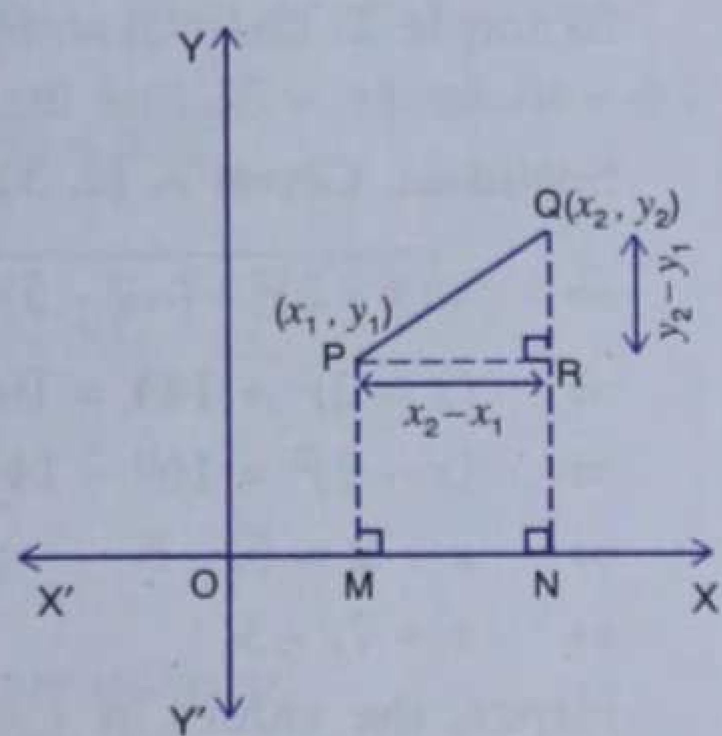
11 Distance and Section Formulae

11.1 DISTANCE FORMULA

Find the distance between two points whose co-ordinates are given.

Let P (x_1, y_1) and Q (x_2, y_2) be two given points in the co-ordinate plane.



Draw PM, QN perpendiculars on x-axis and PR perpendicular on NQ.

From the figure,

$$PR = MN = ON - OM$$
$$= x_2 - x_1 \quad ...(i)$$
$$RQ = NQ - NR = NQ - MP$$
$$= y_2 - y_1 \quad ...(ii)$$

From right-angled ΔPRQ, by Pythagoras Theorem, we get

$$PQ^2 = PR^2 + RQ^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2 \quad \text{(Using (i) and (ii))}$$
$$\therefore \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

(only positive square root is to be taken because PQ being the distance between two points is positive)

Corollary. Distance from origin.

The distance of the point (x, y) from the origin $(0, 0)$

$$= \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}.$$

Remarks

- The formula remains the same if the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are taken in different quadrants. For convenience, we have taken these points in the first quadrant.
- If a point lies on x-axis *i.e.* on $y = 0$, its ordinate is zero, therefore, any point on x-axis can be taken as $(x, 0)$.

- If a point lies on y-axis *i.e.* on $x = 0$, its abscissa is zero, therefore, any point on y-axis can be taken as $(0, y)$.
- To prove that a quadrilateral is a
  - (*i*) *rhombus*, show that all sides are equal.
  - (*ii*) *square*, show that all sides are equal and diagonals are also equal.
  - (*iii*) *parallelogram*, show that opposite sides are equal.
  - (*iv*) *rectangle*, show that opposite sides are equal and diagonals are also equal.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the distance between the points P(3, –5) and Q(8, 7).*

Solution. Let $P(3, -5) \equiv P(x_1, y_1)$ and $Q(8, 7) \equiv Q(x_2, y_2)$.

∴ The distance between the given points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-3)^2 + (7-(-5))^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13 \text{ units.}$$

Example 2. *KM is a straight line of 13 units. If K has the co-ordinates (2, 5) and M has the co-ordinates (x, –7), find the possible values of x.* (2004)

Solution. Given K (2, 5), M $(x, -7)$ and KM = 13

$$\Rightarrow \quad \sqrt{(x-2)^2 + (-7-5)^2} = 13$$

$$\Rightarrow \quad (x-2)^2 + 144 = 169$$

$$\Rightarrow \quad (x-2)^2 = 169 - 144 = 25$$

$$\Rightarrow \quad x - 2 = 5, -5$$

$$\Rightarrow \quad x = 7, -3.$$

Hence, the values of x are 7 or –3.

Example 3. *Point A(1, –5) is mapped as A′ on reflection in the x-axis. Point B(3, 2) is mapped as B′ on reflection in the origin. Write the co-ordinates of A′ and B′. Calculate AB′.*

Solution. Since the point A′ is the reflection of the point A(1, –5) in the x-axis, the co-ordinates of A′ are (1, 5).

Further, as the point B′ is the reflection of the point B(3, 2) in the origin, the co-ordinates of B′ are (–3, –2).

AB′ = distance between the points A(1, –5) and B′(–3, –2)

$$= \sqrt{(-3-1)^2 + (-2-(-5))^2} = \sqrt{(-4)^2 + 3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$

Example 4. *If the point P(–3, 5) is reflected in the line x = 2 on the point P′ and the point Q(a, b) is reflected in the origin on the point Q′ (3, –4), then find the length P′Q.*

Solution. We know that the reflection of the point (x, y) in the line $x = a$ is the point $(-x + 2a, y)$, therefore, the reflection of the point P(–3, 5) in the line $x = 2$ is the point $P'(-(-3) + 2 \cdot 2, 5)$ *i.e.* P′(7, 5).

The reflection of the point Q(a, b) in the origin is the point $(-a, -b)$ but the reflection of the point Q(a, b) in origin is the point Q′(3, –4)

$\Rightarrow \quad -a = 3, -b = -4 \Rightarrow a = -3, b = 4.$

$\therefore$ The point Q is (–3, 4).

$\therefore$ Length P′Q = distance between P(7, 5) and Q(–3, 4)

$$= \sqrt{(-3-7)^2+(4-5)^2} = \sqrt{(-10)^2+(-1)^2}$$

$$= \sqrt{100+1} = \sqrt{101} \text{ units.}$$

Example 5. *What point on the x-axis is at a distance of 5 units from the point (5, – 4) ?*

Solution. Let $(x, 0)$ be any point on the x-axis.

Since the distance between the points $(x, 0)$ and $(5, -4)$ is 5 units,

$\therefore \quad \sqrt{(5-x)^2+(-4-0)^2} = 5 \Rightarrow 25 + x^2 - 10x + 16 = 25$

$\Rightarrow \quad x^2 - 10x + 16 = 0 \Rightarrow (x-2)(x-8) = 0$

$\Rightarrow \quad x - 2 = 0$ or $x - 8 = 0 \Rightarrow x = 2$ or $x = 8$.

$\therefore$ The required point on the x-axis is (2, 0) or (8, 0).

Example 6. *Show that the points (0, 0), (5, 5) and (–5, 5) are the vertices of a right isosceles triangle.*

Solution. Let the points be A(0, 0), B (5, 5) and C (–5, 5), then

$$AB = \sqrt{(5-0)^2+(5-0)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2},$$

$$BC = \sqrt{(-5-5)^2+(5-5)^2} = \sqrt{100+0} = \sqrt{100} = 10 \text{ and}$$

$$CA = \sqrt{(0-(-5))^2+(0-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

$\Rightarrow \quad AB^2 = 50, BC^2 = 100$ and $CA^2 = 50$.

$\therefore \quad AB^2 + CA^2 = 50 + 50 = 100 = BC^2$

$\Rightarrow \quad \Delta$ ABC is right angled and it is right angled at A.

Also $\quad AB = 5\sqrt{2} = CA \Rightarrow \Delta ABC$ is isosceles.

Hence, the given points form a right isosceles triangle.

Example 7. *Show that the points (4, 2), (7, 5) and (9, 7) are collinear.*

Solution. Let the points be A (4, 2), B (7, 5) and C (9, 7), then

$$AB = \sqrt{(7-4)^2+(5-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2},$$

$$BC = \sqrt{(9-7)^2+(7-5)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ and}$$

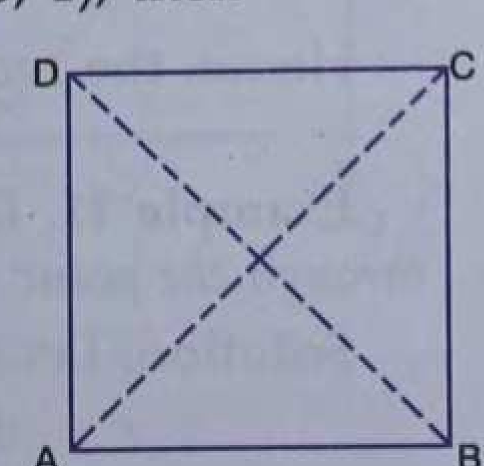
$$AC = \sqrt{(9-4)^2+(7-2)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}.$$

$\therefore \quad AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC.$

Hence, the given points are collinear.

Example 8. *Show that the points (0, –2), (3, 1), (0, 4) and (–3, 1) are the vertices of a square. Also find the area of the square.*

Solution. Let the points be A (0, –2), B (3, 1), C (0, 4) and D (–3, 1), then



$$AB = \sqrt{(3-0)^2+(1-(-2))^2} = \sqrt{9+9} = \sqrt{18},$$

$$BC = \sqrt{(0-3)^2+(4-1)^2} = \sqrt{9+9} = \sqrt{18},$$

$$CD = \sqrt{(-3-0)^2+(1-4)^2} = \sqrt{9+9} = \sqrt{18} \text{ and}$$

$$DA = \sqrt{(0-(-3))^2+(-2-1)^2} = \sqrt{9+9} = \sqrt{18}$$

$\Rightarrow \quad AB = BC = CA = DA \Rightarrow$ all the four sides are equal.

Also $\quad AC = \sqrt{(0-0)^2 + (4-(-2))^2} = \sqrt{0+36} = 6$ and

$$BD = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{36+0} = 6$$

$\Rightarrow \quad AC = BD \Rightarrow$ both the diagonals are equal.

Hence, the given points are the vertices of a square.

Area of the square = $(\text{side})^2 = \left(\sqrt{18}\right)^2$ sq. units = 18 sq. units.

Example 9. *What point on y-axis is equidistant from the points (7, 6) and (–3, 4) ?*

Solution. Let P $(0, y)$ be any point on y-axis.

Let the given points be A(7, 6) and B(–3, 4), then

$$AP = \sqrt{(0-7)^2 + (y-6)^2} \text{ and } BP = \sqrt{(0-(-3))^2 + (y-4)^2}.$$

According to given, $\quad AP = BP$

$\Rightarrow \sqrt{49 + (y-6)^2} = \sqrt{9 + (y-4)^2}$

$\Rightarrow 49 + (y-6)^2 = 9 + (y-4)^2$

$\Rightarrow 49 + y^2 + 36 - 12y = 9 + y^2 + 16 - 8y$

$\Rightarrow -4y = -60 \Rightarrow y = 15.$

$\therefore$ The required point is (0, 15).

Example 10. *Find the points on the x-axis whose distances from the points (2, 3) and* $\left(\frac{3}{2}, -1\right)$ *are in the ratio 2 : 1.*

Solution. Let P $(x, 0)$ be any point on the x-axis. Let the given points be A (2, 3) and B $\left(\frac{3}{2}, -1\right)$, then

$$AP = \sqrt{(x-2)^2 + (0-3)^2} \text{ and } BP = \sqrt{\left(x - \frac{3}{2}\right)^2 + (0-(-1))^2}.$$

According to given, $\frac{AP}{BP} = \frac{2}{1} \Rightarrow AP = 2BP$

$\Rightarrow \quad \sqrt{(x-2)^2 + 9} = 2 . \sqrt{\left(x - \frac{3}{2}\right)^2 + 1}$

$\Rightarrow \quad (x-2)^2 + 9 = 4\left[\left(x - \frac{3}{2}\right)^2 + 1\right]$

$\Rightarrow \quad x^2 - 4x + 4 + 9 = 4\left[x^2 - 3x + \frac{9}{4} + 1\right]$

$\Rightarrow \quad x^2 - 4x + 13 = 4x^2 - 12x + 13$

$\Rightarrow \quad -3x^2 + 8x = 0 \Rightarrow x(-3x + 8) = 0$

$\Rightarrow \quad x = 0$ or $-3x + 8 = 0 \Rightarrow x = 0$ or $\frac{8}{3}$.

Hence, the required points are (0, 0) or $\left(\frac{8}{3}, 0\right)$.

Example 11. *Find the area of a circle whose centre is at the point (5, –1) and which passes through the point (–3, 5). Take $\pi = 3{\cdot}1416$.*

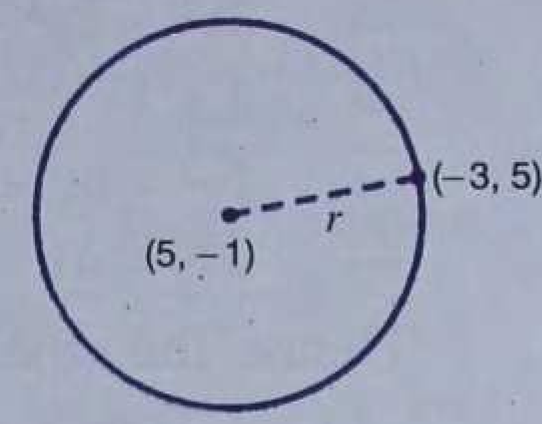
Solution. Let the radius of the circle be r, then

r = distance between the points (5, –1) and (–3, 5)

$= \sqrt{(-3-5)^2 + (5+1)^2}$ units

$= \sqrt{64 + 36}$ units $= \sqrt{100}$ units

$= 10$ units.



$\therefore$ The area of the circle $= \pi r^2 = 3{\cdot}1416 \times 10^2$ sq. units

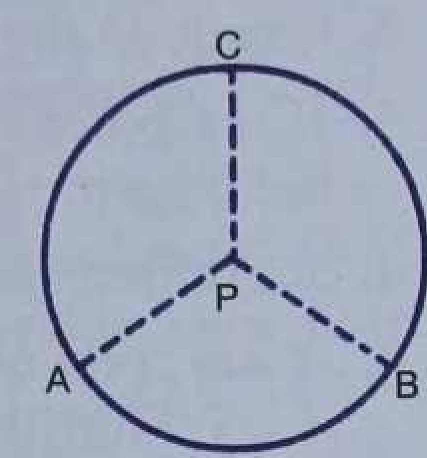
$= 3{\cdot}1416 \times 100$ sq. units

$= 314{\cdot}16$ sq. units.

Example 12. *Find the centre of the circle passing through the points (5, 7), (6, 6) and (2, –2). Also find its radius.*

Solution. Let the points be A(5, 7), B(6, 6) and C (2, –2).

Let P (x, y) be the centre of the circle passing through the given points A, B and C, then



AP = BP = CP (each being radius)

$\Rightarrow$ AP = BP and AP = CP

AP = BP $\Rightarrow \sqrt{(x-5)^2 + (y-7)^2} = \sqrt{(x-6)^2 + (y-6)^2}$

$\Rightarrow (x-5)^2 + (y-7)^2 = (x-6)^2 + (y-6)^2$

$\Rightarrow x^2 + 25 - 10x + y^2 + 49 - 14y = x^2 + 36 - 12x + y^2 + 36 - 12y$

$\Rightarrow 2x - 2y + 2 = 0$

$\Rightarrow x - y + 1 = 0$ ...(*i*)

and AP = CP $\Rightarrow \sqrt{(x-5)^2 + (y-7)^2} = \sqrt{(x-2)^2 + (y+2)^2}$

$\Rightarrow x^2 + 25 - 10x + y^2 + 49 - 14y = x^2 + 4 - 4x + y^2 + 4 + 4y$

$\Rightarrow -6x - 18y + 66 = 0$

$\Rightarrow x + 3y - 11 = 0$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get

$4y - 12 = 0 \Rightarrow y = 3.$

From (*i*), $x - 3 + 1 = 0 \Rightarrow x = 2.$

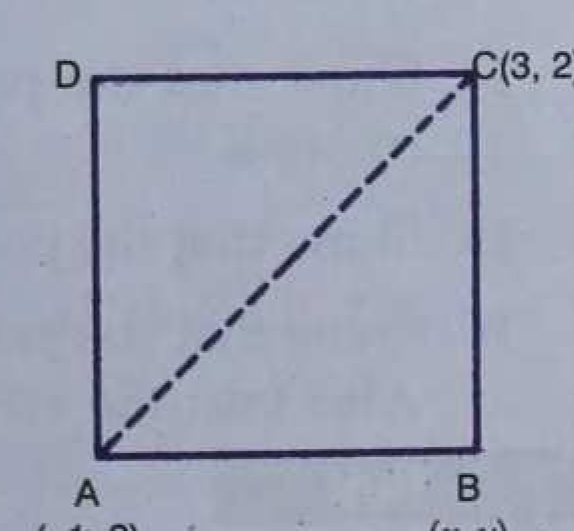
Hence, the centre of the circle is (2, 3).

Radius of the circle $= AP = \sqrt{(2-5)^2 + (3-7)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ units.

Example 13. *The two opposite vertices of a square are (–1, 2) and (3, 2). Find the co-ordinates of the other two.*

Solution. Let A(–1, 2) and C(3, 2) be two opposite vertices of the square ABCD and vertex B be (x, y).



Since ABCD is a square, AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow x^2 + 2x + 1 = x^2 - 6x + 9$

$\Rightarrow 8x = 8 \Rightarrow x = 1.$

Also as ABCD is a square, $\angle B = 90°$. By Pythagoras theorem, $AB^2 + BC^2 = AC^2$

$\Rightarrow ((x+1)^2 + (y-2)^2) + ((x-3)^2 + (y-2)^2) = (3+1)^2 + (2-2)^2$

$\Rightarrow 2x^2 - 4x + 10 + 2y^2 - 8y + 8 = 16$

$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow \quad 2 \times 1^2 + 2y^2 - 4 \times 1 - 8y + 2 = 0 \qquad (\because x = 1)$

$\Rightarrow \quad 2y^2 - 8y = 0 \Rightarrow y^2 - 4y = 0$

$\Rightarrow \quad y(y - 4) = 0 \Rightarrow y = 0, 4.$

Hence, the other two vertices are (1, 0) and (1, 4).

Exercise 11.1

1. Find the distance between the following pairs of points :
   (*i*) (–5, 3), (3, 1) (*ii*) (4, 5), (–3, 2) (*iii*) (–1, –4), (3, 5).
2. Calculate the distance between A(7, 3) and B on the x-axis whose abscissa is 11.
3. A is a point on the y-axis whose ordinate is 5 and B is the point (–3, 1). Calculate the length of AB.
4. Show that the point (4, 4) is equidistant from the points A(1, 0) and B(–1, 4).
5. A is a point on y-axis whose ordinate is 4 and B is a point on x-axis whose abscissa is –3. Find the length of the line segment AB.
6. The distance between A(1, 3) and B (x, 7) is 5. Calculate the possible values of x.
7. Point A (5, –1) on reflection in x-axis is mapped as A′. Also A on reflection in y-axis is mapped as A″. Write the co-ordinates of A′ and A″. Also calculate the distance AA″.
8. The point A(2, –3) is reflected in the origin on A′ and the point B(1, 7) in the y-axis on B′. Write the coordinates of A′ and B′. Also, compare the distances AB and A′B′.
9. B and C have co-ordinates (3, 2) and (0, 3). Find :
   (*i*) the image B′ of B under reflection in the x-axis.
   (*ii*) the image C′ of C under reflection in the line BB′.
   (*iii*) the length of B′C′.
10. What point (or points) on the y-axis are at a distance of 10 units from the point (8, 8) ?
11. Find point (or points) which are at a distance of $\sqrt{10}$ from the point (4, 3), given that the ordinate of the point (or points) is twice the abscissa.
12. A(2, 2), B(–2, 4) and C(2, 6) are the vertices of a triangle ABC. Prove that ABC is an isosceles triangle.
13. Show that the points (1, 1), (–1, –1) and $\left(-\sqrt{3}, \sqrt{3}\right)$ form an equilateral triangle.
14. Show that the points (3, 3), (9, 0) and (12, 21) are the vertices of a right angled triangle.
15. Show that the points (0, –1), (–2, 3), (6, 7) and (8, 3) are the vertices of a rectangle.
16. Show that the points (7, 3), (3, 0), (0, –4) and (4, –1) are the vertices of a rhombus. Also find the area of the rhombus.

Hint

Area of rhombus = $\frac{1}{2}$ (product of diagonals).

17. The points A (0, 3), B(–2, a) and C (–1, 4) are the vertices of a right angled triangle at A, find the value of a.

18. Show by distance formula that the points (–1, –1), (2, 3) and (8, 11) are collinear.

19. What point on the x-axis is equidistant from (7, 6) and (–3, 4) ?

20. Find the points on the y-axis whose distances from the points (6, 7) and (4, – 3) are in the ratio 1 : 2.

21. Find the abscissa of points whose ordinate is 4 and which are at a distance 5 units from (5, 0).

22. Find the value of x such that AB = BC where A, B and C are the points (6, –1), (1, 3) and (x, 8) respectively.

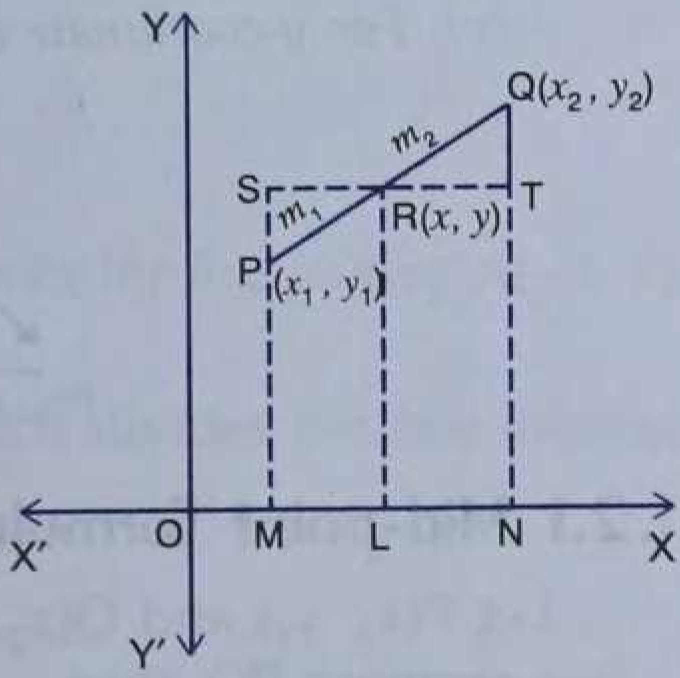
23. If A, B and P are the points (–4, 3), (0, –2) and (α, β) respectively and P is equidistant from A and B, show that 8 α – 10 β + 21 = 0.

24. The centre of a circle is (2α – 1, 3α + 1) and it passes through the point (–3, –1). Find the value (or values) of α if a diameter of the circle is of length 20 units.

25. Find the centre of the circle passing through the points (8, 12), (11, 3) and (0, 14). Also find its radius.

11.2 SECTION FORMULA

Find the co-ordinates of the point which divides (internally) the join of two given points in a given ratio.



Let P (x_1, y_1) and Q (x_2, y_2) be two given points in the co-ordinate plane, and R (x, y) be the point which divides PQ (internally) in the given ratio $m_1 : m_2$

i.e. $\frac{PR}{RQ} = \frac{m_1}{m_2}$ ...(*i*)

Draw PM, QN and RL perpendiculars on x-axis, and through R draw a st. line parallel to x-axis to meet MP (produced) at S and NQ at T.

From the figure,

$$SR = ML = OL - OM = x - x_1 \quad ...(ii)$$

$$RT = LN = ON - OL = x_2 - x \quad ...(iii)$$

$$PS = MS - MP = LR - MP = y - y_1 \quad ...(iv)$$

$$TQ = NQ - NT = NQ - LR = y_2 - y \quad ...(v)$$

Now ΔSPR is similar to ΔTQR,

$$\therefore \quad \frac{SR}{RT} = \frac{PR}{RQ}$$

$$\Rightarrow \quad \frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2} \quad \text{(Using (ii), (iii) and (i))}$$

$$\Rightarrow \quad m_2 x - m_2 x_1 = m_1 x_2 - m_1 x$$

$$\Rightarrow \quad m_1 x + m_2 x = m_1 x_2 + m_2 x_1$$

$$\Rightarrow \quad (m_1 + m_2)\, x = m_1 x_2 + m_2 x_1$$

$$\Rightarrow \quad x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}.$$

Again, as ΔSPR is similar to ΔTQR,

$$\therefore \quad \frac{PS}{TQ} = \frac{PR}{RQ}$$

$$\Rightarrow \quad \frac{y - y_1}{y_2 - y} = \frac{m_1}{m_2} \qquad \text{(Using } (iv), (v) \text{ and } (i))$$

$$\Rightarrow \quad m_2 y - m_2 y_1 = m_1 y_2 - m_1 y$$

$$\Rightarrow \quad m_1 y + m_2 y = m_1 y_2 + m_2 y_1$$

$$\Rightarrow \quad (m_1 + m_2)\, y = m_1 y_2 + m_2 y_1$$

$$\Rightarrow \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

Hence, the co-ordinates of R are $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$.

Rule to write down the co-ordinates of the point which divides the join of two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in a given ratio $m_1 : m_2$.

(i) Draw the line segment joining the given points P and Q.

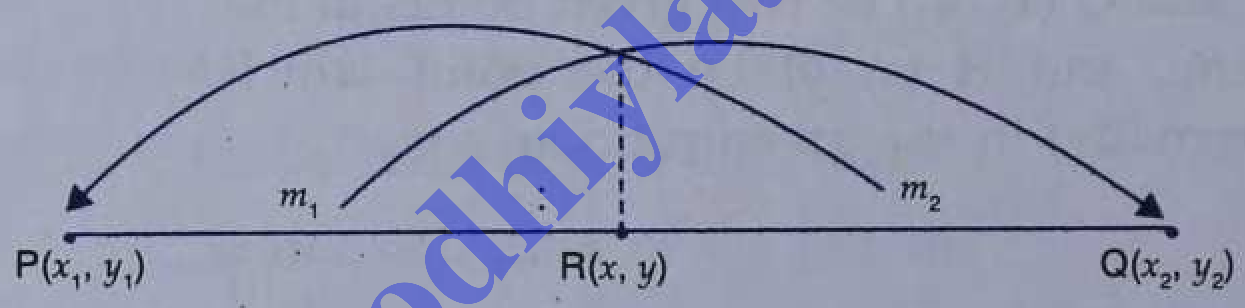
(ii) Write down the co-ordinates of P and Q at extremities.

(iii) Let R (x, y) be the point which divides PQ internally in the ratio $m_1 : m_2$.

(iv) For x-coordinate of R, multiply m_1 with x_2 and m_2 with x_1 as shown in the figure given below by arrows and add the products. Divide the sum by $m_1 + m_2$. Thus

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}.$$

(v) For y-coordinate of R, proceed as in step (iv).



11.2.1 Mid-point formula

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two given points and $R(x, y)$ be the *mid-point* of the line segment PQ, then

PR = RQ, therefore, the ratio is 1 : 1.

$$\therefore \quad x = \frac{1 . x_1 + 1 . x_2}{1 + 1} \text{ and } y = \frac{1 . y_1 + 1 . y_2}{1 + 1}$$

$$\Rightarrow \quad x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}.$$

Hence, the co-ordinates of the mid-point of PQ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Remark

For problems in which it is required to find the ratio when a given point divides the join of two given points, it is convenient to take the ratio as $k : 1$, for, in this way two unknowns (m_1 and m_2) are reduced to one unknown and the section formula becomes

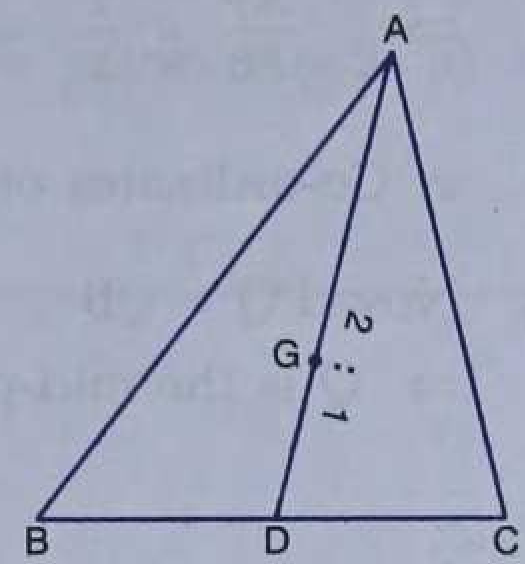
$$x = \frac{kx_2 + x_1}{k + 1} \text{ and } y = \frac{ky_2 + y_1}{k + 1}.$$

Then equate the abscissa or the ordinate of the point so obtained with that of the given point, and find the value of unknown k.

11.2.2 Centroid of a triangle

Definitions. *The straight line joining a vertex of a triangle to the mid-point of the opposite side is called a **median** of the triangle.*

*The point which divides a median of a triangle in the ratio 2 : 1 is called the **centroid** of the triangle. Thus, if AD is a median of the triangle ABC and G is its centroid, then* $\frac{AG}{GD} = \frac{2}{1}$.

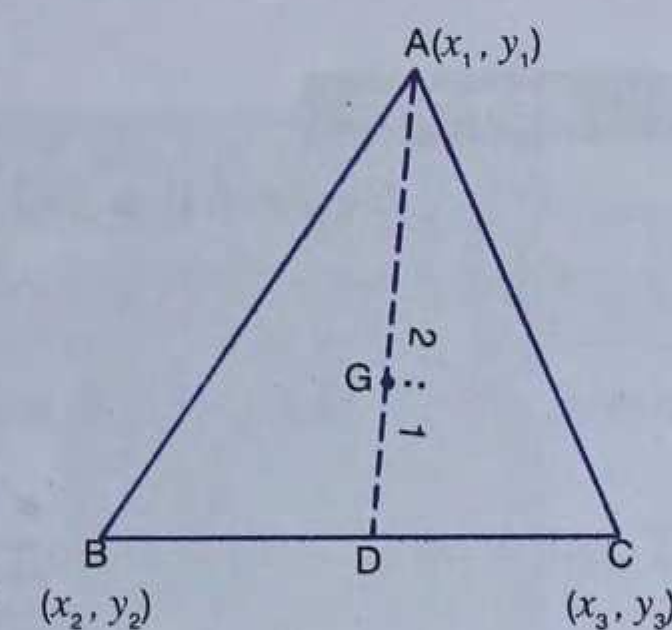


To find the centroid of a triangle whose vertices are given.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the given vertices of a triangle ABC. Let D be the mid-point of BC, then the coordinates of D are $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$. Let G be the centroid of Δ ABC, then G divides the median AD in the ratio 2 : 1. Therefore, co-ordinates of G are

$$\left(\frac{1.x_1 + 2.\frac{x_2 + x_3}{2}}{1 + 2}, \frac{1.y_1 + 2.\frac{y_2 + y_3}{2}}{1 + 2}\right)$$

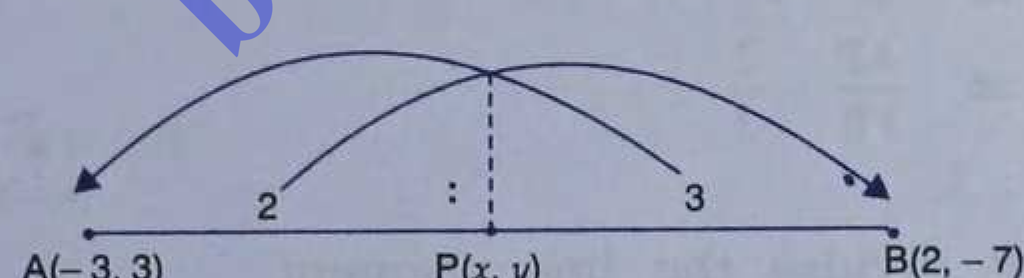
i.e. $$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$



ILLUSTRATIVE EXAMPLES

Example 1. *Calculate the co-ordinates of the point P which divides the line joining A(–3, 3) and B(2, –7) internally in the ratio 2 : 3.*

Solution. Let (x, y) be the co-ordinates of the point P which divides the line joining A(–3, 3) and B(2, –7) internally in the ratio 2 : 3, then



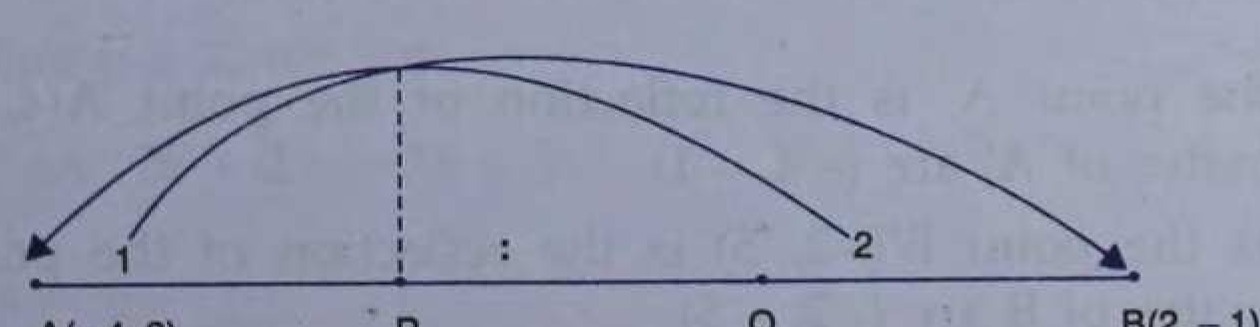
$$x = \frac{2.2 + 3.(-3)}{2 + 3} = \frac{4 - 9}{5} = -\frac{5}{5} = -1 \text{ and}$$

$$y = \frac{2.(-7) + 3.3}{2 + 3} = \frac{-14 + 9}{5} = -\frac{5}{5} = -1.$$

∴ The co-ordinates of P are (–1, –1).

Example 2. *Find the co-ordinates of the points of trisection of the line segment joining the points A(–4, 3) and B(2, –1).*

Solution. Let P and Q be the points of trisection of the line segment AB, then



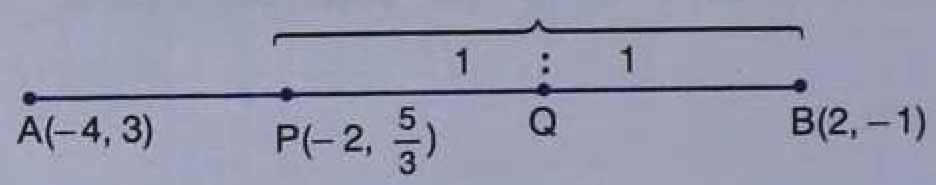
$$AP = PQ = QB \Rightarrow 2AP = PB$$

$\Rightarrow \quad \frac{AP}{PB} = \frac{1}{2} \quad \Rightarrow$ P divides AB in the ratio 1 : 2.

$\therefore$ Co-ordinates of P are $\left(\frac{1.2+2.(-4)}{1+2}, \frac{1.(-1)+2.3}{1+2}\right)$ *i.e.* $\left(\frac{2-8}{3}, \frac{-1+6}{3}\right)$ *i.e.* $\left(-2, \frac{5}{3}\right)$.

Now PQ = QB

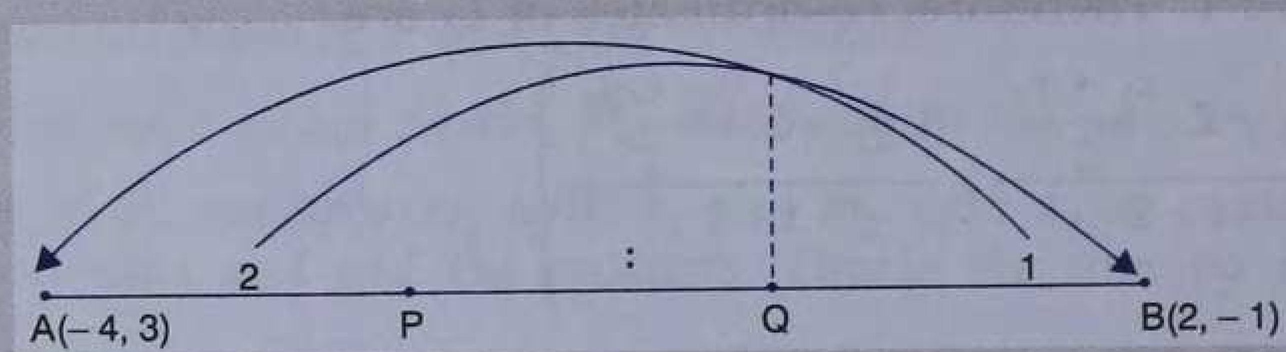
$\Rightarrow$ Q is the mid-point of PB.



$\therefore$ Co-ordinates of Q are $\left(\frac{-2+2}{2}, \frac{\frac{5}{3}+(-1)}{2}\right)$ *i.e.* $\left(0, \frac{\frac{2}{3}}{2}\right)$ *i.e.* $\left(0, \frac{1}{3}\right)$.

Remark

Since AP = PQ = QB, AQ = 2QB $\Rightarrow$ Q divides AB in the ratio 2 : 1.



$\therefore$ Co-ordinates of Q are

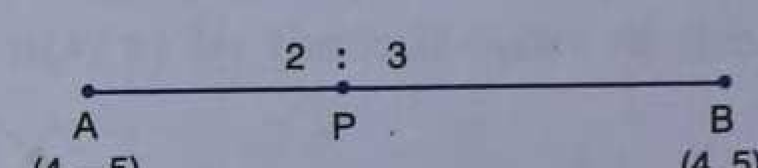
$\left(\frac{2.2+1.(-4)}{2+1}, \frac{2.(-1)+1.3}{2+1}\right)$ *i.e.* $\left(0, \frac{1}{3}\right)$.

Example 3. *If the line joining the points A(4, −5) and B(4, 5) is divided by the point P such that* $\frac{AP}{AB} = \frac{2}{5}$, *find the co-ordinates of P.* (2007)

Solution. Given $\frac{AP}{AB} = \frac{2}{5} \Rightarrow 5AP = 2AB = 2(AP + PB)$

$\Rightarrow \quad 3AP = 2PB \Rightarrow \frac{AP}{PB} = \frac{2}{3}$

$\Rightarrow \quad AP : PB = 2 : 3.$



Thus, the point P divides the line segment joining the points A(4, −5) and B(4, 5) in the ratio 2 : 3 internally.

$\therefore$ The co-ordinates of P are $\left(\frac{2.4+3.4}{2+3}, \frac{2.5+3.(-5)}{2+3}\right)$ *i.e.* (4, −1).

Example 4. *Point A(4, −1) is reflected as A′ in y-axis. Point B on reflection in x-axis is mapped as B′(−2, 5).*

(i) Write the co-ordinates of A′ and B.

(ii) Write the co-ordinates of the middle point of the line segment A′B.

Solution.

(*i*) Since the point A′ is the reflection of the point A(4, −1) in the *y*-axis, the co-ordinates of A′ are (−4, −1).

Also, as the point B′(−2, 5) is the reflection of the point B in the *x*-axis, the co-ordinates of B are (−2, −5).

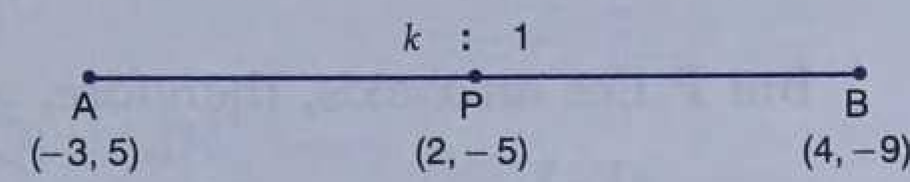
(*ii*) The co-ordinates of the mid-point of A′B are $\left(\frac{-4+(-2)}{2}, \frac{-1+(-5)}{2}\right)$ *i.e.* (−3, −3).

Example 5. *In what ratio does the point $P(2, -5)$ divide the line segment joining the points $A(-3, 5)$ and $B(4, -9)$?*

Solution. Let $P(2, -5)$ divide the line segment joining the points $A(-3, 5)$ and $B(4, -9)$ in the ratio $k : 1$ *i.e.* $AP : PB = k : 1$.

$\therefore$ Co-ordinates of P are

$$\left(\frac{k.4+1.(-3)}{k+1}, \frac{k.(-9)+1.5}{k+1}\right)$$



But P is $(2, -5) \Rightarrow \dfrac{4k-3}{k+1} = 2, \dfrac{-9k+5}{k+1} = -5$.

$$\frac{4k-3}{k+1} = 2 \Rightarrow 4k - 3 = 2k + 2 \Rightarrow 2k = 5 \Rightarrow k = \frac{5}{2}.$$

Similarly $\dfrac{-9k+5}{k+1} = -5 \Rightarrow k = \dfrac{5}{2}$.

$\therefore$ The required ratio is $\dfrac{5}{2} : 1$ *i.e.* $5 : 2$ (internally).

Remark

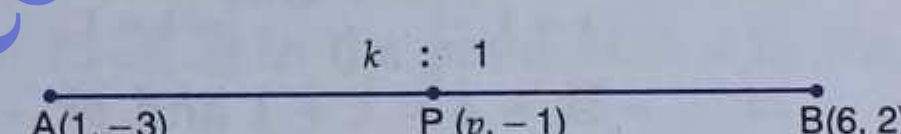
If we get different values of k from the two equations, it means that point P does not lie on the line AB, and the question of finding the dividing ratio will not arise.

Example 6. *In what ratio does the point $P(p, -1)$ divide the line segment joining the points $A(1, -3)$ and $B(6, 2)$? Hence, find the value of p.*

Solution. Let $P(p, -1)$ divide the line segment joining the points $A(1, -3)$ and $B(6, 2)$ in the ratio $k : 1$ *i.e.* $AP : PB = k : 1$.

$\therefore$ Co-ordinates of P are $\left(\dfrac{k.6+1.1}{k+1}, \dfrac{k.2+1.(-3)}{k+1}\right)$.

But P is $(p, -1) \Rightarrow \dfrac{2k-3}{k+1} = -1$



$\Rightarrow \quad 2k - 3 = -k - 1 \Rightarrow 3k = 2 \Rightarrow k = \dfrac{2}{3}$.

$\therefore$ The required ratio is $\dfrac{2}{3} : 1$ *i.e.* $2 : 3$ (internally).

Also $\dfrac{6k+1}{k+1} = p$ ...(*i*)

Putting $k = \dfrac{2}{3}$ in (*i*), we get

$$p = \frac{6 . \frac{2}{3}+1}{\frac{2}{3}+1} = \frac{5}{\frac{5}{3}} = \frac{5}{1} \times \frac{3}{5} = 3. \text{ Hence, } p = 3.$$

Example 7. *Calculate the ratio in which the line joining $A(6, 5)$ and $B(4, -3)$ is divided by the line $y = 2$.* (2000)

Solution. Let the line segment joining the points $A(6, 5)$ and $B(4, -3)$ be divided by the line $y = 2$ in the ratio $k : 1$ at P. By section formula, the co-ordinates of P are $\left(\dfrac{4k+6}{k+1}, \dfrac{-3k+5}{k+1}\right)$.

As P lies on the line $y = 2$, we get

$$\frac{-3k+5}{k+1} = 2 \Rightarrow 2k + 2 = -3k + 5$$

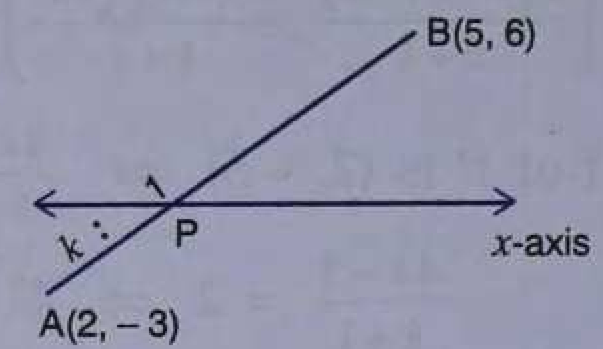
$$\Rightarrow \quad 5k = 3 \Rightarrow k = \frac{3}{5}.$$

$\therefore$ The required ratio is $\dfrac{3}{5} : 1$ *i.e.* $3 : 5$.

Example 8. *In what ratio is the line joining the points (2, –3) and (5, 6) divided by the x-axis ? Also find the co-ordinates of the point of intersection.*

Solution. Let the line segment joining the points A(2, – 3) and B(5, 6) be divided by the x-axis in the ratio $k : 1$ at P. By section formula, co-ordinates of P are $\left(\frac{5k+2}{k+1}, \frac{6k+(-3)}{k+1}\right)$.

But P lies on x-axis, therefore, y-co-ordinate of P = 0



$\Rightarrow \quad \frac{6k-3}{k+1} = 0 \Rightarrow 6k - 3 = 0$

$\Rightarrow \quad k = \frac{1}{2}.$

$\therefore$ The required ratio is $\frac{1}{2} : 1$ *i.e.* 1 : 2.

And the co-ordinates of the point of intersection are $\left(\frac{5.\frac{1}{2}+2}{\frac{1}{2}+1}, 0\right)$ *i.e.* $\left(\frac{\frac{9}{2}}{\frac{3}{2}}, 0\right)$ *i.e.* (3, 0).

Example 9. *If the points A(4, –5), B(1, 1) and C(–2, p) are collinear with B lying between A and C, then find the value of p.*

Solution. As the points A, B and C are collinear and B lies in between A and C, let the point B divide the line segment AC in the ratio $k : 1$, then the coordinates of the point B are $\left(\frac{-2k+4}{k+1}, \frac{pk-5}{k+1}\right)$.

But the point B is (1, 1)

$\Rightarrow \quad \frac{-2k+4}{k+1} = 1$ and $\frac{pk-5}{k+1} = 1$

$\Rightarrow \quad -2k + 4 = k + 1$ and $pk - 5 = k + 1$

$\Rightarrow \quad 3k = 3$ and $pk = k + 6$

$\Rightarrow \quad k = 1$ and $p \,.\, 1 = 1 + 6 \quad \Rightarrow p = 7.$

Hence, $p = 7$.

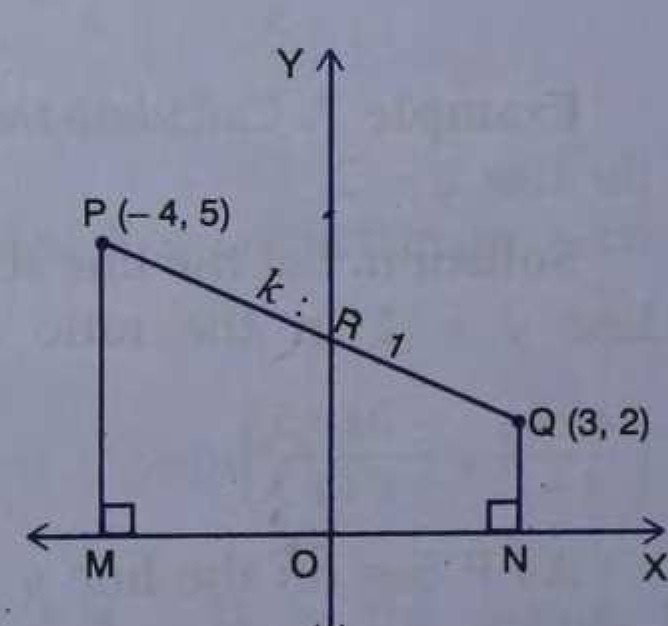
Example 10. *The line joining P (– 4, 5) and Q (3, 2) intersects the y-axis at R. PM and QN are perpendiculars from P and Q on the x-axis. Find :*

(i) the ratio PR : RQ.

(ii) the co-ordinates of R.

(iii) the area of the quadrilateral PMNQ. (2004)

Solution. (*i*) Let the line segment joining the points P (–4, 5) and Q (3, 2) be divided by the y-axis in the ratio $k : 1$ at R. By section formula, the co-ordinates of the point R are $\left(\frac{3k-4}{k+1}, \frac{2k+5}{k+1}\right)$. As the point R lies on the y-axis, x-co-ordinate of R = 0



$\Rightarrow \quad \frac{3k-4}{k+1} = 0 \Rightarrow 3k - 4 = 0$

$\Rightarrow \quad k = \frac{4}{3}.$

$\therefore \quad$ The required ratio is $\frac{4}{3} : 1$ *i.e.* 4 : 3.

(*ii*) The co-ordinates of the point R are $\left(0, \frac{2 \cdot \frac{4}{3}+5}{\frac{4}{3}+1}\right)$ *i.e.* $\left(0, \frac{23}{7}\right)$.

(*iii*) The quadrilateral PMNQ is a trapezium.

$\therefore$ The area of quad. PMNQ $= \frac{1}{2}$ (MP + NQ) × MN

$$= \frac{1}{2}(5 + 2) \times 7 \text{ sq. units}$$

$$= 24 \cdot 5 \text{ sq. units.}$$

Example 11. *The mid-point of the line segment joining (2a, 4) and (–2, 2b) is (1, 2a + 1). Find the values of a and b.* *(2007)*

Solution. Let A, B be the points $(2a, 4)$, $(-2, 2b)$ respectively.

The co-ordinates of the mid-points of AB are

$\left(\frac{2a+(-2)}{2}, \frac{4+2b}{2}\right)$ *i.e.* $(a - 1, 2 + b)$.

But the mid-point of AB is $(1, 2a + 1)$,

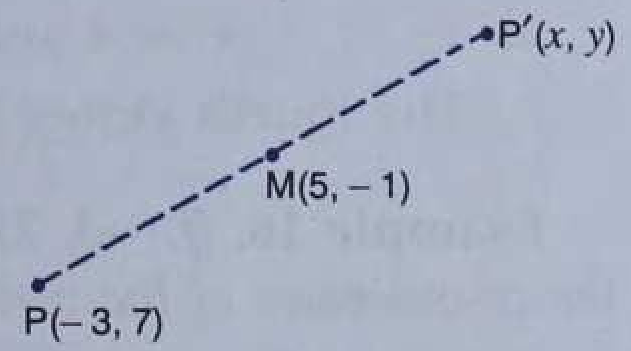
$\therefore \quad a - 1 = 1, \quad 2 + b = 2a + 1$

$\Rightarrow \quad a = 2, \quad 2 + b = 2 . 2 + 1 \Rightarrow a = 2, b = 3.$

Hence, $a = 2$ and $b = 3$.

Example 12. *Find the reflection of the point P(– 3, 7) in the point M(5, – 1).*

Solution. Let $P'(x, y)$ be the reflection of the point P(–3, 7) in the point M(5, –1), then M is the mid-point of the segment PP′



$\Rightarrow \quad \frac{-3+x}{2} = 5, \frac{7+y}{2} = -1$

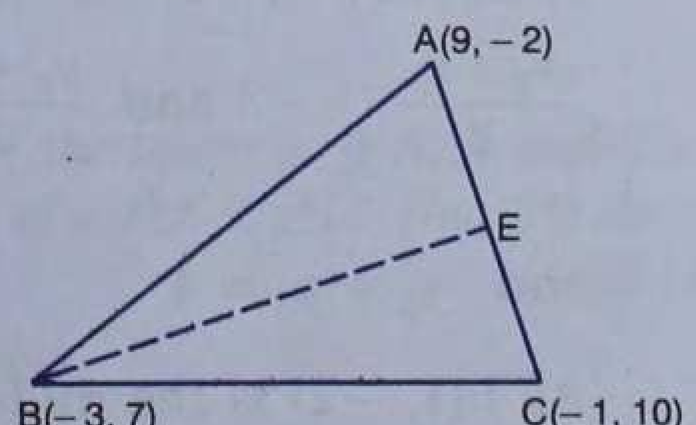
$\Rightarrow \quad -3 + x = 10, 7 + y = -2$

$\Rightarrow \quad x = 13, y = -9.$

$\therefore$ The reflection of the point P in the point M is the point (13, –9).

Example 13. *Find the length of the median through the vertex B of the triangle ABC with vertices A(9, –2), B(–3, 7) and C(–1, 10).*

Solution. Let E be the mid-point of AC, then BE is the median through B.



As E is mid-point of AC, co-ordinates of E are

$\left(\frac{9+(-1)}{2}, \frac{-2+10}{2}\right)$ *i.e.* (4, 4).

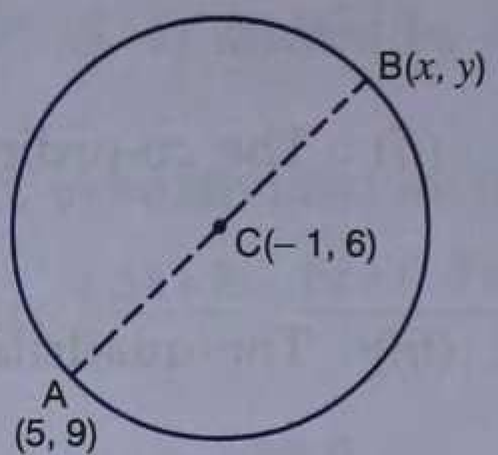
$\therefore$ Length of median BE $= \sqrt{(4-(-3))^2+(4-7)^2}$

$$= \sqrt{(7)^2+(-3)^2} = \sqrt{49+9} = \sqrt{58} \text{ units.}$$

Example 14. *The centre of a circle is C(–1, 6) and one end of a diameter is A(5, 9). Find the co-ordinates of the other end.*

Solution. Let the other end of the diameter of the circle be $B(x, y)$ whose one end is the point A(5, 9).

$\therefore$ The mid-point of AB is $\left(\frac{5+x}{2}, \frac{9+y}{2}\right)$.



The centre of the circle is C(–1, 6).

Since the centre of the circle is the mid-point of AB,

$$\frac{5+x}{2} = -1 \text{ and } \frac{9+y}{2} = 6$$

$\Rightarrow$ $5 + x = -2$ and $9 + y = 12$

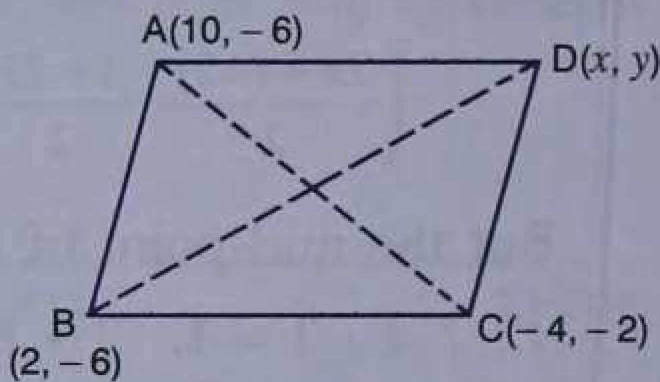
$\Rightarrow$ $x = -7$ and $y = 3$.

$\therefore$ The co-ordinates of the other end of the diameter are (–7, 3).

Example 15. *Three consecutive vertices of a parallelogram ABCD are A(10, –6), B(2, –6) and C(–4, –2), find the fourth vertex D.*

Solution. In the parallelogram ABCD, the three consecutive vertices are A(10, – 6), B(2, – 6) and C(–4, –2). Let the fourth vertex D be (x , y).

The mid-point of AC is $\left(\frac{10+(-4)}{2}, \frac{-6+(-2)}{2}\right)$



i.e. (3, – 4) ...(*i*)

The mid-point of BD is

$\left(\frac{2+x}{2}, \frac{-6+y}{2}\right)$ ...(*ii*)

Since the diagonals of a parallelogram bisect each other, the mid-points of AC and BD are same.

$\therefore$ From (*i*) and (*ii*), $\frac{2+x}{2} = 3$ and $\frac{-6+y}{2} = -4$

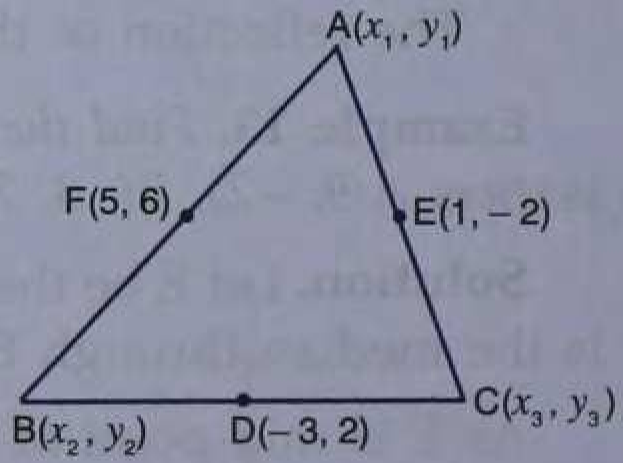
$\Rightarrow$ $2 + x = 6$ and $-6 + y = -8$

$\Rightarrow$ $x = 4$ and $y = -2$.

$\therefore$ The fourth vertex of the parallelogram ABCD is D(4, –2).

Example 16. *If (–3, 2), (1, –2) and (5, 6) are the mid-points of the sides of a triangle, find the co-ordinates of the vertices of the triangle.*

Solution. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC, such that D(–3, 2), E(1, –2) and F(5, 6) be the mid-points of the sides BC, CA and AB respectively.



Since D(–3, 2) is the mid-point of BC,

$$\frac{x_2 + x_3}{2} = -3 \text{ and } \frac{y_2 + y_3}{2} = 2$$

$\Rightarrow$ $x_2 + x_3 = -6$ ...(*i*)

and $y_2 + y_3 = 4$ ...(*ii*)

As E(1, – 2) is the mid-point of CA, $\frac{x_3 + x_1}{2} = 1$ and $\frac{y_3 + y_1}{2} = -2$

$\Rightarrow$ $x_3 + x_1 = 2$ ...(*iii*)

and $y_3 + y_1 = -4$ ...(*iv*)

Also F(5, 6) is the mid-point of AB,

$$\frac{x_1 + x_2}{2} = 5 \text{ and } \frac{y_1 + y_2}{2} = 6$$

$\Rightarrow$ $x_1 + x_2 = 10$ ...(*v*)

and $y_1 + y_2 = 12$ ...(*vi*)

Adding (*i*), (*iii*) and (*v*), we get

$$2(x_1 + x_2 + x_3) = 6 \Rightarrow x_1 + x_2 + x_3 = 3 \quad \ldots(vii)$$

Subtracting (*i*), (*iii*) and (*v*) from (*vii*) *in turn*, we get

$x_1 = 9$, $x_2 = 1$ and $x_3 = -7$.

Adding (*ii*), (*iv*) and (*vi*), we get

$$2(y_1 + y_2 + y_3) = 12 \Rightarrow y_1 + y_2 + y_3 = 6 \quad \ldots(viii)$$

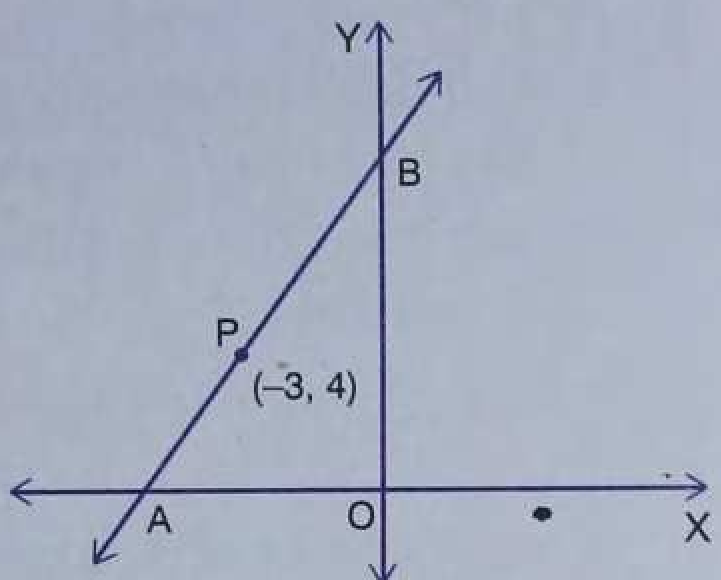
Subtracting (*ii*), (*iv*) and (*vi*) from (*viii*) *in turn*, we get

$y_1 = 2$, $y_2 = 10$ and $y_3 = -6$.

∴ The vertices of ΔABC are A(9, 2), B(1, 10) and C(−7, −6).

Example 17. *In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(–3, 4) on AB divides it in the ratio 2 : 3. Find the co-ordinates of A and B.* (2013)

Solution. Let the co-ordinates of the points A, B be (*a*, 0), (0, *b*) respectively.



Given point P(–3, 4) divides the segment AB in the ratio 2 : 3

i.e. AP : PB = 2 : 3.

∴ Co-ordinates of P are

$$\left(\frac{2.0 + 3.a}{2+3}, \frac{2.b + 3.0}{2+3}\right) \text{ i.e. } \left(\frac{3a}{5}, \frac{2b}{5}\right).$$

But P is (− 3, 4) $\Rightarrow \frac{3a}{5} = -3, \frac{2b}{5} = 4$

$\Rightarrow a = -5, b = 10$.

∴ The co-ordinates of A and B are (−5, 0) and (0, 10) respectively.

Example 18. *Two vertices of a triangle are (−1, 4) and (5, 2). If the centroid is (0, −3), find the third vertex.*

Solution. Two vertices of a triangle are (−1, 4) and (5, 2). Let the third vertex be (*x*, *y*), then the centroid of the triangle is

$$\left(\frac{-1+5+x}{3}, \frac{4+2+y}{3}\right) \qquad \left|\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)\right.$$

i.e. $\left(\frac{x+4}{3}, \frac{y+6}{3}\right)$.

But the centroid of the triangle is (0, −3)

$\Rightarrow \frac{x+4}{3} = 0$ and $\frac{y+6}{3} = -3$

$\Rightarrow x + 4 = 0$ and $y + 6 = -9$

$\Rightarrow x = -4$ and $y = -15$.

∴ The third vertex of the triangle is (−4, −15).

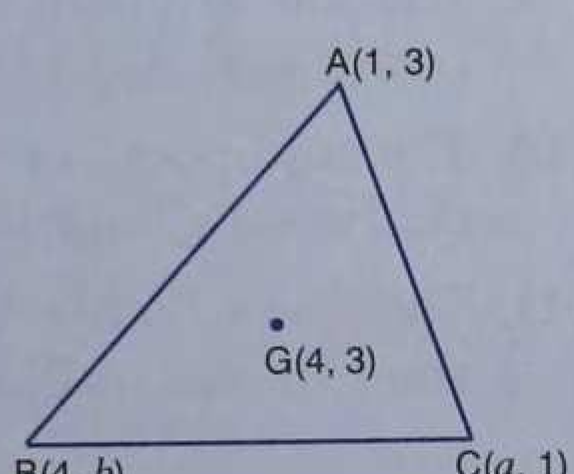
Example 19. *ABC is a triangle and G (4, 3) is the centroid of the triangle. If A, B and C are the points (1, 3), (4, b) and C (a, 1) respectively, find the values of a and b. Also find the length of side BC.* (2011)

Solution. Since G (4, 3) is the centroid of ΔABC, we have

$\frac{1+4+a}{3} = 4$ and $\frac{3+b+1}{3} = 3$

$\Rightarrow 5 + a = 12$ and $4 + b = 9$

$\Rightarrow a = 7$ and $b = 5$.



Hence, the values of *a* and *b* are; *a* = 7, *b* = 5.

The co-ordinates of the point B and C are (4, 5) and (7, 1) respectively.

Length of side BC = $\sqrt{(7-4)^2 + (1-5)^2}$ units

$= \sqrt{9+16}$ units = 5 units.

Exercise 11.2

1. Find the co-ordinates of the mid-points of the line segments joining the following pairs of points :

   (*i*) (2, – 3), (– 6, 7) (*ii*) (5, – 11), (4, 3) (*iii*) (a + 3, 5b), (2a – 1, 3b + 4).

2. Given that the co-ordinates of points A and B are (– 3, 2) and (9, 7) respectively, find :

   (*i*) the co-ordinates of mid-point of AB.

   (*ii*) the distance between A and B.

3. The co-ordinates of two points A and B are (–3, 3) and (12, –7) respectively. P is a point on the line segment AB such that AP : PB = 2 : 3. Find the co-ordinates of P.

4. P divides the distance between A(–2, 1) and B(1, 4) in the ratio 2 : 1. Calculate the co-ordinates of the point P.

5. (*i*) Find the co-ordinates of the points of trisection of the line segment joining the points (3, –3) and (6, 9).

   (*ii*) The line segment joining the points (2, 1) and (5, –8) is trisected at the points P and Q. If the point P lies on the line $2x - y + k = 0$, find the value of k.

6. Find the co-ordinates of the point which is three-fourth of the way from A(3, 1) to B(–2, 5).

Hint

Let P be the required point, then AP = $\frac{3}{4}$AB.

7. Point P(3, –5) is reflected to P′ in the x-axis. Also P on reflection in the y-axis is mapped as P″.

   (*i*) Find the co-ordinates of P′ and P″.

   (*ii*) Compute the distance P′P″.

   (*iii*) Find the middle point of the line segment P′P″.

   (*iv*) On which co-ordinate axis does the middle point of the line segment PP″ lie ?

8. Use graph paper for this question. Take 1 cm = 1 unit on both axes. Plot the points A(3, 0) and B(0, 4).

   (*i*) Write down the co-ordinates of A_1, the reflection of A in the y-axis.

   (*ii*) Write down the co-ordinates of B_1, the reflection of B in the x-axis.

   (*iii*) Assign the special name to quadrilateral ABA_1B_1.

   (*iv*) If C is the mid-point of AB, write down the co-ordinates of C_1, the reflection of C in the origin.

   (*v*) Assign the special name to quadrilateral ABC_1B_1.

9. The line segment joining A(– 3, 1) and B(5, – 4) is a diameter of a circle whose centre is C. Find the co-ordinates of the point C.

10. The mid-point of the line segment joining the points (3m, 6) and (–4, 3n) is (1, 2m – 1). Find the values of m and n.

11. The co-ordinates of the mid-point of the line segment PQ are (1, –2). The co-ordinates of P are (–3, 2). Find the co-ordinates of Q.

12. AB is a diameter of a circle with centre C(–2, 5). If point A is (3, –7), find
(*i*) the length of radius AC.
(*ii*) the co-ordinates of B. (*2013*)

13. Find the reflection (image) of the point (5, – 3) in the point (– 1, 3).

14. The line segment joining $A\left(-1, \frac{5}{3}\right)$ and B(a, 5) is divided in the ratio 1 : 3 at P, the point where the line segment AB intersects y-axis. Calculate :
(*i*) the value of a. (*ii*) the co-ordinates of P.

15. The point P(–4, 1) divides the line segment joining the points A(2, –2) and B in the ratio 3 : 5. Find the point B.

16. (*i*) In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6) ?
(*ii*) In what ratio does the point (– 4, b) divide the line segment joining the points P(2, –2), Q(–14, 6) ? Hence, find the value of b.

17. The line segment joining A(2, 3) and B(6, – 5) is intersected by x-axis at a point K. Write down the ordinate of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K. (*2006*)

18. If A = (–4, 3) and B = (8, –6),
(*i*) find the length of AB.
(*ii*) in what ratio is the line joining AB, divided by the x-axis? (*2008*)

19. (*i*) Calculate the ratio in which the line segment joining (3, 4) and (–2, 1) is divided by the y-axis.
(*ii*) In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points (3, –1) and (8, 9)? Also find the co-ordinates of the point of division.

20. Given a line segment AB joining the points A (– 4, 6) and B (8, – 3). Find :
(*i*) the ratio in which AB is divided by the y-axis.
(*ii*) the coordinates of the point of intersection.
(*iii*) the length of AB. (*2012*)

21. (*i*) Write down the co-ordinates of the point P that divides the line joining A(–4, 1) and B(17, 10) in the ratio 1 : 2.
(*ii*) Calculate the distance OP where O is the origin.
(*iii*) In what ratio does the y-axis divide the line AB ?

22. Calculate the length of the median through the vertex A of the triangle ABC with vertices A(7, –3), B(5, 3) and C(3, –1).

23. Prove by section formula that the points (10, –6), (2, –6), (–4, –2) and (4, –2), taken in this order, are the vertices of a parallelogram.

24. Three consecutive vertices of a parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). Find the fourth vertex D.

25. If the points A (–2, –1), B (1, 0), C (p, 3) and D (1, q) form a parallelogram ABCD, find the values of p and q.

26. Prove that the points A(–5, 4), B(–1, –2) and C(5, 2) are the vertices of an isosceles right-angled triangle. Find the co-ordinates of D so that ABCD is a square.

27. Find the third vertex of a triangle if its two vertices are (–1, 4) and (5, 2) and mid-point of one side is (0, 3).

Hint

Let A, B be the given vertices (–1, 4), (5, 2) respectively and C(α, β) be the third vertex of a triangle. Note that (0, 3) is not the mid-point of AB, therefore, it is the mid-point of either AC or BC.

28. Find the co-ordinates of the vertices of the triangle, the middle points of whose sides are $\left(0, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 0\right)$.

29. Show by section formula that the points (3, –2), (5, 2) and (8, 8) are collinear.

30. Find the value of p for which the points (–5, 1), (1, p) and (4, –2) are collinear.

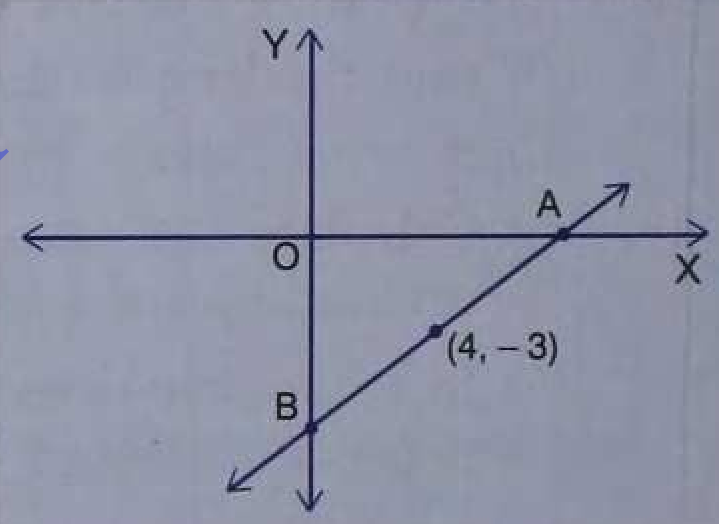
31. A(10, 5), B(6, –3) and C(2, 1) are the vertices of a triangle ABC. L is the mid-point of AB and M is the mid-point of AC. Write down the co-ordinates of L and M. Show that LM = $\frac{1}{2}$BC. *(2001)*

32. A (2, 5), B (–1, 2) and C (5, 8) are the vertices of a triangle ABC. P and Q are points on AB and AC respectively such that AP : PB = AQ : QC = 1 : 2.

(*i*) Find the co-ordinates of P and Q.

(*ii*) Show that PQ = $\frac{1}{3}$BC.

33. The mid point of the line segment AB shown in the adjoining diagram is (4, –3). Write down the co-ordinates of A and B.



34. Find the co-ordinates of the centroid of a triangle whose vertices are :

A(–1, 3), B(1, –1) and C(5, 1). *(2006)*

35. Two vertices of a triangle are (3, –5) and (–7, 4). Find the third vertex, given that the centroid is (2, –1).

36. The vertices of a triangle are A(–5, 3), B(p, –1) and C(6, q). Find the values of p and q if the centroid of the triangle ABC is the point (1, –1).

CHAPTER TEST

1. What point (or points) on the y-axis are at a distance of 10 units from the point (8, 8) ?
2. A(−4, −1), B(−1, 2) and C(α, 5) are the vertices of an isosceles triangle. Find the value of α, given that AB is the unequal side.
3. If A(−3, 2), B(α, β) and C(−1, 4) are the vertices of an isosceles triangle, prove that $\alpha + \beta = 1$, given AB = BC.
4. (*i*) Show that the points (2, 1), (0, 3), (−2, 1) and (0, −1), taken in order, are the vertices of a square. Also find the area of the square.

   (*ii*) Show that the points (−3, 2), (−5, −5), (2, −3) and (4, 4), taken in order, are the vertices of rhombus. Also find its area. Do the given points form a square ?
5. The ends of a diagonal of a square have co-ordinates (−2, p) and (p, 2). Find p if the area of the square is 40 sq. units.

Hint

$\sqrt{2}$ (length of side) = length of diagonal = $\sqrt{(p+2)^2 + (2-p)^2}$

$\Rightarrow$ 2(length of side)^2 = $2p^2 + 8$ $\Rightarrow$ (length of side)^2 = $p^2 + 4$

$\Rightarrow$ area of square = $p^2 + 4 = 40$ (given).

6. Find the co-ordinates of the circumcentre of the triangle whose vertices are (2, −2), (8, 6) and (8, −2). Also find the circumradius.
7. A and B have co-ordinates (4, 3) and (0, 1). Find :

   (*i*) the image A′ of A under reflection in the y-axis.

   (*ii*) the image B′ of B under reflection in the line AA′.

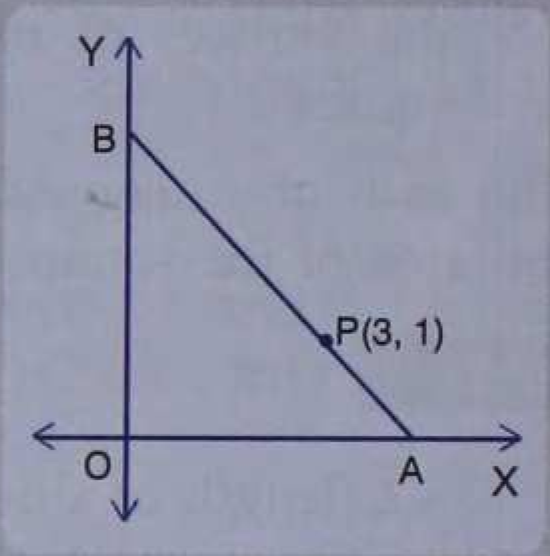
   (*iii*) the length of A′B′.
8. Find the co-ordinates of the point that divides the line segment joining the points P(5, −2) and Q(9, 6) internally in the ratio 3 : 1.
9. P and Q are the points on the line segment joining the points A (3, −1) and B(−6, 5) such that AP = PQ = QB. Find the co-ordinates of P and Q.
10. The centre of a circle is ($\alpha + 2$, $\alpha - 5$). Find the value of α, given that the circle passes through the points (2, −2) and (8, −2).
11. The mid-point of the line joining A(2, p) and B(q, 4) is (3, 5). Calculate the numerical values of p and q.
12. The ends of a diameter of a circle have the co-ordinates (3, 0) and (−5, 6). PQ is another diameter where Q has the co-ordinates (−1, −2). Find the co-ordinates of P and the radius of the circle.
13. Find the ratio in which the point P(−3, p) divides the line segment joining the points (−5, −4) and (−2, 3). Hence find the value of p.
14. In what ratio is the line joining the points (4, 2) and (3, −5) divided by the x-axis ? Also find the co-ordinates of the point of division.
15. If the abscissa of a point P is 2, find the ratio in which it divides the line segment joining the points (−4, 3) and (6, 3). Hence, find the co-ordinates of P.
16. Determine the ratio in which the line $2x + y - 4 = 0$ divide the line segment joining the points A (2, −2) and B (3, 7). Also find the co-ordinates of the point of division.

17. The point A(2, −3) is reflected in the x-axis onto the point A′. Then the point A′ is reflected in the line $x = 4$ onto the point A″.
   (*i*) Write the coordinates of A′ and A″.
   (*ii*) Find the ratio in which the line segment AA″ is divided by the x-axis. Also find the coordinates of the point of division.

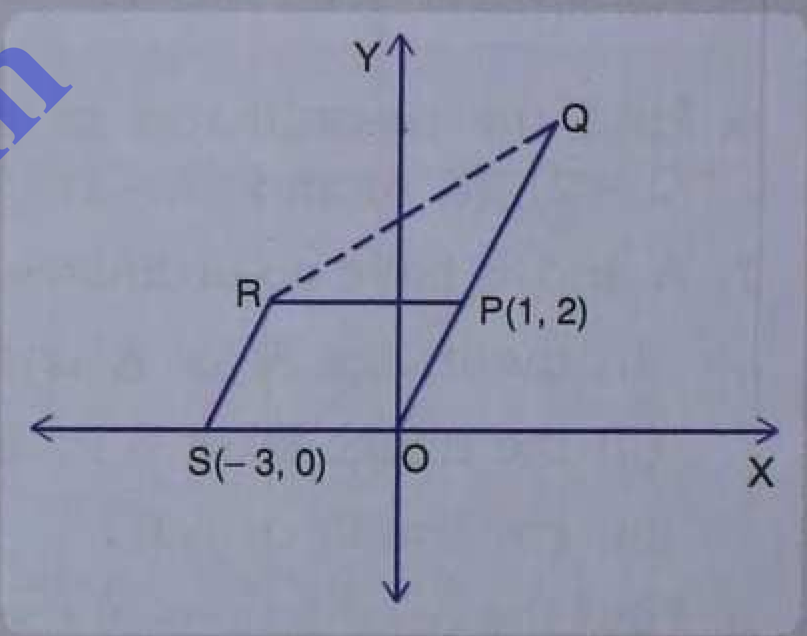
18. ABCD is a parallelogram. If the co-ordinates of A, B and D are (10, −6), (2, −6) and (4, −2) respectively, find the co-ordinates of C.

19. ABCD is a parallelogram whose vertices A and B have co-ordinates (2, −3) and (−1, −1) respectively. If the diagonals of the parallelogram meet at the point M (1, −4), find the co-ordinates of C and D. Hence, find the perimeter of the parallelogram.

20. In the adjoining figure, P(3, 1) is a point on the line segment AB such that AP : PB = 2 : 3. Find the co-ordinates of A and B.



21. Given O(0, 0), P(1, 2), S(−3, 0). P divides OQ in the ratio 2 : 3 and OPRS is a parallelogram. Find :
   (*i*) the co-ordinates of Q.
   (*ii*) the co-ordinates of R.
   (*iii*) the ratio in which RQ is divided by the y-axis.



22. If A (5, −1), B (−3, −2) and C (−1, 8) are the vertices of a triangle ABC, find the length of the median through A and the co-ordinates of the centroid of triangle ABC.