No. of pupils	12	16	30	20	14	8

Draw the ogive for the above data and from it determine the median (use graph paper).

23.4 MODE

Mode (or modal value) *of a statistical data is the variate which occurs most frequently. In other words,* ***mode*** *of a statistical data is the variate which has maximum frequency.*

Modal class. The class with maximum frequency is called the ***modal class.***

ILLUSTRATIVE EXAMPLES

Example 1. *Find the mode of the following data* : 3, 5, 1, 2, 4, 6, 0, 2, 2, 3.

Solution. In the given data, 2 is repeated more number of times than any other number,

$\therefore$ mode = 2.

Example 2. *Find the mode for the following frequency distribution* :

Marks obtained (out of 10)	0	2	3	4	6	7	9	10
No. of students	3	5	12	18	21	8	2	1

Solution. In the given distribution, the variate 6 has the maximum frequency,

$\therefore$ mode = 6.

Remarks

- A data may not have a mode. For example, the data 2, 5, 0, 7, 4, 6 has no mode because no number occurs more number of times than any other number.
- A data may have more than one modes. For example, the data 2, 5, 0, 3, 5, 7, 6, 3, 8 has two modes 3 and 5 because each of 3 and 5 is repeated twice.
- An approximate relation between three measures, *mean, median* and *mode* of central tendency of a statistical data is given by the formula :

  Mode = 3 Median – 2 Mean.

  This is known as ***Empirical Relation.***

23.4.1 Estimation of mode from histogram

In a continuous frequency distribution, the mode can be determined from the histogram of the given (continuous) frequency distribution.

Procedure :

1. Draw histogram for the given data.
2. Inside the highest rectangle (which represents the class with maximum frequency *i.e.* modal class), draw two st. lines from the corners of the rectangles on either side of the highest rectangle to the opposite corners of the highest rectangle.
3. Through the point of intersection of the two st. lines drawn in step 2, draw a vertical line to meet the x-axis at the point M (say).

   The variate at the point M is the required ***mode.***

Remark

If, in a problem, frequency distribution is discontinuous, first convert it into continuous distribution and then find the mode as explained above.

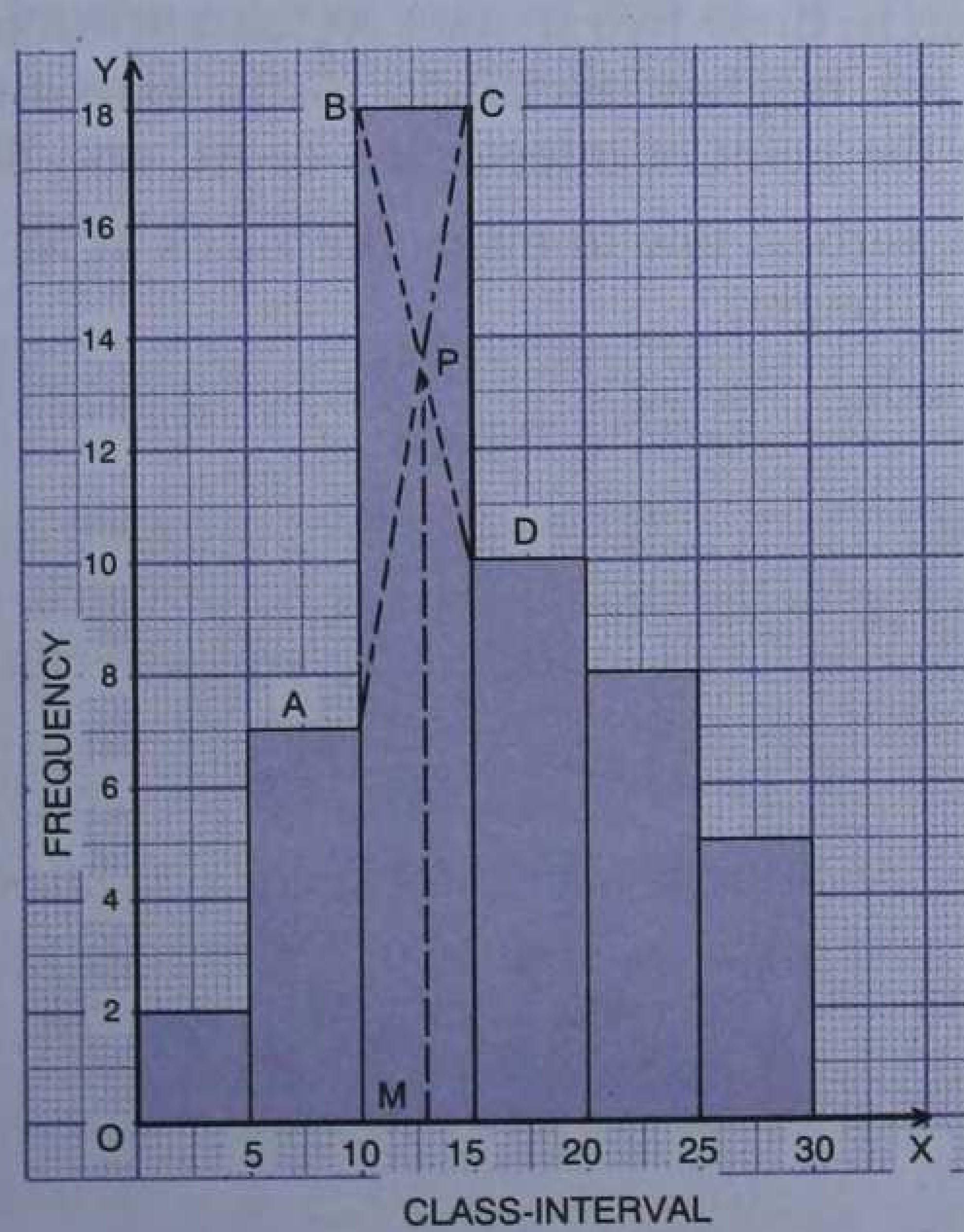
ILLUSTRATIVE EXAMPLES

Example 1. *For the following frequency distribution, draw a histogram. Hence calculate the mode :*

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Frequency	2	7	18	10	8	5

(2004)

Solution. *Steps :*



1. Draw histogram by taking
   1 cm along x-axis = 5 (class-interval),
   1 cm along y-axis = 2 (frequency).
2. In the highest rectangle, draw two straight lines AC and BD from corners of the rectangles on either side of the highest rectangle to the opposite corners of the highest rectangle. Let P be the point of intersection of the lines AC and BD.
3. Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents 13.

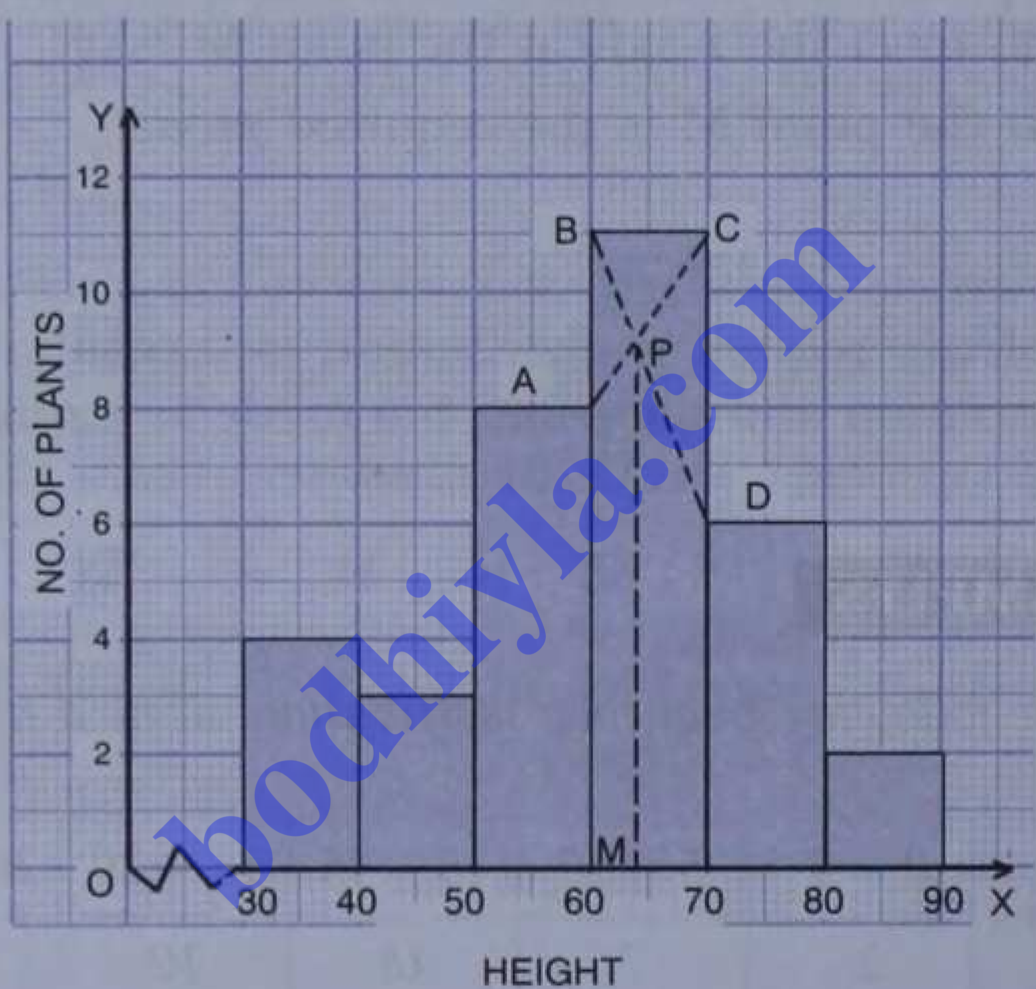
   ∴ The required mode = 13.

Example 2. *Find the mode of the following distribution by drawing a histogram :*

Height (in cm)	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of plants	4	3	8	11	6	2

Solution. *Steps :*

1. Draw histogram by taking
   1 cm along x-axis = 10 cm (height),
   1 cm along y-axis = 2 plants.



2. In the highest rectangle, draw two st. lines AC and BD from corners of the rectangles on either side of the heighest rectangle to the opposite corners of the highest rectangle. Let P be the point of intersection of AC and BD.
3. Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents 64 cm (height).

   ∴ The required mode = 64.

Remark

In the above example, the modal class is 60 – 70. Therefore, to find the mode of the given distribution, it is sufficient to consider the three classes 50 – 60, 60 – 70 and 70 – 80.

Example 3. *Find the mode of the following distribution by drawing a histogram :*

Daily wages (in ₹)	*31 – 36*	*37 – 42*	*43 – 48*	*49 – 54*	*55 – 60*	*61 – 66*
No. of workers	*6*	*12*	*20*	*15*	*9*	*4*

Solution. *Steps*

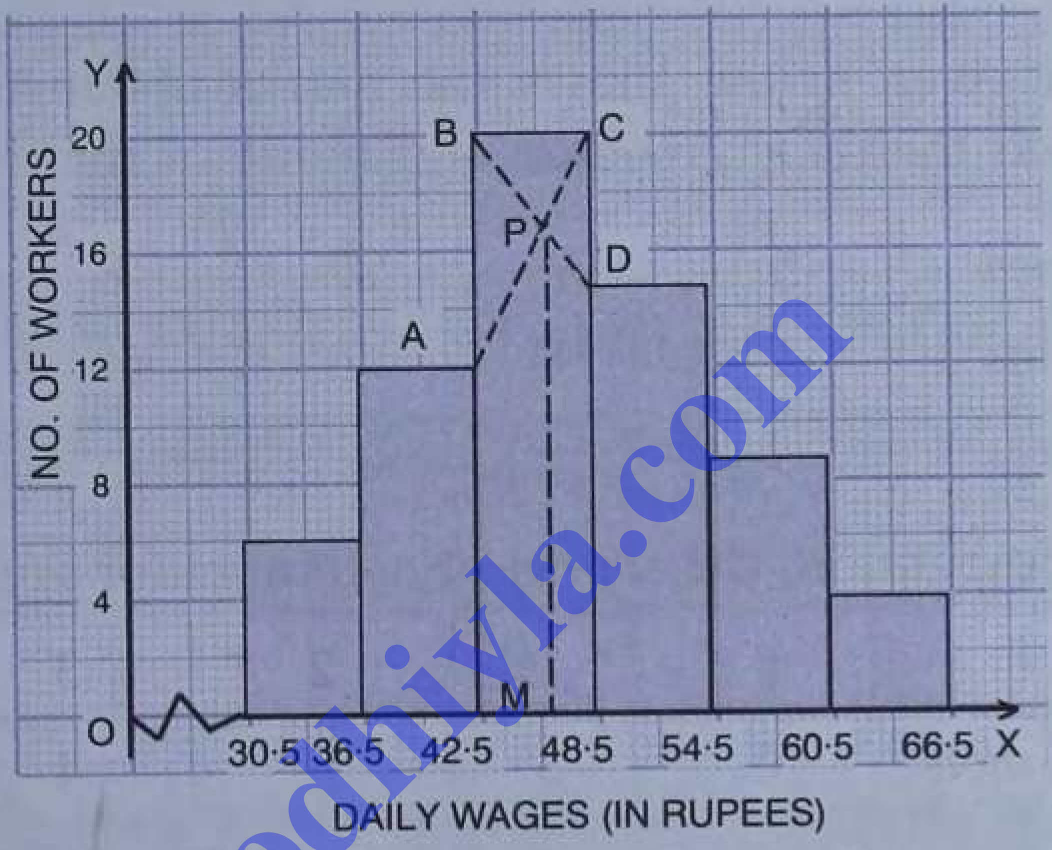
1. The given frequency distribution is discontinuous, to convert it into continuous distribution,

$$\text{adjustment factor} = \frac{37-36}{2} = \frac{1}{2} = \cdot 5.$$

Continuous frequency table for the given data is

Classes before adjustment	*Classes after adjustment*	*Frequency*
31 – 36	30·5 – 36·5	6
37 – 42	36·5 – 42·5	12
43 – 48	42·5 – 48·5	20
49 – 54	48·5 – 54·5	15
55 – 60	54·5 – 60·5	9
61 – 66	60·5 – 66·5	4

Draw the histogram by taking 1 cm along x-axis = ₹ 6 and 1 cm along y-axis = 4 workers.



2. In the highest rectangle, draw two st. line AC and BD from corners of the rectangles on either side of the highest rectangle to the opposite corners of the highest rectangle. Let P be the point of intersection of AC and BD.
3. Through P, draw a vertical line to meet the x-axis at M. The abscissa of the point M represents ₹ 46·5.

∴ The required mode = ₹ 46·5.

Exercise 23.3

1. Find the mode of the following sets of numbers :

(*i*) 3, 2, 0, 1, 2, 3, 5, 3 (*ii*) 5, 7, 6, 8, 9, 0, 6, 8, 1, 8

(*iii*) 9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7

2. Calculate the mean, the median and the mode of the numbers :

3, 2, 6, 3, 3, 1, 1, 2

3. Find the mean, median and mode of the following distribution :

8, 10, 7, 6, 10, 11, 6, 13, 10. (2009)

4. Calculate the mean, the median and the mode of the following numbers :

3, 1, 5, 6, 3, 4, 5, 3, 7, 2. (2000)

5. A boy scored the following marks in various class tests during a term, each test being marked out of 20 :

15, 17, 16, 7, 10, 12, 14, 16, 19, 12, 16.

(*i*) What are his modal marks ? (*ii*) What are his median marks ?

(*iii*) What are his mean marks ?

6. Find the mean, median and mode of the following marks obtained by 16 students in a class test marked out of 10 marks :

0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8.

7. Find the mode and the median of the following frequency distribution :

x	10	11	12	13	14	15
f	1	4	7	5	9	3

(2012)

8. Calculate the mean, the median and the mode of the following distribution :

No. of goals	0	1	2	3	4	5
No. of matches	2	4	7	6	8	3

9. The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.

Marks obtained	5	6	7	8	9	10
No. of students	3	9	6	4	2	1

(2010)

10. At a shooting competition, the scores of a competitor were as given below :

Score	0	1	2	3	4	5
Number of shots	0	3	6	4	7	5

(*i*) What was his modal score ? (*ii*) What was his median score ?

(*iii*) What was his total score ? (*iv*) What was his mean score ?

11. (*i*) Using step-deviation method, calculate the mean marks of the following distribution.

(*ii*) State the modal class.

Class intervals	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80	80 – 85	85 – 90
Frequency	5	20	10	10	9	6	12	8

(2011)

12. The following table gives the weekly wages (in ₹) of workers in a factory :

Weekly wages (in ₹)	50–55	55–60	60–65	65–70	70–75	75–80	80–85	85–90
No. of workers	5	20	10	10	9	6	12	8

Calculate :

(*i*) the mean.

(*ii*) the modal class.

(*iii*) the number of workers getting weekly wages below ₹ 80.

(*iv*) the number of workers getting ₹ 65 or more but less than ₹ 85 as weekly wages. (2002)

13. Calculate the mean of the distribution given below :

Marks	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Frequency	4	6	12	6	7	5

Also state (*i*) the median class (*ii*) the modal class.

14. Draw a histogram from the following frequency distribution and find the mode from the graph :

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Frequency	2	5	18	14	8	5

(*2013*)

15. Find the modal height of the following distribution by drawing a histogram :

Height (in cm)	140 – 150	150 – 160	160 – 170	170 – 180	180 – 190
No. of students	7	6	4	10	2

16. A Mathematics aptitude test of 50 students was recorded as follows :

Marks	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	4	8	14	19	5

Draw a histogram for the above date using a graph paper and locate the mode. (*2011*)

17. Draw a histogram and estimate the mode for the following frequency distribution :

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	2	8	10	5	4	3

(*2003*)

18. In a school, the weekly pocket money of 50 students is as follows :

Weekly pocket money in ₹	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	2	8	12	14	8	6

Draw a histogram on a graph paper and find the mode from the graph.

(*2009*)

19. IQ of 50 students was recorded as follows :

IQ score	80 – 90	90 – 100	100 – 110	110 – 120	120 – 130	130 – 140
No. of students	6	9	16	13	4	2

Draw a histogram for the above data and estimate the mode.

20. The daily profits (in ₹), of 100 shops in a market, are distributed as follows :

Profit per shop (in ₹)	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600
No. of shops	12	18	27	20	17	6

Draw a histogram of the data given above, on graph paper, and estimate the mode.

21. Draw a histogram for the following distribution :

Wt. (in kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64	65 – 69
No. of students	2	8	12	10	6	4

Hence estimate the modal weight.

22. Find the mode of the following distribution by drawing a histogram.

Mid value	12	18	24	30	36	42	48
Frequency	20	12	8	24	16	8	12

Also state the modal class.

Hint

Convert the given data into continuous classes.

CHAPTER TEST

1. Arun scored 36 marks in English, 44 marks in Civics, 75 marks in Mathematics and x marks in Science. If he has scored an average of 50 marks, find x.
2. The mean of 20 numbers is 18. If 3 is added to each of the first ten numbers, find the mean of new set of 20 numbers.
3. The average height of 30 students is 150 cm. It was detected later that one value of 165 cm was wrongly copied as 135 cm for the computation of mean. Find the correct mean.
4. There are 50 students in a class of which 40 are boys and the rest girls. The average weight of the students in the class is 44 kg and the average weight of the girls is 40 kg. Find the average weight of boys.
5. The contents of 50 boxes of matches were counted giving the following results :

No. of matches	41	42	43	44	45	46
No. of boxes	5	8	13	12	7	5

Calculate the mean number of matches per box.

6. The heights of 50 children were measured (correct to the nearest cm) giving the following results :

Height (in cm)	65	66	67	68	69	70	71	72	73
No. of children	1	4	5	7	11	10	6	4	2

Calculate the mean height for this distribution correct to one place of decimal.

7. Find the value of p for the following distribution whose mean is 20·6 :

Variate (x_i)	10	15	20	25	35
Frequency (f_i)	3	10	p	7	5

8. Find the value of p, if the mean of the following distribution is 18.

Variate (x)	13	15	17	19	$20 + p$	23
Frequency (f)	8	2	3	4	$5p$	6

9. A trader takes a sample of 50 eggs and weighs them. The following table gives his findings :

Weight of eggs (in gm)	80 – 84	85 – 89	90 – 94	95 – 99	100 – 104	105 – 109	110 – 114
Frequency	2	6	12	14	10	5	1

Calculate the mean weight to the nearest gm.

10. Calculate the Arithmetic mean, correct to one decimal place, for the following frequency distribution :

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Students	02	04	05	16	20	10	06	08	04

continued

11. The mean of the following frequency distribution is 62·8. Find the value of p :

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	p	12	7	8

12. The measures of the diameters of the heads of 150 screws is given in the following table. If the mean diameter of the heads of the screws is 51·2 mm, find the values of p and q :

Diameter (in mm)	32 – 36	37 – 41	42 – 46	47 – 51	52 – 56	57 – 61	62 – 66
No. of screws	15	17	p	25	q	20	30

13. The median of the following numbers, arranged in ascending order, is 25. Find x :

11, 13, 15, 19, $x + 2$, $x + 4$, 30, 35, 39, 46.

14. If the median of 5, 9, 11, 3, 4, x, 8 is 6, find the value of x.

15. Find the median of :

17, 26, 60, 45, 33, 32, 29, 34, 56.

If 26 is replaced by 62, find the new median.

16. The marks scored by 16 students in a class test are :

3, 6, 8, 13, 15, 5, 21, 23, 17, 10, 9, 1, 20, 21, 18, 12.

Find : (*i*) the median (*ii*) lower quartile (*iii*) upper quartile (*iv*) inter quartile range.

17. Using the data given below, construct the cumulative frequency table and draw the ogive. From the ogive, estimate the median.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	3	8	12	14	10	6	5	2

Also state the median class.

18. Draw a cumulative frequency curve for the following data :

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students	8	10	22	40	20

Hence determine :

(*i*) the median.

(*ii*) the pass marks if 85% of the students pass.

(*iii*) the marks which 45% of the students exceed.

19. Find the median and mode for the set of numbers :

2, 2, 3, 5, 5, 5, 6, 8, 9.

20. Calculate the mean, the median and the mode of the following distribution :

Age in years	12	13	14	15	16	17	18
No. of students	2	3	5	6	4	3	2

21. The daily wages of 30 employees in an establishment are distributed as follows :

Daily wages (in ₹)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of employees	1	8	10	5	4	2

Estimate the modal daily wages for this distribution by a graphical method. (You may only consider the classes 10 – 20, 20 – 30 and 30 – 40).