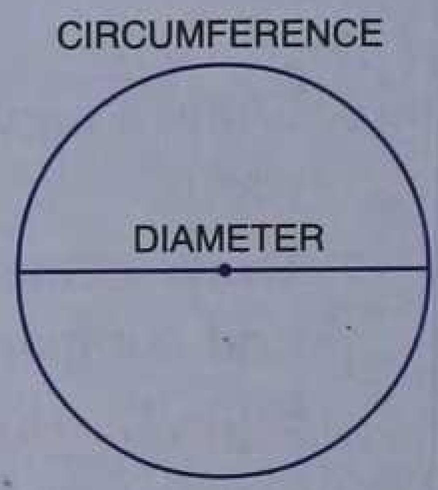
18 Mensuration

18.1 CIRCUMFERENCE AND AREA OF A CIRCLE

The ratio of circumference of any circle to its diameter is constant, and this constant ratio is denoted by π (Pi, a Greek letter) *i.e.*



$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

$\Rightarrow$ circumference = $\pi \times d$, where d is diameter of the circle.

Note

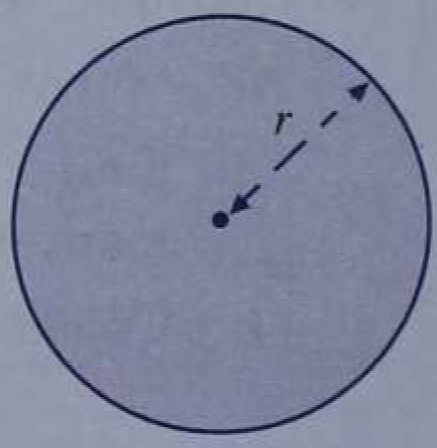
Approximate value of π is $\frac{22}{7}$ or 3·14 or $\frac{355}{113}$ or 3·1416.

1. Circumference and area of a circle.

If r is the radius of a circle, then

(*i*) *the circumference of the circle* = $2\pi r$.

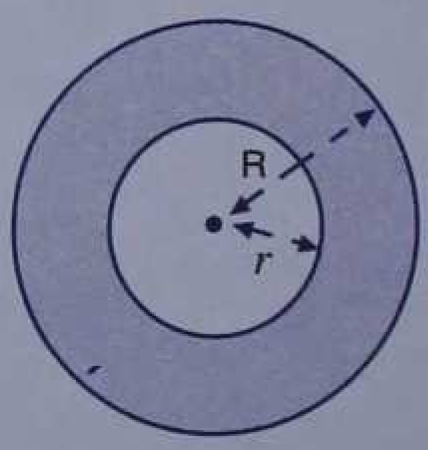
(*ii*) *the area of the circle* = πr^2.



2. Area of a circular ring.

If R and r are the radii of the bigger and smaller (concentric) circles, then

area of ring (shaded portion) = $\pi(R^2 - r^2)$.

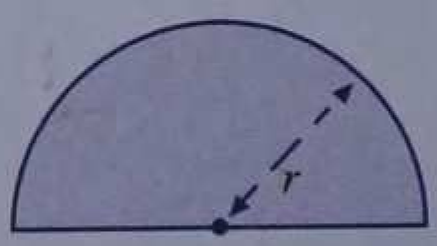


3. Perimeter and area of a semicircle.

If r is the radius of a circle, then

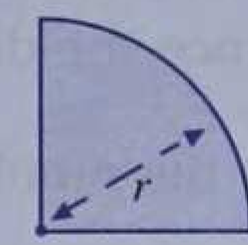
(*i*) *the perimeter of the semicircle* $= \frac{1}{2} \times 2\pi r + 2r$
$= (\pi + 2)\, r$.

(*ii*) *the area of the semicircle* $= \frac{1}{2}\pi r^2$.



4. Perimeter and area of a quadrant of a circle.

If r is the radius of a circle, then



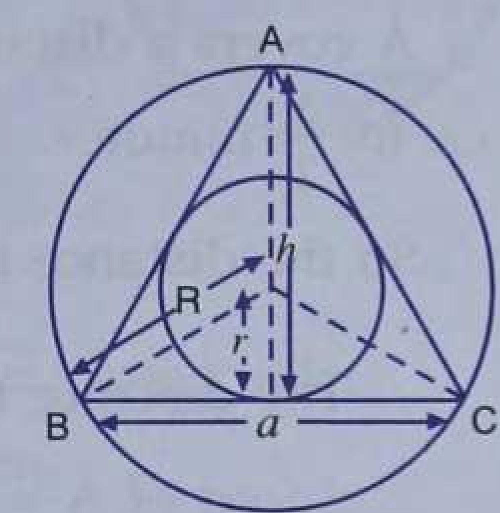
(*i*) *the perimeter of the quadrant* $= \frac{1}{4} \times 2\pi r + 2r$

$= \left(\frac{\pi}{2} + 2\right) r.$

(*ii*) *the area of the quadrant* $= \frac{1}{4} \pi r^2.$

5. Circumference and area of circumscribed and inscribed circles of an equilateral triangle.

Let ABC be an equilateral triangle of side a and height h, then $h = \frac{\sqrt{3}}{2} a.$



If R and r are the radii of the circumscribed and inscribed circles of Δ ABC, then

$R = \frac{2}{3} h$ and $r = \frac{1}{3} h$, and

(*i*) *the circumference of the circumscribed circle* $= 2\pi R = \frac{4}{3} \pi h.$

(*ii*) *the area of the circumscribed circle* $= \pi R^2 = \frac{4}{9} \pi h^2.$

(*iii*) *the circumference of the inscribed circle* $= 2\pi r = \frac{2}{3} \pi h.$

(*iv*) *the area of the inscribed circle* $= \pi r^2 = \frac{1}{9} \pi h^2.$

6. Circumference and area of circumscribed and inscribed circles of a regular hexagon.

Let a be the side of a regular hexagon, and R, r be the radii of the circumscribed, inscribed circles respectively of the regular hexagon, then $R = a$ and $r = \frac{\sqrt{3}}{2} a$, and

(*i*) *the circumference of the circumscribed circle*

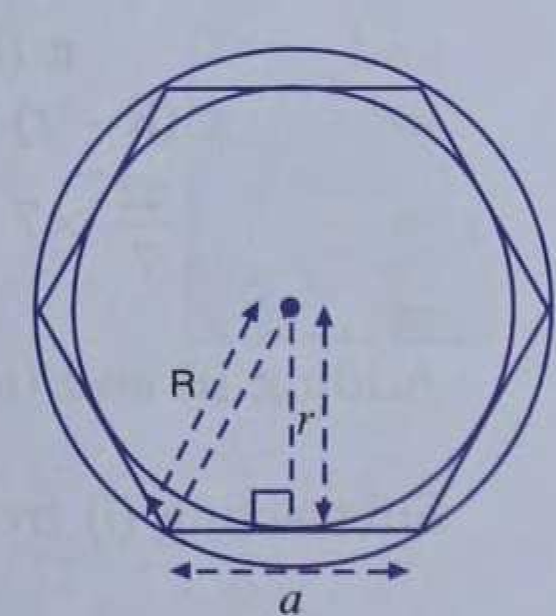
$= 2\pi R = 2\pi a.$

(*ii*) *the area of the circumscribed circle* $= \pi R^2 = \pi a^2.$

(*iii*) *the circumference of the inscribed circle*

$= 2\pi r = \sqrt{3}\ \pi a.$

(*iv*) *the area of the inscribed circle* $= \pi r^2 = \frac{3}{4} \pi a^2.$



ILLUSTRATIVE EXAMPLES

Example 1. *How many times will the wheel of a car rotate in a journey of 88 km if it is known that the diameter of the wheel is 56 cm ?* $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution. Given the diameter of the wheel = 56 cm,

$\therefore$ the radius of the wheel $= \frac{1}{2} \times 56$ cm $= 28$ cm.

$\therefore$ Circumference of the wheel $= 2\pi r$

$= 2 \times \frac{22}{7} \times 28$ cm $= 176$ cm.

$\therefore$ Distance covered by the wheel in one revolution = 176 cm.

Since the distance covered = 88 km = 88 × 1000 × 100 cm,

$\therefore$ the number of times the wheel will rotate = $\frac{88 \times 1000 \times 100}{176}$ = 50000.

Example 2. *There are two concentric circular tracks of radii 100 metres and 102 metres respectively. A runs on the inner track and goes once round the track in 1 minute 30 seconds; while B runs on the outer track in 1 minute 32 seconds. Who runs faster ?*

Solution. Circumference of the inner track = $(2\pi \times 100)$ m = 200π m.

Circumference of the outer track = $(2\pi \times 102)$ m = 204π m.

A covers a distance equal to circumference of the inner track in 1 minute 30 seconds *i.e.* in $\frac{3}{2}$ minutes.

So the distance travelled by A in $\frac{3}{2}$ minutes = 200π m

$\therefore$ The distance travelled by A in 1 minute = $\left(\frac{2}{3} \times 200\pi\right)$ m = $133{\cdot}33\pi$ m.

$\therefore$ Speed of A = $133{\cdot}33\pi$ m/min.

B covers a distance equal to circumference of the outer track in 1 minute 32 seconds *i.e.* in $\left(1 + \frac{32}{60}\right)$ min *i.e.* in $\frac{23}{15}$ min.

$\therefore$ The distance travelled by B in 1 minute = $\left(\frac{15}{23} \times 204\pi\right)$ m = $133{\cdot}04\pi$ m.

$\therefore$ Speed of B = $133{\cdot}04\pi$ m/min.

Since speed of A is greater than speed of B, therefore, A runs faster.

Example 3. *The area of a circular ring enclosed between two concentric circles is 286 cm². Find the radii of the two circles, given that their difference is 7 cm.* $\left(\textit{Take } \pi = \frac{22}{7}\right)$

Solution. Let the radii of the outer and the inner circles be R cm and r cm respectively.

According to the given information,

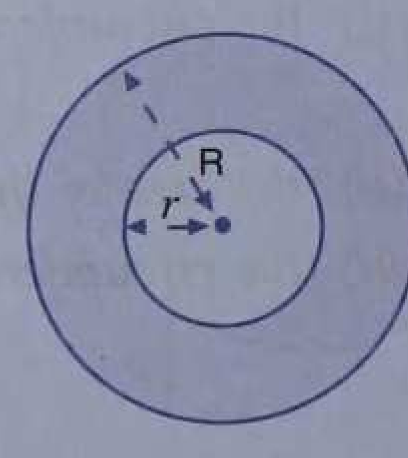
$$R - r = 7 \quad \ldots(i)$$

and $$\pi(R^2 - r^2) = 286$$

$$\Rightarrow \quad \pi(R - r)(R + r) = 286$$

$$\Rightarrow \quad \frac{22}{7} \times 7\,(R + r) = 286 \quad [\text{using } (i)]$$

$$\Rightarrow \quad R + r = 13 \quad \ldots(ii)$$



Adding (*i*) and (*ii*), we get

$$2R = 20 \Rightarrow R = 10.$$

Subtracting (*i*) from (*ii*), we get

$$2r = 6 \Rightarrow r = 3.$$

$\therefore$ The radii of the two circles are 10 cm and 3 cm.

Example 4. *Two circles touch externally. The sum of their areas is 58 π cm² and the distance between their centres is 10 cm. Find the radii of the two circles.*

Solution. Let R cm and r cm be the radii of two circles, then

$$R + r = 10$$

$$\Rightarrow \quad r = 10 - R \quad \ldots(i)$$

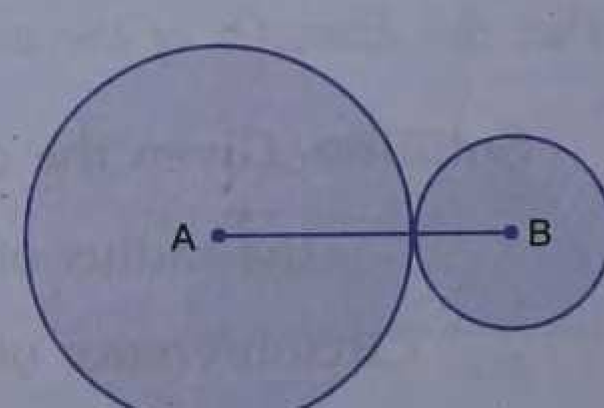
Also $\pi R^2 + \pi r^2 = 58\pi$

$$\Rightarrow \quad R^2 + r^2 = 58$$

$$\Rightarrow \quad R^2 + (10 - R)^2 = 58 \quad (\text{using } (i))$$

$$\Rightarrow \quad R^2 + 100 + R^2 - 20R - 58 = 0$$

$$\Rightarrow \quad 2R^2 - 20R + 42 = 0$$



$\Rightarrow \quad R^2 - 10R + 21 = 0 \Rightarrow (R - 7)(R - 3) = 0$

$\Rightarrow \quad R - 7 = 0$ or $R - 3 = 0 \Rightarrow R = 7$ or $R = 3.$

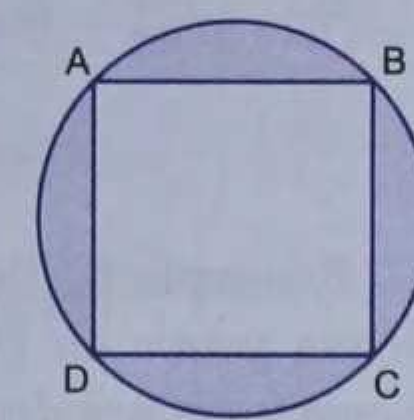
When $R = 7$, then $r = 10 - 7 = 3$ and when $R = 3$, $r = 10 - 3 = 7$.

Hence, the radii of the two circles are 7 cm and 3 cm.

Example 5. *In the adjoining figure, ABCD is a square inscribed in a circle of radius 7 cm. Calculate :*

(i) the area of the circle.

(ii) the area of the shaded portion. $\left(\text{Take } \pi = \frac{22}{7}\right)$



Solution.

(*i*) Area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 7^2 \text{ cm}^2 = 154 \text{ cm}^2.$$

(*ii*) Let a cm be the side of the square ABCD inscribed in the given circle.

Join BD, then BD = diameter of circle $= 2 \times 7$ cm $= 14$ cm.

From right angled Δ BCD, by Pythagoras Th., we get

$BC^2 + CD^2 = BD^2$

$\Rightarrow \quad a^2 + a^2 = (14)^2 \Rightarrow 2a^2 = 196 \Rightarrow a^2 = 98.$

Area of the square ABCD $= a^2 \text{ cm}^2 = 98 \text{ cm}^2$.

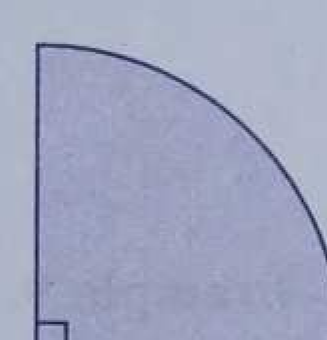
$\therefore$ The area of the shaded portion = area of circle – area of square ABCD

$= 154 \text{ cm}^2 - 98 \text{ cm}^2 = 56 \text{ cm}^2.$

Example 6. *The perimeter of a sheet of tin in the shape of a quadrant of a circle is 12·5 cm. Find its area.* $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution. Let r cm be the radius of the circle, then perimeter of a quadrant of the circle

$$= \frac{1}{4} \times 2\pi r + 2r = \left(\frac{\pi}{2} + 2\right) r.$$



According to given, $\left(\frac{\pi}{2} + 2\right) r = 12{\cdot}5$

$$\Rightarrow \quad \left(\frac{1}{2} \times \frac{22}{7} + 2\right) r = \frac{25}{2}$$

$$\Rightarrow \quad \frac{25}{7} r = \frac{25}{2} \Rightarrow r = \frac{7}{2}.$$

$$\therefore \text{ Area of quadrant} = \frac{1}{4} \pi r^2 \text{ cm}^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2$$

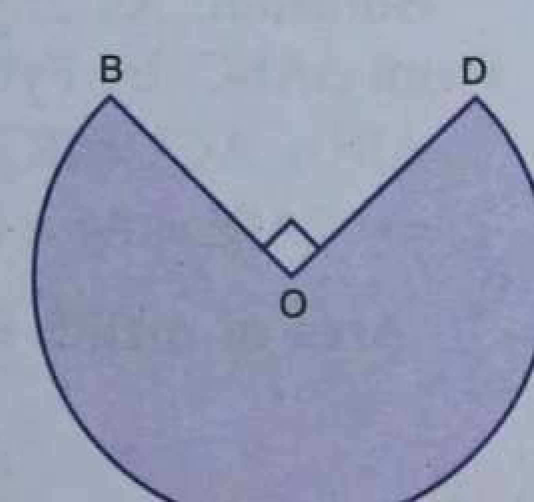
$$= \frac{77}{8} \text{ cm}^2 = 9{\cdot}625 \text{ cm}^2.$$

Example 7. *The shape of the top of a table in a restaurant is that of a sector of a circle with centre O and* $\angle BOD = 90°$. *If* $BO = OD = 60$ *cm, find :*

(i) the area of the top of the table

(ii) the perimeter of the table. Take $\pi = 3{\cdot}14$

(2002)



Solution. Given $\angle BOD = 90°$, so the top of the table is $\frac{3}{4}$ of a circle of radius 60 cm.

(*i*) The area of the top of the table $= \frac{3}{4}\pi \times 60^2 \text{ cm}^2$

$= (3 \times 60 \times 15 \times 3{\cdot}14) \text{ cm}^2$

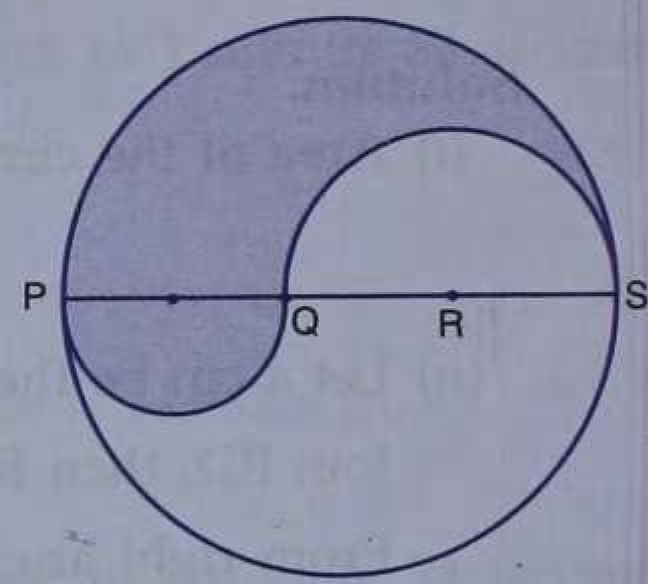
$= (2700 \times 3{\cdot}14) \text{ cm}^2 = 8478 \text{ cm}^2.$

(*ii*) The perimeter of the table $= \left(\frac{3}{4} \times 2\pi \times 60\right) \text{ cm} + (2 \times 60) \text{ cm}$

$= (90 \times 3{\cdot}14) \text{ cm} + 120 \text{ cm}$

$= (282{\cdot}6 + 120) \text{ cm} = 402{\cdot}6 \text{ cm}.$

Example 8. *PS is a diameter of a circle of radius 6 cm. Q and R are points on the diameter that PQ, QR and RS are equal. Semicircles are drawn with PQ and QS as diameters, as shown in the figure. Find the perimeter of the shaded region. π = 3·14.* (2003)



Also find the area of the shaded region.

Solution. PS = 2 × radius = (2 × 6) cm = 12 cm.

Given PQ = QR = RS

$\Rightarrow$ $PQ = \frac{1}{3}$ of $PS = \frac{1}{3}$ of 12 cm = 4 cm

and QS = 8 cm.

∴ The semicircles with PQ and QS as diameters have radii 2 cm and 4 cm respectively.

∴ Perimeter of the shaded region

$= \left(\frac{1}{2} \times 2\pi \times 6 + \frac{1}{2} \times 2\pi \times 4 + \frac{1}{2} \times 2\pi \times 2\right) \text{ cm}$

$= \pi(6 + 4 + 2) \text{ cm} = 12\pi \text{ cm}$

$= (12 \times 3{\cdot}14) \text{ cm} = 37{\cdot}68 \text{ cm}.$

Area of the shaded region

$= \left(\frac{1}{2} \times \pi \times 6^2 - \frac{1}{2} \times \pi \times 4^2 + \frac{1}{2} \times \pi \times 2^2\right) \text{ cm}^2$

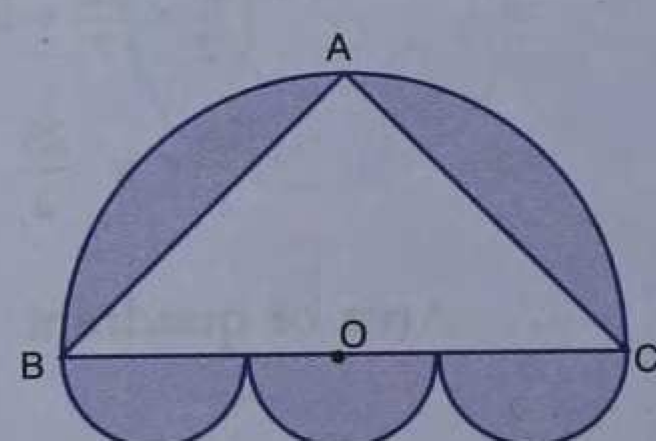
$= \frac{\pi}{2}(36 - 16 + 4) \text{ cm}^2 = 12\pi \text{ cm}^2$

$= (12 \times 3{\cdot}14) \text{ cm}^2 = 37{\cdot}68 \text{ cm}^2.$

Example 9. *A doorway is decorated as shown in the adjoining figure. There are four semicircles. BC, the diameter of the larger semicircle is of length 84 cm. The centres of the three equal semicircles lie on BC. ABC is an isosceles triangle with AB = AC.*

If BO = OC, find the area of the shaded region.

Take $\pi = \frac{22}{7}$. (2010)



Solution. As angle in a semicircle is 90°, ∠A = 90°.
From ΔABC, by Pythagoras theorem, we get

$AB^2 + AC^2 = BC^2 \Rightarrow AB^2 + AB^2 = (84)^2$

$\Rightarrow 2AB^2 = 84 \times 84 \Rightarrow AB^2 = 84 \times 42.$

Area of ΔABC $= \frac{1}{2} AB \times AC = \frac{1}{2} AB \times AB = \frac{1}{2} AB^2$

$= \frac{1}{2} \times 84 \times 42 \text{ cm}^2 = 1764 \text{ cm}^2.$

Radius of semicircle with BC as diameter $= \frac{1}{2} \times 84 \text{ cm} = 42 \text{ cm}.$

Diameter of each of three equal semicircles = $\frac{1}{3} \times 84$ cm = 28 cm

$\Rightarrow$ radius of each of three equal semicircles = 14 cm.

The area of the shaded region = area of semicircle with 42 cm as radius + area of three equal semicircles of radius 14 cm – area of ΔABC

$$= \frac{1}{2}\pi \times 42^2 \text{ cm}^2 + 3 \times \frac{1}{2}\pi \times 14^2 \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= \frac{1}{2}\pi (42^2 + 3 \times 14^2) \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 14^2 (9 + 3) \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= (22 \times 14 \times 12) \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= 3696 \text{ cm}^2 - 1764 \text{ cm}^2 = 1932 \text{ cm}^2.$$

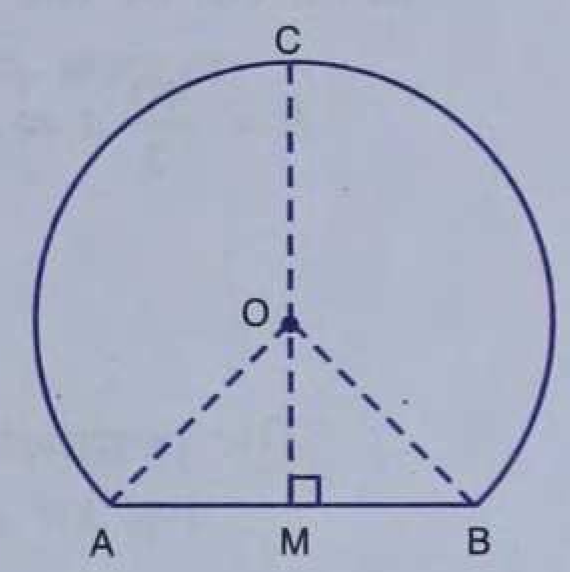
Example 10. *The given figure shows a cross-section of a railway tunnel. The radius OA of the circular part is 2 m. If $\angle AOB = 90°$, calculate :*

(i) the height of the tunnel.

(ii) the perimeter of the cross-section.

(iii) the area of the cross-section.

(Leave the answer in π and surds.)



Solution.

(*i*) Given $\angle AOB = 90°$.

$OA = OB$ (radii of the same circle) $\Rightarrow \angle OAB = \angle OBA$

$\Rightarrow \quad \angle OAB = \frac{1}{2}(180° - 90°) = 45°.$

In ΔOAM, $OM \perp AB$.

$\therefore \quad \angle AOM = 180° - (90° + 45°) = 45°.$

Thus $\angle OAM = \angle AOM \Rightarrow OM = AM.$

From right triangle OAM, by Pythagoras theorem, we get

$OA^2 = AM^2 + OM^2 \quad \Rightarrow 2^2 = OM^2 + OM^2$

$\Rightarrow \quad 2\,OM^2 = 4 \Rightarrow OM^2 = 2 \Rightarrow OM = \sqrt{2}$ m.

$\therefore$ The height of the tunnel = $(2 + \sqrt{2})$ m.

(*ii*) Since $OM \perp AB$, M is mid-point of AB.

$\therefore \quad AB = 2\,AM = 2\,OM = 2\sqrt{2}$ m.

Length of the major arc ACB = $\frac{3}{4}$ of circumference

$$= \left(\frac{3}{4} \times 2\pi \times 2\right) \text{ m} = 3\pi \text{ m}.$$

$\therefore$ The perimeter of the cross-section = $(3\pi + 2\sqrt{2})$ m.

(*iii*) Area of $\Delta OAB = \frac{1}{2} \times AB \times OM = \left(\frac{1}{2} \times 2\sqrt{2} \times \sqrt{2}\right) \text{m}^2 = 2 \text{ m}^2.$

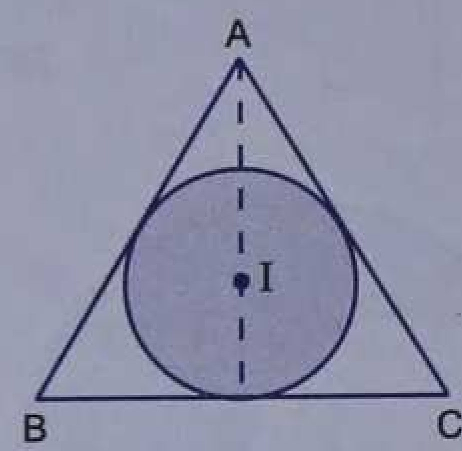
Area of the sector OACB = $\frac{3}{4}$ of area of circle of radius 2 m

$$= \left(\frac{3}{4} \times \pi \times 2^2\right) \text{ m}^2 = 3\pi \text{ m}^2.$$

$\therefore$ The area of the cross-section = $(3\pi + 2)$ m^2.

Example 11. *The area of a circle inscribed in an equilateral triangle is 154 cm². Find the perimeter of the triangle. (Take π to be $\frac{22}{7}$ and $\sqrt{3} = 1{\cdot}73$). Give your answer correct to 1 decimal place.*

Solution. Let r cm be the radius of the inscribed circle in the equilateral triangle ABC (shown in the adjoining figure), then according to given



$$\pi r^2 = 154$$

$$\Rightarrow \frac{22}{7} r^2 = 154 \quad \Rightarrow \quad r^2 = 49$$

$$\Rightarrow \quad r = 7 \text{ cm.}$$

Let h cm be the height of the Δ ABC, then $r = \frac{1}{3}h$

$\Rightarrow h = 3r = 3 \times 7$ cm = 21 cm.

Let a cm be the side of the equilateral Δ ABC, then

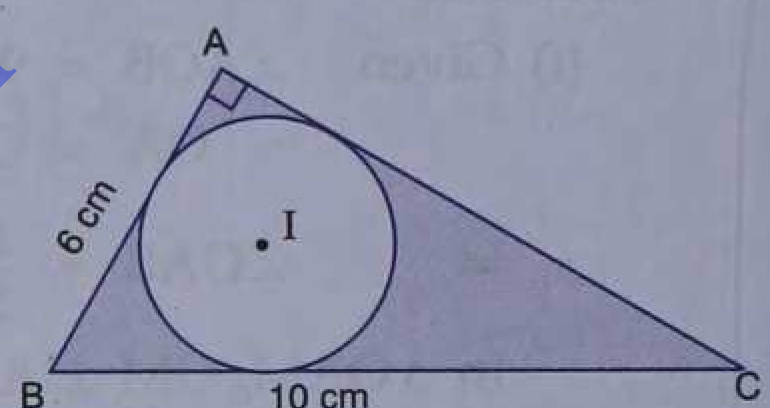
$$h = \frac{\sqrt{3}}{2} a \Rightarrow a = \frac{2}{\sqrt{3}} h = \frac{2}{\sqrt{3}} \times 21 \text{ cm} = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ cm}$$

$$= \frac{42}{3} \times \sqrt{3} \text{ cm} = 14\sqrt{3} \text{ cm.}$$

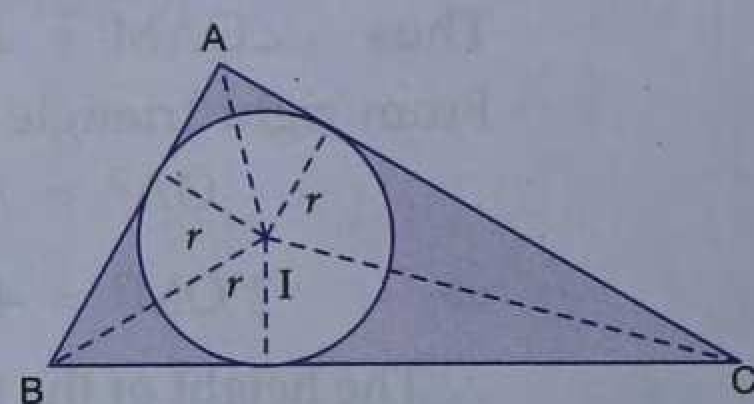
∴ The perimeter of the equilateral Δ ABC $= 3a = 3 \times 14\sqrt{3}$ cm

$= 42 \times 1{\cdot}73$ cm $= 72{\cdot}7$ cm.

Example 12. *In the adjoining figure, ABC is a right angled triangle at A. Find the area of the shaded region if AB = 6 cm, BC = 10 cm and I is the centre of incircle of Δ ABC. Take π = $\frac{22}{7}$.*



Solution. In Δ ABC, ∠A = 90°. By Pythagoras theorem, we get



$$AC^2 = BC^2 - AB^2 = 10^2 - 6^2 = 64$$

$$\Rightarrow \quad AC = 8 \text{ cm.}$$

∴ Area of Δ ABC $= \frac{1}{2} \times 6 \times 8$ cm²

$= 24$ cm².

Let r cm be the radius of the incircle. From figure,

area of Δ IBC + area of Δ ICA + area of Δ IAB = area of Δ ABC

$$\Rightarrow \quad \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r = 24$$

$$\Rightarrow \quad 12r = 24 \Rightarrow r = 2$$

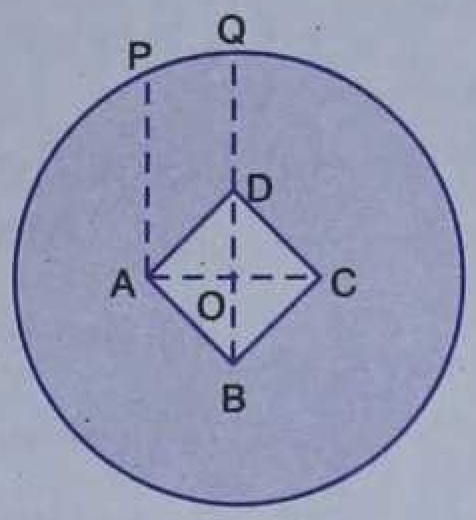
∴ Radius of incircle = 2 cm.

Area of shaded region = area of Δ ABC – area of incircle

$$= (24 - \pi \times 2^2) \text{ cm}^2 = \left(24 - \frac{22}{7} \times 4\right) \text{ cm}^2$$

$$= \frac{80}{7} \text{ cm}^2 = 11\frac{3}{7} \text{ cm}^2.$$

Example 13. *In the adjoining figure, ABCD is a square drawn inside a circle with centre O. The centre of the square coincides with O and the diagonal AC is horizontal. If AP, DQ are vertical and AP = 45 cm, DQ = 25 cm, find*



(i) the radius of the circle.

(ii) the side of the square.

(iii) the area of the shaded region.

Take $\sqrt{2} = 1{\cdot}41$ *and* $\pi = 3{\cdot}14$.

Solution.

(*i*) Let AO = x cm, then OD = x cm and radius of the circle

= OP = OQ = OD + DQ = $(x + 25)$ cm.

Since Δ PAO is right angled at A, by Pythagoras theorem, we get

$$OP^2 = AP^2 + AO^2 \Rightarrow (x + 25)^2 = 45^2 + x^2$$

$$\Rightarrow x^2 + 50x + 625 = 2025 + x^2 \Rightarrow 50x = 2025 - 625$$

$$\Rightarrow 50x = 1400 \Rightarrow x = 28,$$

$\therefore$ the radius of the circle = OP = (28 + 25) cm = 53 cm.

(*ii*) From right angled triangle AOD, by Pythagoras theorem, we get

$$AD^2 = AO^2 + OD^2 = x^2 + x^2 = 2x^2 = 2 \times 28^2$$

$$\Rightarrow AD = 28\sqrt{2} \text{ cm} = 28 \times 1{\cdot}41 \text{ cm} = 39{\cdot}48 \text{ cm}.$$

(*iii*) Area of the shaded region = area of the circle – area of the square

$$= \pi \times 53^2 - (28\sqrt{2})^2 = 3{\cdot}14 \times 2809 - 1568 = 8820{\cdot}26 - 1568$$

$$= 7252{\cdot}26 \text{ cm}^2.$$

Exercise 18.1

Take $\pi = \frac{22}{7}$, *unless stated otherwise.*

1. Find the length of the diameter of a circle whose circumference is 44 cm.
2. Find the radius and the area of a circle if its circumference is 18π cm.
3. Find the perimeter of semi-circular plate of radius 3·85 cm.
4. Find the radius and circumference of a circle whose area is 144π cm^2.
5. A sheet is 11 cm long and 2 cm wide. Circular pieces 0·5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared. (2004)

Hint

First we have to cut squares of side 0·5 cm.

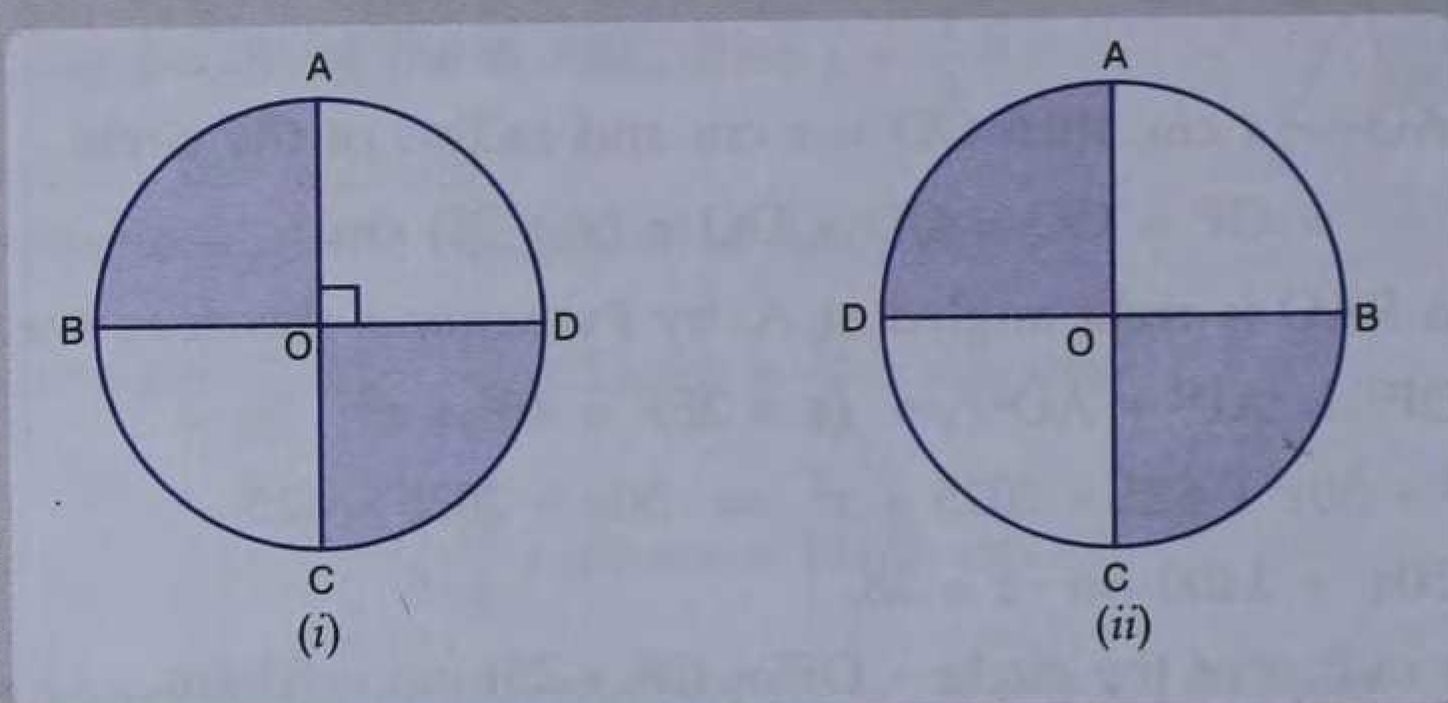
6. If the area of a semi-circular region is 77 cm^2, find its perimeter.

7. (*a*) In the figure (*i*) given below, AC and BD are two perpendicular diameters of a circle ABCD. Given that the area of the shaded portion is 308 cm^2, calculate :

(*i*) the length of AC and

(*ii*) the circumference of the circle. (2004)

(*b*) In the figure (*ii*) given below, AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16 cm, calculate the area and perimeter of the shaded part. (Take $\pi = 3{\cdot}14$) (2009)



8. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 minute 28 seconds with a uniform speed of 1·1 m/sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

9. The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Give your answer, correct to the nearest km.

10. The circumference of a circle is 123·2 cm. Calculate :

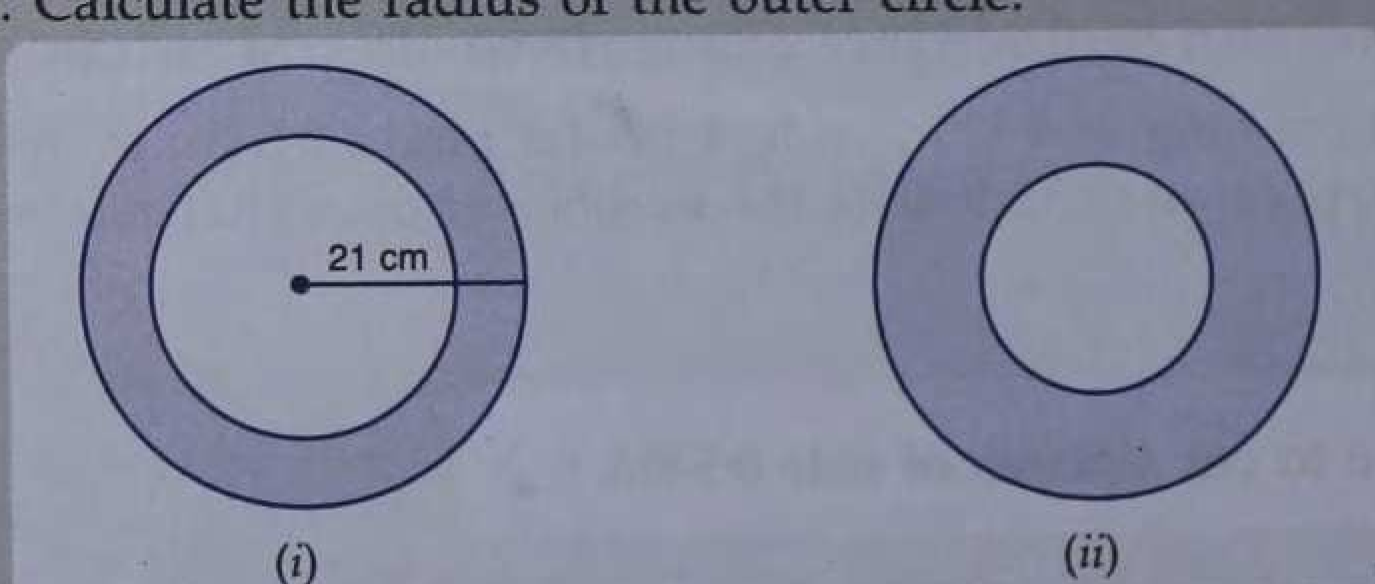
(*i*) the radius of the circle in cm.

(*ii*) the area of the circle in cm^2, correct to the nearest cm^2.

(*iii*) the effect on the area of the circle if the radius is doubled.

11. (*a*) In the figure (*i*) given below, the area enclosed between the concentric circles is 770 cm^2. Given that the radius of the outer circle is 21 cm, calculate the radius of the inner circle. (2001)

(*b*) In the figure (*ii*) given below, the area enclosed between the circumferences of two concentric circles is 346·5 cm^2. The circumference of the inner circle is 88 cm. Calculate the radius of the outer circle.



12. A road 3·5 m wide surrounds a circular plot whose circumference is 44 m. Find the cost of paving the road at ₹ 50 per m^2.

13. The sum of diameters of two circles is 14 cm and the difference of their circumferences is 8 cm. Find the circumferences of the two circles.

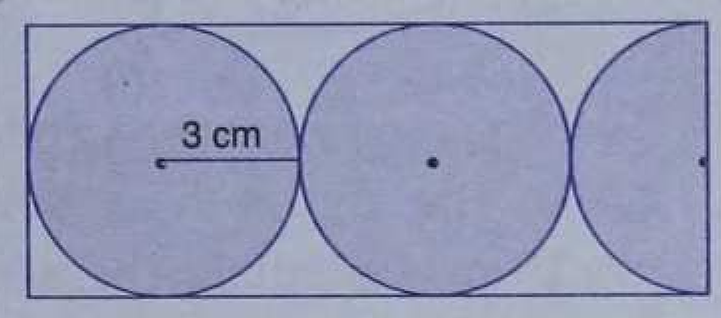
14. Find the circumference of the circle whose area is equal to the sum of the areas of three circles with radius 2 cm, 3 cm and 6 cm.

15. A copper wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent into the form of a circle, find the area of the circle.

16. A copper wire when bent in the form of an equilateral triangle has area $121\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle, find the area enclosed by the wire.

17. (*a*) Find the circumference of the circle whose area is 16 times the area of the circle with diameter 7 cm.

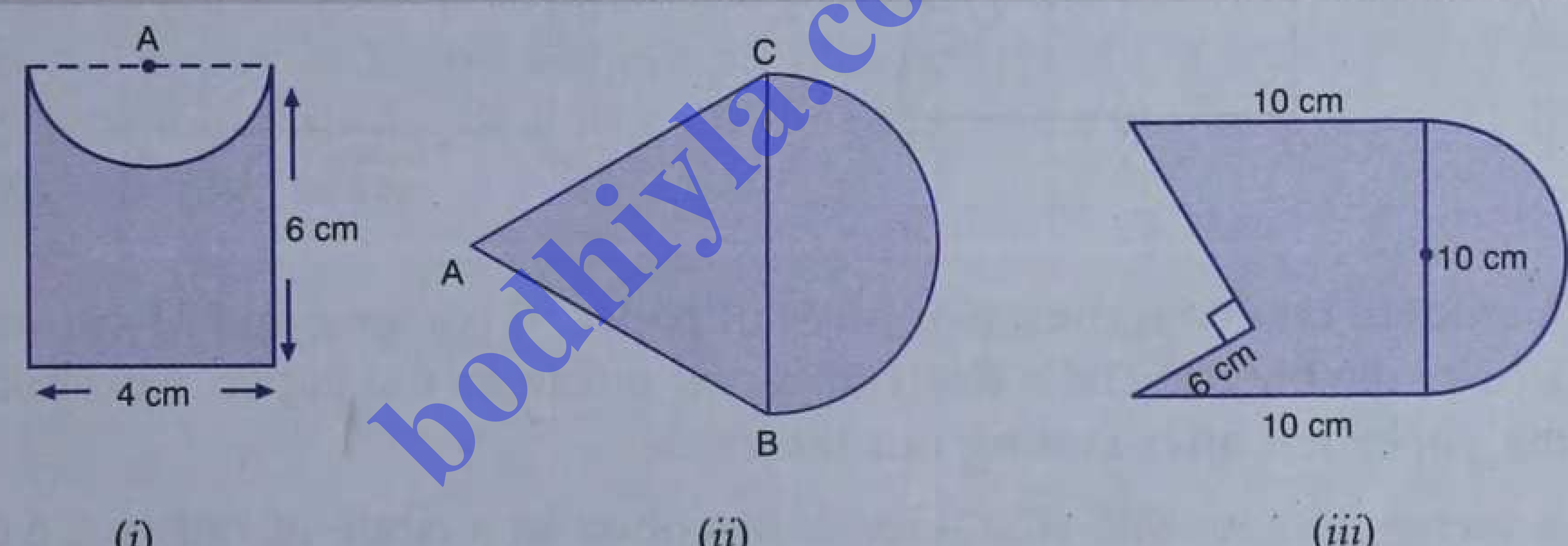
(*b*) In the given figure, find the area of the unshaded portion within the rectangle. (Take $\pi = 3{\cdot}14$) (*2008*)



18. (*a*) In the figure (*i*) given below, A is the centre of the arc of the circle. Find the perimeter and the area of the shaded region.

(*b*) In the figure (*ii*) given below, ABC is an equilateral triangle of side 14 cm, side BC is the diameter of a semicircle. Find the area of the shaded region. Take $\sqrt{3} = 1{\cdot}732$. (*2007*)

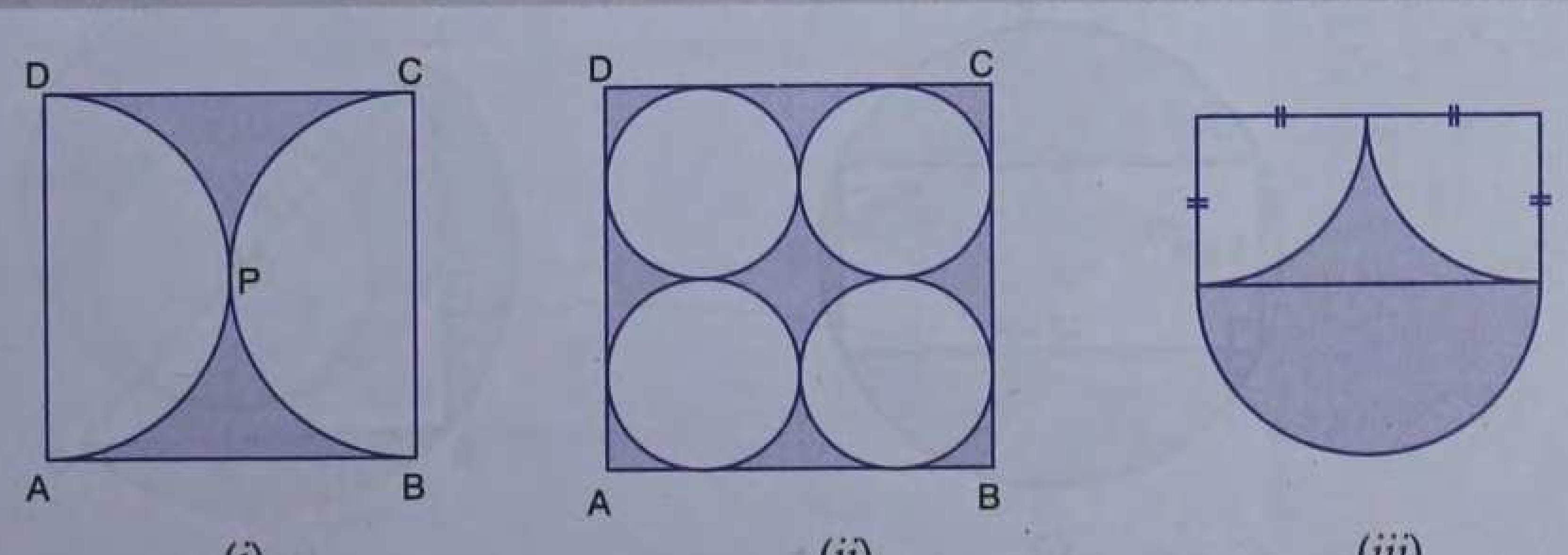
(*c*) Find the perimeter and area of the shaded region in figure (*iii*) given below in square cm correct to one place of decimal.



19. (*a*) In the figure (*i*) given below, ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area and the perimeter of the shaded region.

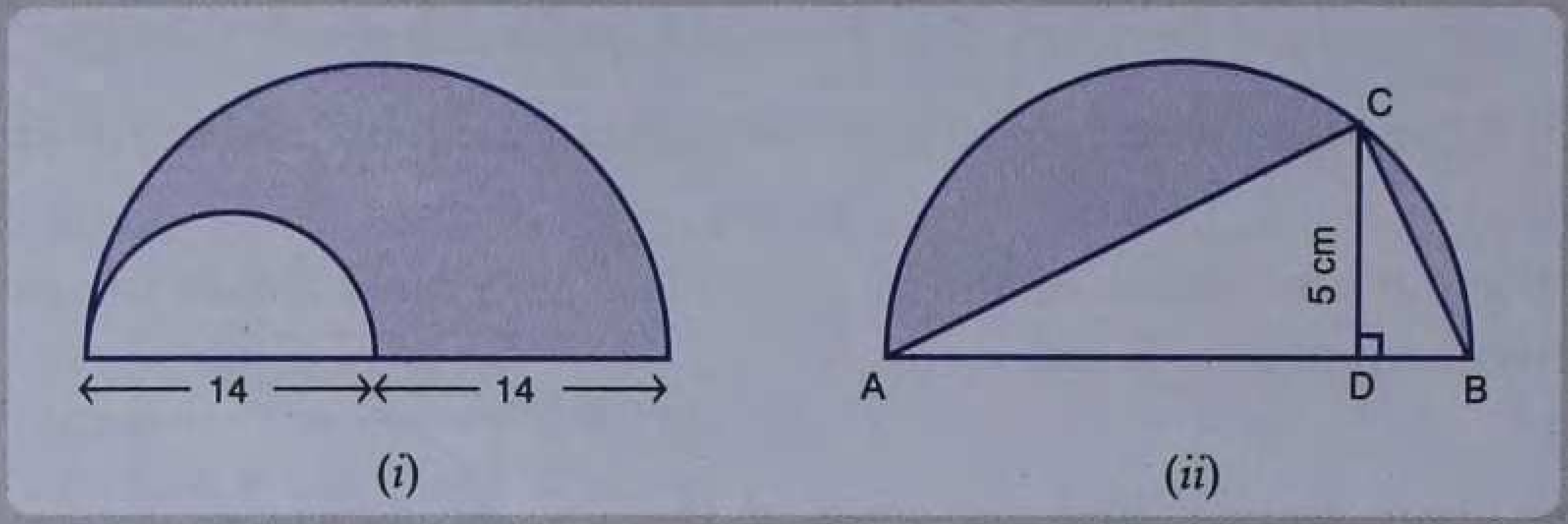
(*b*) In the figure (*ii*) given below, ABCD is a square of side 14 cm. Find the area and the perimeter of the shaded region.

(*c*) In the figure (*iii*) given below, the diameter of the semicircle is equal to 14 cm. Calculate the area of the shaded region. Take $\pi = \frac{22}{7}$. (*2011*)



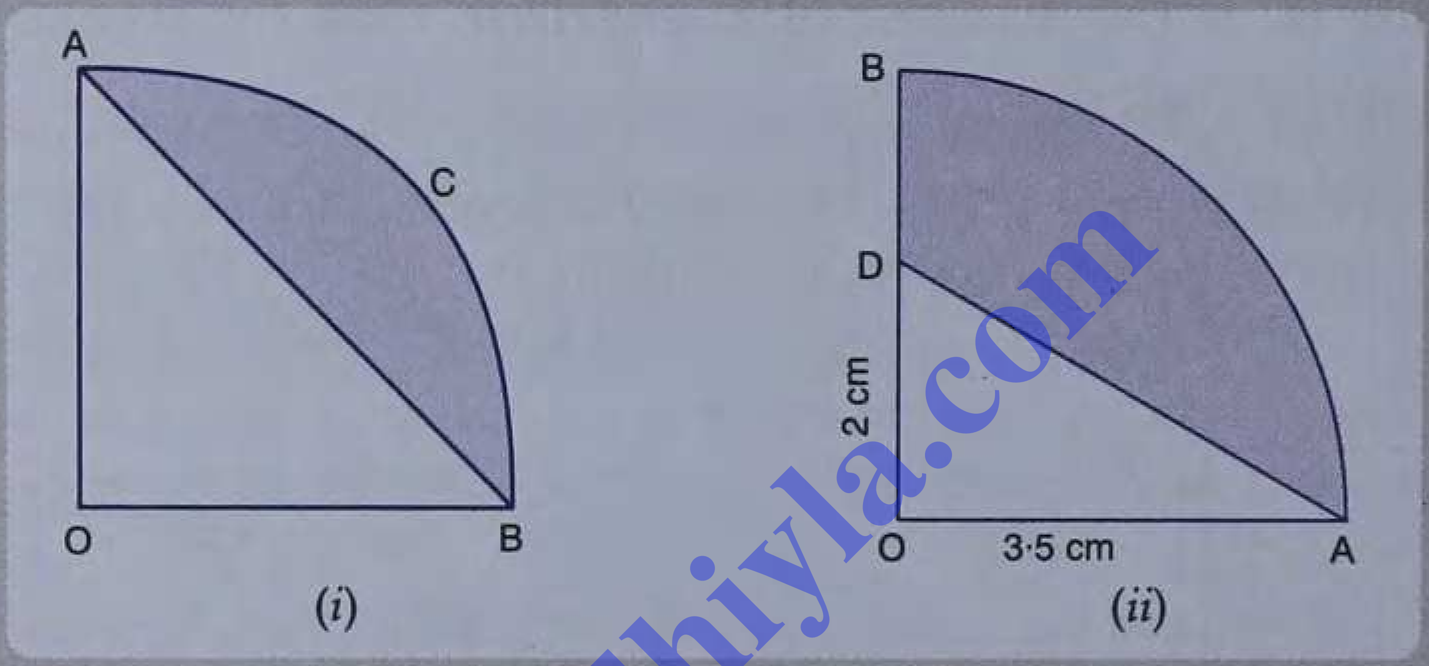
20. (*a*) Find the area and the perimeter of the shaded region in figure (*i*) given below. The diamensions are in centimetres.

(*b*) In the figure (*ii*) given below, area of $\Delta ABC = 35 \text{ cm}^2$. Find the area of the shaded region.



21. (*a*) In the figure (*i*) given below, AOBC is a quadrant of a circle of radius 10 m. Calculate the area of the shaded portion. Take $\pi = 3{\cdot}14$ and give your answer correct to two significant figures.

(*b*) In the figure (*ii*) given below, OAB is a quadrant of a circle. The radius OA = 3·5 cm and OD = 2 cm. Calculate the area of the shaded portion. *(2013, 05)*

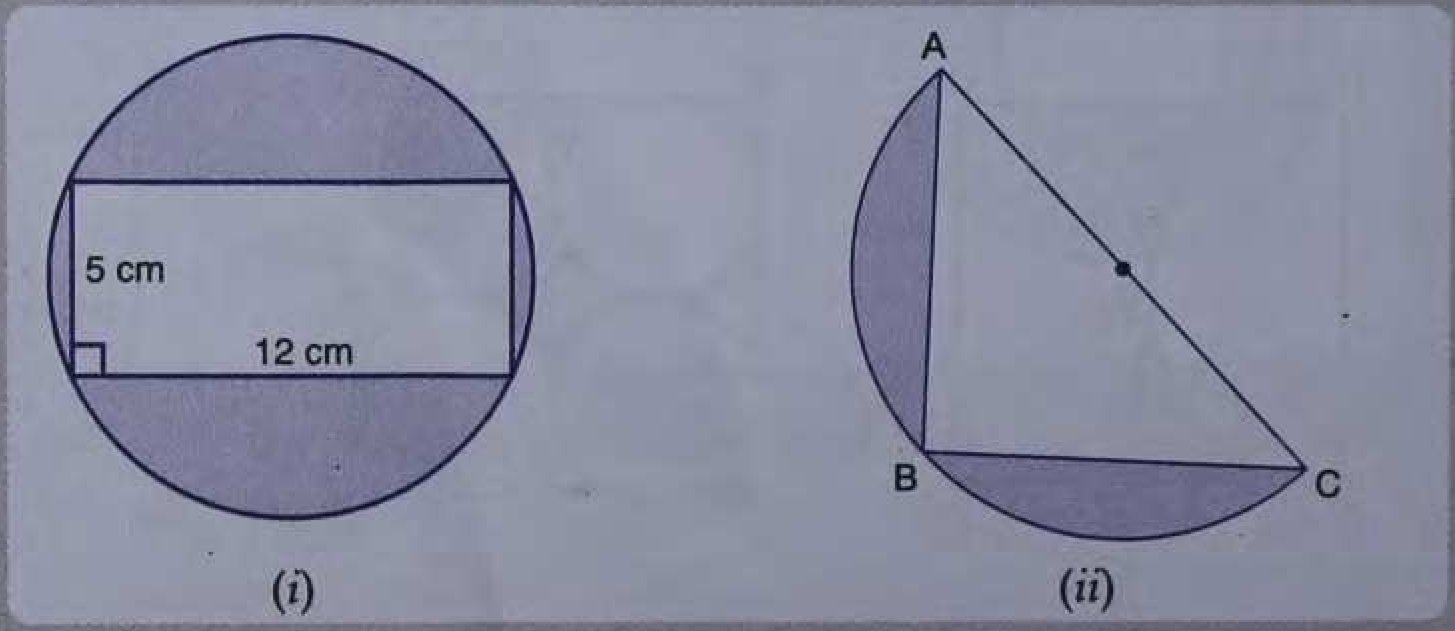


22. A student takes a rectangular piece of paper 30 cm long and 21 cm wide. Find the area of the biggest circle that can be cut out from the paper. Also find the area of the paper left after cutting out the circle.

23. A rectangle with one side 4 cm is inscribed in a circle of radius 2·5 cm. Find the area of the rectangle.

24. (*a*) In the figure (*i*) given below, calculate the area of the shaded region correct to two decimal places. (Take $\pi = 3{\cdot}142$).

(*b*) In the figure (*ii*) given below, ABC is an isosceles right angled triangle with $\angle ABC = 90°$. A semicircle is drawn with AC as diameter. If AB = BC = 7 cm, find the area of the shaded region. Take $\pi = \frac{22}{7}$. *(2012)*

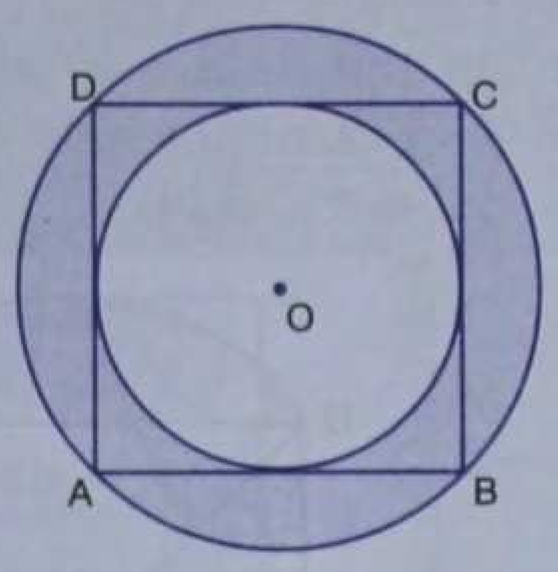


25. A circular field has perimeter 660 m. A plot in the shape of a square having its vertices on the circumference is marked in the field. Calculate the area of the square field.

26. In the adjoining figure, ABCD is a square. Find the ratio between

(i) the circumferences

(ii) the areas of the incircle and the circumcircle of the square.



Hint

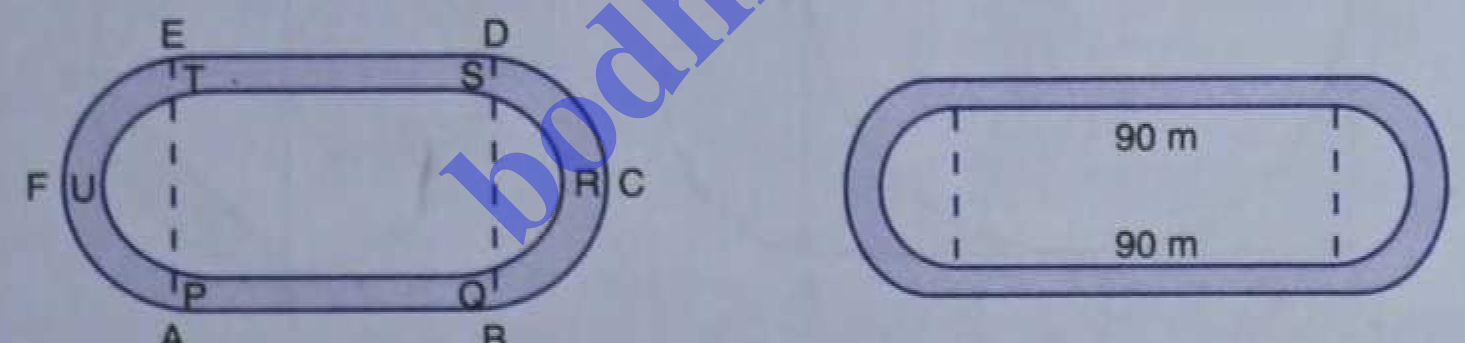
Let a side of the square be $2a$ units, then radius of incircle = a units and radius of circumcircle = $\sqrt{2}\,a$ units (why?)

27. (a) The figure (i) given below shows a running track surrounding a grassed enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semicircular region at each end. PQ = 200 m; PT = 70 m.

(i) Calculate the area of the grassed enclosure in m^2.

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF of the track.

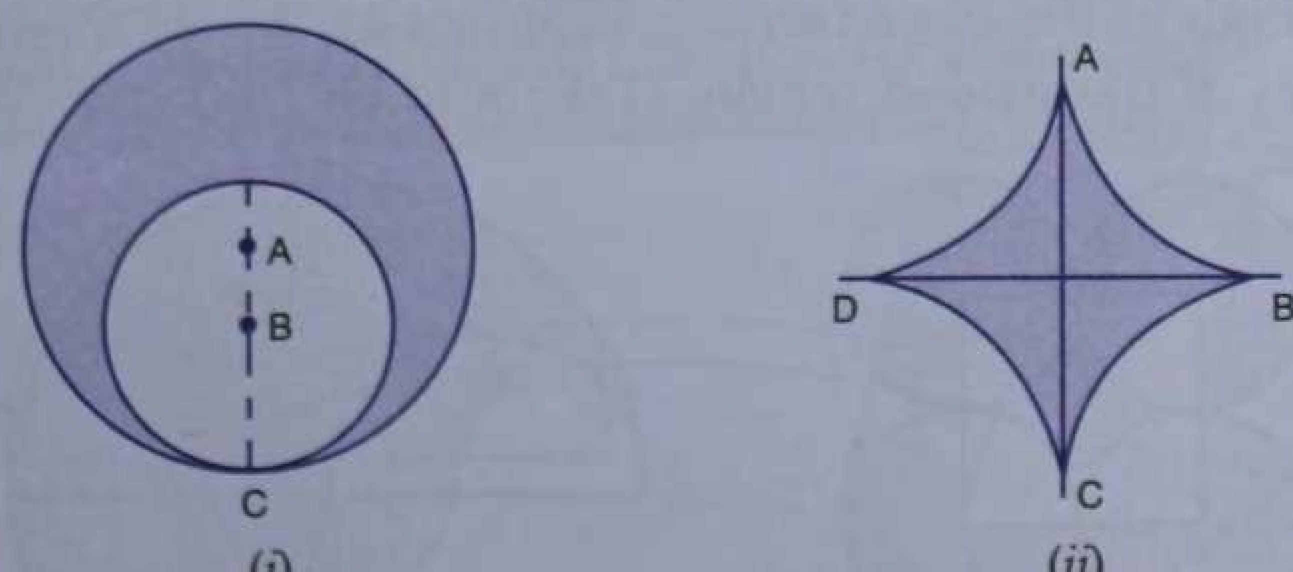
(b) In the figure (ii) given below, the inside perimeter of a practice running track with semi-circular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



(i) (ii)

28. (a) In the figure (i) given below, two circles with centres A and B touch each other at the point C. If AC = 8 cm and AB = 3 cm, find the area of the shaded region.

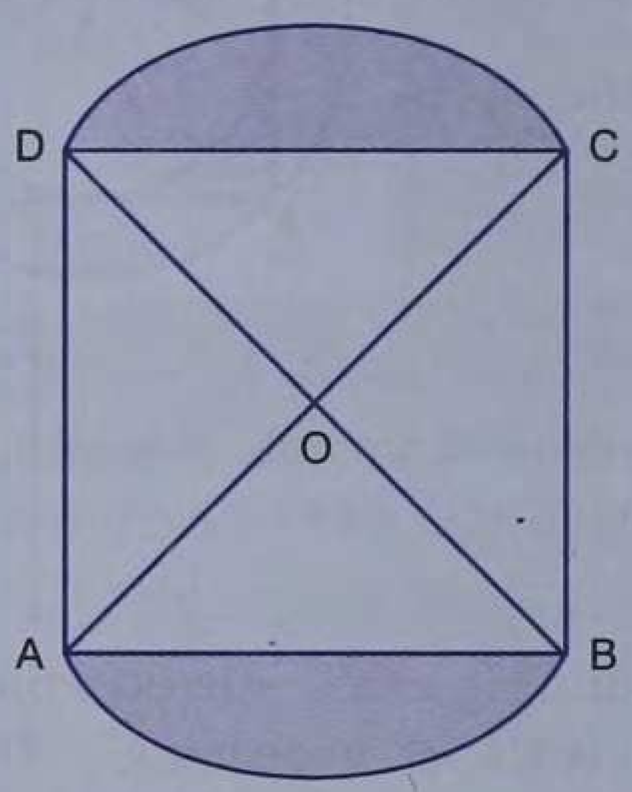
(b) The quadrants shown in the figure (ii) given below are each of radius 7 cm. Calculate the area of the shaded portion. (2000)



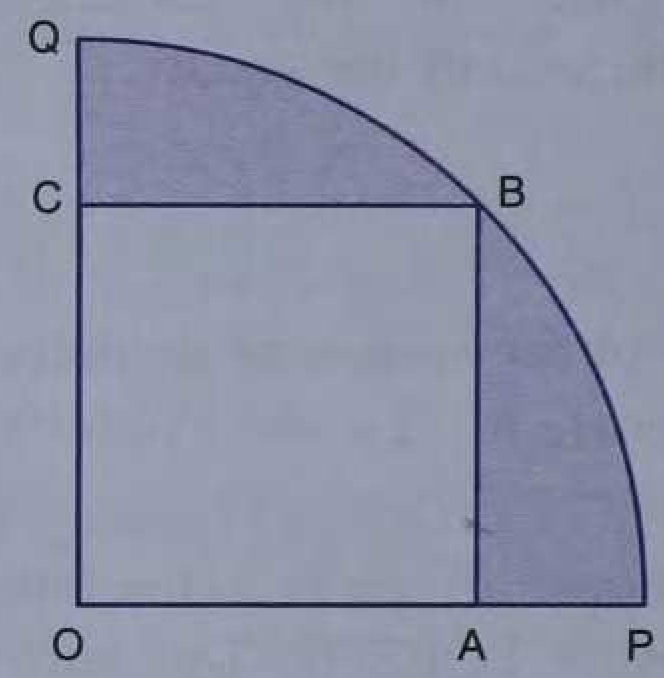
(i) (ii)

29. (*a*) In the figure (*i*) given below, two circular flower beds have been shown on the two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.

(*b*) In the figure (*ii*) given below, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3{\cdot}14$)



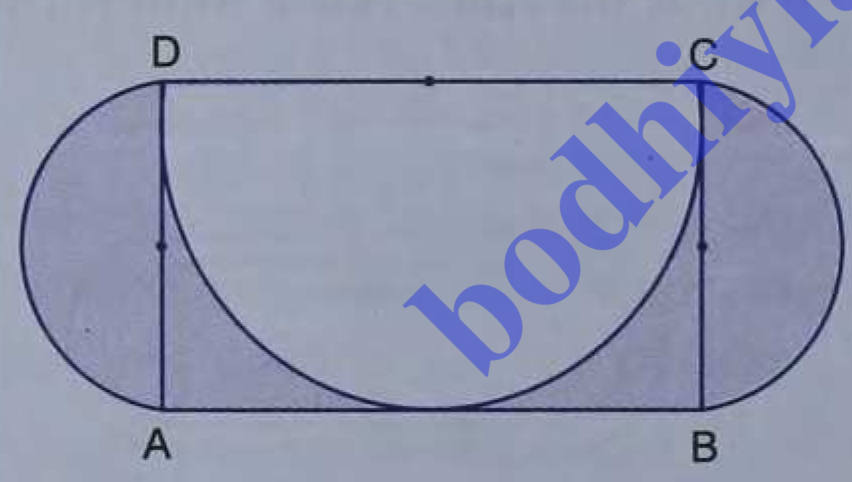
(*i*)



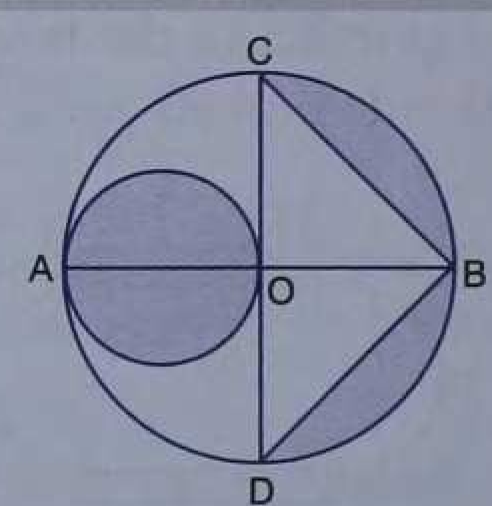
(*ii*)

30. (*a*) In the figure (*i*) given below, ABCD is a rectangle, AB = 14 cm and BC = 7 cm. Taking DC, BC and AD as diameters, three semicircles are drawn as shown in the figure. Find the area of the shaded portion.

(*b*) In the figure (*ii*) given below, AB and CD are diameters of the circle with centre O. If OA = 7 cm, find the area of the shaded region. (*2006*)



(*i*)



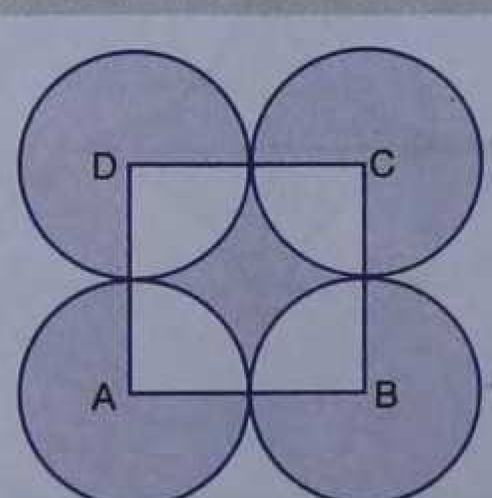
(*ii*)

31. (*a*) In the figure (*i*) given below, ABCD is a square of side 14 cm. A, B, C and D are centres of the equal circles which touch externally in pairs. Find the area of the shaded region.

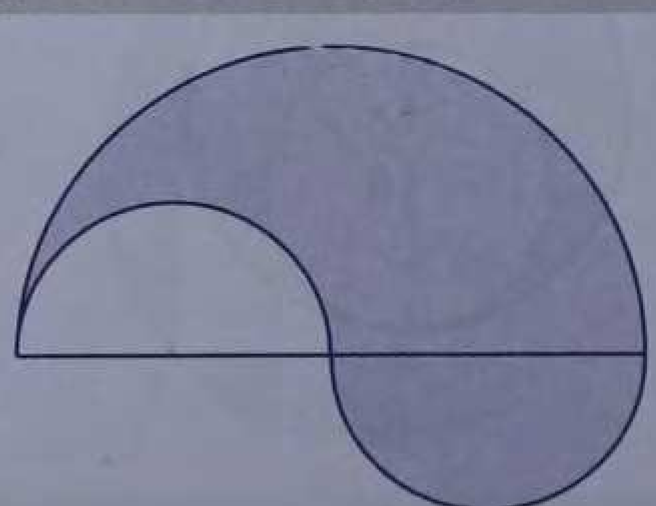
(*b*) In the figure (*ii*) given below, the boundary of the shaded region in the given diagram consists of three semi-circular arcs, the smaller being equal. If the diameter of the larger one is 10 cm, calculate.

(*i*) the length of the boundary.

(*ii*) the area of the shaded region. (Take π to be $3{\cdot}14$)

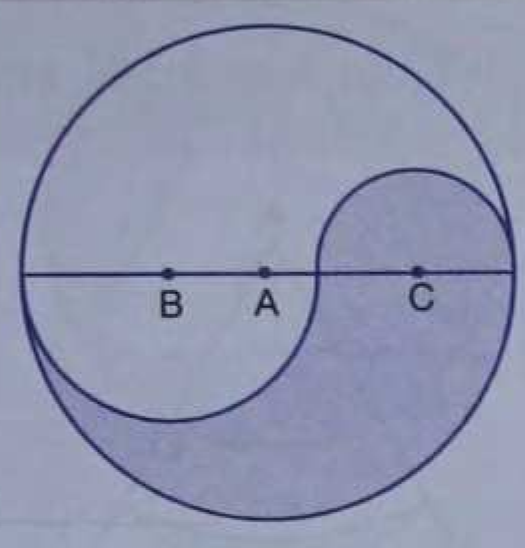


(*i*)

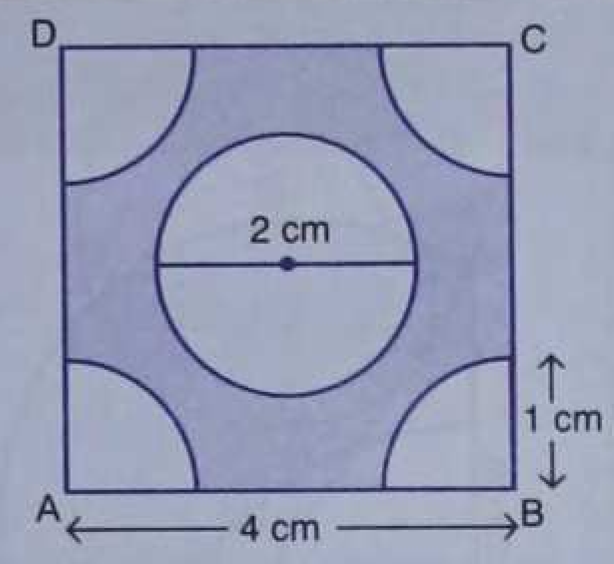

(*ii*)

32. (*a*) In the figure (*i*) given below, the points A, B and C are centres of arcs of circles of radii 5 cm, 3 cm and 2 cm respectively. Find the perimeter and the area of the shaded region. (Take $\pi = 3{\cdot}14$)

(*b*) In the figure (*ii*) given below, ABCD is a square of side 4 cm. At each corner of the square a quarter circle of radius 1 cm, and at the centre a circle of diameter 2 cm are drawn. Find the perimeter and the area of the shaded region. Take $\pi = 3{\cdot}14$.



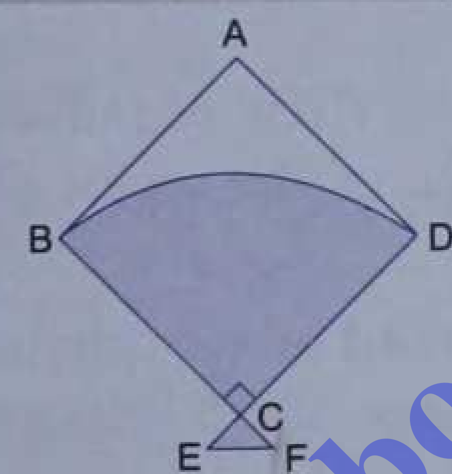
(*i*)



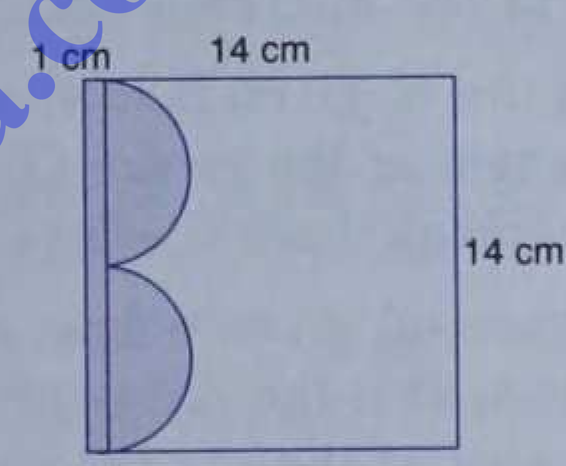
(*ii*)

33. (*a*) The figure (*i*) given below shows a kite, in which BCD is in the shape of a quadrant of a circle of radius 42 cm. ABCD is a square and ΔCEF is an isosceles right angled triangle whose equal sides are 6 cm long. Find the area of the shaded region.

(*b*) In the figure (*ii*) given below, from a sheet of cardboard in the shape of a square of side 14 cm, a piece in the shape of the letter B is cut off. The curved side of the letter consists of two equal semicircles and the breadth of the rectangular piece is 1 cm. Find the area of the remaining part of the cardboard.



(*i*)



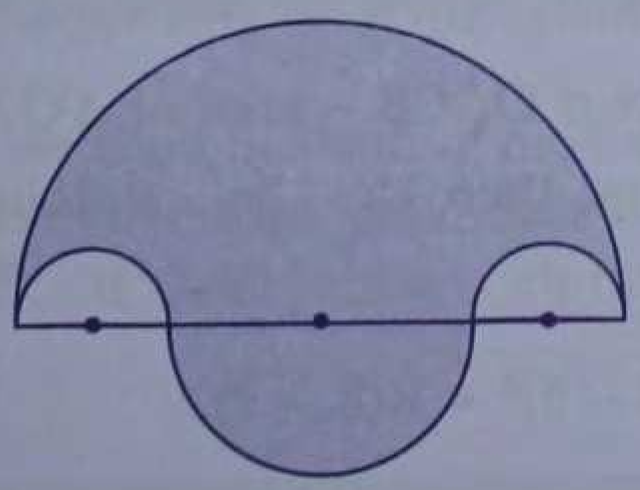
(*ii*)

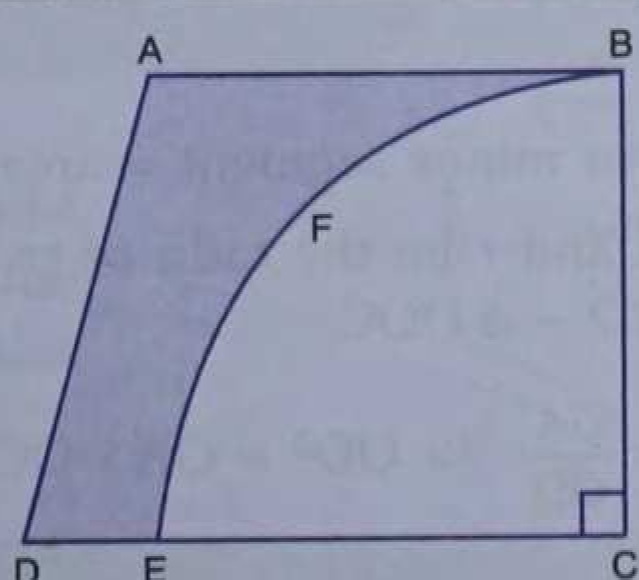
34. (*a*) In the figure (*i*) given below, the boundary of the shaded region in the given diagram consists of four semi-circular arcs, the smallest two being equal. If the diameter of the largest is 14 cm and of the smallest is 3·5 cm, calculate

(*i*) the length of the boundary.

(*ii*) the area of the shaded region.

(*b*) In the figure (*ii*) given below, a piece of cardboard, in the shape of a trapezium ABCD, and AB $\parallel$ DC and $\angle$BCD = 90°, quarter circle BFEC is removed. Given AB = BC = 3·5 cm and DE = 2 cm. Calculate the area of the remaining piece of the cardboard.


(*i*)

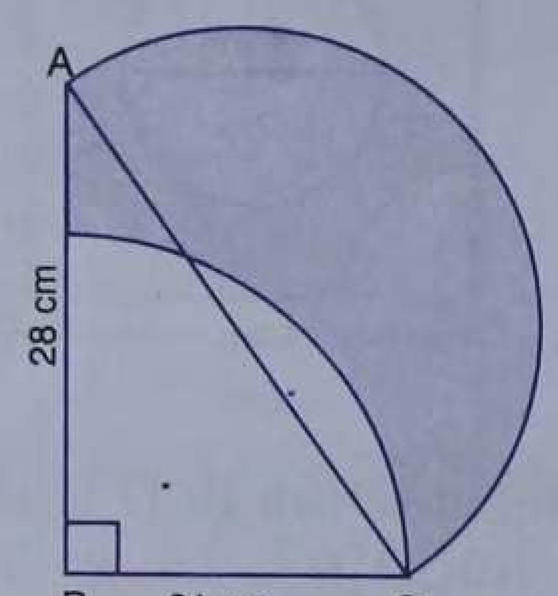


(*ii*)

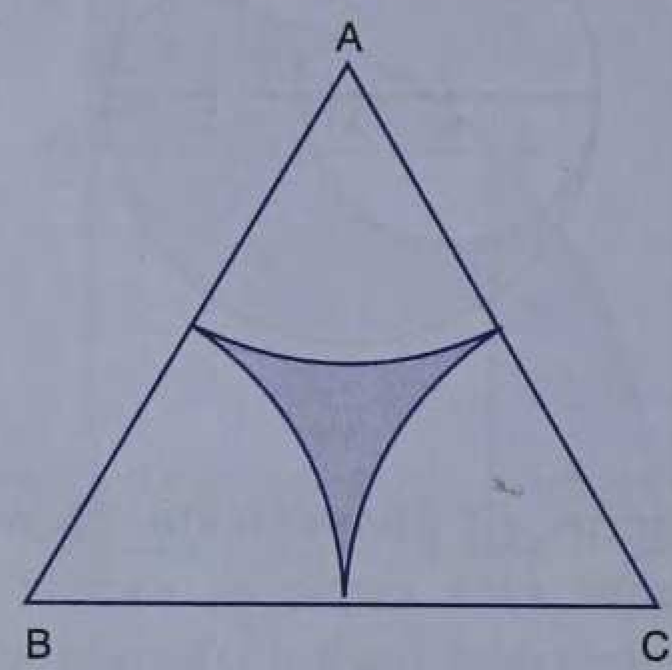
35. (*a*) In the figure (*i*) given below, ABC is a right angled triangle, $\angle B = 90°$, AB = 28 cm and BC = 21 cm. With AC as diameter a semicircle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region correct to two decimal places.

(*b*) In the figure (*ii*) given below, ABC is an equilateral triangle of side 8 cm. A, B and C are the centres of circular arcs of equal radius. Find the area of the shaded region correct upto 2 decimal places.

(Take $\pi = 3{\cdot}142$ and $\sqrt{3} = 1{\cdot}732$).



(*i*)



(*ii*)

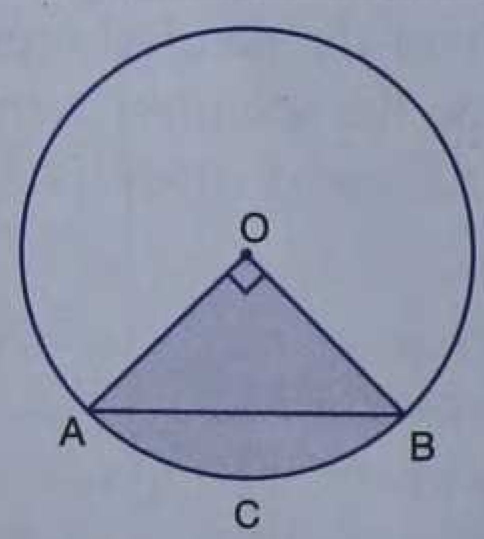
36. A circle is inscribed in a regular hexagon of side $2\sqrt{3}$ cm. Find

(*i*) the circumference of the inscribed circle.

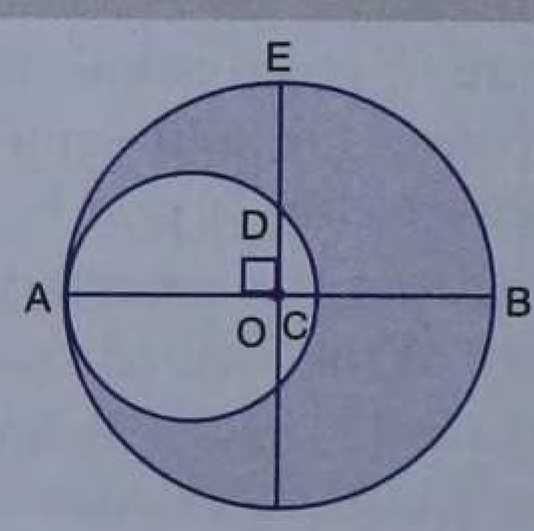
(*ii*) the area of the inscribed circle.

37. (*a*) In the figure (*i*) given below, a chord AB of a circle of radius 10 cm subtends a right angle at the centre O. Find the area of the sector OACB and of the major segment. Take $\pi = 3{\cdot}14$.

(*b*) In the figure (*ii*) given below, *crescent* is formed by two circles which touch at the point A, O is the centre of the bigger circle. If CB = 9 cm and ED = 5 cm, find the area of the shaded region. Take $\pi = 3{\cdot}14$.



(*i*)



(*ii*)

Hint

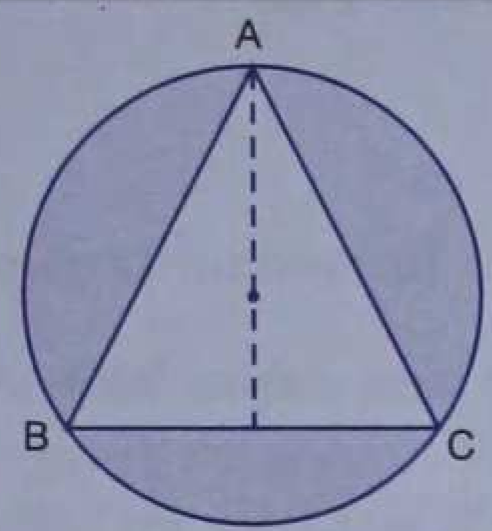
(*a*) Area of minor segment = area of sector OACB – area of ΔOAB.

(*b*) Let R and *r* be the radii of two circles, then $2R - 2r = 9$. Join AD and CD, $\Delta AOD \sim \Delta DOC$

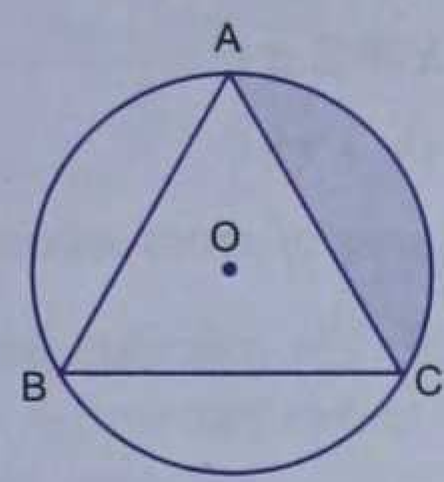
$$\Rightarrow \frac{OD}{OC} = \frac{OA}{OD} \Rightarrow OD^2 = OA \times OC \Rightarrow (R-5)^2 = R(R-9).$$

38. (*a*) In the figure (*i*) given below, ABC is an equilateral triangle inscribed in a circle of radius 4 cm. Find the area of the shaded region. Leave the answer in π and surds.

(*b*) In the figure (*ii*) given below, ABC is an equilateral triangle inscribed in a circle of radius 4 cm with centre O. Find the area of the shaded region. Leave the answer in π and surds.



(*i*)



(*ii*)

Hint

(*a*) R (radius of circumcircle) $= \frac{2}{3}$ h (height of Δ) and $h = \frac{\sqrt{3}}{2}$ a (side of Δ)

$\Rightarrow R = \frac{2}{3} \cdot \frac{\sqrt{3}}{2} a = \frac{1}{\sqrt{3}} a \Rightarrow 4 = \frac{1}{\sqrt{3}} a \Rightarrow a = 4\sqrt{3}$.

Required area = area of circle – area of Δ ABC $= \pi.4^2 - \frac{\sqrt{3}}{4}(4\sqrt{3})^2$.

(*b*) $\angle AOC = 2.\angle ABC = 2 \times 60° = 120°$

$\Rightarrow$ required area $= \frac{1}{3}$(area of circle – area of Δ ABC).

18.2 SURFACE AREA AND VOLUME OF CYLINDER

Cylinder. *A solid obtained by revolving a rectangular lamina about one of its sides is called a* ***right circular cylinder****. In other words, a solid whose cross-sections (perpendicular to its height) are circles congruent to each other is called a* ***right circular cylinder****.*

(*i*) The radius of any circular cross-section is called the *radius of the cylinder*.

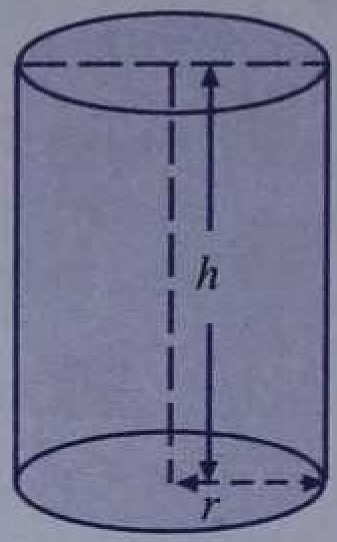
(*ii*) The line joining the centres of circular cross-sections at the ends is called the *axis of the cylinder*.

(*iii*) The distance between the centres of circular cross-sections at the ends is called the *height (or length) of the cylinder*.

In this book, we shall be dealing only with right circular cylinders. Therefore, whenever we use the word 'cylinder', it will mean right circular cylinder.

Solid cylinder

Let r be the radius and h be the height of a solid cylinder, then



(i) *Curved (lateral) surface area*

$= \text{perimeter of cross-section} \times \text{height} = 2\pi r h.$

(ii) *Total surface area*

$= \text{curved surface area} + \text{area of two circular ends}$

$= 2\pi r h + 2.\pi r^2$

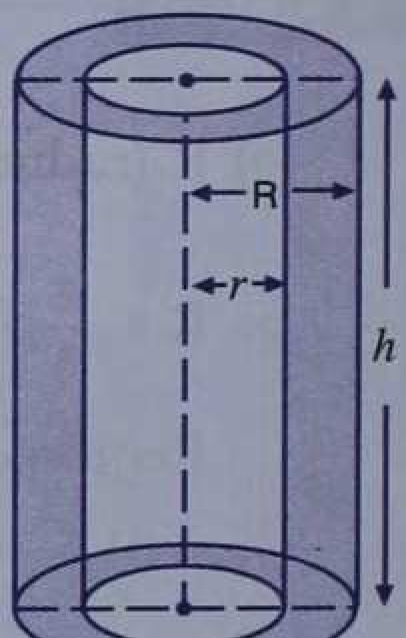
$= 2\pi r\,(h + r).$

(iii) *Volume* $= \text{area of cross-section} \times \text{height}$

$= \pi r^2 h.$

Hollow cylinder

Let R and r be the external and internal radii of a hollow cylinder, and h be its height, then



(i) *Thickness of cylinder* $= R - r.$

(ii) *Area of a cross-section* $= \pi (R^2 - r^2).$

(iii) *External curved surface area* $= 2\pi R h.$

(iv) *Internal curved surface area* $= 2\pi r h.$

(v) *Total surface area* = *external curved area* + *internal curved area* + *area of two ends*

$= 2\pi R h + 2\pi r h + 2\pi (R^2 - r^2)$

$= 2\pi (Rh + rh + R^2 - r^2).$

(vi) *Volume of the material* $= \pi R^2 h - \pi r^2 h$

$= \pi (R^2 - r^2) h.$

ILLUSTRATIVE EXAMPLES

Example 1. *The volume of a right circular cylinder is 616 cm³ and its height is 16 cm. Find :*

(i) *its radius* (ii) *its total surface area.*

Solution.

(i) Let r cm be the radius of the circular cylinder, then its

volume $= \pi r^2 h = 616 \text{ cm}^3$ (given)

$\Rightarrow \frac{22}{7} \times r^2 \times 16 = 616 \Rightarrow r^2 = \frac{616 \times 7}{22 \times 16} = \frac{49}{4} \Rightarrow r = \frac{7}{2}.$

$\therefore$ Radius of cylinder $= 3{\cdot}5$ cm.

(ii) Total surface area $= 2\pi r (h + r)$

$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(16 + \frac{7}{2}\right) \text{ cm}^2 = 22 \times \frac{39}{2} \text{ cm}^2$

$= 429 \text{ cm}^2.$

Example 2. *Find the number of coins, 2·4 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 12 cm and diameter 6 cm.*

Solution. Each coin is a right circular cylinder with radius $= \frac{2{\cdot}4}{2}$ cm $= \frac{6}{5}$ cm and height $= 2$ mm $= \frac{1}{5}$ cm.

$\therefore$ Volume of one coin $= \pi \times \left(\frac{6}{5}\right)^2 \times \frac{1}{5}$ cm^3 $= \frac{36\pi}{125}$ cm^3

Radius of required cylinder $= \frac{6}{2}$ cm = 3 cm and its height = 12 cm.

$\therefore$ Volume of the required cylinder $= \pi \times 3^2 \times 12$ cm^3 $= 108\pi$ cm^3

$\therefore$ The number of coins $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 coin}}$

$$= \frac{108\pi}{\frac{36}{125}\pi} = \frac{108 \times 125}{36} = 375.$$

Hence, the required number of coins = 375.

Example 3. *The total surface area of a hollow metal cylinder, open at both ends, of external radius 8 cm and height 10 cm is 338π cm². Taking r cm to be inner radius, write down an equation in r and use it to find the thickness of the metal in the cylinder.*

Solution. External radius of hollow cylinder = R = 8 cm and its height = h = 10 cm.

$$\begin{aligned}\text{Total surface area} &= (2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)) \text{ cm}^2\\ &= 2\pi[h(R + r) + (R^2 - r^2)] \text{ cm}^2\\ &= 2\pi(R + r)(h + R - r) \text{ cm}^2\\ &= 2\pi(8 + r)(10 + 8 - r) \text{ cm}^2\\ &= 2\pi(8 + r)(18 - r) \text{ cm}^2\end{aligned}$$

According to given

$$\begin{aligned} 2\pi(8 + r)(18 - r) &= 338\pi\\ \Rightarrow \quad (8 + r)(18 - r) &= 169\\ \Rightarrow \quad 144 + 10r - r^2 &= 169\\ \Rightarrow \quad r^2 - 10r + 25 &= 0 \Rightarrow (r - 5)^2 = 0 \Rightarrow r = 5.\end{aligned}$$

$\therefore$ Internal radius = 5 cm.

$\therefore$ Thickness of metal in the cylinder = (8 − 5) cm = 3 cm.

Exercise 18.2

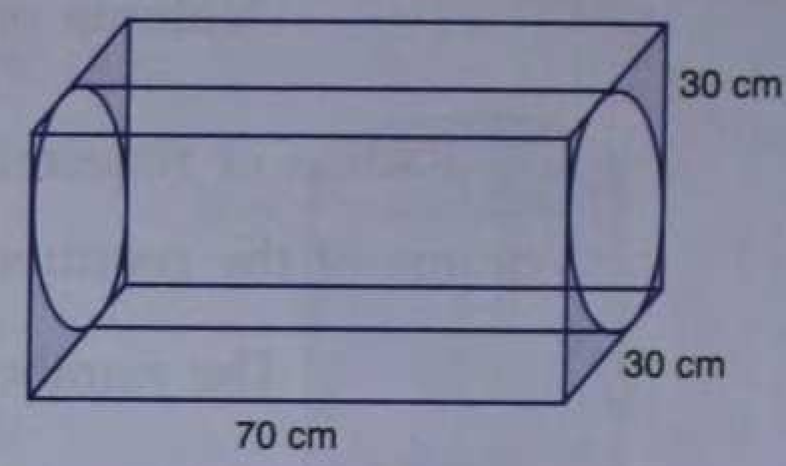
Take $\pi = \frac{22}{7}$, *unless stated otherwise.*

1. The diameter of the base of a right circular cylinder is 28 cm and its height is 21 cm. Find its
   (*i*) curved surface area (*ii*) total surface area (*iii*) volume.
2. The height of a circular cylindrical pillar is 10·5 m and the diameter of its base is 90 cm. What will be the cost of painting the curved surface of the pillar at ₹ 40 per m² ?
3. A solid metallic circular cylinder of radius 14 cm and height 12 cm is melted and recast into small cubes of edge 2 cm. How many such cubes can be made from the solid cylinder ?
4. A hollow copper pipe of inner diameter 6 cm and outer diameter 10 cm is melted and changed into a solid circular cylinder of the same height as that of the pipe. Find the diameter of the solid cylinder.

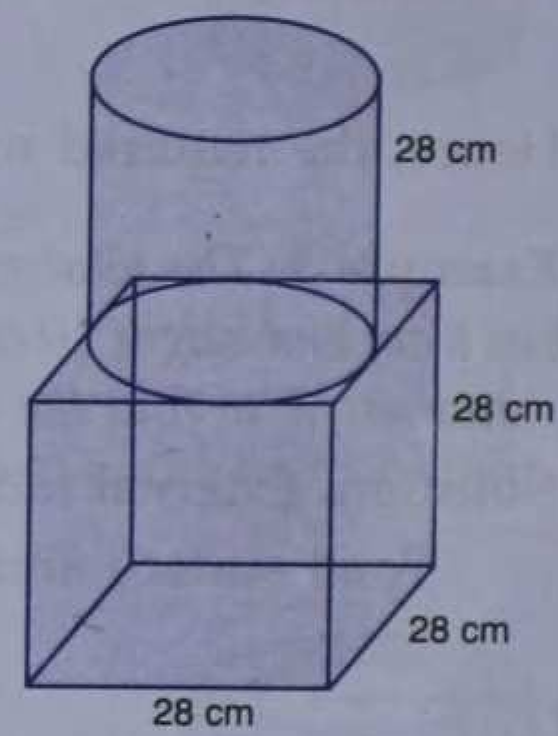
Hint

Let h cm be the height of the copper pipe and r cm be the radius of the solid cylinder, then $\pi(5^2 - 3^2)h = \pi r^2 h$.

5. The adjoining figure shows a cuboidal block of wood through which a circular cylindrical hole of the biggest size is drilled. Find the volume of the wood left in the block.



6. The adjoining figure shows a solid trophy made of shinning glass. If one cubic centimetre of glass costs ₹ 0·75, find the cost of the glass for making the trophy.



7. The difference between the outer and the inner curved surfaces of a circular cylinder 14 cm long is 88 cm^2. Find the outer and the inner radii of the cylinder, given that the volume of the metal is 176 cm^3.

18.3 SURFACE AREA AND VOLUME OF CONE

Cone. *A solid obtained by revolving a right angled triangular lamina about any side (other than the hypotenuse) is called a* ***right circular cone.***

The solid shown in the figure given below is a right circular cone.

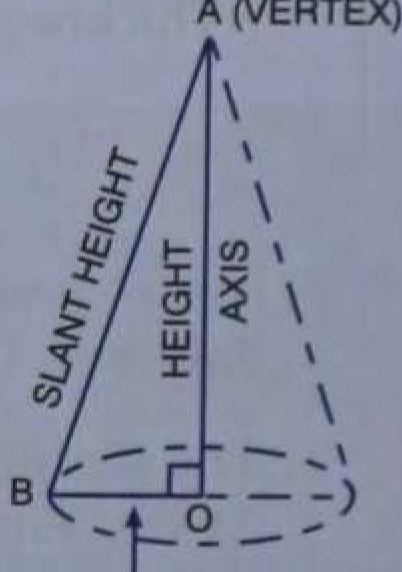
(*i*) A cone has a *vertex* and a *circular* base.

(*ii*) The radius of circular base is called the *radius of the cone.*

(*iii*) The line joining the centre of the circular base and the vertex is called the *axis of the cone.*

(*iv*) The distance between the centre of the circular base and the vertex is called the *height of the cone.*

(*v*) The distance between the vertex and any point on the circular base is called the *slant height of the cone.*



In this book, we shall be dealing only with right circular cones. Therefore, whenever we use the word *'cone'*, it will mean right circular cone.

Surface area and volume of a cone

Let r be the radius of the cone, h be its height and l be the slant height, then

(*i*) *Slant height* $= l = \sqrt{r^2 + h^2}$.

(*ii*) *Curved (lateral) surface area* $= \pi rl$.

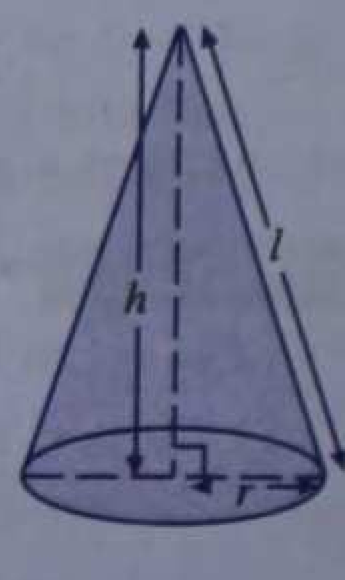
(*iii*) *Total surface area*

$= $ *curved surface area + area of circular base*

$= \pi r l + \pi r^2$

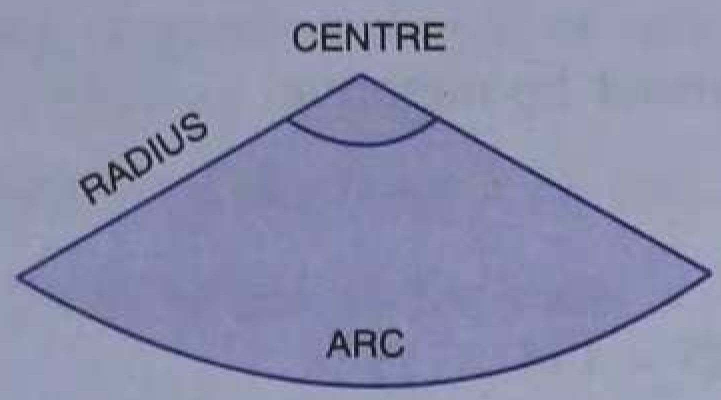
$= \pi r (l + r)$.

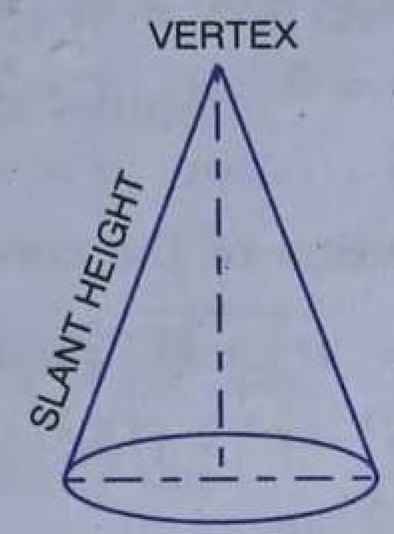
(*iv*) *Volume* $= \frac{1}{3} \pi r^2 h$.



Remark

If a sector of a circle is folded to make the radii coincide, the surface so formed is a *hollow right circular cone.*





We note the following :

(i) *The vertex of the cone is the centre of the circle.*

(ii) *The slant height of the cone is equal to the radius of the circle.*

(iii) *The circumference of the circular base of the cone is equal to the length of the arc of the sector.*

(iv) *Curved surface area of the cone is equal to the area of the sector.*

ILLUSTRATIVE EXAMPLES

Example 1. *A cone of height 24 cm has base diameter 14 cm. Find :*

(i) the slant height of the cone.

(ii) the lateral surface area of the cone.

(iii) the total surface area of the cone.

(iv) the volume of the cone. $\left(\text{Take } \pi = \frac{22}{7}\right)$.

Solution. Let r be the radius, h be the height and l be the slant height of the given cone, then

$$\text{diameter of base} = 2r = 14 \text{ cm (given)}$$

$$\Rightarrow \quad r = 7 \text{ cm.}$$

$$\text{Also height of cone} = h = 24 \text{ cm (given).}$$

(i) Slant height $= l = \sqrt{r^2 + h^2} = \sqrt{7^2 + (24)^2}$ cm $= \sqrt{49 + 576}$ cm

$= \sqrt{625}$ cm $= 25$ cm.

(ii) Lateral surface area of the cone $= \pi rl$

$= \frac{22}{7} \times 7 \times 25 \text{ cm}^2 = 550 \text{ cm}^2.$

(iii) Total surface area of the cone $= \pi r(l + r)$

$= \frac{22}{7} \times 7 \times (25 + 7) \text{ cm}^2 = 704 \text{ cm}^2.$

(iv) Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24 \text{ cm}^3 = 1232 \text{ cm}^3.$

Example 2. *The total surface area of a right circular cone of slant height 13 cm is 90 π cm². Calculate :*

(i) its radius in cm.

(ii) its volume in cm³. Take π = 3·1416.

Solution. Let r be the radius, h be the height and l be the slant height of the given cone, then $l = 13$ cm (given).

(i) Total surface area = $\pi r (l + r) = \pi r (13 + r)$ ($\because$ l = 13 cm)

According to the given data,

$\pi r (13 + r) = 90\pi \Rightarrow 13r + r^2 = 90$

$\Rightarrow r^2 + 13r - 90 = 0 \Rightarrow (r - 5)(r + 18) = 0$

$\Rightarrow r - 5 = 0$ or $r + 18 = 0$

$\Rightarrow r = 5$ or $r = -18$ but r cannot be negative.

$\therefore$ The radius of the cone = 5 cm.

(ii) Since $l = \sqrt{r^2 + h^2} \Rightarrow l^2 = r^2 + h^2$

$\Rightarrow h^2 = l^2 - r^2 = (13)^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow h = 12$ cm.

$\therefore$ The volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3{\cdot}1416 \times 5^2 \times 12 \text{ cm}^3$

$= 314{\cdot}6 \text{ cm}^3$.

Example 3. *The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the :*

(i) the radius of the floor. *(ii) height of the tent.*

(iii) length of the canvas required to cover this conical tent if its width is 2 m. (2008)

Solution.

(i) Let r metres be the radius of the base of the conical tent, then area of the base floor = $\pi r^2 \text{ m}^2$

$\therefore \pi r^2 = 154$ (given)

$\Rightarrow \frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = 49 \Rightarrow r = 7$.

Hence, the radius of the floor = 7 m.

(ii) Let h metres be the height of the conical tent, then the volume of the tent $= \frac{1}{3}\pi r^2 h \text{ m}^3$

$\therefore \frac{1}{3}\pi r^2 h = 1232$ (given)

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 7^2 \times h = 1232 \Rightarrow h = \frac{1232 \times 3}{22 \times 7} = 24.$

Hence, the height of the tent = 24 m.

(iii) Let l metres be the slant height of the conical tent, then

$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25.$

$\therefore$ The area of the canvas required to make the tent

= curved surface area of the conical tent

$= \pi r l \text{ m}^2 = \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 550 \text{ m}^2$

The width of the canvas = 2 metres (given)

$\therefore$ The length of the canvas $= \frac{\text{area}}{\text{width}} = \frac{550}{2}$ m = 275 m.

Hence, the length of the canvas required to make the tent = 275 m.

Example 4. *A solid cylinder of silver 9 cm high and 4 cm in diameter is melted and recast into a right circular cone of diameter 6 cm. Find the height and the total surface area of the cone. Give your answer correct to one decimal place. (Take $\pi = 3{\cdot}14$).*

Solution. Radius of cylinder $= \frac{1}{2} \times 4$ cm = 2 cm, and its height is 9 cm.

$\therefore$ Volume of the cylinder $= \pi r^2 h = \pi \times 2^2 \times 9 \text{ cm}^3 = 36\pi \text{ cm}^3$.

Radius of the cone = $\frac{1}{2} \times 6$ cm = 3 cm.

Let h cm be the height of the cone.

$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3^2 \times h \text{ cm}^3 = 3\pi h \text{ cm}^3$.

According to given, volume of cone = volume of cylinder

$\Rightarrow 3\pi h = 36\pi \Rightarrow h = 12$.

$\therefore$ The height of the cone = 12 cm.

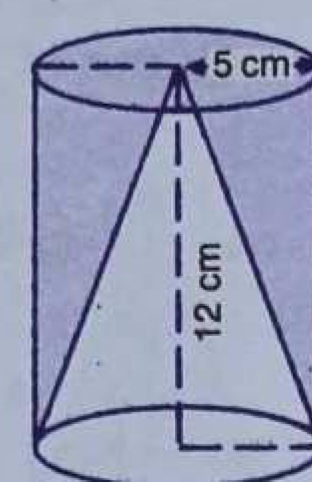
Slant height of the cone = $\sqrt{r^2 + h^2} = \sqrt{3^2 + 12^2}$ cm

= $\sqrt{153}$ cm = 12·37 cm.

$\therefore$ Total surface area of the cone = $\pi r (l + r)$

= $3{\cdot}14 \times 3\,(12{\cdot}37 + 3)\text{ cm}^2 = 144{\cdot}8\text{ cm}^2$.

Example 5. *From a circular cylinder of diameter 10 cm and height 12 cm, a conical cavity of the same base radius and of the same height is hollowed out. Find the volume and the whole surface of the remaining solid. (Take π = 3·14).*



Solution. Radius of the cylinder = $\frac{1}{2} \times 10$ cm

= 5 cm.

Height of the cylinder = 12 cm.

$\therefore$ Volume of the cylinder = $\pi r^2 h$

= $\pi \times 5^2 \times 12\text{ cm}^3 = 300\pi\text{ cm}^3$.

Radius of the cone = 5 cm.

Height of the cone = 12 cm.

Slant height of the cone = $\sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2}$ cm = 13 cm.

Volume of the cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi \times 5^2 \times 12\text{ cm}^3 = 100\pi\text{ cm}^3$.

$\therefore$ The volume of the remaining solid

= $300\pi\text{ cm}^3 - 100\pi\text{ cm}^3$

= $200 \times 3{\cdot}14\text{ cm}^3 = 628\text{ cm}^3$.

Curved surface of the cylinder = $2\pi rh = 2\pi \times 5 \times 12\text{ cm}^2$

= $120\pi\text{ cm}^2$.

Curved surface of the cone = $\pi rl = \pi \times 5 \times 13\text{ cm}^2$

= $65\pi\text{ cm}^2$.

Area of (upper) circular base of cylinder = $\pi r^2 = \pi \times 5^2\text{ cm}^2$

= $25\pi\text{ cm}^2$.

$\therefore$ The whole surface area of the remaining solid

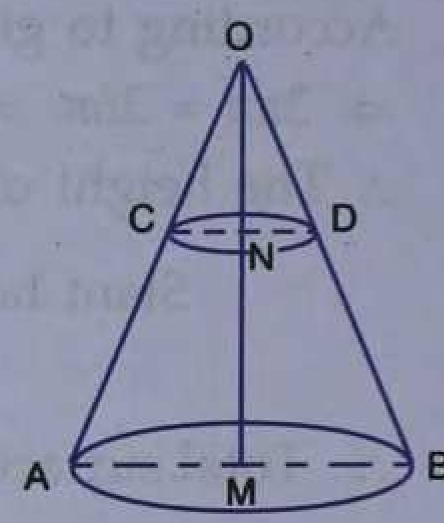
= curved surface area of cylinder + curved surface area of cone + area of (upper) circular base of cylinder

= $120\pi\text{ cm}^2 + 65\pi\text{ cm}^2 + 25\pi\text{ cm}^2$

= $210 \times 3{\cdot}14\text{ cm}^2 = 659{\cdot}4\text{ cm}^2$.

Example 6. *The height of a cone is 40 cm. A small cone is cut off at the top of a plane parallel to its base. If its volume be $\frac{1}{64}$ of the volume of the given cone, at what height above the base is the section cut ?*

Solution. Let OAB be the given cone of height 40 cm and base radius R cm. Let this cone be cut by the plane CND (parallel to the base plane AMN) to obtain the cone OCN with height h cm and base radius r cm.



Then ΔOND ~ ΔOMB.

$$\therefore \quad \frac{r}{R} = \frac{h}{40} \qquad \ldots(i)$$

According to given,

volume of cone OCD = $\frac{1}{64}$ × volume of cone OAB

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{1}{64} \times \frac{1}{3}\pi R^2 \times 40$$

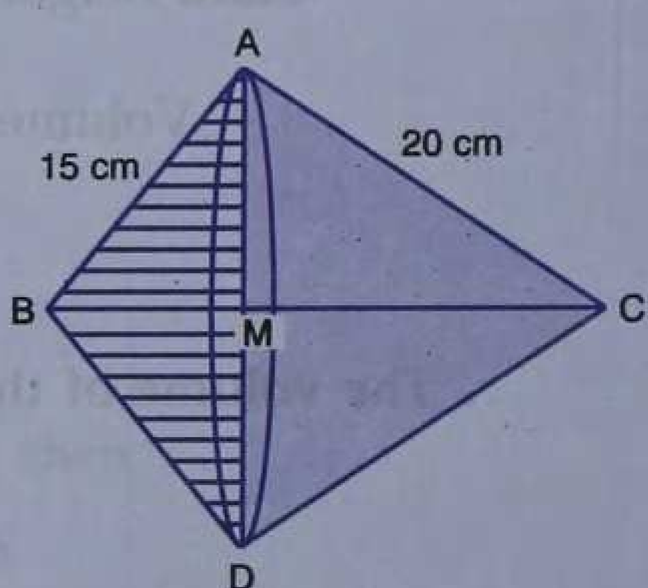
$$\Rightarrow \quad \left(\frac{r}{R}\right)^2 = \frac{5}{8h} \quad \Rightarrow \quad \left(\frac{h}{40}\right)^2 = \frac{5}{8h} \qquad \text{(using } (i))$$

$$\Rightarrow \quad h^3 = \frac{40 \times 40 \times 5}{8} = 1000 \quad \Rightarrow h = 10.$$

The height of the cone OCD = 10 cm.

$\therefore$ The section is cut at the height of (40 – 10) cm *i.e.* 30 cm from the base.

Example 7. *A right triangle, with sides 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. Take π = 3·14.*



Solution. Let ABC be right triangle right angled at A with sides AB, AC of 15 cm, 20 cm respectively.

By Pythagoras theorem,

$$BC^2 = AB^2 + AC^2 = 15^2 + 20^2 = 625$$

$$\Rightarrow \quad BC = 25 \text{ cm}.$$

On revolving ΔABC about hypotenuse BC, double cone is formed with AM (or MD) as radius of the common base.

Height of cone ABD is BM and its slant height is 15 cm.

Height of cone ACD is CM and its slant height is 20 cm.

Now ΔABM ~ ΔCBA (A.A. axiom of similarity)

$$\therefore \quad \frac{BM}{AB} = \frac{AM}{AC} = \frac{AB}{BC} \Rightarrow \frac{BM}{15} = \frac{AM}{20} = \frac{15}{25}$$

$$\Rightarrow \quad BM = 9 \text{ cm and } AM = 12 \text{ cm}.$$

$$\therefore \quad CM = BC - BM = (25 - 9) \text{ cm} = 16 \text{ cm}.$$

$$\text{Volume of the double cone} = \left(\frac{1}{3}\pi \times (12)^2 \times 9 + \frac{1}{3}\pi \times (12)^2 \times 16\right) \text{cm}^3$$

$= \frac{1}{3}\pi \times (12)^2 \times (9 + 16) \text{ cm}^3$

$= \frac{3 \cdot 14}{3} \times 144 \times 25 \text{ cm}^3 = 3768 \text{ cm}^3.$

Surface area of the double cone $= (\pi \times 12 \times 15 + \pi \times 12 \times 20) \text{ cm}^2$

$= \pi \times 12 \times 35 \text{ cm}^2 = 3{\cdot}14 \times 420 \text{ cm}^2$

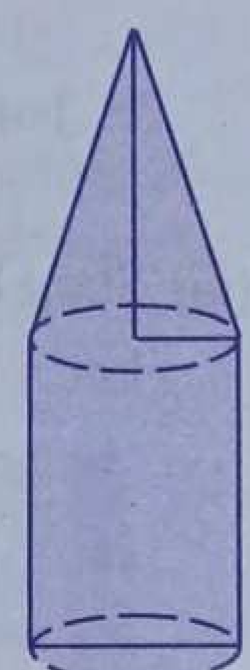
$= 1318{\cdot}8 \text{ cm}^2.$

Example 8. *The diagram (given below) shows a model of a rocket consisting of a cylinder surmounted by a cone at one end. The dimensions of the model are : common radius 3 cm, height of cone = 4 cm and total height = 14 cm. If the model is drawn to a scale of 1 : 500, find*

(i) the total surface area of the rocket in π m².

(ii) the total volume of the rocket in π m³.

Solution. Since the model is drawn to the scale 1 : 500, and the dimensions of the model are :



common radius = 3 cm, height of conical part = 4 cm,

height of cylindrical part = 14 cm – 4 cm = 10 cm,

therefore, the actual dimensions of the rocket are :

common radius = 3 × 500 cm = 15 m,

height of conical part = 4 × 500 cm = 20 m,

height of cylindrical part = 10 × 500 = 50 m.

Slant height of rocket $= \sqrt{15^2 + 20^2}$ m $= \sqrt{625}$ m = 25 m.

(*i*) The total surface area of the rocket = curved surface area of cone + curved surface area of cylinder + area of base of cylinder

$= \pi \times 15 \times 25 + 2\pi \times 15 \times 50 + \pi \times 15^2$

$= \pi(375 + 1500 + 225) \text{ m}^2 = 2100\pi \text{ m}^2.$

(*ii*) The total volume of the rocket = volume of conical part + volume of cylindrical part

$= \frac{1}{3}\pi \times 15^2 \times 20 + \pi \times 15^2 \times 50 = 225\pi\left(\frac{20}{3} + 50\right) \text{ m}^3$

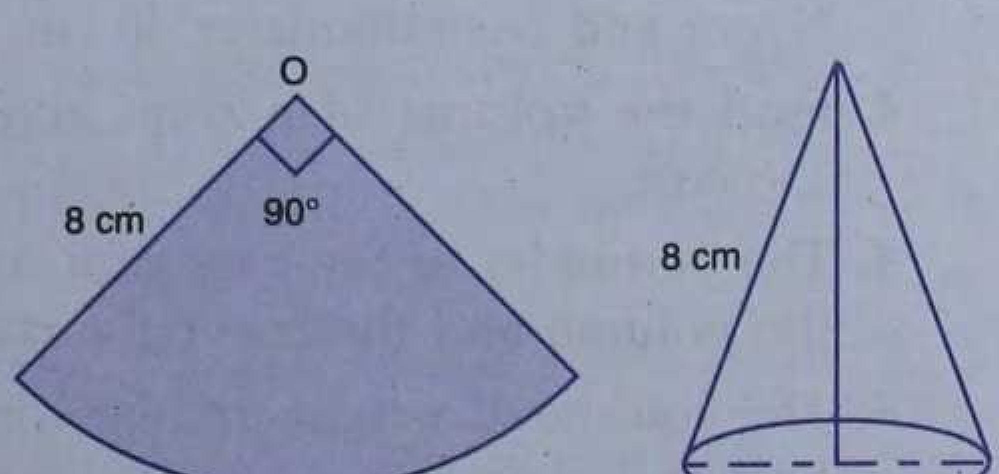
$= 225\pi \times \frac{170}{3} \text{ m}^3 = 12750\pi \text{ m}^3.$

Example 9. *A sector of a circle of radius 8 cm has an angle of 90°. It is rolled up so that the two bounding radii are joined together to form a cone. Find :*

(i) the radius of the cone.

(ii) the total surface area of the cone.

(iii) the volume of the cone. Leave the answer in surds and π.



Solution.

(*i*) The length of the arc of sector $= \frac{1}{4} \times 2\pi \times 8$ cm $= 4\pi$ cm.

The circumference of the circular base of the cone

= length of the arc of the sector

$= 4\pi$ cm.

Let r be the radius of the cone, then

$$2\pi r = 4\pi \text{ cm} \Rightarrow r = 2 \text{ cm}.$$

(*ii*) Area of the sector $= \frac{1}{4} \times \pi \times (\text{radius})^2$

$$= \frac{1}{4} \pi \times 8^2 \text{ cm}^2 = 16\pi \text{ cm}^2.$$

$\therefore$ The curved surface area of the cone = area of the sector $= 16\pi$ cm^2.

Area of the circular base of the cone $= \pi r^2$

$$= \pi \times 2^2 \text{ cm}^2 = 4\pi \text{ cm}^2.$$

$\therefore$ Total surface area of the cone $= 16\pi$ cm^2 $+ 4\pi$ cm^2

$$= 20\pi \text{ cm}^2.$$

(*iii*) Slant height of the cone = radius of the sector = 8 cm.

Let h be the height of the cone, then

$$h^2 = l^2 - r^2 = 8^2 - 2^2 = 64 - 4 = 60$$

$$\Rightarrow \quad h = \sqrt{60} \text{ cm} = 2\sqrt{15} \text{ cm}.$$

$\therefore$ Volume of the cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \pi \times 2^2 \times 2\sqrt{15} \text{ cm}^3 = \frac{8\sqrt{15}\,\pi}{3} \text{ cm}^3.$$

Exercise 18.3

Take $\pi = \frac{22}{7}$, unless stated otherwise.

1. Calculate the curved surface area of a cone whose perpendicular height is 4·8 cm and the radius of whose base is 3·6 cm. Take $\pi = 3{\cdot}14$ and give your answer correct to 2 decimal places.
2. Find the volume of the cone given that its slant height is 17 cm and radius is 8 cm. (Take $\pi = 3{\cdot}142$)
3. Find the volume and the total surface area of a cone having slant height 17 cm and base diameter 30 cm. Take $\pi = 3{\cdot}14$.
4. Find the volume of a cone given that its height is 8 cm and area of base is 156 cm^2.
5. The perimeter of the base of a cone is 44 cm and the slant height is 25 cm. Find the volume and the curved surface of the cone.
6. The volume of a right circular cone is 660 cm^3. If the diameter of its base is 12 cm, find
   (*i*) the height of the cone.
   (*ii*) the slant height of the cone.

7. The curved surface of a cone is 550 cm^2. Find the volume of the cone given that its base diameter is 14 cm.

8. The volume of a right circular cone is 9856 cm^3 and the area of the base is 616 cm^2. Find

   (*i*) the slant height of the cone.

   (*ii*) total surface area of the cone.

9. The radius and the height of a cone are in the ratio 1 : 3, and its volume is 1078 cm^3. Find its diameter and the lateral surface area correct to two decimal places.

10. (*a*) The ratio of the base radii of two right circular cones of the same height is 3 : 4. Find the ratio of their volumes.

    (*b*) The ratio of the heights of two right circular cones is 5 : 2 and that of their base radii is 2 : 5. Find the ratio of their volumes.

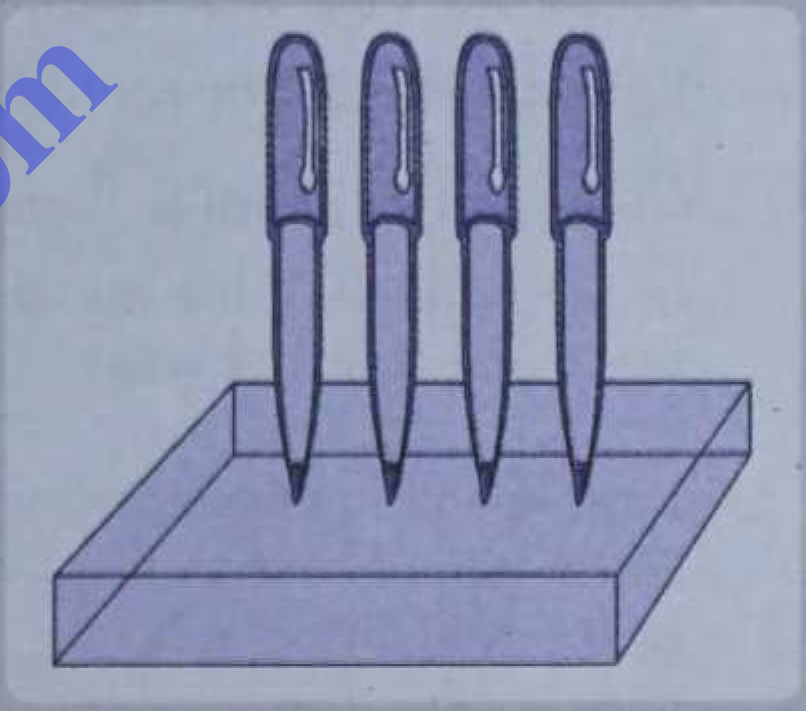
    (*c*) The height and the radius of the base of a right circular cone is half the corresponding height and radius of another bigger cone. Find :

      (*i*) the ratio of their volumes.

      (*ii*) the ratio of their lateral surface areas.

11. Find what length of canvas 2 m in width is required to make a conical tent 20 m in diameter and 42 m in slant height allowing 10% for folds and the stitching. Also find the cost of the canvas at the rate of ₹ 80 per metre.

12. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3·5 cm. The radius of each of the depression is 0·5 cm and the depth is 1·4 cm. Find the volume of the wood in the entire stand, correct to 2 decimal places.



13. The volume of a cone is the same as that of the cylinder whose height is 9 cm and diameter 40 cm. Find the radius of the base of the cone if its height is 108 cm.

14. A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find

    (*i*) the radius and

    (*ii*) the slant height of the heap.

    Leave your answer in square root form. (*2004*)

15. A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16·8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one third of the water in the original cone overflows. What is the volume of each of the solid cone submerged? (*2006*)

16. A hollow metallic cylindrical tube has an internal radius of 3 cm and height 21 cm. The thickness of the metal of the tube is $\frac{1}{2}$ cm. The tube is melted and cast into a right circular cone of height 7 cm. Find the radius of the cone correct to one decimal place.

17. Two right circular cones X and Y are made, X having three times the radius of Y and Y having half the volume of X. Find the ratio of heights of X and Y.

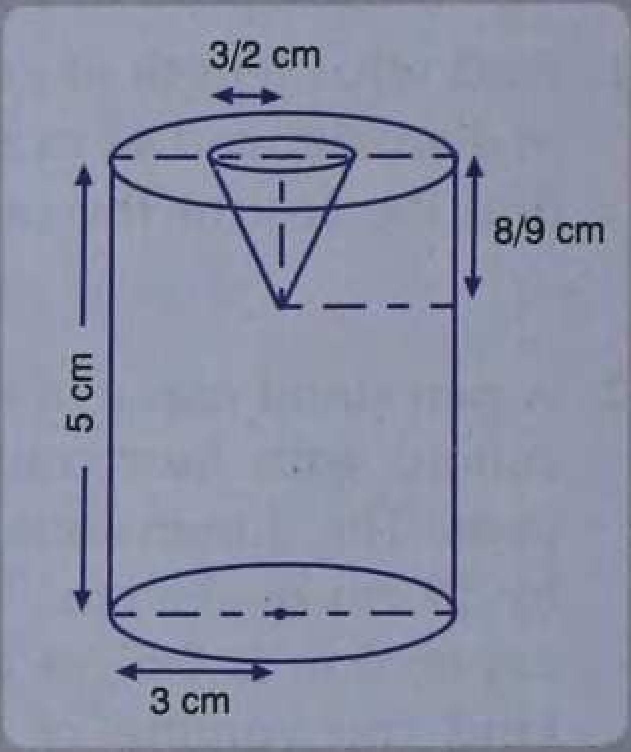
18. A right circular cone of height 20 cm and base diameter 30 cm is cast into smaller cones of equal sizes with base radius 10 cm and height 9 cm. Find how many cones are made.

19. A circus tent is in the shape of a cylinder surmounted by a cone. The diameter of the cylindrical portion is 24 m and its height is 11 m. If the vertex of the cone is 16 m above the ground, find the area of the canvas used to make the tent.

20. An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching. Give your answer to the nearest m^2. *(2001)*

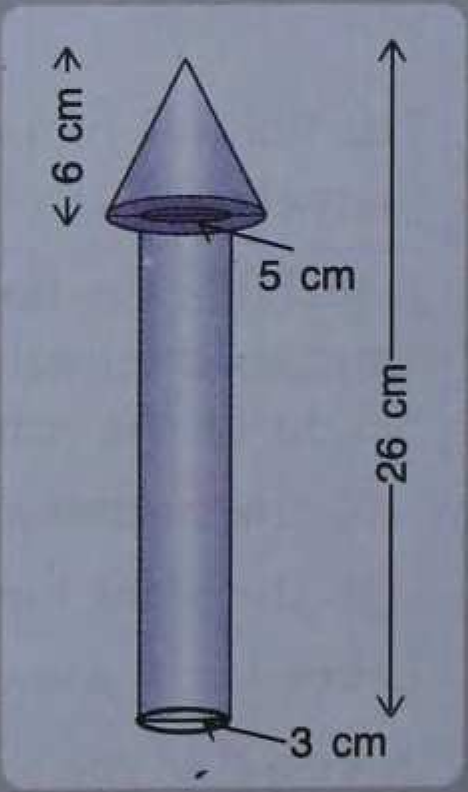
21. From a solid cylinder whose height is 8 cm and radius is 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to 4 significant figures. (π = 3·1416). Also find the total surface area of the remaining solid.

22. A metallic cylinder has radius 3 cm and height 5 cm. It is made of a metal A. To reduce its weight, a conical hole is drilled in the cylinder as shown and it is completely filled with a lighter metal B. The conical hole has a radius of $\frac{3}{2}$ cm and its depth is $\frac{8}{9}$ cm. Calculate the ratio of the volume of the metal A to the volume of the metal B in the solid.



23. From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and of base radius 7 cm is drilled out. Find the volume and the total surface of the remaining solid.

24. The adjoining figure shows a wooden toy rocket which is in the shape of a circular cone mounted on a circular cylinder. The total height of the rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of cylindrical portion is 3 cm. If the conical portion is to be painted green and the cylindrical portion red, find the area of the rocket painted with each of these colours. Also find the volume of the wood in the rocket. Use π = 3·14 and give answers correct to 2 decimal places.



25. An open cylindrical vessel of internal diameter 7 cm and height 8 cm stands on a horizontal table. Inside this is placed a solid metallic right circular cone, the diameter of whose base is $\frac{7}{2}$ cm and height 8 cm. Find the volume of water required to fill the vessel. If the cone is replaced by another cone, whose height is $1\frac{3}{4}$ cm and the radius of whose base is 2 cm, find the drop in the water level.

26. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section cut ?

27. A semi-circular lamina of radius 35 cm is folded so that the two bounding radii are joined together to form a cone. Find :

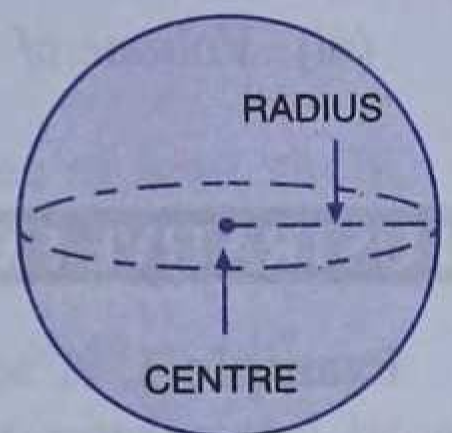
(*i*) the radius of the cone. (*ii*) the (lateral) surface area of the cone.

18.4 SURFACE AREA AND VOLUME OF SPHERE

Sphere. *A solid obtained by revolving a circular lamina about any of its diameters is called a* ***sphere.***

(*i*) The centre of the circle revolved is called the *centre of the sphere.*

(*ii*) The radius of the circle revolved is called the *radius of the sphere.*



Remark

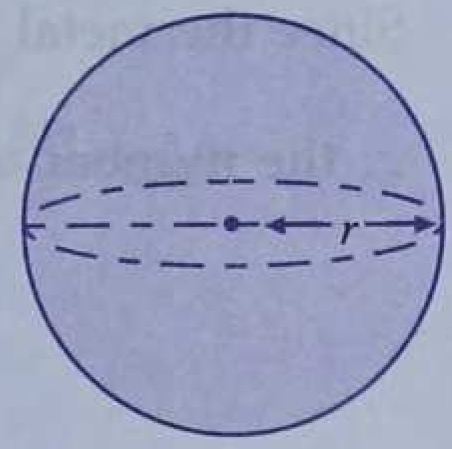
If a circle is revolved about any of its diameters, a *hollow sphere* is generated.

Solid sphere

Let r be the radius of solid sphere, then

(*i*) *Surface area* = $4\pi r^2$.

(*ii*) *Volume* = $\frac{4}{3}\pi r^3$.



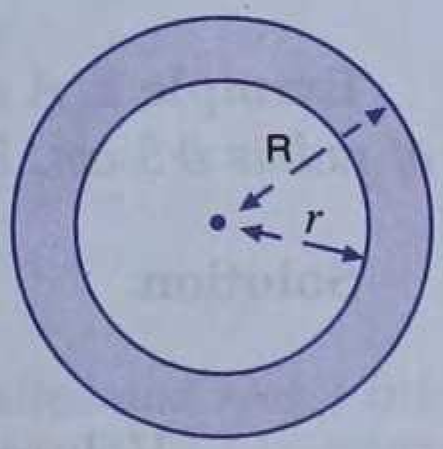
Spherical shell

The solid enclosed between two concentric spheres is called *a* ***spherical shell.***

Let R and r be the radii of the outer and the inner spheres, then

(*i*) *Thickness of shell* = R – r.

(*ii*) *Volume of the material* = $\frac{4}{3}\pi(R^3 - r^3)$.



Hemisphere

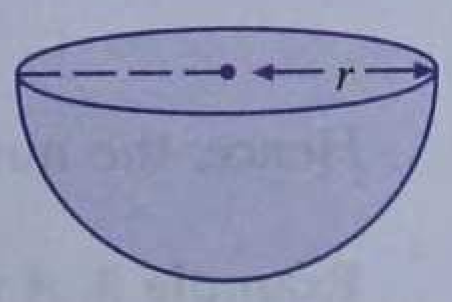
When a sphere is cut by a plane through its centre into two parts, then each part is called a ***hemisphere.***

Let r be the radius of the sphere *i.e.* of the hemi-sphere, then

(*i*) *Curved surface area*

$= \frac{1}{2}$ *(surface area of sphere)*

$= \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$.



(*ii*) *Total surface area*

= *curved surface area + area of circular base*

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$.

(*iii*) *Volume* = $\frac{1}{2}$ *(volume of sphere)* $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$.

Hemispherical shell

The solid enclosed between two concentric hemispheres is called a *hemisperical shell.*

Let R and r be the radii of the outer and the inner hemispheres, then

(i) *Thickness of shell* = $R - r$.

(ii) *Area of base* = $\pi(R^2 - r^2)$.

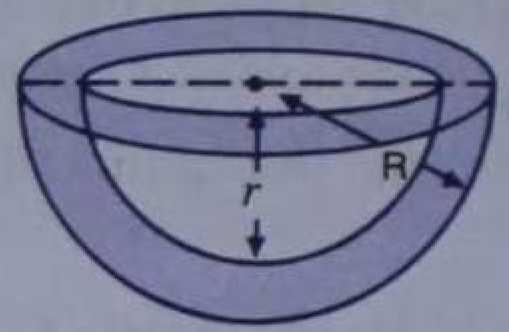
(iii) *External curved surface area* = $2\pi R^2$.

(iv) *Internal curved surface area* = $2\pi r^2$.

(v) *Total surface area* = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi(3R^2 + r^2)$.

(vi) *Volume of material* = $\frac{2}{3}\pi(R^3 - r^3)$.



ILLUSTRATIVE EXAMPLES

Example 1. *A solid metal cylinder of radius 14 cm and height 21 cm is melted down and recast into spheres of radius 3·5 cm. Calculate the number of spheres that can be made.*

Solution. Volume of cylinder = $\pi \times (14)^2 \times 21$ cm³.

Volume of one sphere = $\frac{4}{3}\pi \times \left(\frac{7}{2}\right)^3$ cm³.

Since the metal of cylinder is to be converted into spheres,

$\therefore$ the number of spheres that can be made = $\frac{\text{volume of cylinder}}{\text{volume of one sphere}}$

$$= \frac{\pi \times 14 \times 14 \times 21}{\frac{4}{3}\pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}} = \frac{3 \times 14 \times 14 \times 21 \times 2}{7 \times 7 \times 7}$$

$= 72.$

Example 2. *A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0·5 cm. Find the number of spheres formed.* (2011)

Solution. Volume of cone = $\frac{1}{3}\pi \times 5^2 \times 8$ cm³.

Volume of one small sphere = $\frac{4}{3}\pi \times \left(\frac{1}{2}\right)^3$ cm³.

Since the metal of the cone is to be converted in small spheres,

$\therefore$ the number of small spheres that can be formed = $\frac{\text{volume of cone}}{\text{volume of one sphere}}$

$$= \frac{\frac{1}{3}\pi \times 5^2 \times 8}{\frac{4}{3}\pi \times \left(\frac{1}{2}\right)^3} = 5^2 \times 8 \times 2 = 400.$$

Hence, the number of small spheres that can be formed = 400.

Example 3. *A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top, which is open, is 2·5 cm. It is filled with water upto rim. When some lead shots, each of which is a sphere of radius 0·25 cm, are dropped into the vessel, $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel.* (2003)

Solution. Radius of the cone = 2·5 cm = $\frac{5}{2}$ cm and its height = 11 cm.

Volume of water in the cone = volume of the cone

$$= \frac{1}{3}\pi \times \left(\frac{5}{2}\right)^2 \times 11 \text{ cm}^3 = \frac{275}{12}\pi \text{ cm}^3$$

∴ Volume of water which flows out of the cone = $\frac{2}{5} \times \frac{275}{12}\pi$ cm³ = $\frac{55}{6}\pi$ cm³.

Radius of lead shot (sphere) = 0·25 cm = $\frac{1}{4}$ cm.

∴ Volume of one lead shot = $\frac{4}{3}\pi \times \left(\frac{1}{4}\right)^3$ cm³ = $\frac{\pi}{48}$ cm³.

Since the volume of water which flows out is equal to the volume of lead shots,

∴ the number of lead shots dropped into the vessel = $\dfrac{\frac{55}{6}\pi}{\frac{\pi}{48}} = \frac{55}{6} \times 48 = 440.$

Example 4. *A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Find the number of cones recast.* (2013)

Solution. Volume of sphere = $\frac{4}{3}\pi \times (15)^3$ cm³,

volume of one cone = $\frac{\pi}{3} \times (2{\cdot}5)^2 \times 8$ cm³ = $\frac{\pi}{3} \times \left(\frac{5}{2}\right)^2 \times 8$ cm³ = $\frac{50\pi}{3}$ cm³.

Since the metal of sphere is to be converted into cones,

∴ the number of cones that can be made = $\dfrac{\text{volume of sphere}}{\text{volume of one cone}}$

$$= \frac{\frac{4}{3}\pi \times (15)^3}{\frac{50\pi}{3}} = \frac{2}{25} \times 3375 = 270.$$

Hence, the number of cones recasted = 270.

Example 5. *The surface area of a solid metallic sphere is 616 cm². It is melted and recast into smaller spheres of diameter 3·5 cm. How many such spheres can be obtained ?* (2007)

Solution. Let the radius of the original sphere be r cm.

∴ $4\pi r^2 = 616$

⇒ $4 \times \frac{22}{7} \times r^2 = 616$ ⇒ $r^2 = \frac{616 \times 7}{4 \times 22} = 49$

⇒ $r = 7$.

∴ Volume of the original sphere = $\frac{4}{3}\pi \times 7^3$ cm³.

Radius of the small sphere = $\frac{1}{2} \times 3{\cdot}5$ cm = $\frac{7}{4}$ cm.

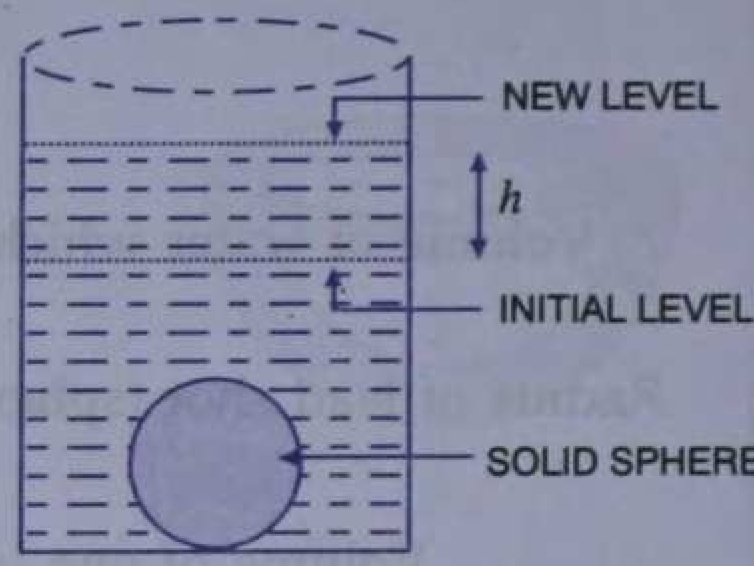
∴ Volume of one small sphere = $\frac{4}{3}\pi \times \left(\frac{7}{4}\right)^3$.

∴ The number of small spheres = $\dfrac{\frac{4}{3}\pi \times 7^3}{\frac{4}{3}\pi \times \left(\frac{7}{4}\right)^3} = 64.$

Example 6. *A cylindrical can of internal diameter 21 cm contains water. A solid sphere whose diameter is 10·5 cm is lowered into the cylindrical can. The sphere is completely immersed in water. Calculate the rise in water level, assuming that no water overflows.*

Solution. Radius of sphere $= \frac{1}{2} \times 10{\cdot}5$ cm

$= \frac{21}{4}$ cm.



$\therefore$ Volume of the sphere $= \frac{4}{3}\pi \left(\frac{21}{4}\right)^3$ cm³.

Radius of the cylinder $= \frac{1}{2} \times 21 \text{ cm} = \frac{21}{2}$ cm.

Let the rise in water level in the cylinder be h cm.

$\therefore$ The volume of the water risen in the cylinder $= \pi \times \left(\frac{21}{2}\right)^2 \times h$ cm³.

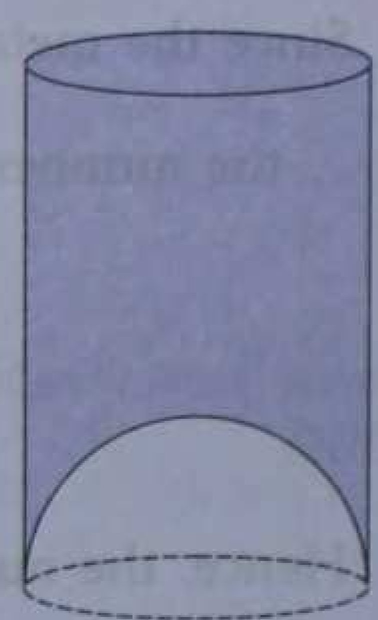
Since the volume of water risen = volume of sphere,

$$\therefore \quad \pi \times \frac{21}{2} \times \frac{21}{2} \times h = \frac{4}{3}\pi \times \frac{21}{4} \times \frac{21}{4} \times \frac{21}{4}$$

$$\Rightarrow \quad h = \frac{7}{4} = 1{\cdot}75$$

$\therefore$ Rise in water level = 1·75 cm.

Example 7. *A juice seller was serving his customers using glasses as shown in the adjoining figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical portion raised which reduces the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3{\cdot}14$)*



Solution. Inner diameter of the cylindrical glass = 5 cm.

$\therefore$ Inner radius of the glass $= \frac{5}{2}$ cm $= 2{\cdot}5$ cm

Height of cylindrical glass = 10 cm.

$\therefore$ Apparent capacity of the glass = inner volume of glass

$= \pi r^2 h = 3{\cdot}14 \times (2{\cdot}5)^2 \times 10$ cm³

$= 31{\cdot}4 \times 6{\cdot}25$ cm³

$= 196{\cdot}25$ cm³

Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3{\cdot}14 \times (2{\cdot}5)^3$

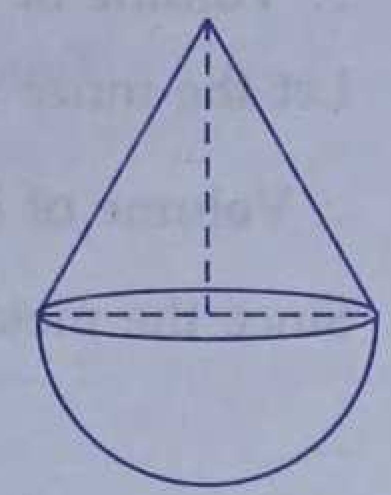
$= 32{\cdot}71$ cm³

$\therefore$ Actual capacity of the glass = apparent capacity of glass − volume of hemisphere

$= 196{\cdot}25$ cm³ $- 32{\cdot}71$ cm³

$= 163{\cdot}54$ cm³.

Example 8. *The adjoining figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate :*



(i) the height of the cone.

(ii) the volume of the solid. (Take $\pi = 3{\cdot}14$) (2009)

Solution.

(*i*) Diameter of the base of cone = 6 cm (given),

$\therefore$ radius of the base of cone = $\frac{1}{2} \times 6$ cm = 3 cm.

Let the height of the cone be h cm, then

slant height = $\sqrt{h^2 + r^2}$

$\Rightarrow$ $5 = \sqrt{h^2 + 3^2}$ $\Rightarrow$ $25 = h^2 + 9$

$\Rightarrow$ $h^2 = 16$ $\Rightarrow$ $h = 4$.

$\therefore$ Height of the cone = 4 cm.

(*ii*) Radius of the hemisphere = $\frac{1}{2} \times 6$ cm = 3 cm.

Volume of solid = volume of cone + volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \left(\frac{1}{3}\pi \times 3^2 \times 4 + \frac{2}{3}\pi \times 3^3\right) \text{cm}^3$$

$$= (12\pi + 18\pi) \text{ cm}^3 = 30 \times 3{\cdot}14 \text{ cm}^3$$

$$= 94{\cdot}2 \text{ cm}^3.$$

Example 9. *A hollow sphere of internal and external diameters 4 cm and 8 cm respectively, is melted into a cone of base diameter 8 cm. Find the height of the cone.* (2002)

Solution. Internal radius of sphere = $\frac{1}{2} \times 4$ cm = 2 cm.

External radius of sphere = $\frac{1}{2} \times 8$ cm = 4 cm.

$\therefore$ Volume of the metal in the spherical shell = $\frac{4}{3}\pi(4^3 - 2^3)$ cm^3

$$= \frac{224}{3}\pi \text{ cm}^3.$$

Radius of the cone = $\frac{1}{2} \times 8$ cm = 4 cm.

Let h cm be the height of the cone.

$\therefore$ Volume of the cone = $\frac{1}{3}\pi \times 4^2 \times h$ cm^3 = $\frac{16}{3}\pi h$ cm^3.

Since the metal of the spherical shell is to be converted into a conical solid,

$\therefore$ $\frac{16}{3}\pi h = \frac{224}{3}\pi \Rightarrow 16h = 224 \Rightarrow h = 14$

$\therefore$ The height of the cone = 14 cm.

Example 10. *A spherical shell of iron whose external radius is 9 cm is melted into a conical solid of 28 cm in diameter and of $4\frac{3}{7}$ cm height. Find the inner diameter of the shell.*

Solution. Radius of the cone = $\frac{1}{2} \times 28$ cm = 14 cm.

Height of the cone = $4\frac{3}{7}$ cm = $\frac{31}{7}$ cm.

$\therefore$ Volume of the cone = $\frac{1}{3}\pi \times (14)^2 \times \frac{31}{7}$ cm³ = $\frac{31 \times 28}{3}$ π cm³.

Let the inner radius of the spherical shell be r cm.

$\therefore$ Volume of iron in the spherical shell = $\frac{4}{3}$ π $(9^3 - r^3)$ cm³.

Since the metal of the spherical shell is to be converted into a conical solid,

$$\therefore \quad \frac{4}{3}\pi(9^3 - r^3) = \frac{31 \times 28}{3}\pi$$

$$\Rightarrow \quad 9^3 - r^3 = 31 \times 7 \quad \Rightarrow \quad 729 - r^3 = 217$$

$$\Rightarrow \quad r^3 = 512 \quad \Rightarrow \quad r^3 = (8)^3 \Rightarrow r = 8.$$

$\therefore$ The inner diameter of the shell = 2 × 8 cm = 16 cm.

Example 11. *A solid sphere and a solid hemisphere have the same total surface areas. Find the ratio of their volumes.*

Solution. Let the radius of the sphere be R and that of the hemisphere be r.

Surface area of sphere = $4\pi R^2$,

total surface area of hemisphere = $3\pi r^2$.

According to given, $4\pi R^2 = 3\pi r^2$

$$\Rightarrow \quad \frac{R^2}{r^2} = \frac{3}{4} \quad \Rightarrow \frac{R}{r} = \frac{\sqrt{3}}{2} \qquad \ldots(i)$$

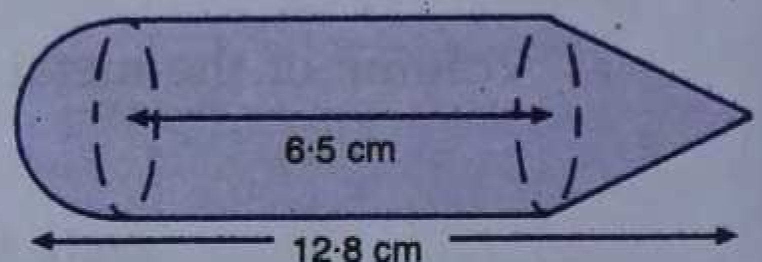
$$\frac{\text{Volume of sphere}}{\text{Volume of hemisphere}} = \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = 2\left(\frac{R}{r}\right)^3$$

$$= 2 . \left(\frac{\sqrt{3}}{2}\right)^3 \qquad \text{(using } (i))$$

$$= \frac{3\sqrt{3}}{4}.$$

Hence, the ratio of their volumes = $3\sqrt{3} : 4$.

Example 12. *The adjoining figure represents a solid consisting of a cylinder surmounted by a cone at one end and a hemisphere at the other end. Given that common radius = 3·5 cm, the height of the cylinder = 6·5 cm and the total height = 12·8 cm, calculate the volume of the solid, correct to the nearest cm.*



Solution. Radius of the hemisphere = 3·5 cm = $\frac{7}{2}$ cm.

Height of the cylinder = 6·5 cm = $\frac{13}{2}$ cm.

Given total height = 12·8 cm,

$\therefore$ Height of the cone = (12·8 − 3·5 − 6·5) cm = 2·8 cm = $\frac{14}{5}$ cm.

Radius of cylinder = $\frac{7}{2}$ cm = radius of cone.

Volume of the hemisphere = $\frac{2}{3}\pi\left(\frac{7}{2}\right)^3$ cm³.

Volume of the cylinder = $\pi\left(\frac{7}{2}\right)^2\left(\frac{13}{2}\right)$ cm³.

Volume of the cone = $\frac{1}{3}\pi\left(\frac{7}{2}\right)^2\left(\frac{14}{5}\right)$ cm³.

∴ The volume of the given solid

$$= \left(\frac{2}{3}\pi \times \left(\frac{7}{2}\right)^3 + \pi \times \left(\frac{7}{2}\right)^2 \times \frac{13}{2} + \frac{1}{3}\pi \times \left(\frac{7}{2}\right)^2 \times \frac{14}{5}\right) \text{cm}^3$$

$$= \pi\left(\frac{7}{2}\right)^2 \left[\frac{2}{3} \times \frac{7}{2} + \frac{13}{2} + \frac{1}{3} \times \frac{14}{5}\right] \text{cm}^3$$

$$= \frac{22}{7} \times \frac{49}{4}\left[\frac{7}{3} + \frac{13}{2} + \frac{14}{15}\right] \text{cm}^3$$

$$= \frac{77}{2} \times \frac{70+195+28}{30} \text{cm}^3 = \frac{77 \times 293}{60} \text{cm}^3 = 376 \text{ cm}^3 \text{ (approx.)}.$$

Example 13. *The internal and external diameters of a hollow hemispherical vessel are 14 cm and 21 cm respectively. The cost of silver plating of 1 cm² surface area is ₹ 0·40. Find the total cost of silver plating the vessel all over.*

Solution. External radius $= \frac{21}{2}$ cm,

internal radius $= \frac{14}{2}$ cm = 7 cm.

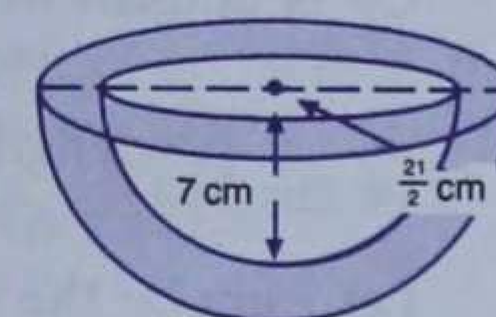
Total surface area of the vessel

$$= \pi\,(3R^2 + r^2)$$

$$= \pi\left(3 \times \frac{21}{2} \times \frac{21}{2} + 7 \times 7\right) \text{cm}^2$$

$$= \frac{22}{7} \times 7 \times 7\left(\frac{27}{4} + 1\right) \text{cm}^2$$

$$= 154 \times \frac{31}{4} \text{cm}^2 = \frac{77 \times 31}{2} \text{cm}^2.$$



Since the cost of silver plating of 1 cm² of surface area = ₹ 0·40 = ₹ $\frac{4}{10}$,

∴ total cost of silver plating the vessel all over

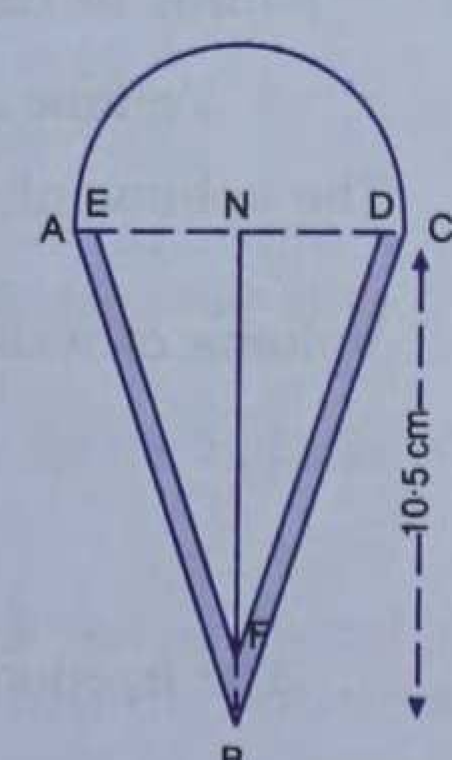
$$= ₹\, \frac{77 \times 31}{2} \times \frac{4}{10} = ₹\, 477\cdot40.$$

Example 14. *The figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3·5 cm and the height of the cone is 10·5 cm. The outer shell ABCDFE is shaded and is not filled with ice-cream. If AE = DC = ·5 cm, AB ∥ EF and BC ∥ FD, calculate*

(i) the volume of the ice-cream in the cone (the unshaded portion including the hemisphere) in cm³.

(ii) the volume of the outershell (the shaded portion) in cm³.

Give your answers correct to the nearest cm³.



Solution. Radius of the inner cone EFD = EN = AN – AE

= 3·5 cm – ·5 cm = 3 cm.

Δ ABN ~ Δ EFN (check)

$$\therefore \frac{FN}{BN} = \frac{EN}{AN} \Rightarrow \frac{FN}{10\cdot5} = \frac{3}{3\cdot5} \Rightarrow FN = 9 \text{ cm}$$

⇒ height of the inner cone EFD = 9 cm.

(*i*) The volume of the ice-cream = volume of hemisphere + volume of inner cone EFD

$$= \left(\frac{2}{3}\pi \times \left(\frac{7}{2}\right)^3 + \frac{1}{3}\pi \times 3^2 \times 9\right) \text{cm}^3 = \frac{\pi}{3}\left(\frac{343}{4} + 81\right) \text{cm}^3$$

$$= \frac{22}{7} \times \frac{1}{3} \times \frac{667}{4} \text{cm}^3 = \frac{7337}{42} \text{cm}^3$$

$$= 174\cdot69 \text{ cm}^3 = 175 \text{ cm}^3 \text{ (to the nearest cm}^3\text{)}.$$

(*ii*) The volume of the shell = volume of the outer cone ABC

– volume of the inner cone EFD

$$= \left(\frac{1}{3}\pi \times \left(\frac{7}{2}\right)^2 \times \frac{21}{2} - \frac{1}{3}\pi \times 3^2 \times 9\right) \text{cm}^3 = \frac{\pi}{3}\left(\frac{1029}{8} - 81\right) \text{cm}^3$$

$$= \frac{22}{7} \times \frac{1}{3} \times \frac{381}{8} \text{ cm}^3 = \frac{11 \times 127}{28} \text{ cm}^3 = \frac{1397}{28} \text{ cm}$$

$$= 49{\cdot}89 \text{ cm}^3 = 50 \text{ cm}^3 \text{ (to the nearest cm}^3\text{)}.$$

Example 15. *A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides it is just immersed. What fraction of the water overflows ?*

Solution. A vertical section of the conical vessel and the sphere when immersed are shown in the figure.

From right angled Δ AMB,

$$AB^2 = AM^2 + MB^2 = 8^2 + 6^2$$
$$= 64 + 36 = 100$$
$$\Rightarrow AB = 10 \text{ cm}.$$

CB is tangent to the circle at M and AB is tangent to it at P.

$$PB = MB = 6$$

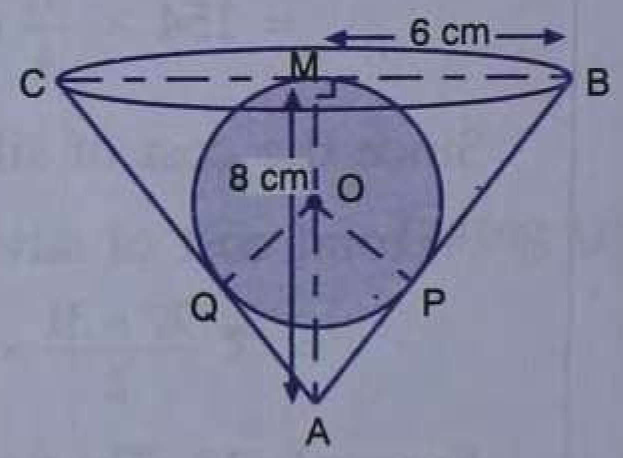
(∵ lengths of tangents from an external point to a circle are equal in length)

$$\therefore AP = AB - PB = (10 - 6) \text{ cm} = 4 \text{ cm}.$$

Let r cm be the radius of the circle, then OP = OM = r

$$\therefore AO = AM - OM = (8 - r) \text{ cm}.$$

From right angled Δ OAP,



$$OA^2 = AP^2 + OP^2$$
$$\Rightarrow (8 - r)^2 = 4^2 + r^2$$
$$\Rightarrow 64 - 16r + r^2 = 16 + r^2$$
$$\Rightarrow 48 = 16r \Rightarrow r = 3.$$

∴ Radius of circle *i.e.* of the sphere = 3 cm.

$$\therefore \text{Volume of sphere} = \frac{4}{3}\pi \times 3^3 \text{ cm}^3 = 36\,\pi \text{ cm}^3.$$

The volume of water which overflows = volume of the sphere

$$= 36\pi \text{ cm}^3.$$

Volume of water in the cone before immersing the sphere

$$= \text{volume of the cone} = \frac{1}{3}\pi \times 6^2 \times 8 \text{ cm}^3$$
$$= 96\pi \text{ cm}^3.$$

$$\therefore \text{The fraction of water which overflows} = \frac{\text{Volume of water overflows}}{\text{Total volume of water}} = \frac{36\pi}{96\pi} = \frac{3}{8}.$$

Exercise 18.4

Take $\pi = \frac{22}{7}$, *unless stated otherwise.*

1. Find the volume and the curved surface of the sphere if its diameter is 14 cm.
2. If the volume of a sphere is $179\frac{2}{3}$ cm³, find its radius and the surface area.
3. 8 metallic spheres, each of radius 2 cm, are melted and cast into a single sphere. Calculate the radius of the new (single) sphere.

4. The radius of a sphere is 9 cm. It is melted and drawn into a wire of diameter 2 mm. Find the length of the wire in metres.
5. A metallic disc, in the shape of a right circular cylinder, is of height 2·5 mm and base radius 12 cm. Metallic disc is melted and made into a sphere. Calculate the radius of the sphere.
6. The radius of the base of a cone and the radius of a sphere are the same, each being 8 cm. Given that the volume of these two solids are also the same, calculate the slant height of the cone, correct to one place of decimal.
7. Two spheres of the same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the big sphere.

Hint

Weight of single big sphere = (1 + 7) kg = 8 times weight of small sphere
⇒ volume of single big sphere = 8 times volume of small sphere.

8. (*a*) A metallic sphere of radius 10·5 cm is melted and then recast into small cones, each of radius 3·5 cm and height 3 cm. Find the number of cones thus obtained. *(2005)*
   (*b*) A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones formed. *(2012)*
9. What is the least number of solid metallic spheres, each 6 cm in diameter, that should be melted to cast a solid metallic cylinder whose height is 53 cm and diameter 6 cm ? Also find the total surface area of the cylinder.
10. The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Calculate :
    (*i*) the radius of the solid sphere.
    (*ii*) the number of cones recast. (Take π = 3·14). *(2000)*
11. (*a*) If the ratio of the radii of two spheres is 3 : 7, find :
    (*i*) the ratio of their volumes.
    (*ii*) the ratio of their surface areas.
    (*b*) If the ratio of the volumes of the two spheres is 125 : 64, find the ratio of their surface areas.
12. A sphere and a cube have the same surface. Show that the ratio of the volume of the sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$.
13. An iron sphere of diameter 12 cm is dropped into a cylindrical can of diameter 24 cm containing water. Find the rise in the level of water when the sphere is completely immersed.
14. (*a*) There is water to a height of 14 cm in a cylindrical glass jar of radius 8 cm. Inside the water there is a sphere of diameter 12 cm completely immersed. By what height will the water go down when the sphere is removed ?
    (*b*) A cylindrical can whose base is horizontal and of radius 3·5 cm contains sufficient water so that when a sphere is placed in the can, the water *just* covers the sphere. Given that the sphere *just* fits into the can, calculate :
    (*i*) the total surface area of the can in contact with water when the sphere is in it.
    (*ii*) the depth of the water in the can before the sphere was put into the can. Give your answer as proper fractions.

15. Marbles of diameter 1·4 cm are dropped into a cylindrical beaker containing some water and are fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5·6 cm.

16. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top which is open is 5 cm. It is filled with water upto the rim. When lead shots, each of which is a sphere of radius 0·5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped into the vessel.

17. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4 cm and height is 72 cm, find the uniform thickness of the cylinder.

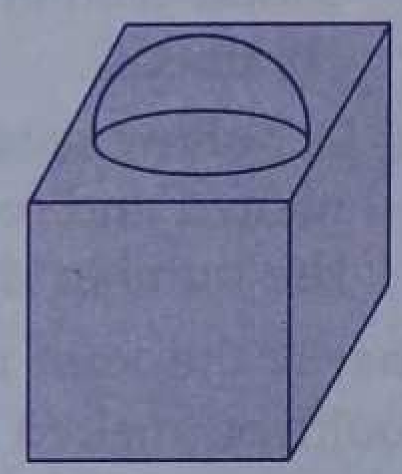
18. A hollow copper ball has an external diameter of 12 cm, and a thickness of 0·1 cm. Find :

(*i*) the outer surface area of the ball.

(*ii*) the weight of the ball if 1 cm^3 of copper weighs 8·8 g. (Take π to be 3·14)

19. Find the total surface area of a hemisphere of diameter 42 cm.

20. The adjoining figure shows a decorative block, which is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter 4·2 cm. Find the total volume and the total surface area of the block.

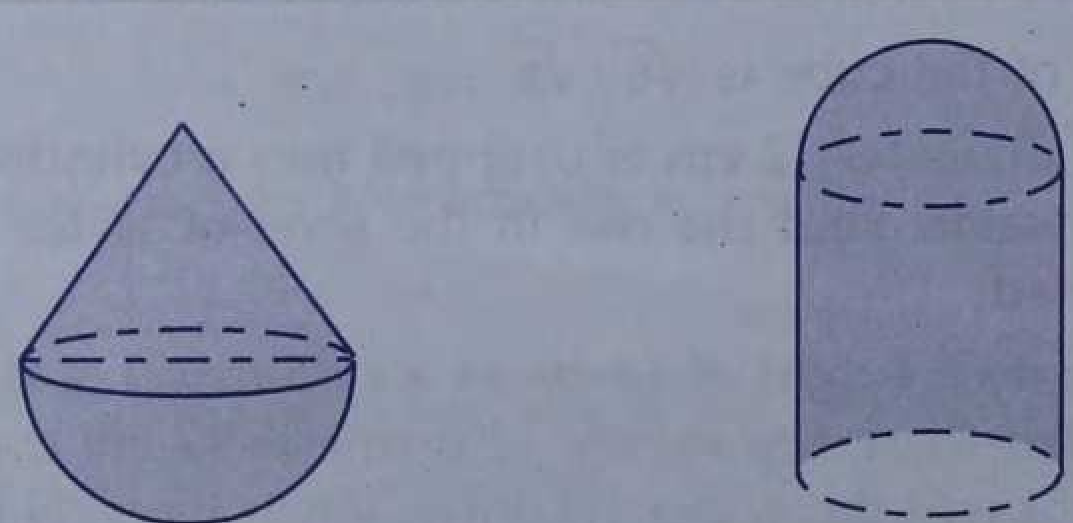


21. (*a*) The figure (*i*) given below shows a hemisphere of radius 5 cm surmounted by a right circular cone of base radius 5 cm. Find the volume of the solid if the height of the cone is 7 cm. Give your answer correct to two places of decimal.

(*b*) The figure (*ii*) given below shows a metal container in the form of a cylinder surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3·5 m. Calculate :

(*i*) the total area of the internal surface, excluding the base.

(*ii*) the internal volume of the container in m^3.



(*i*) (*ii*)

22. A hemispherical bowl of diameter 7·2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4·8 cm. Find the height of the cone. *(2010)*

23. A buoy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of the hemisphere. The radius of the base of the cone is 3·5 metres and its volume is $\frac{2}{3}$ of the hemisphere. Calculate the height of the cone and the surface area of the buoy correct to 2 places of decimal.

24. A circular hall (big room) has a hemispherical roof. The greatest height is equal to the inner diameter. Find the area of the floor, given that the capacity of the hall is 48510 m^3.

25. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3·5 cm and the height of the cylindrical and conical portions are 10 cm and 6 cm respectively. Find the volume of the solid. (Take π = 3·14).

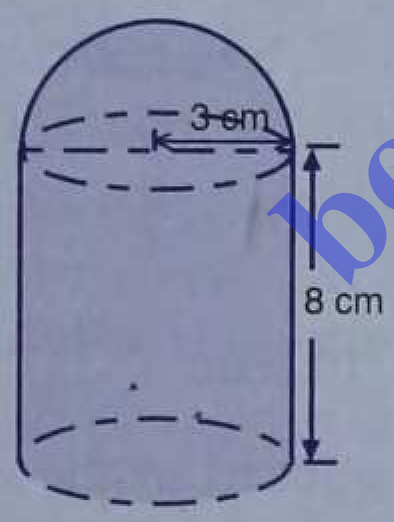
26. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of the conical part is 12 cm.

27. (*a*) The figure (*i*) given below shows a model of a solid consisting of a cylinder surmounted by a hemisphere at one end. If the model is drawn to a scale of 1 : 200, find

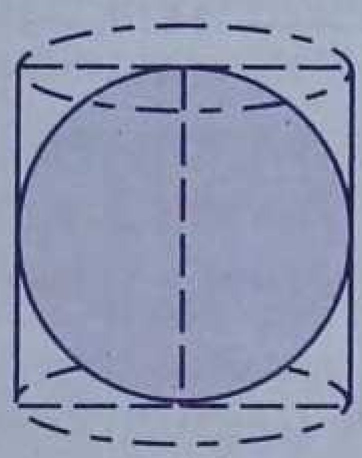
(*i*) the total surface area of the solid in π m^2.

(*ii*) the volume of the solid in π litres.

(*b*) In the figure (*ii*) given below, a sphere is inscribed in a cylinder, prove that the surface of the sphere is equal to the curved surface of the cylinder.



(*i*)


(*ii*)

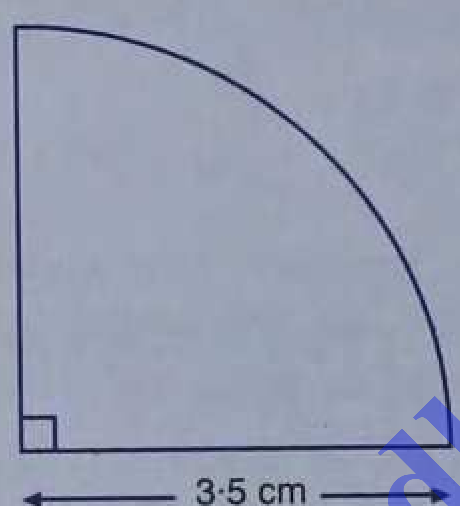
28. A hemisphere, a cylinder and a cone have equal base diameters and have the same height. Prove that their volumes are in the ratio 2 : 3 : 1.

29. A solid consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of the water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cubic centimetre.

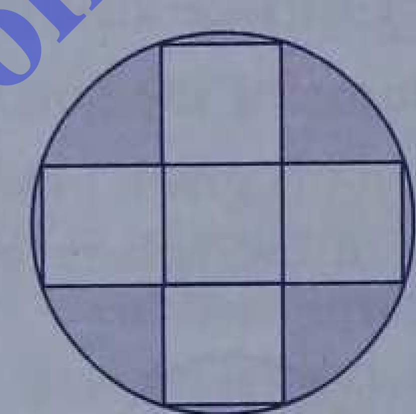
CHAPTER TEST

Take $\pi = \frac{22}{7}$, *unless stated otherwise.*

1. A tonga is being driven at 11 km/hr. If each wheel of the tonga is 140 cm in diameter, find the number of revolutions made by each wheel per minute.
2. A wire is looped in the form of a circle of radius 28 cm. It is rebent into the form of a square. Find the area of the square.
3. (*a*) From a sheet of paper of dimensions 2 m × 1·5 m, how many circles of radius 5 cm can be cut? Also find the area of the paper wasted. Take $\pi = 3{\cdot}14$.
   (*b*) If the diameter of a semi-circular protractor is 14 cm, then find its perimeter.
4. A race track is in the form of a ring whose inner circumference is 352 m and the outer circumference is 396 m. Find the area of the track.
5. Find the area enclosed between two concentric circles of radii 3·5 cm and 7 cm. A third concentric circle is drawn outside the 7 cm circle so that the area enclosed between it and the 7 cm circle is the same as that between the two inner circles. Find the radius of the third circle correct to one decimal place.
6. (*a*) In the figure (*i*) given below, the radius is 3·5 cm. Find the perimeter of the quarter of the circle.
   (*b*) In the figure (*ii*) given below, there are five squares each of side 2 cm.
   (*i*) Find the radius of the circle.
   (*ii*) Find the area of the shaded region. (Take $\pi = 3{\cdot}14$).



(*i*)

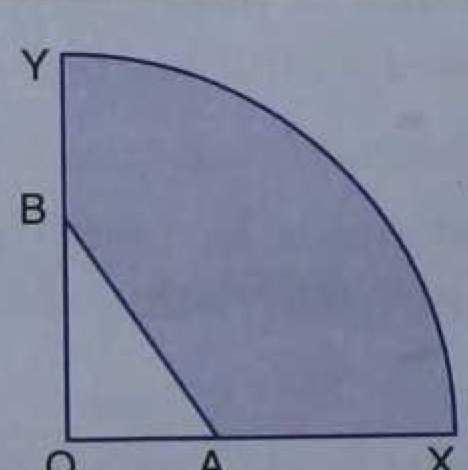


(*ii*)

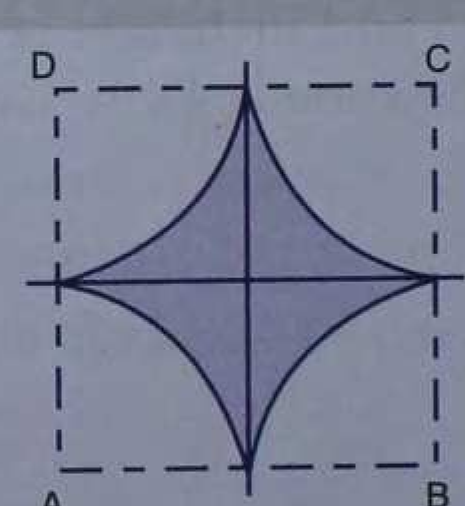
Hint

(*b*) (*i*) radius of the circle = $\sqrt{3^2 + 1^2}$ cm = $\sqrt{10}$ cm.

7. (*a*) In the figure (*i*) given below, a piece of cardbord in the shape of a quadrant of a circle of radius 7 cm is bounded by perpendicular radii OX and OY. Points A and B lie on OX and OY respectively such that OA = 3 cm and OB = 4 cm. The triangular part OAB is removed. Calculate the area and the perimeter of the remaining piece.
   (*b*) In the figure (*ii*) given below, ABCD is a square. Points A, B, C and D are centres of quadrants of circles of the same radius. If the area of the shaded portion is $21\frac{3}{7}$ cm², find the radius of the quadrants.



(*i*)



(*ii*)

8. A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze ?

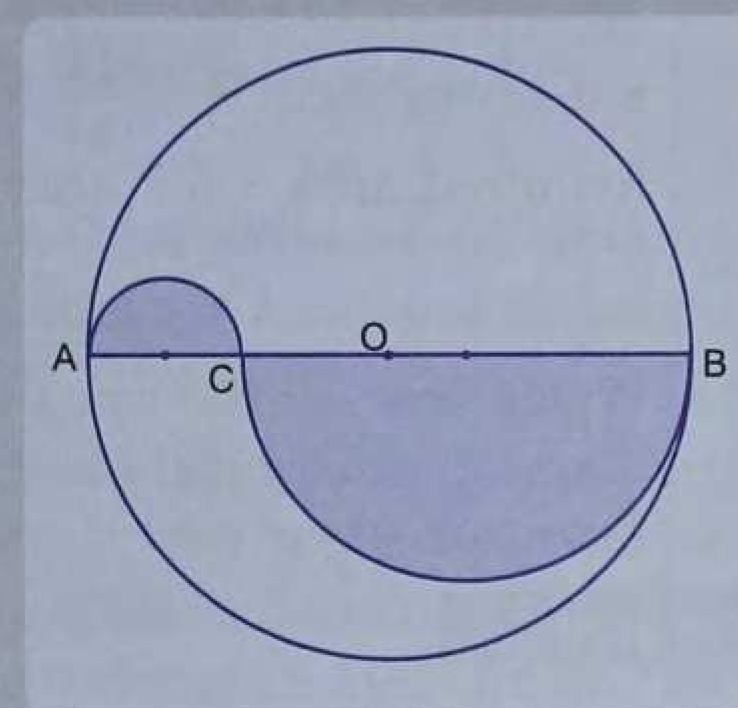
9. The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in 15 minutes.

10. Find the radius of a circle if a 90° arc has a length of $3{\cdot}5\,\pi$ cm. Hence, find the area of the sector formed by this arc.

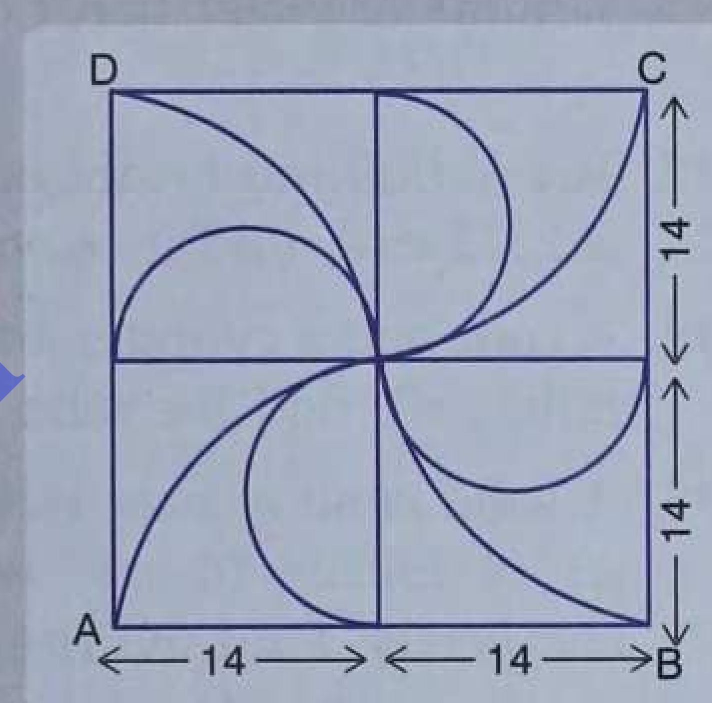
11. In the adjoining figure, if the diameter of the circle with centre O is 28 cm and AC = $\frac{1}{4}$ AB, find

(*i*) the area of the shaded region.

(*ii*) perimeter of the shaded region.



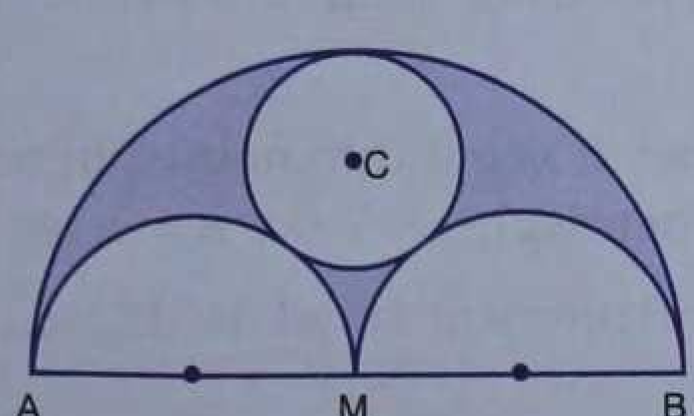
12. Mary is wearing a set of ear-rings made of line segments, semicircular arcs and arcs of quadrants of circles (shown in the adjoining figure). Calculate the total length of the metal wire required to make a set of ear-rings. All dimensions are given in millimetres.



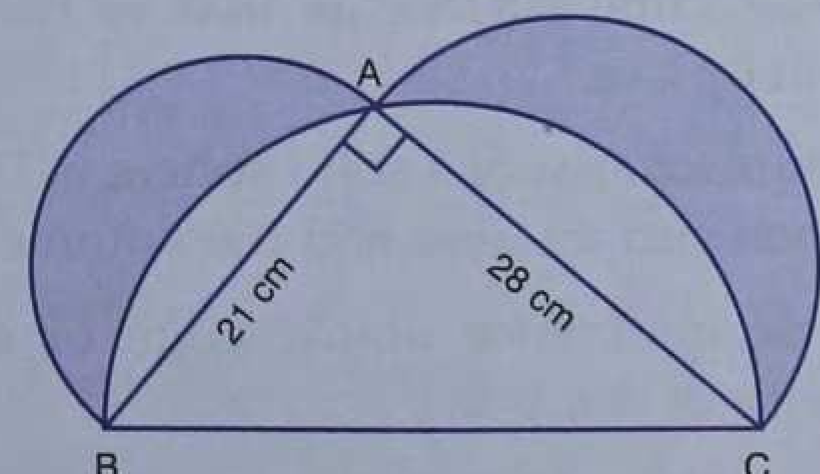
13. A cube whose each edge is 28 cm long has a circle of maximum radius on each of its face painted red. Find the total area of the unpainted surface of the cube.

14. (*a*) In the figure (*i*) given below, AB = 36 cm and M is mid-point of AB. Semicircles are drawn on AB, AM and MB as diameters. A circle with centre C touches all the three circles as shown. Find the area of the shaded region.

(*b*) In the figure (*ii*) given below, ABC is a right angled triangle, ∠A = 90°, AB = 21 cm and AC = 28 cm. Semicircles are described on AB, BC and AC as diameters. Find the area of the shaded region.



(*i*)

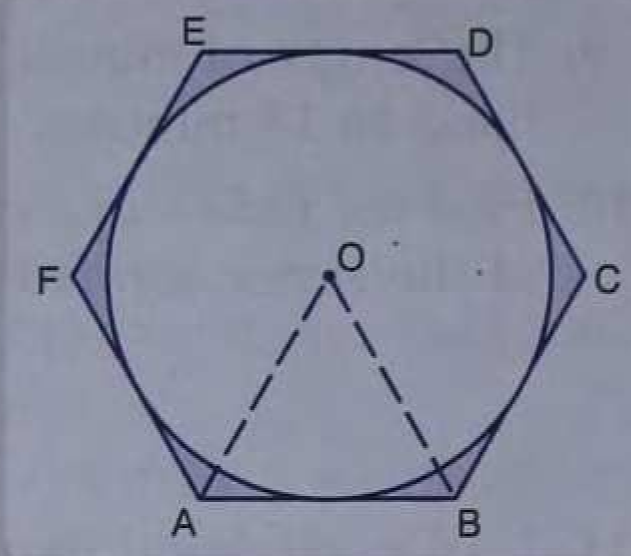


(*ii*)

Hint

(*a*) It will be found that radius of circle with centre C = $\frac{1}{6} \times 36$ cm = 6 cm.

15. In the adjoining figure ABCDEF is a regular hexagon of side 10 cm. O is the centre of the inscribed circle. Find the area of the shaded region.
(Leave the answer in π and surds.)



Hint

Radius of incircle= $\frac{\sqrt{3}}{2} \times 10$ cm $= 5\sqrt{3}$ cm.
Required area = 6 × area of Δ OAB
− area of incircle.

16. Water flows at the rate of 10 m per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel of base diameter 40 cm and depth 24 cm ?

Hint

Volume of water that flows out in one minute $= \pi \times \left(\frac{1}{4}\right)^2 \times 10 \times 100$ cm^3.

17. The radius and height of a right circular cone are in the ratio 5 : 12. If its volume is 2512 cm^3, find its slant height. (Take $\pi = 3{\cdot}14$)

18. A cone and a cylinder are of the same height. If diameters of their bases are in the ratio 3 : 2, find the ratio of their volumes.

19. A solid cone of base radius 9 cm and height 10 cm is lowered into a cylindrical jar of radius 10 cm, which contains water sufficient to submerge the cone competely. Find the rise in water level in the jar.

20. An iron pillar has some part in the form of a right circular cylinder and the remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if one cu. cm of iron weighs 7·8 grams.

21. A circus tent is made of canvas and is in the form of right circular cylinder and a right circular cone above it. The diameter and height of the cylindrical part of the tent are 126 m and 5 m respectively. The total height of the tent is 21 m. Find the total cost of the tent if the canvas used costs ₹ 36 per square metre.

22. The diameter of a sphere is 21 cm. Find the length of the edge of a cube which has the same surface as that of the sphere. Give your answer correct to one decimal place.

23. The surface area of a solid sphere is 1256 cm^2. It is cut into hemispheres. Calculate the total surface area and the volume of a hemisphere. Take $\pi = 3{\cdot}14$.

24. The circumference of the edge of a hemispherical bowl is 132 cm. Find the capacity of the bowl.

25. A hemisphere of lead of radius 8 cm is cast into a right circular cone of base radius 6 cm. Determine the height of the cone correct to 2 places of decimal.

26. A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

27. The diameter of a metallic sphere is 42 cm. It is metled and drawn into a cylindrical wire of 28 cm diameter. Find the length of the wire.

28. A sphere of diameter 6 cm is dropped into a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

29. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and the total surface area of the solid.

30. A solid sphere of radius 6 cm is metled into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.

31. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3·5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical vessel, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylindrical vessel is 5 cm and its height is 10·5 cm, find the volume of water left in the cylindrical vessel.