ALGEBRA

5 Linear Inequations

5.1 INEQUALITIES AMONG REAL NUMBERS

Let a, b be any real numbers, then

(1) a is **less than** b, written as $a < b$, if and only if $b - a$ is positive. For example,

(*i*) $2 < 7$, since $7 - 2 = 5$ which is +ve.

(*ii*) $-5 < -2$, since $-2 - (-5) = -2 + 5 = 3$ which is +ve.

(*iii*) $\frac{2}{3} < \frac{4}{5}$, since $\frac{4}{5} - \frac{2}{3} = \frac{12-10}{15} = \frac{2}{15}$ which is +ve.

(2) a is **less than or equal** to b, written as $a \leq b$, if and only if $b - a$ is either positive or zero. For example,

(*i*) $-3 \leq 5$, since $5 - (-3) = 5 + 3 = 8$ which is +ve.

(*ii*) $\frac{2}{7} \leq \frac{2}{7}$, since $\frac{2}{7} - \frac{2}{7} = 0$.

(3) a is **greater than** b, written as $a > b$, if and only if $a - b$ is positive. For example,

(*i*) $7 > 2$, since $7 - 2 = 5$ which is +ve.

(*ii*) $-2 > -5$, since $-2 - (-5) = -2 + 5 = 3$ which is +ve.

(*iii*) $\frac{4}{5} > \frac{2}{3}$, since $\frac{4}{5} - \frac{2}{3} = \frac{12-10}{15} = \frac{2}{15}$ which is +ve.

(4) a is **greater than or equal** to b, written as $a \geq b$, if and only if $a - b$ is either positive or zero. For example,

(*i*) $5 \geq -3$, since $5 - (-3) = 5 + 3 = 8$ which is +ve.

(*ii*) $\frac{2}{7} \geq \frac{2}{7}$, since $\frac{2}{7} - \frac{2}{7} = 0$.

CARL FRIEDRICH GAUSS

Carl Friedrich Gauss (1777 – 1855), a French mathematician, was a pioneer in the development of theory of numbers. Mathematics is the queen of all the sciences and number theory is the queen of mathematics.

Geometrically, let A, B be the points on the number line representing the numbers a, b respectively, then

(1) $a < b$ if and only if the point A is towards the left of B.

(2) $a \leq b$ if and only if either the point A is towards the left of B or A coincides with B.

(3) $a > b$ if and only if the point A is towards the right of B.

(4) $a \geq b$ if and only if either the point A is towards the right of B or it coincides with B.

Note

The statements $a < b$ and $a \leq b$ are equivalent to the statements $b > a$ and $b \geq a$ respectively. For example,

(*i*) $2 < 7$ is equivalent to $7 > 2$. (*ii*) $-3 \leq 5$ is equivalent to $5 \geq -3$.

5.2 LINEAR INEQUATIONS

Statements such as

$$x < 3,\ x + 5 \leq 7,\ 2x - 3 > 8,\ 3x + 5 \geq 11,\ \frac{x-3}{2} < 2x + 1$$

are called **linear inequations**. In general, a linear inequation can always be written as

$$ax + b < 0,\ ax + b \leq 0,\ ax + b > 0 \text{ or } ax + b \geq 0$$

where a and b are real numbers, $a \neq 0$.

Replacement set. *The set from which values of the variable (involved in the inequation) are chosen is called the* ***replacement set.***

Solution set. *A solution to an inequation is a number (chosen from replacement set) which, when substituted for the variable, makes the inequation true. The set of all solutions of an inequation is called the* ***solution set*** *of the inequation.*

For example, consider the inequation $x < 4$.

	Replacement set	*Solution set*
(*i*)	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}	{1, 2, 3}
(*ii*)	{–1, 0, 1, 2, 5, 8}	{–1, 0, 1, 2}
(*iii*)	{–5, 10}	{–5}
(*iv*)	{5, 6, 7, 8, 9, 10}	ϕ.

Note that the solution set depends upon the replacement set.

5.3 SOLVING LINEAR INEQUATIONS IN ONE VARIABLE

Solving an inequation in one variable is much like solving an equation in one variable because most of the basic rules apply. Of course, there is one exception.

Two permissible rules

1. Addition – Subtraction Rule.

If the same number or expression is added to or subtracted from both sides of an inequation, the resulting inequation has the same solution (or solutions) as the original.

2. Multiplication – Division Rule.

(*i*) *If both sides of an inequation are multiplied or divided by the same positive number, the resulting inequation has the same solution (or solutions) as the original.*

(*ii*) *If both sides of an inequation are multiplied or divided by the same negative number, the resulting inequation has the same solution (or solutions) as the original if the symbol of the inequality is reversed.*

Thus, the only difference between solving a linear equation and solving an inequation concerns multiplying or dividing both sides by a negative number. Therefore, **always reverse the symbol of an inequation when multiplying or dividing by a negative number.**

5.3.1 Procedure to solve a linear inequation in one variable

(*i*) Simplify both sides by removing group symbols and collecting like terms.

(*ii*) Remove fractions (or decimals) by multiplying both sides by an appropriate factor (L.C.M. of fractions or a power of 10 in case of decimals).

(*iii*) Isolate all variable terms on one side and all constants on the other side. Collect like terms when possible.

(*iv*) Make the coefficient of the variable 1.

(*v*) Choose the solution set from the replacement set.

Note

If the replacement set is not given, then it is to be taken as **R**.

ILLUSTRATIVE EXAMPLES

Example 1. *Given $x \in \{-3, -4, -5, -6\}$ and $9 \le 1 - 2x$, find the possible values of x. Also represent its solution set on the number line.*

Solution. Given $9 \le 1 - 2x$

$\Rightarrow \quad 2x + 9 \le 1 - 2x + 2x$ (Add $2x$)

$\Rightarrow \quad 2x + 9 \le 1$

$\Rightarrow \quad 2x + 9 + (-9) \le 1 + (-9)$ (Add -9)

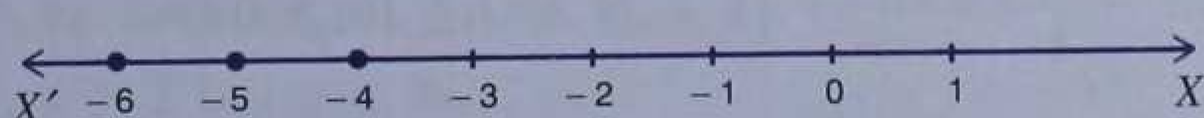
$\Rightarrow \quad 2x \le -8$

$\Rightarrow \quad x \le -4$ (Divide by 2)

But $\quad x \in \{-3, -4, -5, -6\}$,

$\therefore \quad$ the solution set is $\{-4, -5, -6\}$.

The graph of the solution set is shown by thick dots on the number line.



Example 2. *Solve the inequation $3 - 2x \ge x - 12$, given that $x \in$* **N**.

Solution. Given $3 - 2x \ge x - 12$

$\Rightarrow \quad (-3) + 3 - 2x \ge x - 12 + (-3)$ (Add -3)

$\Rightarrow \quad -2x \ge x - 15$

$\Rightarrow \quad (-x) - 2x \ge (-x) + x - 15$ (Add $-x$)

$\Rightarrow \quad -3x \geq -15$

$\Rightarrow \quad \left(-\frac{1}{3}\right)(-3x) \leq \left(-\frac{1}{3}\right)(-15)$ [Multiply by $-\frac{1}{3}$ and reverse the symbol]

$\Rightarrow \quad x \leq 5$ but $x \in \mathbf{N}$ *i.e.* $x \in \{1, 2, 3, 4, 5, \ldots\}$,

$\therefore$ the solution set is $\{1, 2, 3, 4, 5\}$.

Example 3. *If the replacement set is the set of integers lying between -3 and 10, find the solution set of $14 - 5x \geq 3x - 40$. Also represent the solution set on the number line.*

Solution. Given $14 - 5x \geq 3x - 40$

$\Rightarrow \quad 14 - 5x + 5x \geq 3x - 40 + 5x$ (Add $5x$)

$\Rightarrow \quad 14 \geq 8x - 40$

$\Rightarrow \quad 14 + 40 \geq 8x - 40 + 40$ (Add 40)

$\Rightarrow \quad 54 \geq 8x$

$\Rightarrow \quad 8x \leq 54$ ($\because a \geq b \Rightarrow b \leq a$)

$\Rightarrow \quad x \leq \frac{54}{8}$ (Divide by 8)

$\Rightarrow \quad x \leq 6{\cdot}75$

But the replacement set is the set of integers lying between -3 and 10

i.e. $\{-2, -1, 0, 1, 2, \ldots, 9\}$,

$\therefore$ the solution set is $\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$.

The graph of the solution set is shown by thick dots on the number line.

Example 4. *Find the solution set of $2 \leq 3(x - 2) + 5 < 8$, $x \in W$. Also represent its solution on the number line.*

Solution. Given $2 \leq 3(x - 2) + 5 < 8$

$\Rightarrow \quad 2 \leq 3x - 6 + 5 < 8$

$\Rightarrow \quad 2 \leq 3x - 1 < 8$

$\Rightarrow \quad 2 + 1 \leq 3x - 1 + 1 < 8 + 1$ (Add 1)

$\Rightarrow \quad 3 \leq 3x < 9$

$\Rightarrow \quad 1 \leq x < 3$ (Divide by 3)

But $x \in \mathbf{W}$ *i.e.* $x \in \{0, 1, 2, 3, 4, \ldots\}$,

$\therefore$ the solution set is $\{1, 2\}$.

The graph of the solution set is shown by thick dots on the number line.

Example 5. *Solve* $\frac{2x+1}{3} \geq \frac{3x-2}{5}$, $x \in \mathbf{R}$. *Graph the solution set on the number line.*

Solution. Given $\frac{2x+1}{3} \geq \frac{3x-2}{5}$.

Multiplying both sides by 15, L.C.M. of fractions, we get

$5(2x + 1) \geq 3(3x - 2) \Rightarrow 10x + 5 \geq 9x - 6$

$\Rightarrow \quad (-9x) + 10x + 5 \geq (-9x) + 9x - 6$ (Add $-9x$)

$\Rightarrow \quad x + 5 \geq -6$

$\Rightarrow \quad x + 5 + (-5) \geq -6 + (-5)$ (Add – 5)

$\Rightarrow \quad x \geq -11.$

Hence, the solution set is $\{x : x \in \mathbf{R}, x \geq -11\}$.

The graph of the solution set is shown by the thick portion of the number line. The solid circle at –11 indicates that the number –11 is included among the solutions.

Example 6. *Find the range of values of x, which satisfy the inequation* $-\frac{1}{5} \leq \frac{3x}{10} + 1 < \frac{2}{5}$, $x \in R$. *Graph the solution set on the number line.*

Solution. Given $-\frac{1}{5} \leq \frac{3x}{10} + 1 < \frac{2}{5}$.

Multiplying throughout by 10, L.C.M. of fractions, we get

$-2 \leq 3x + 10 < 4$

$\Rightarrow \quad -2 - 10 \leq 3x + 10 + (-10) < 4 + (-10)$ (Add – 10)

$\Rightarrow \quad -12 \leq 3x < -6$

$\Rightarrow \quad -4 \leq x < -2$ (Dividing by 3)

Hence, the solution set is $\{x : x \in \mathbf{R}, -4 \leq x < -2\}$.

The graph of the solution set is shown by the thick portion of the number line. The solid circle at – 4 indicates that the number – 4 is included among the solutions and the open circle at – 2 indicates that the number – 2 is not included among the solutions.

Example 7. *List the elements of the solution set of the inequation* $-3 < x - 2 \leq 9 - 2x$, $x \in N$.

Solution. Given $-3 < x - 2 \leq 9 - 2x$, $x \in \mathbf{N}$

$\Rightarrow \quad -3 < x - 2 \text{ and } x - 2 \leq 9 - 2x$

$\Rightarrow \quad -3 + 2 < x - 2 + 2 \text{ and } 2x + x - 2 \leq 9 - 2x + 2x$

$\Rightarrow \quad -1 < x \text{ and } 3x - 2 \leq 9 \quad \Rightarrow \quad -1 < x \text{ and } 3x - 2 + 2 \leq 9 + 2$

$\Rightarrow \quad -1 < x \text{ and } 3x \leq 11$

$\Rightarrow \quad -1 < x \text{ and } x \leq \frac{11}{3}$

$\Rightarrow \quad -1 < x \leq \frac{11}{3}$ but $x \in \mathbf{N}$,

$\therefore$ the solution set is $\{1, 2, 3\}$.

Example 8. *P is the solution set of* $8x - 1 > 5x + 2$ *and Q is the solution set of* $7x - 2 \geq 3(x + 6)$, *where* $x \in N$. *Find the set* $P \cap Q$.

Solution. Given $8x - 1 > 5x + 2$

$\Rightarrow \quad 8x - 1 + (-5x + 1) > 5x + 2 + (-5x + 1)$ (Add – 5x + 1)

$\Rightarrow \quad 3x > 3 \Rightarrow x > 1$ but $x \in \mathbf{N}$,

$\therefore \quad P = \{2, 3, 4, 5, \ldots\}$.

Also $7x - 2 \geq 3(x + 6) \Rightarrow 7x - 2 \geq 3x + 18$

$\Rightarrow \quad 7x - 2 + (-3x + 2) \geq 3x + 18 + (-3x + 2)$ (Add – 3x + 2)

$\Rightarrow \quad 4x \geq 20 \Rightarrow x \geq 5$ but $x \in \mathbf{N}$,

$\therefore \quad Q = \{5, 6, 7, 8, \ldots\}$.

$\therefore \quad P \cap Q = \{5, 6, 7, 8, \ldots\}$.

Example 9. *Solve the following inequation, and graph the solution on the number line :*

$2x - 5 \le 5x + 4 < 11, x \in R.$ (2002)

Solution. Given $2x - 5 \le 5x + 4 < 11, x \in \mathbf{R}$

$\Rightarrow \quad 2x - 5 \le 5x + 4$ and $5x + 4 < 11$

$\Rightarrow \quad 2x - 5 + (-5x + 5) \le 5x + 4 + (-5x + 5)$ and $5x + 4 + (-4) < 11 + (-4)$

$\Rightarrow \quad -3x \le 9$ and $5x < 7$

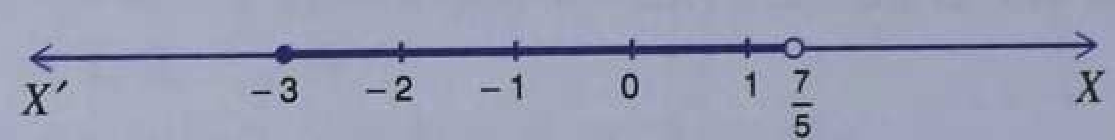
$\Rightarrow \quad x \ge -3$ and $x < \frac{7}{5}$

$\Rightarrow \quad -3 \le x$ and $x < \frac{7}{5}$

$\Rightarrow \quad -3 \le x < \frac{7}{5}$ where $x \in \mathbf{R}$

$\therefore$ The solution set $= \left\{x : x \in \mathbf{R}, -3 \le x < \frac{7}{5}\right\}$.

The graph of the solution set is shown by the thick position of the number line. The solid circle at -3 indicates that the number -3 is included among the solutions and the open circle at $\frac{7}{5}$ indicates that the number $\frac{7}{5}$ is not included among the solutions.



Example 10. *Solve the following inequation, write the solution set and represent it on the number line:* $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R.$ (2013)

Solution. Given $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in \mathbf{R}$

$\Rightarrow \quad -\frac{x}{3} \le \frac{x}{2} - \frac{4}{3} < \frac{1}{6} \Rightarrow -2x \le 3x - 8 < 1$

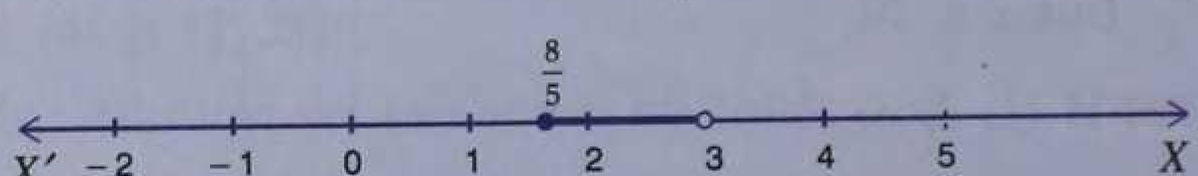
$\Rightarrow \quad -2x \le 3x - 8$ and $3x - 8 < 1$

$\Rightarrow \quad 8 \le 5x$ and $3x < 9$

$\Rightarrow \quad \frac{8}{5} \le x$ and $x < 3 \Rightarrow \frac{8}{5} \le x < 3,$

$\therefore$ The solution set $= \left\{x : x \in \mathbf{R}, \frac{8}{5} \le x < 3\right\}$.

The graph of the solution set is shown by the thick portion of the number line.



Example 11. *Find the set of values of x satisfying*

$7x + 3 \ge 3x - 5$ *and* $\frac{x}{4} - 5 \le \frac{5}{4} - x$, *where* $x \in \mathbf{N}$.

Also graph the solution set on the number line.

Solution. Given $7x + 3 \ge 3x - 5$ and $\frac{x}{4} - 5 \le \frac{5}{4} - x$

$\Rightarrow \quad 7x - 3x \ge -5 - 3$ and $x - 20 \le 5 - 4x$

$\Rightarrow \quad 4x \ge -8$ and $x + 4x \le 5 + 20$

$\Rightarrow \quad x \ge -2$ and $5x \le 25$

$\Rightarrow \quad -2 \le x$ and $x \le 5$

$\Rightarrow \quad -2 \le x \le 5.$

But $x \in \mathbf{N}$ *i.e.* $x \in \{1, 2, 3, 4, \ldots\}$,

$\therefore$ the solution set is $\{1, 2, 3, 4, 5\}$.

The graph of the solution set is shown by thick dots on the number line.

Example 12. *John needs a minimum of 360 marks in four tests in a Mathematics course to obtain an A grade. On his first three tests, he scored 88, 96, 79 marks. What should his score be in the fourth test so that he can make an A grade ?*

Solution. Let John score x marks in the fourth test. Then the sum of John's test scores should be greater than or equal to 360 *i.e.*

$$88 + 96 + 79 + x \geq 360 \Rightarrow 263 + x \geq 360$$

$$\Rightarrow x \geq 360 + (-263) \Rightarrow x \geq 97.$$

$\therefore$ John should score 97 marks or greater than 97 marks in the fourth test to obtain A grade.

Example 13. *An integer is such that one-third of the next integer is atleast 2 more than one-fourth of the previous integer. Find the smallest value of the integer.*

Solution. Let the integer be x, then one-third of the next integer is $\frac{x+1}{3}$ and one-fourth of the previous integer is $\frac{x-1}{4}$.

According to given, $\frac{x+1}{3} \geq \frac{x-1}{4} + 2$

$\Rightarrow \quad 4(x + 1) \geq 3(x - 1) + 2 \times 12$

$\Rightarrow \quad 4x + 4 \geq 3x - 3 + 24$

$\Rightarrow \quad 4x - 3x \geq -3 + 24 - 4$

$\Rightarrow \quad x \geq 17.$

$\therefore$ The smallest value of $x = 17$.

Example 14. *Find three largest consecutive natural numbers such that the sum of one-third of first, one-fourth of second and one-fifth of the third is atmost 25.*

Solution. Let three required consecutive natural numbers be x, $x + 1$ and $x + 2$.

According to given, $\frac{x}{3} + \frac{x+1}{4} + \frac{x+2}{5} \leq 25$

$\Rightarrow \quad 20x + 15(x + 1) + 12(x + 2) \leq 25 \times 60$

$\Rightarrow \quad 20x + 15x + 15 + 12x + 24 \leq 1500$

$\Rightarrow \quad 47x + 39 \leq 1500$

$\Rightarrow \quad 47x \leq 1500 - 39$

$\Rightarrow \quad 47x \leq 1461 \Rightarrow x \leq \frac{1461}{47}$

$\Rightarrow \quad x \leq 31\frac{4}{47}.$

$\therefore$ The largest value of $x = 31$.

When $x = 31$, $x + 1 = 31 + 1 = 32$ and $x + 2 = 31 + 2 = 33$.

Hence, the required numbers are 31, 32 and 33.

Exercise 5

1. Solve the inequation $3x - 11 < 3$, where $x \in \{1, 2, 3, \ldots, 10\}$. Also represent its solution on a number line.
2. Solve $2(x - 3) < 1$, $x \in \{1, 2, 3, \ldots, 10\}$.
3. Solve $5 - 4x > 2 - 3x$, $x \in \mathbf{W}$. Also represent its solution on the number line.
4. List the solution set of $30 - 4(2x - 1) < 30$, given that x is a positive integer.
5. Solve $2(x - 2) < 3x - 2$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.
6. If x is a negative integer, find the solution set of $\frac{2}{3} + \frac{1}{3}(x + 1) > 0$.
7. Solve $\frac{2x - 3}{4} \geq \frac{1}{2}$, $x \in \{0, 1, 2, \ldots, 8\}$.
8. Solve $x - 3(2 + x) > 2(3x - 1)$, $x \in \{-3, -2, -1, 0, 1, 2\}$. Also represent its solution on the number line.
9. Given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$, solve $x - 3 < 2x - 1$.
10. Given $A = \{x ; x \in \mathbf{I}, -4 \leq x \leq 4\}$, solve $2x - 3 < 3$ where x has the domain A. Graph the solution set on the number line.
11. List the solution set of the inequation $\frac{1}{2} + 8x > 5x - \frac{3}{2}$, $x \in \mathbf{Z}$.
12. List the solution set of $\frac{11 - 2x}{5} \geq \frac{9 - 3x}{8} + \frac{3}{4}$, $x \in \mathbf{N}$.
13. Find the values of x, which satisfy the inequation :

    $-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$, $x \in \mathbf{N}$.

    Graph the solution set on the number line. *(2001)*
14. If $x \in \mathbf{W}$, find the solution set of $\frac{3}{5}x - \frac{2x - 1}{3} > 1$. Also graph the solution set on the number line, if possible.
15. Solve :

    (*i*) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$, where x is a positive odd integer.

    (*ii*) $\frac{2x + 3}{3} \geq \frac{3x - 1}{4}$, where x is positive even integer.
16. Given that $x \in \mathbf{I}$, solve the inequation and graph the solution on the number line :

    $3 \geq \frac{x - 4}{2} + \frac{x}{3} \geq 2$. *(2004)*
17. Given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$, find the values of x for which

    $-3 < 2x - 1 < x + 4$.
18. Solve $1 \geq 15 - 7x > 2x - 27$, $x \in \mathbf{N}$.
19. If $x \in \mathbf{Z}$, solve $2 + 4x < 2x - 5 \leq 3x$. Also represent its solution on the number line.
20. Solve the inequation $12 + 1\frac{5}{6}x \leq 5 + 3x$, $x \in \mathbf{R}$. Represent the solution on a number line.
21. Solve $\frac{4x - 10}{3} \leq \frac{5x - 7}{2}$, $x \in \mathbf{R}$ and represent the solution set on the number line.
22. Solve $\frac{3x}{5} - \frac{2x - 1}{3} > 1$, $x \in \mathbf{R}$ and represent the solution set on the number line.
23. Solve the inequation : $-3 \leq 3 - 2x < 9$, $x \in \mathbf{R}$. Represent your solution on a number line. *(2000)*

24. Solve $2 \leq 2x - 3 \leq 5$, $x \in \mathbf{R}$ and mark it on number line. (2003)

25. Given that $x \in \mathbf{R}$, solve the following inequality and graph the solution on the number line :

$-1 \leq 3 + 4x < 23.$ (2006)

26. Solve the following inequation and graph the solution on the number line :

$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3}$, $x \in \mathbf{R}$. (2007)

27. Solve the following inequation and represent the solution set on the number line :

$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$, $x \in \mathbf{R}$. (2010)

28. Solve $\frac{2x+1}{2} + 2(3 - x) \geq 7$, $x \in \mathbf{R}$. Also graph the solution set on the number line.

29. Find the range of values of x, which satisfy $-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$, $x \in \mathbf{R}$. Graph these values of x on the real number line.

30. Solve the inequation : $-2\frac{1}{2} + 2x \leq \frac{4x}{3} \leq \frac{4}{3} + 2x$, $x \in \mathbf{W}$. Graph the solution set on the number line.

31. Solve the inequation $2x - 5 \leq 5x + 4 < 11$, where $x \in \mathbf{I}$. Also represent the solution set on the number line. (2011)

32. If $x \in \mathbf{I}$, A is the solution set of $2(x - 1) < 3x - 1$ and B is the solution set of $4x - 3 \leq 8 + x$, find $A \cap B$.

33. If P is the solution set of $-3x + 4 < 2x - 3$, $x \in \mathbf{N}$, and Q is the solution set of $4x - 5 < 12$, $x \in \mathbf{W}$, find

(*i*) $P \cap Q$ (*ii*) $Q - P$.

34. $A = \{x : 11x - 5 > 7x + 3, x \in \mathbf{R}\}$ and

$B = \{x : 18x - 9 \geq 15 + 12x, x \in \mathbf{R}\}$.

Find the range of set $A \cap B$ and represent it on a number line. (2005)

35. Given : $P = \{x : 5 < 2x - 1 \leq 11, x \in \mathbf{R}\}$,

$Q = \{x : -1 \leq 3 + 4x < 23, x \in \mathbf{I}\}$ where $\mathbf{R}$ = {real numbers}, $\mathbf{I}$ = {integers}.

Represent P and Q on number lines. Write down the elements of $P \cap Q$.

36. If $x \in \mathbf{I}$, find the smallest value of x which satisfies the inequation

$2x + \frac{5}{2} > \frac{5x}{3} + 2.$

37. Given $20 - 5x < 5(x + 8)$, find the smallest value of x when

(*i*) $x \in \mathbf{I}$ (*ii*) $x \in \mathbf{W}$ (*iii*) $x \in \mathbf{N}$.

38. Solve the following inequation and represent the solution set on the number line :

$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$, $x \in \mathbf{R}$. (2012)

39. Solve the given inequation and graph the solution on the number line :

$2y - 3 < y + 1 \leq 4y + 7$; $y \in \mathbf{R}$. (2008)

40. Solve the inequation and represent the solution set on the number line :

$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x$, where $x \in \mathbf{I}$. (2009)

41. Find the greatest integer which is such that if 7 is added to its double, the resulting number becomes greater than three times the integer.

42. One-third of a bamboo pole is buried in mud, one-sixth of it is in water and the part above the water is greater than or equal to 3 metres. Find the length of the shortest pole.

CHAPTER TEST

1. Solve the inequality : $5x - 2 \leq 3\ (3 - x)$, where $x \in \{-2, -1, 0, 1, 2, 3, 4\}$. Also represent its solution on the number line.
2. Solve the inequation : $6x - 5 < 3x + 4$, $x \in \mathbf{I}$.
3. Find the solution set of the inequation : $x + 5 \leq 2x + 3$, $x \in \mathbf{R}$. Graph the solution set on the number line.
4. If $x \in \mathbf{R}$ (real numbers) and $-1 < 3 - 2x \leq 7$, evaluate x and represent it on a number line.
5. Solve the inequation : $\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}$, $x \in \mathbf{R}$.
6. Find the range of values of x, which satisfy $7 \leq -4x + 2 < 12$, $x \in \mathbf{R}$. Graph these values of x on the real number line.
7. If $x \in \mathbf{R}$, solve $2x - 3 \geq x + \frac{1-x}{3} > \frac{2}{5}x$. Also represent the solution on the number line.

Hint

On solving $2x - 3 \geq x + \frac{1-x}{3}$, we get $x \geq \frac{5}{2}$, and on solving

$$x + \frac{1-x}{3} > \frac{2}{5}x, \text{ we get } x > -\frac{5}{4} \Rightarrow x \geq \frac{5}{2}.$$

8. Find positive integers which are such that if 6 is subtracted from five times the integer then the resulting number cannot be greater than four times the integer.
9. Find three smallest consecutive natural numbers such that the difference between one-third of the largest and one-fifth of the smallest is atleast 3.