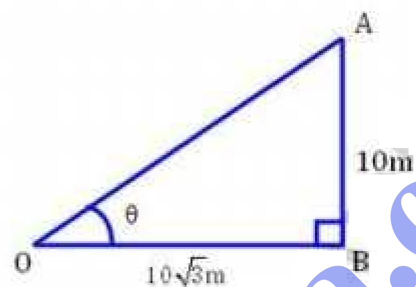
Height And Distance

EXERCISE – 21.1

Q1. An electric pole is 10m high. If its shadow is $10\sqrt{3}$ m in length, find the elevation of the sun.

Sol. let AB be the pole and OB is its shadow



$AB = 10m$, $OB = 10\sqrt{3}$

and θ is the angle of elevation of the sun.

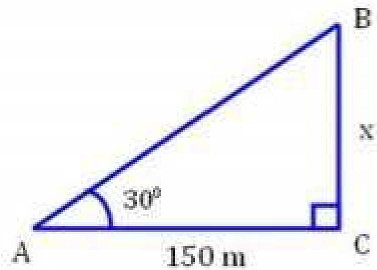
$$\tan\theta = \frac{AB}{OB} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$\therefore \theta = 30^\circ$

Q2. The angle of elevation of the top of a tower, from a point on the ground and at a distance of 150m from its foot is 30°. find the height of the tower correct to one decimal place.

Sol. let BC be the tower and A is the point on the ground such that $\angle A = 30^\circ$ and $AC = 150m$.

let the height of the tower $= x$ m.



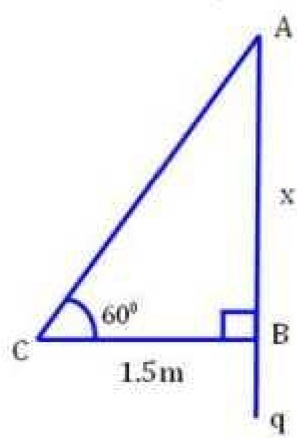
$\tan\theta = \dfrac{BC}{AC} \Rightarrow \tan 30^\circ = \dfrac{x}{150} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{x}{150}$

$\Rightarrow x = \dfrac{150}{\sqrt{3}} \Rightarrow x = \dfrac{150 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 50\sqrt{3}$ m.

$\Rightarrow x = 50(1.732) = 86.6600 = 86.6$ m.

Q3. A ladder is placed against a wall such that it just reaches the top of the wall. The foot of the ladder is 1.5m away from the wall and the ladder is inclined at an angle of 60° with the ground. Find the height of the wall.

Sol.



let AB be the wall and AC be the ladder whose foot C is 1.5m away from B.

let AB = x m and angle of inclination is 60°.

$\tan\theta = \frac{AB}{CB} \Rightarrow \tan 60^\circ = \frac{x}{1.5}$

$\Rightarrow \sqrt{3} = \frac{x}{1.5} \Rightarrow x = \sqrt{3} \times 1.5 = 1.732 \times 1.5$

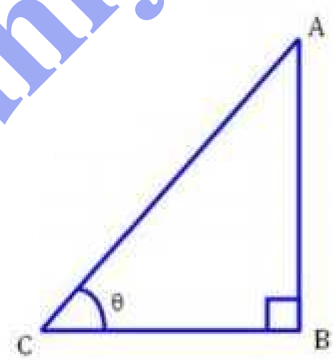
$\Rightarrow x = 2.596 = 2.6m$

$\therefore$ Height of the wall = 2.6m.

Q4. what is the angle of elevation of the sun when the length of shadow of a vertical pole is equal to its height.

Sol. let AB be the pole and CB be its shadow and θ is the angle of elevation of the sun.

let AB = x m, BC = x m.



$\tan\theta = \frac{AB}{CB} = \frac{x}{x} = 1 \Rightarrow \theta = 45^\circ$ ($\because \tan 45^\circ = 1$)

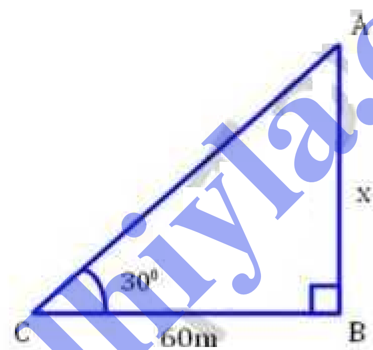
$\therefore$ Angle of elevation = 45°.

Q5. A river is 60m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to the foot of the tree, on the other bank is 30°. Find the height of the tree.

Sol. let AB be the tree and BC is the width of the river and C is the point exactly opposite to B on the other bank and angle of elevation is 30°.

let height of the tree $AB = x$ m and

width of the river $BC = 60$m.



$\tan\theta = \frac{AB}{CB} \Rightarrow \tan 30^\circ = \frac{x}{60}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{60} \Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}}$

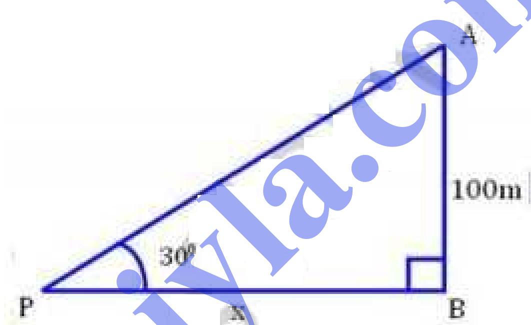
$\Rightarrow x = 20\sqrt{3} = 20(1.732) = 34.64$m

$\therefore$ Height of the tree $= 34.64$m.

Q6. From a point P on level ground the angle of elevation of the top of ~~a~~ tower is 30°. If the tower is 100m high, how far is P from the foot of the tower?

Sol. let AB be the tower and P is at a distance of x m from B, the foot of the tower.

while height of the tower AB = 100m and angle of elevation = 30°.



$$\tan\theta = \frac{AB}{PB} \Rightarrow \tan 30^\circ = \frac{100}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x} \Rightarrow x = 100\sqrt{3}$$

$$\Rightarrow x = 100(1.732) = 173.2 \text{ m}$$

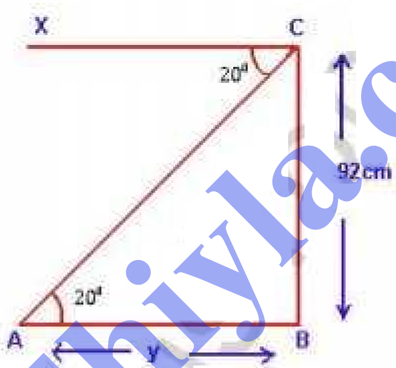
$\therefore$ Distance of P from the foot of the tower = 173.2m.

Q7. From the top of a cliff 92m high, the angle of depression of a buoy is 20°. Calculate the nearest meter, the distance of the boy from the foot of the cliff.

Sol. let BC be the height of the cliff

i.e, BC = 92m and A be the boy

let the distance of the boy from the foot of cliff be y.



Now in ΔABC,

$$\tan 20^\circ = \frac{BC}{AB} = \frac{92}{y}$$

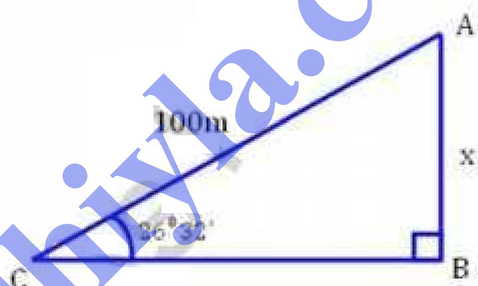
$$\Rightarrow y = \frac{92}{\tan 20^\circ} = 253\,m.$$

$$\Rightarrow y = 253\,m.$$

Q8. A boy is flying a kite with a string of length 100m. If the string is tight and the angle of elevation of the kite is $26°32'$. find the height of the kite correct to one decimal place (ignore the height of the boy)

Sol. let AB be the height of the kite A and AC is the ~~and~~ string and angle of elevation of the kite is $26°32'$

let $AB = x$ m and $AC = 100$ m.



$$\sin\theta = \frac{AB}{AC} \rightarrow \sin 26°32' = \frac{x}{100}$$

$$\Rightarrow 0.4467 = \frac{x}{100} \Rightarrow x = 100 \times 0.4467$$

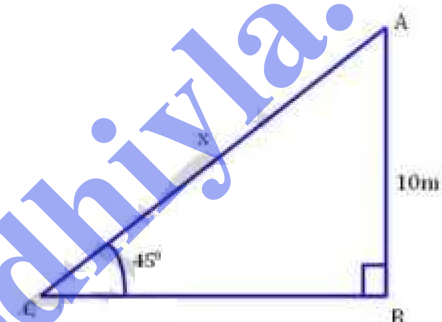
$$\Rightarrow x = 44.67 = 44.7$$

$\therefore$ Height of the kite $= 44.7$ m..

Q9. An electric pole is 10m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Sol. let AB be the pole and AC be the wire which makes an angle of 45° with the ground.

Height of the pole AB = 10m and
let length of wire AC = x m.



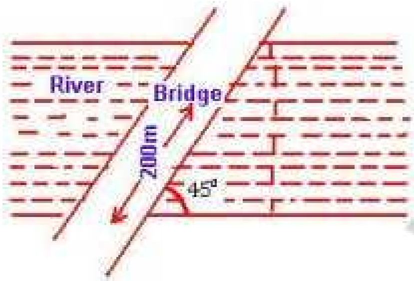
$\sin\theta = \frac{AB}{AC} \Rightarrow \sin 45° = \frac{10}{x}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{x} \Rightarrow x = 10\sqrt{2}$

$\Rightarrow x = 10(1.414) = 14.14$ m.

$\therefore$ length of wire = 14.14m.

Q10. A bridge across a river makes an angle of 45° with the river bank. If the length of the bridge across the river is 200m, what is the breadth of the river?



Sol. let AB be the width of river = x m

length of the bridge AC = 200 m

angle with the river bank = 45°

$\sin\theta = \frac{AB}{AC} \Rightarrow \sin 45° = \frac{x}{200}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{x}{200} \Rightarrow x = \frac{200}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow x = \frac{200(1.414)}{2}$

$\Rightarrow x = 100(1.414)$

$\Rightarrow x = 141.4$ m

$\therefore$ width of the river = 141.4 m.

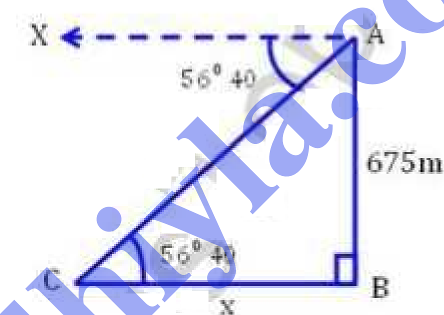
Q11. An Aeroplane is 675m directly above one end of a bridge. The angle of depression of the other end of the bridge is 56°40'. How long is the bridge?

Sol. let AB be the height of aeroplane

A and CB is the bridge of a river

$\therefore AB = 675m$ and angle of depression $= 56°40'$

let length of bridge $CB = x$.



$\tan\theta = \frac{AB}{CB} \Rightarrow \tan 56°40' = \frac{675}{CB}$

$\Rightarrow 1.5204 = \frac{675}{x}$

$\Rightarrow x = \frac{675}{1.5204} = 443.96$

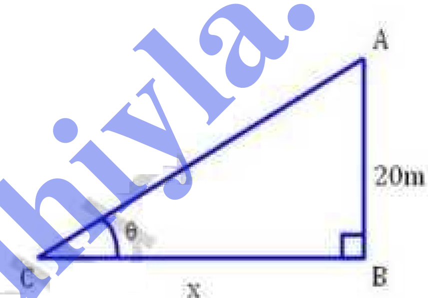
$\therefore$ length of the bridge $= 443.96 = 444$ m.

Q12. A vertical tower is 20m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0.53. How far is he standing from the foot of the tower?

Sol. let AB be the tower and let a man C stands at a distance from the foot of the tower = x m

and $\cos\theta = 0.53$.

Height of the tower AB = 20m.



$\cos\theta = 0.53 \Rightarrow \theta = 58^\circ$ (from tables)

Now $\tan\theta = \frac{AB}{CB} \Rightarrow \tan 58^\circ = \frac{20}{x}$

$\Rightarrow 1.6003 = \frac{20}{x} \Rightarrow x = \frac{20}{1.6003}$

$\Rightarrow x = 12.49 = 12.5m$

$\therefore$ Height of the tower $= 12.5m$.

Q13. The upper part of a tree broken by a wind falls to the ground without being detached. The top of the broken part touches the ground at an angle of $38°30'$ at a point 6m from the foot of the ~~Ceot~~tree.

Calculate :

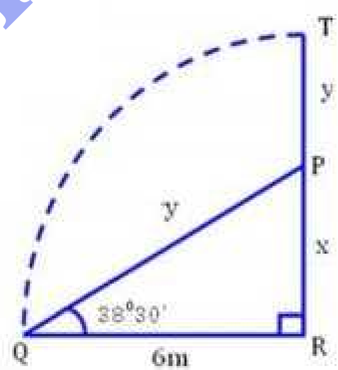
(i) The height at which the tree is broken.

(ii) The original height of the tree correct to two decimal places.

Sol. let TR be the total height of the tree and TP is broken part which touches the ground at the distance of 6m from the foot of the tree making an angle of $38°30'$ with the ground.

let $PR = x$ and $TR = x + y$

$\therefore PQ = PT = y$



In right ΔPQR,

$$\tan\theta = \frac{PR}{QR} \Rightarrow \tan 38°30' = \frac{x}{6}$$

$$\Rightarrow x = 6 \times 0.7954 = 4.7724$$

and $\quad \sin\theta = \dfrac{PR}{PQ} \Rightarrow \sin 38^\circ 30' = \dfrac{x}{y}$

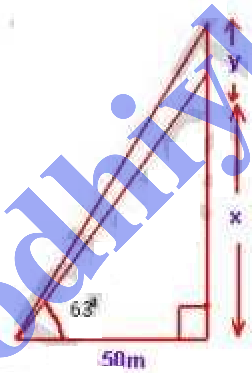
$$\Rightarrow 0.6225 = \frac{4.7724}{y}$$

$$\Rightarrow y = \frac{4.7724}{0.6225} = 7.6665$$

$\therefore$ Heigh of the tree $= 4.7724 + 7.6665 = 12.4389 = 12.44\,m$

and height of the tree at which it is broken $= 4.77\,m$.

Q14. Some students wished to find the height x of a building and the height y of the flag pole on the building. They made the measurements as shown in the diagram. find x and y. Give your answer to nearest m.



Sol. In ΔBDC,

$$\tan\theta = \frac{BC}{DC} \Rightarrow \tan 63^\circ = \frac{x}{50}$$

$$\Rightarrow x = 50 \times 1.9626 = 98.13\,m = 98\,m.$$

In ΔADC,

$$\tan 66^\circ = \frac{AC}{DC} = \frac{x+y}{50}$$

$$\Rightarrow 2.2460 = \frac{x+y}{50}$$

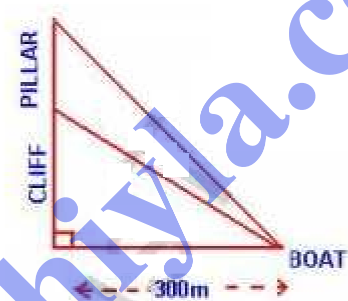
$\Rightarrow x+y = 2.2460 \times 5$

$\Rightarrow x+y = 11.23$ m

$y = 11.23 - x = 11.23 - 9.813 = 14.17 = 14$ m

Hence $x = 98$ m and $y = 14$ m

Q15 From a boat 300m away from a vertical cliff, the angles of elevation of the top and the foot of a vertical concrete pillar at the edge of the cliffs are $55^\circ 40'$ and $54^\circ 20'$ respectively. find the height of the pillar correct to nearest m.



sol. let CB be the cliff and AC be the pillar and D be the boat which is 300m away from the foot of the cliff i.e, BD = 300m.

Angles of elevation of the top & foot of the pillar are $55^\circ 40'$ and $54^\circ 20'$ respectively.

let $CB = x$ and $AC = y$

In right ΔCBD,

$\tan\theta = \frac{CB}{BD} \Rightarrow \tan 54^\circ 20' = \frac{x}{300}$

$\Rightarrow x = 300 \times 1.3933 = 417.99$ m

Again in right ΔABD,

$$\tan 55^\circ 40' = \frac{AB}{BD} = \frac{x+y}{300}$$

$$\Rightarrow \frac{x+y}{300} = 1.4641$$

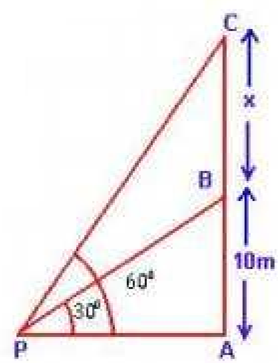
$$\Rightarrow x+y = 1.4641 \times 300 = 439.23$$

$$\Rightarrow y = 439.23 - 417.99 = 21.24$$

$\therefore$ Height of the pillar $= 21.24\,m = 21\,m$.

Q16. From point P on the ground, the angle of elevation of the 10m tall building and a helicopter hovering over the top of the building are 30° and 60° respectively. find the height of the helicopter above the ground.

Sol. let AB be the building and C is the helicopter and $BC = x$ is the height of the helicopter above the building.



In ΔPAB,

$$\tan 30^\circ = \frac{10}{PA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{PA} \Rightarrow PA = 10\sqrt{3}\,m.$$

Now in ΔPAC,

$$\tan 60^\circ = \frac{AC}{PA} \Rightarrow \sqrt{3} = \frac{x+10}{10\sqrt{3}}$$

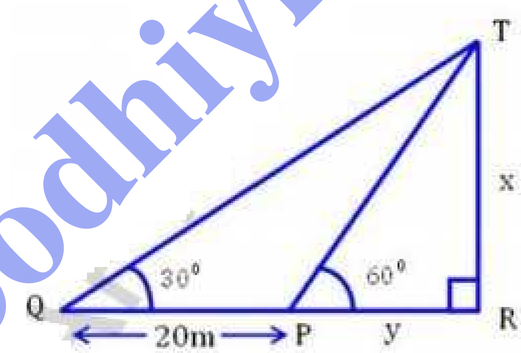
$$\Rightarrow x+10 = 30 \Rightarrow x = 20m.$$

$\therefore$ Total height of helicopter from ground = 20+10 = 30m

Q17. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°; when he retries 20m from the bank, he finds the angle to be 30°. find the height of the tree and the breadth of the river.

Sol. let TR be the tree and PR be width of river

let TR = x and PR = y.



In right ΔTPR,

$$\tan\theta = \frac{TR}{PR} \Rightarrow \tan 60^\circ = \frac{x}{y}$$

$$\Rightarrow x = y\sqrt{3} \quad \text{——— (1)}$$

Again in right ΔTQR,

$$\tan 30^\circ = \frac{TR}{QR} = \frac{x}{y+20}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y+20}$$

$$\Rightarrow x = \frac{y+20}{\sqrt{3}} \quad \text{---} \quad (ii)$$

From (i) & (ii),

$$y\sqrt{3} = \frac{y+20}{\sqrt{3}}$$

$$\Rightarrow 3y = y + 20$$

$$\Rightarrow y = 10$$

Substituting the value of y in (i)

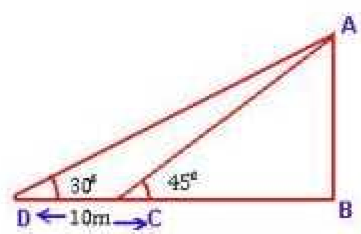
$$x = 10 \times \sqrt{3} = 10\,(1.732) = 17.32 \text{ m}.$$

$\therefore$ Height of the tree $= 17.32$ m.

and width of the river $= 10$ m.

Q18. The shadow of a vertical tower on a level ground increases by 10m when the altitude of sun changes from 45° to 30°. Find the height of the tower, correct to two decimal places.

Sol. let AB be the height of the tower.



In ΔABC,

$$\tan 45^\circ = \frac{AB}{BC} \Rightarrow AB = BC \quad \text{——— (1)}$$

In ΔABD,

$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow AB = \frac{BD}{\sqrt{3}} = \frac{DC + BC}{\sqrt{3}}$$

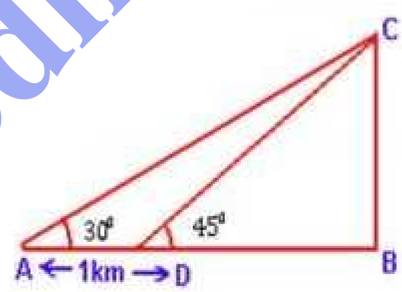
$$\Rightarrow AB = \frac{10 + AB}{\sqrt{3}} \quad \{\text{from (1)}\}$$

$$\Rightarrow AB\sqrt{3} = 10 + AB$$

$$\Rightarrow AB(\sqrt{3}-1) = 10 \Rightarrow AB = \frac{10}{\sqrt{3}-1} = 13.66\text{ m}.$$

Q19. From the top of a hill, the angles of depression of two consecutive km stones, due east are found to be 30° and 45° respectively. Find the distance of two stones form the foot of the hill.

Sol.



let BC be the hill and A and D be the two consecutive km stones.

In ΔBCD, $\tan 45^\circ = \frac{BC}{BD}$

$$\Rightarrow BC = BD \quad \text{——— (1)}$$

In ΔABC,

$$\tan 30^\circ = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{AB}$$

$$\Rightarrow AB = \sqrt{3}\,BC$$

$$\Rightarrow AD + DB = \sqrt{3}\,BC$$

$$\Rightarrow AD + BC = \sqrt{3}\,BC$$

$$\Rightarrow AD = (\sqrt{3}-1)\,BC$$

$$\Rightarrow 1 = (\sqrt{3}-1)\,BC$$

$$\Rightarrow BC = \frac{1}{\sqrt{3}-1} = 1.366 \text{ km}$$

$$\Rightarrow BD = 1.366 \text{ km} \quad \{\text{from}(1)\}$$

and $AB = AD + BD = 1 + 1.366 = 2.366$ km.

Q20. At a point on level ground, the angle of elevation of a Vertical tower is found to be such that its target is $\frac{5}{12}$. on walking 192m towards the tower, the tangent of the angle is found to be $\frac{3}{4}$. find the height of the tower.

Sol. let TR be the tower and P is the point on the ground such that $\tan\theta = \frac{5}{12}$ and $\tan\alpha = \frac{3}{4}$.

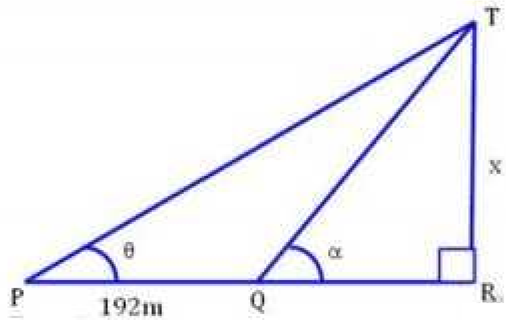
$PQ = 192$ m

let $TR = x$ and $QR = y$.

Now in right ΔTQR

$$\tan\alpha = \frac{TR}{QR} = \frac{x}{y} \Rightarrow \frac{3}{4} = \frac{x}{y}$$

$$\Rightarrow y = \frac{4}{3}x \quad \text{——(i)}$$



Again in right ΔTPR,

$$\tan\theta = \frac{TR}{PR} \Rightarrow \frac{5}{12} = \frac{x}{y+192}$$

$$\Rightarrow x = \frac{12}{5}(y+192) \quad \text{—— (ii)}$$

from (i) & (ii),

$$x = \left(\frac{4}{3}x + 192\right)\frac{5}{12}$$

$$\Rightarrow x = \frac{5}{9}x + 80 \Rightarrow x - \frac{5}{9}x = 80$$

$$\Rightarrow \frac{4x}{9} = 80 \Rightarrow x = 180$$

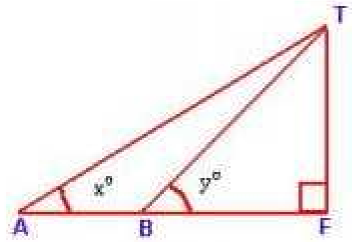
$\therefore$ Height of the tower $= 180m$.

Q21. In the fig. not drawn to scale, TF is a tower. The elevation of T from A is x° where $\tan x = \frac{2}{5}$ and $AF = 200m$. The elevation of T from B. where $AB = 80m$ is y°. Calculate :

(i) The height of the tower TF.

(ii) The angle y, correct to the nearest degree.

Sol. let height of the tower $TF = x$.



$\tan x = \frac{2}{5}$, AF = 200m, AB = 80m

(i) In right ΔATF,

$$\tan x^\circ = \frac{TF}{AF} \Rightarrow \frac{2}{5} = \frac{x}{200}$$

$$\Rightarrow x = \frac{2 \times 200}{5} = 80 \text{ m.}$$

$\therefore$ Height of the tower = 80m.

(ii) In right ΔTBF,

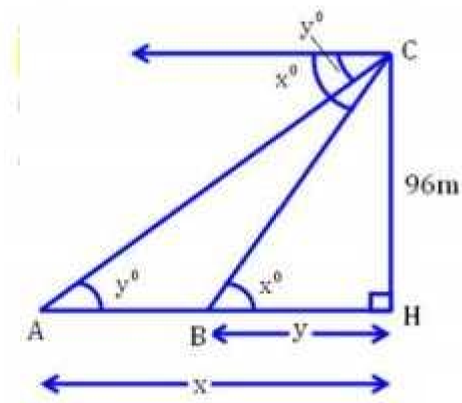
$$\tan y = \frac{TF}{BF} = \frac{80}{200-80} = \frac{80}{120} = \frac{2}{3} = 0.6667$$

$$\Rightarrow y = 33^\circ 41' \approx 34^\circ.$$

Q22. From the top of a church spire 96m high, the angle depression of two vehicles on a road, at the same level as the base of the spire and on the same side of it are x° and y°, where $\tan x^\circ = \frac{1}{4}$ and $\tan y^\circ = \frac{1}{7}$. Calculate the distance between the vehicles.

Sol. Height of the church CH. let A and B are two vehicles which make the angle of depression from C are x° and y° respectively.

let $AH = x$ and $BH = y$



In right ΔCBH,

$\tan x^0 = \frac{CH}{AH} = \frac{96}{y} \Rightarrow \frac{1}{4} = \frac{96}{y}$

$\Rightarrow y = 96 \times 4 = 384\ m.$

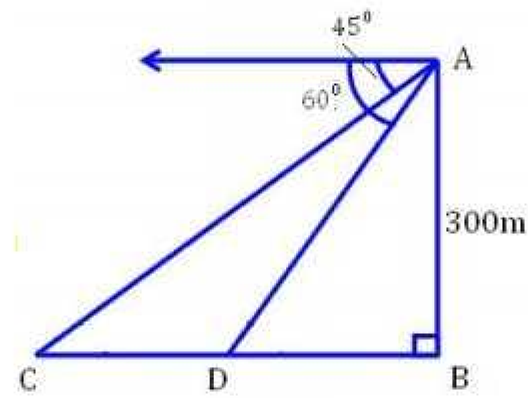
Again in right ΔCAH,

$\tan y^0 = \frac{CH}{AH} = \frac{96}{x} \Rightarrow \frac{1}{7} = \frac{96}{x}$

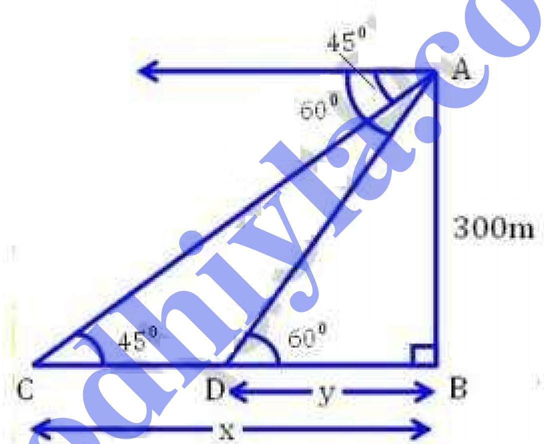
$\Rightarrow x = 96 \times 7 = 672\ m.$

$\therefore AB = x - y = 672 - 384 = 288\ m.$

Q23 In the adjoining fig. not drawn to the scale. AB is a tower and two objects C and D are located on the ground, on the same side of AB. When observed from the top A of the tower, their angles of depression are 45° and 60°. Find the distance between the two objects, if the height of the tower is 300m. Give your answer to the nearest meter.



Sol. let $CB = x$ and $DB = y$, $AB = 300m$.



In right ΔACB,

$$\tan\theta = \frac{AB}{CB} \Rightarrow \tan 45^\circ = \frac{AB}{CB} = \frac{300}{x}$$

$$\Rightarrow 1 = \frac{300}{x} \Rightarrow x = 300 \text{ m.}$$

In right ΔADB,

$$\tan 60^\circ = \frac{AB}{DB} = \frac{300}{y}$$

$$\Rightarrow \sqrt{3} = \frac{300}{y} \Rightarrow y = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{300\sqrt{3}}{3}$$

$\Rightarrow y = 100(1.732) = 173.2\,m.$

$CD = x - y = 300 - 173.2 = 126.8 = 127\,m.$

Distance between two objects $= 127\ m.$

Q24. The horizontal distance between two towers is 140m. The angle of elevation of the top of the first tower, when seen from the top of the second tower is 30°. If the height of the second tower is 60m, find the height of the first tower.

sol. let the height of first tower $TR = x$

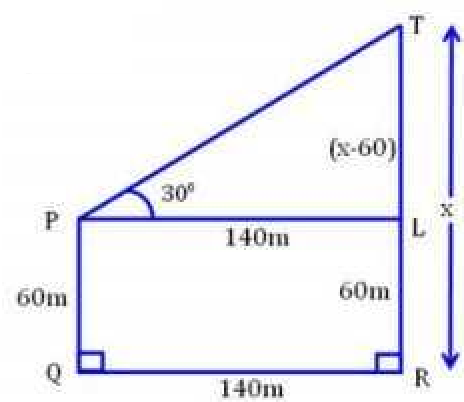
height of second tower $PQ = 60\ m$

Distance between two towers $QR = 140m.$

Draw $PL \parallel QR$, then

$LR = PQ = 60m$, $PL = QR = 140m$

$\therefore TL = (x - 60)\ m.$



Now in right ΔTPL,

$$\tan\theta = \frac{TL}{PL} \Rightarrow \tan 30^\circ = \frac{x-60}{140}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x-60}{140} \Rightarrow x-60 = \frac{140}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

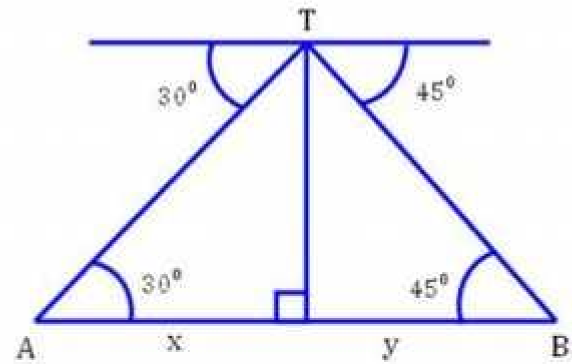
$$\Rightarrow x-60 = \frac{140\sqrt{3}}{3} \Rightarrow x = \frac{140\sqrt{3}}{3} + 60$$

$$\Rightarrow x = \frac{140(1.732)}{3} + 60 = 140.83$$

$\therefore$ Height of first tower $= 140.83$ m.

Q25. From a light house the angle of depression of two ships on opposite sides of the light house were observed to be 30° and 45°. If the height of the light house is 90m and the line joining the two ships passes through the foot of the light house, find the distance between two ships. Give your answer correct to two decimal places.

Sol. TR is the light house and A & B are two ships. From the top of the light house, angles of depression with the ships are 30° and 45° respectively.



Height of light house $TR = 90m$

let distance between $AR = x$ and $RB = y$

Now in right ΔTAR,

$$\tan\theta = \frac{TR}{AR} \Rightarrow \tan 30° = \frac{90}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{90}{x} \Rightarrow x = 90\sqrt{3} \quad \text{——— (i)}$$

and in right ΔTAB,

$$\tan 45° = \frac{TR}{RB} \Rightarrow 1 = \frac{TR}{RB} = \frac{90}{y}$$

$$\Rightarrow y = 90 \quad \text{——— (ii)}$$

$$\therefore AB = x+y = 90\sqrt{3}+90 = 90(1.732)+90$$

$$= 155.88+90 = 245.88\ m$$

$\therefore$ Distance between two ships $= 245.88\ m.$

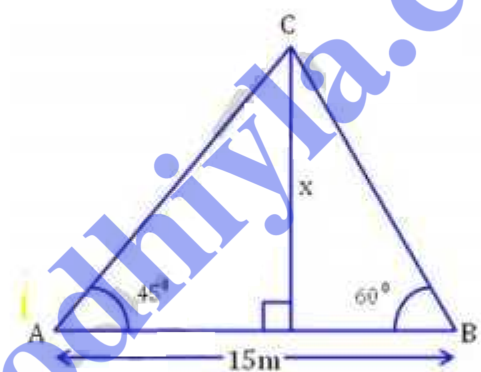
Q26. The angle of elevation of a pillar from a point A on the ground is 45° and from a point B diametrically opposite to A and on the other side of the pillar is 60°. find the height of the pillar, given that the distance between A and B is 15m.

Sol. let CD be the pillar and let CD = x

Angles of elevation of points A and B are 45° and 60° respectively.

$AB = 15m$

let $AD = y$ then $DB = 15 - y$



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Now in right ΔCAD,

$$\tan\theta = \frac{CD}{AD} \Rightarrow \tan 45^\circ = \frac{x}{y}$$

$$\Rightarrow x = y \quad \text{——(i)}$$

In right ΔCDB,

$$\tan 60^\circ = \frac{x}{15-y} \Rightarrow \sqrt{3} = \frac{x}{15-y}$$

$$\Rightarrow x = \sqrt{3}(15-y) \quad \text{——(ii)}$$

from (i) & (ii),

$$x = \sqrt{3}(15 - x)$$

$\Rightarrow \quad x = 15\sqrt{3} - \sqrt{3}x$

$\Rightarrow \quad x + \sqrt{3}x = 15\sqrt{3}$

$\Rightarrow \quad x = \dfrac{15\sqrt{3}}{1+\sqrt{3}} = \dfrac{15(1.732)}{1+1.732} = 9.51$

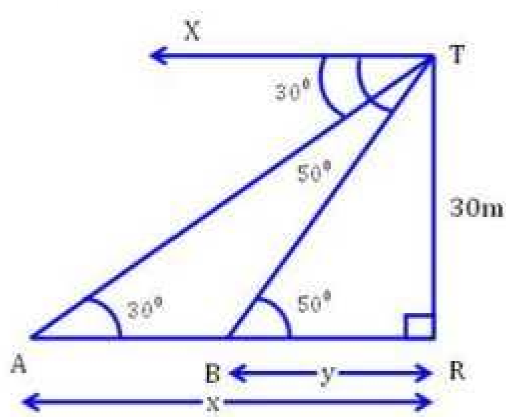
$\therefore$ Height of pillar $= 9.51$ m.

Q27. The angles of depression of two boats on a river from the top of a tower on the bank of the river are 30° and 50°. If the height of the tower is 30 m and the boats are in line with the tower on the same side of it, find the distance between the boats.

Sol. let TR be the tree and TR = 30m.

let A and B be the two boats in the same line and angles of depression from the top of the tree are 30° and 50° respectively.

let AR = x and BR = y.



Now in right ΔTAR,

$$\tan\theta = \frac{TR}{AR} \Rightarrow \tan 30^\circ = \frac{30}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{x} \Rightarrow x = 30\sqrt{3} = 51.96$$

In right ΔTBR,

$$\tan 50^\circ = \frac{TR}{BR} \Rightarrow 1.1917 = \frac{30}{y}$$

$$\Rightarrow y = \frac{30}{1.1917} = 25.17$$

$$AB = AR - RB = x - y = 51.96 - 25.17 = 26.79$$

$\therefore$ Distance between the boats $= 26.79$ m.

Q28. From a tower 126m high, the angles of depression of two rocks which are in a horizontal line through the base of the tower are 16° and $12^\circ 20'$. find the distance between the rocks if they are on

(i) The same side of the tower.

(ii) The opposite side of the tower.

Sol. let CD be the tower and CD = 126m.

let A and B be the two rocks on the same line and angles of depression are 16° and $12^\circ 20'$ respectively.

In right ΔCAD,

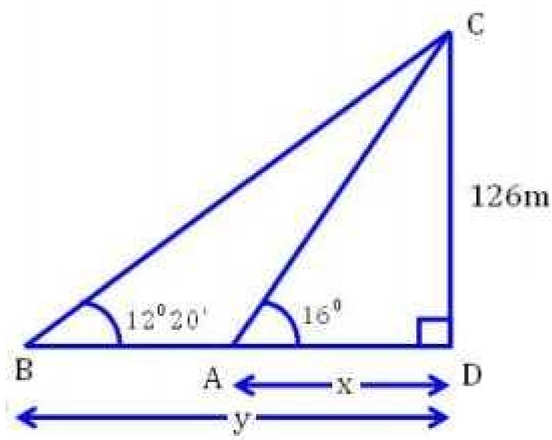
$$\tan\theta = \frac{CD}{AD} \Rightarrow \tan 16^\circ = \frac{126}{x}$$

$$\Rightarrow 0.2867 = \frac{126}{x} \Rightarrow x = \frac{126}{0.2867} = 439.48$$

In ΔCBD,

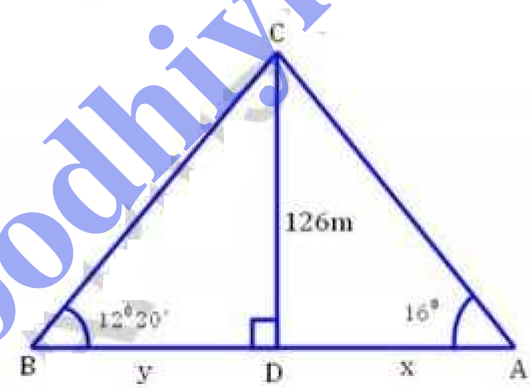
$$\tan 12^\circ 20' = \frac{126}{y} \Rightarrow y = \frac{126}{0.2186} = 576.40 \text{ m}$$

(i) In first case (on the same side of the tower)



$AB = BD - AD = y - x = 576.40 - 439.48 = 136.92 m.$

(ii) In second case (on the opposite side of the tower)



$AB = BD + AD = y + x = 576.40 + 439.48$

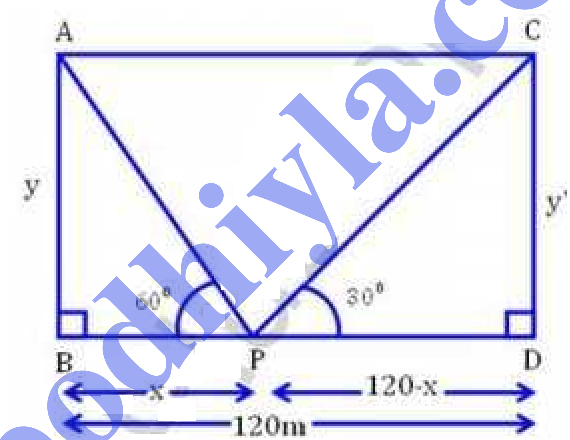
$= 1015.88\ m.$

Q29. Two pillars of equal height stand on either side of roadway which is 120 m wide. at a point in the road between pillars, the elevations of the pillars are 60° and 30°. Find the height of the pillars and the position of the point.

Sol. let the height of each pillar $= y$

let the point from the foot of first pillar $AB = x$

and then the distance from the foot of the second pillar $= 120 - x$ $\quad (\because BD = 120\,m)$



In right ΔABP,

$$\tan\theta = \frac{AB}{BP} \Rightarrow \tan 60^\circ = \frac{y}{x}$$

$$\Rightarrow \sqrt{3} = \frac{y}{x} \Rightarrow y = \sqrt{3}\,x \quad \text{—— (i)}$$

In right ΔCPD,

$$\tan 30^\circ = \frac{CD}{PD} = \frac{y}{120 - x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{120 - x}$$

$$\Rightarrow y = \frac{120 - x}{\sqrt{3}} \quad \text{—— (ii)}$$

from (i) & (ii),

$$\sqrt{3}x = \frac{120 - x}{\sqrt{3}} \Rightarrow 3x = 120 - x$$

$$\Rightarrow 4x = 120 \Rightarrow x = 30 \text{ m}$$

Substitute the value of x in (i)

$$y = \sqrt{3} \times 30 = 30(1.732) = 51.96 \text{ m}$$

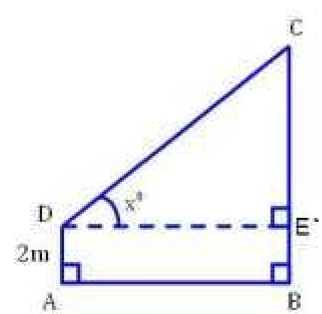
$\therefore$ Height of each pillar = 51.96 m

and point P is 30m for the first pillar.

Q30. With reference to the pillar figure given alongside, a man stands on the ground at point A, which is on the same horizontal plane as B, the foot of the vertical pole BC. The height of the pole is 10m. The man eye is 2m above the ground. He observes the angle of elevation at C, the top of the pole as x°, where $\tan x^\circ = \frac{2}{5}$. Calculate:

(i) The distance AB in metres.

(ii) The angle of elevation of the top of the pole when he is standing 15m from the pole. Give your answer to the nearest degree.



Sol. In the fig., $DE \parallel AB$

$EB = AD = 2m$; $CE = 10 - 2 = 8m$.

let $DE = AB = x$, $\tan x^\circ = \frac{2}{5}$

Now in right ΔCDE,

$$\tan x^\circ = \frac{CE}{DE} = \frac{8}{x} \Rightarrow \frac{2}{5} = \frac{8}{x}$$

$$\Rightarrow x = \frac{8 \times 5}{2} = 20\,m.$$

(i) Distance of $AB = 20\,m$.

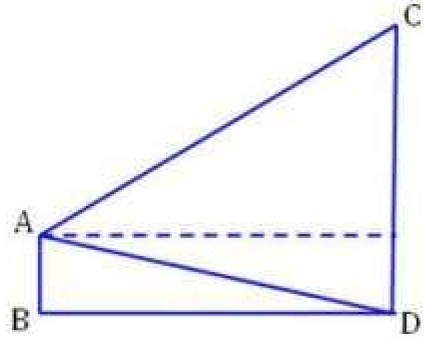
(ii) when $DE = AB = 15m$ and $CE = 8m$

$$\tan\theta = \frac{CE}{DE} = \frac{8}{15} = 0.5333$$

$$\Rightarrow \theta = 28^\circ 4' = 28^\circ$$

Hence angle of elevation $= 28^\circ$.

Q31. From a window A, 10m above the ground the angle of elevation of the top C of a tower is x°, where $\tan x^\circ = \frac{5}{2}$ and the angle of depression of the foot D of the tower is y°, where $\tan y^\circ = \frac{1}{4}$. Calculate the height CD of the tower in meters.



let $CD = x$, $BD = AE = y$

$AE \parallel BD$. then $ED = AB = 10m$

$CE = x - 10$

In right $\triangle CAE$,

$\tan x^\circ = \frac{CE}{AE} \Rightarrow \frac{5}{2} = \frac{x-10}{y}$

$\Rightarrow 5y = 2(x-10) \Rightarrow y = \frac{2}{5}(x-10)$ —— (i)

In right $\triangle ABD$,

$\tan y = \frac{AB}{BD} \Rightarrow \frac{1}{4} = \frac{10}{y} \Rightarrow y = 40$ —— (ii)

From (i) & (ii),

$\frac{2}{5}(x-10) = 40 \Rightarrow 2x - 20 = 200$

$\Rightarrow 2x = 220 \Rightarrow x = 110$

$\therefore$ Height of $CD = 110 m$.

Q32. A man 1.8m high stands at a distance of 3.6 m from a lamp post and casts a shadow of 5.4 m on the ground. find the height of the lamp post.

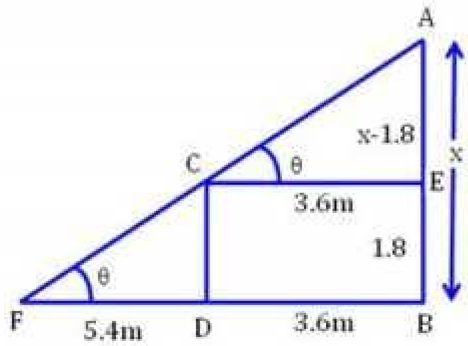
Sol. AB is the lamp post. CD is the height of man. BD is the distance of man from the foot of the lamp and FD is the shadow of man

$CE \parallel DB$

let $AB = x$, $CD = 1.8 m$

$EB = CD = 1.8m$, then $AE = x - 1.8$

Shadow $FD = 5.4m$



Now in right ΔACE,

$$\tan\theta = \frac{AE}{CE} = \frac{x-1.8}{3.6} \quad \text{——(i)}$$

Again in right ΔCFD,

$$\tan\theta = \frac{CD}{FD} = \frac{1.8}{5.4} = \frac{1}{3} \quad \text{——(ii)}$$

From (i) & (ii),

$$\frac{x-1.8}{3.6} = \frac{1}{3} \Rightarrow 3x - 5.4 = 3.6$$

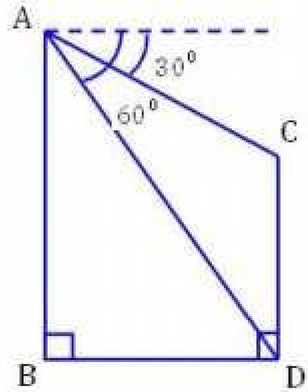
$$\Rightarrow 3x = 9.0 \Rightarrow x = 3$$

$\therefore$ Height of Lamppost = 3m.

Q33. In the given figure alongside, from the top of a building AB, 60m height, the angle of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. find:

(i) The horizontal distance between AB and CD.

(ii) The height of the lamp post.



Sol. AB is the building and AB = 60m

CD is the lamp post. let CD = x

and angles of the depression from the top of the building to the top and bottom of the lamp posts are 30° and 60° respectively.

Draw $CE \parallel DB$

let $BD = CE = y$, $EB = CD = x$

$\therefore AE = 60 - x$

Now in right $\triangle ABD$,

$\tan\theta = \frac{AB}{BD} \Rightarrow \tan 60^\circ = \frac{60}{y} \Rightarrow \sqrt{3} = \frac{60}{y}$

$\Rightarrow y = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} = 20(1.732) = 34.64m.$

Now in right $\triangle AEC$,

$\tan 30^\circ = \frac{AE}{EC} = \frac{AE}{BD} = \frac{60-x}{y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-x}{y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{(60-x)\sqrt{3}}{60} \Rightarrow 60 = 3(60-x)$

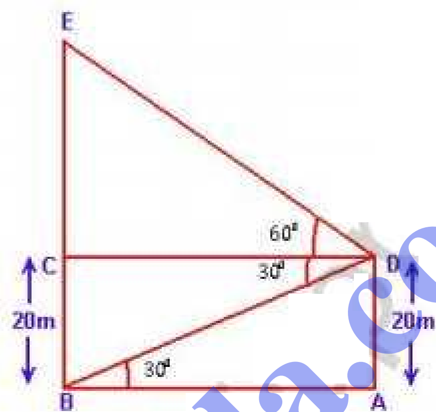
$\Rightarrow 3x = 120 \Rightarrow x = 40.$

Hence (i) the distance between AB and CD = 34.64 m.

(ii) Height of the lamp post = 40m.

Q34. A vertical pole and vertical tower are on the same level ground. From the top of the pole, the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20m.

Sol.



Let AD = 20m be the height of pole and BE be the height of the tower from the same ground level.

In ΔABD,

$$\tan 30^\circ = \frac{20}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{AB} \Rightarrow AB = 20\sqrt{3}$$

Now in ΔCDE,

$$\tan 60^\circ = \frac{CE}{CD} \Rightarrow \sqrt{3} = \frac{CE}{20\sqrt{3}} \quad [\because AB = CD]$$

$$\Rightarrow CE = 20\sqrt{3} \times \sqrt{3} = 60\,m.$$

$\therefore$ The height of the tower is $BC + CE = 20 + 60$

$= 80\,m.$

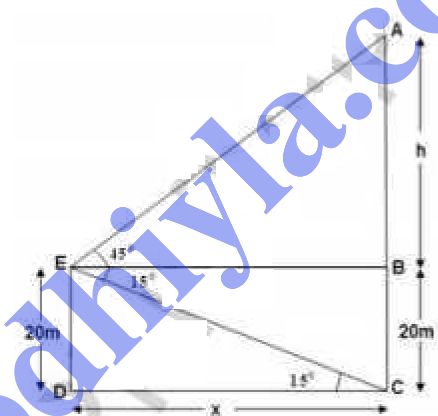
Q35. From the top of a building 20m high, the angle of elevation of the top of a monument is 45° and the angle of depression of its foot is 15°. find the height of the monument.

Sol. let DE be the building 20m high, the angle of elevation of the top of a monument is 45° and the angle of depression of its foot is 15°.

let DE be the building and AC be the monument.

then height of monument $= h + 20$ m

$BE = DC = x$



In ΔDEC, $\tan 15° = \frac{DE}{DC} = \frac{20}{x} \Rightarrow x = \frac{20}{\tan 15°}$

$\Rightarrow x = \frac{20}{0.2679} = 74.65$ m.

$BE = DC = 74.65$ m.

In ΔABE, $\tan 45° = \frac{AB}{BE} = \frac{h}{74.65} \Rightarrow 1 = \frac{h}{74.65}$

$\Rightarrow h = 74.65$ m.

Hence, height of monument $= h + 20 = 74.65 + 20$

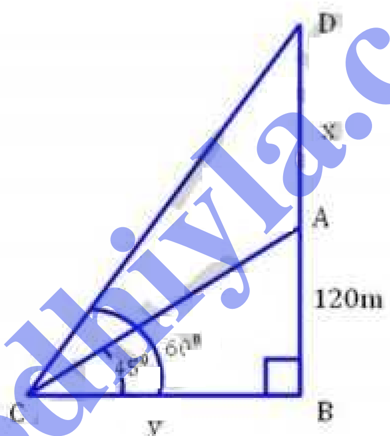
$= 94.65$ m.

Q36. The angle of elevation of the top of an unfinished tower at point distant 120m from its base is 45°. how much higher must the tower be raised so that its angle of elevation at the same point may be 60°?

Sol. let AB be the unfinished tower and $AB = 120m$ and angle of elevation $= 45^\circ$.

let x be higher raised so that the angle of elevation becomes 60°.

let $BC = y$.



In right ΔABC,

$$\tan\theta = \frac{AB}{CB} \Rightarrow \tan 45^\circ = \frac{AB}{CB} = \frac{120}{y} \Rightarrow 1 = \frac{120}{y}$$

$$\Rightarrow y = 120m.$$

Now in right ΔDBC,

$$\tan 60^\circ = \frac{DB}{CB} \Rightarrow \sqrt{3} = \frac{120+x}{120} \Rightarrow x = 120\sqrt{3} - 120$$

$$\Rightarrow x = 120(1.732-1) = 120 \times 0.732 = 87.84\ m.$$

$\therefore$ Raised tower $= 87.84\ m$.