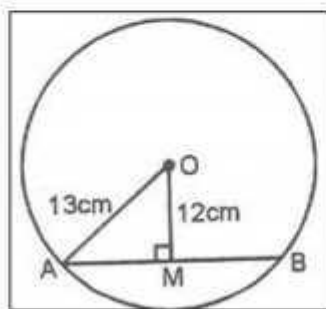


Circles

EXERCISE - 16.1

- Q1. Calculate the length of a chord which is at a distance of 12cm. from the centre of a circle of radius 13cm.

Sol.



AB is chord of a circle with centre O and OA is its radius $OM \perp AB$.

$$\therefore OA = 13\text{cm}, OM = 12\text{cm}.$$

Now in right $\triangle OAM$, $OA^2 = OM^2 + AM^2$

$$\Rightarrow 169 = 144 + 25 \Rightarrow 169 - 144 = 25 \Rightarrow (5)^2$$

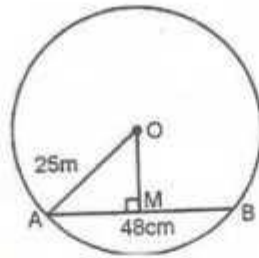
$$\Rightarrow AM = 5\text{cm}.$$

$$\therefore OM \perp AB$$

$$\therefore M \text{ is the midpoint of } AB. \quad AB = 2AM = 2 \times 5 = 10\text{cm}.$$

- Q2. A chord of length 48cm is drawn in a circle of radius 25cm. calculate its distance from the centre of the circle.

Sol.



AB is the chord of the circle with centre O and radius OA and $OM \perp AB$.

$$AB = 48\text{cm}, \quad OA = 25\text{cm}$$

$$\therefore OM \perp AB$$

M is the midpoint of AB.

$$\therefore AM = \frac{1}{2} AB = \frac{1}{2} \times 48 = 24\text{cm}$$

Now in right $\triangle OAM$, $OA^2 = OM^2 + AM^2$

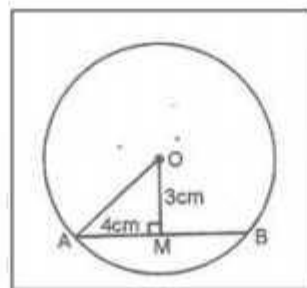
$$\Rightarrow (25)^2 = (OM)^2 + (24)^2 \Rightarrow OM^2 = (25)^2 - (24)^2$$

$$\Rightarrow OM^2 = 625 - 576 = 49 = 7^2$$

$$\therefore OM = 7\text{cm}$$

3. A chord of length 8cm is at a distance of 3cm from the centre of the circle. Calculate the radius of the circle.

Sol.



AB is the chord of a circle with centre O and radius OA and $OM \perp AB$.

$$\therefore AB = 8\text{cm}, OM = 3\text{cm}.$$

$$\therefore OM \perp AB.$$

$\therefore M$ is the midpoint of AB.

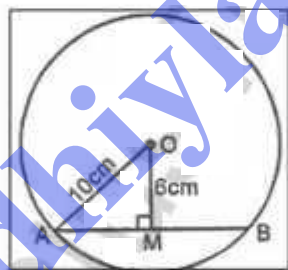
$$AM = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4\text{cm}.$$

Now in right $\triangle OAM$, $OA^2 = OM^2 + AM^2$

$$\Rightarrow (3)^2 + (4)^2 = OA^2 \Rightarrow OA = 5\text{cm}.$$

4. Calculate the length of the chord which is at a distance of 6cm from the centre of a circle of diameter 20cm.

Sol.



AB is the chord of the circle with centre O and radius OA and $OM \perp AB$.

$$\text{Diameter of the circle} = 20\text{cm} \Rightarrow \text{Radius} = 10\text{cm}.$$

$$\therefore OA = 10\text{cm}, OM = 6\text{cm}.$$

Now in right $\triangle OAM$, $OA^2 = AM^2 + OM^2 \Rightarrow (10)^2 = AM^2 + (6)^2$

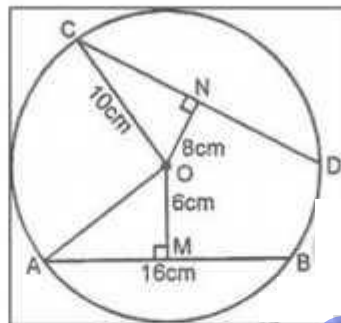
$$\Rightarrow AM^2 = 64 \Rightarrow AM = 8\text{cm}.$$

$OM \perp AB$. M is the midpoint of AB

$$\therefore AB = 2AM = 2 \times 8 = 16\text{cm}.$$

5. A chord of length 16cm is at a distance of 6cm from the centre of the circle. find the length of the chord of the same circle which is at a distance of 8cm from the centre.

Sol.



AB is a chord of a circle with centre O and OA is the radius of the circle and $OM \perp AB$.
 $\therefore AB = 16\text{cm}$, $OM = 6\text{cm}$ $OM \perp AB$.

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8\text{cm}.$$

Now in right $\triangle OAM$, $OA^2 = AM^2 + OM^2$

$$\Rightarrow OA^2 = 8^2 + 6^2 = 100 \Rightarrow OA = 10\text{cm}.$$

Now CD is another chord of the same circle $ON \perp CD$ and OC is the radius.

In right $\triangle ONC$, $OC^2 = ON^2 + NC^2$

$$\Rightarrow (10)^2 = (8)^2 + NC^2 \Rightarrow NC^2 = 100 - 64 = 36$$

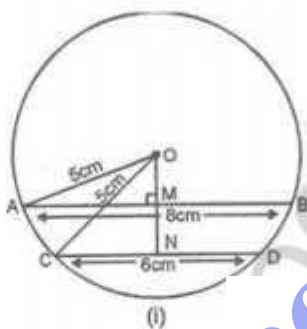
$$\Rightarrow NC = 6\text{cm}.$$

But $ON \perp AB$ $\therefore N$ is the midpoint of CD

$$\therefore CD = 2NC = 2 \times 6 = 12\text{cm}.$$

6. In a circle of radius 5cm, AB and CD are two parallel chords of length 8cm and 6cm respectively. Calculate the distance between the chords if they are on
- The same side of the centre.
 - The opposite side of the centre.

Sol.



Two chords AB and CD of a circle with centre O and radius OA or OC.

$$\therefore OA = OC = 5\text{cm} \quad \therefore AB = 8\text{cm}, \quad CD = 6\text{cm}$$

OM and ON are $\perp$ from O to AB and CD respectively.

$\therefore$ M and N are the midpoints of AB and CD respectively.

$$\text{In right } \triangle OAM, \quad OA^2 = AM^2 + OM^2$$

$$\Rightarrow 5^2 = 4^2 + OM^2 \Rightarrow OM^2 = 9 \Rightarrow OM = 3\text{cm}.$$

$$\text{Again in right } \triangle OCN, \quad OC^2 = CN^2 + ON^2$$

$$\Rightarrow 5^2 = 3^2 + ON^2 \quad (\because CN = \frac{1}{2} CD)$$

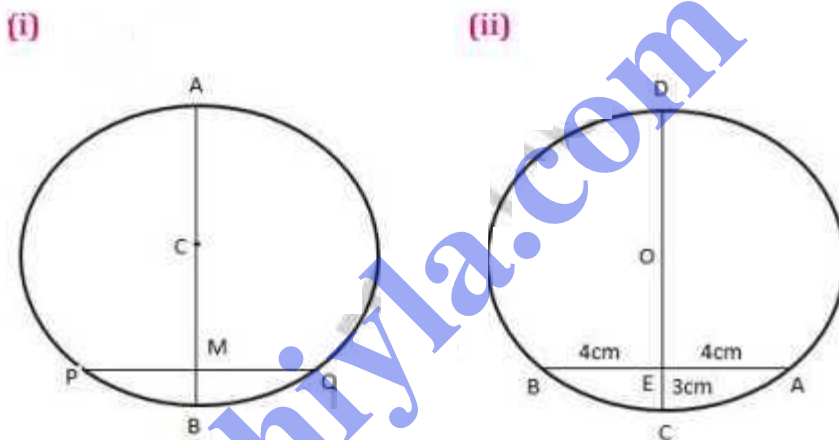
$$\Rightarrow ON^2 = 16 \Rightarrow ON = 4\text{cm}.$$

$$(i) \text{ Distance } MN = ON - OM = 4 - 3 = 1\text{cm}.$$

$$(ii) \text{ Distance } MN = OM + ON = 3 + 4 = 7\text{cm}.$$

7. (a) In the figure (i) given below, a chord PQ of a circle with centre C and radius 15cm is bisected at M by a diameter AB. If CM = 9cm. Calculate the lengths of (i) PQ (ii) AB (iii) BP.

- (b) In the fig (ii) given below, CD is the diameter which meets the chord AB in E such that $AE = BE = 4$ cm. If $CE = 3$ cm, find the radius of the circle.



- Sol. (a) In fig (i), Radius $CP = CB = CA = 15$ cm.
 $CM = 9$ cm.

$$\therefore MB = CB - MC = 15 - 9 = 6 \text{ cm.}$$

$\therefore$ M is the midpoint of chord PQ. $\therefore CM \perp PQ$.

(i) Now in right $\triangle CPM$, $CP^2 = PM^2 + CM^2$

$$\Rightarrow (15)^2 = PM^2 + (9)^2 \Rightarrow PM^2 = (15)^2 - (9)^2 = 144$$

$$\Rightarrow PM = 12 \text{ cm.}$$

But M is the midpoint of PQ. $PQ = 2PM$

$$PQ = 2 \times 12 = 24 \text{ cm.}$$

(ii) In right $\triangle APM$, $PM = 12\text{cm}$, $AM = 15 + 9 = 24\text{cm}$.

$$\therefore AP^2 = PM^2 + AM^2 = (12)^2 + (24)^2 = 144 + 576 = 720$$

$$\therefore AP^2 = 144 \times 5 \Rightarrow AP = 12\sqrt{5}\text{ cm}.$$

(iii) In right $\triangle PMB$, $BP^2 = PM^2 + MB^2 = (12)^2 + (6)^2$

$$\Rightarrow BP^2 = 144 + 36 = 180 \Rightarrow PB = \sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}\text{cm}.$$

(b) $AB = 8\text{cm}$, $EC = 3\text{cm}$.

Let radius $OB = OC = r$

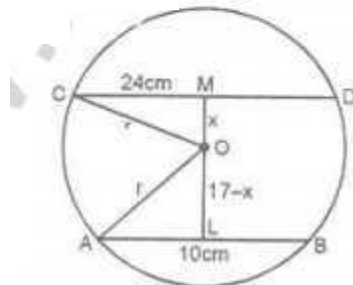
$$\therefore OE = (r - 3)\text{ cm}.$$

Now in right $\triangle OBE$, $OB^2 = BE^2 + OE^2$

$$\Rightarrow r^2 = 4^2 + (r - 3)^2 \Rightarrow r^2 = 16 + r^2 - 6r + 9$$

$$\Rightarrow 6r = 25 \Rightarrow r = \frac{25}{6} = 4\frac{1}{6}\text{ cm}.$$

8. AB and CD are two parallel chords of a circle of lengths 10cm and 24cm respectively. If the chords are on the opposite sides of the centre and the distance between them is 17cm , find the radius of the circle.



Sol.

AB and CD are two chords of a circle such that $AB \parallel CD$, $AB = 10\text{cm}$ and $CD = 24\text{cm}$ and distance $LM = 17\text{cm}$.

Let radius $OA = OC = r$, and $OM = x$.

$$\therefore OL = 17 - x.$$

Now in right $\triangle OAL$, $OA^2 = AL^2 + OL^2 \Rightarrow r^2 = 5^2 + (17 - x)^2$

$$\Rightarrow r^2 = 25 + (17 - x)^2 \quad \text{--- (i)} \quad (\because L \text{ is midpoint of } AB)$$

similarly, in right $\triangle OCM$, $OC^2 = CM^2 + OM^2 = 12^2 + x^2$

(M is midpoint of CD)

$$\therefore r^2 = 144 + x^2 \quad \text{--- (ii)}$$

From (i) & (ii),

$$25 + (17 - x)^2 = 144 + x^2$$

$$\Rightarrow 25 + 289 + x^2 - 34x = 144 + x^2 \Rightarrow 314 - 34x = 144$$

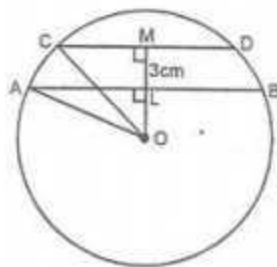
$$\Rightarrow 34x = 314 - 144 \Rightarrow x = \frac{170}{34} = 5.$$

Substituting the value of x in (ii)

$$\Rightarrow r^2 = 144 + x^2 = 144 + 25 = 169 \Rightarrow r = 13\text{cm}.$$

9.

AB and CD are two parallel chords of a circle of length 10cm and 4cm respectively. If the chords lie on the same sides of the circle and the distance between them is 3cm , find the diameter of the circle.



Sol. AB and CD are two parallel chords and $AB = 10\text{cm}$,
 $CD = 4\text{cm}$ and the distance between AB & CD = 3cm .
 let radius of circle $OA = OC = r$. $OM \perp CD$ which
 intersects AB in L.

$\therefore$ let $OL = x$, then $OM = x + 3$.

Now in right $\triangle OLA$, $OA^2 = AL^2 + OL^2$

$$\Rightarrow r^2 = 5^2 + x^2 \Rightarrow r^2 = 25 + x^2 \quad \text{--- (i)}$$

Again in right $\triangle OCM$, $OC^2 = CM^2 + OM^2$

$$\Rightarrow r^2 = (2)^2 + (x+3)^2 \Rightarrow r^2 = 4 + (x+3)^2 \quad \text{--- (ii)}$$

from (i) & (ii), $25 + x^2 = 4 + (x+3)^2$

$$\Rightarrow 25 + x^2 = 4 + x^2 + 9 + 6x \Rightarrow 6x = 12$$

$$\Rightarrow x = 2\text{cm}.$$

Substituting the value of x in (i)

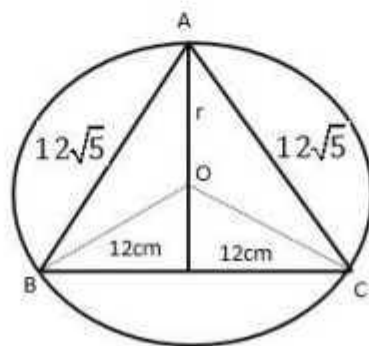
$$r^2 = 25 + x^2 = 25 + 4 = 29 \Rightarrow r = \sqrt{29}\text{ cm}.$$

$$\therefore \text{Diameter} = 2r = 2\sqrt{29}\text{ cm}.$$

10. ABC is an isosceles triangle inscribed in a circle.

If $AB = AC = 12\sqrt{5}\text{cm}$ and $BC = 24\text{cm}$. find the radius
 of the circle.

Sol.



$$AB = AC = 12\sqrt{5} \text{ cm and } BC = 24 \text{ cm.}$$

Join OB and OC and OA. Draw $AD \perp BC$ which will pass through centre O.

$$\therefore OD \text{ bisects } BC \text{ in } D. \therefore BD = DC = 12 \text{ cm.}$$

$$\text{In right } \triangle ABD, AB^2 = AD^2 + BD^2$$

$$\Rightarrow (12\sqrt{5})^2 = AD^2 + (12)^2 \Rightarrow AD^2 = 720 - 144 = 576$$

$$\Rightarrow AD = 24 \text{ cm.}$$

$$\text{let radius of the circle} = OA = OB = OC = r.$$

$$\therefore OD = AD - AO = 24 - r.$$

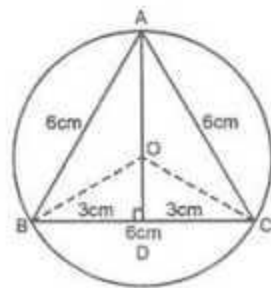
$$\text{Now in right } \triangle OBD, OB^2 = BD^2 + OD^2$$

$$\Rightarrow r^2 = (12)^2 + (24 - r)^2 \Rightarrow r^2 = 144 + 576 + r^2 - 48r$$

$$\Rightarrow 48r = 720 \Rightarrow r = 15 \text{ cm.}$$

11. A equilateral triangle of side 6cm is inscribed in a circle. find the radius of the circle.

Sol. ABC is an equilateral $\triangle$ inscribed in a circle with centre O. join OB and OC.



From A, draw $AD \perp BC$ which will pass through the centre O of the circle.

$\therefore$ Each side of $\triangle ABC = 6\text{cm}$.

$$AD = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}\text{cm}.$$

$$OD = AD - AO = 3\sqrt{3} - r$$

Now in right $\triangle OBD$, $OB^2 = BD^2 + OD^2$

$$\Rightarrow r^2 = 3^2 + (3\sqrt{3} - r)^2 = 9 + 27 + r^2 - 6\sqrt{3}r$$

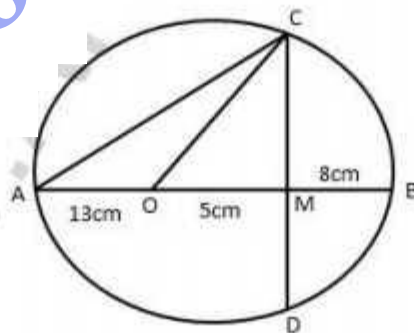
($\because$ D is the midpoint of BC)

$$\Rightarrow 6\sqrt{3}r = 36 \Rightarrow r = \frac{36}{6\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 2\sqrt{3}\text{cm}.$$

$\therefore$ Radius = $2\sqrt{3}\text{cm}$.

12. AB is a diameter of a circle. M is a ^{mid} point of AB such that $AM = 18\text{cm}$ and $MB = 8\text{cm}$. find the length of the shortest chord through M.

Sol.



In a circle with centre O, AB is the diameter and M is a point on AB such that $AM = 18\text{cm}$ and $MB = 8\text{cm}$.

$$\therefore AB = AM + MB = 18 + 8 = 26\text{cm}.$$

$\therefore$ Radius of the circle = 13cm .

Let CD be the shortest chord drawn through M.

$\therefore CD \perp AB$. Join OC. $\Rightarrow OM = AM - AO$.

$$OM = 18 - 13 = 5\text{cm}.$$

$$OC = OA = 13\text{cm}.$$

Now in right $\triangle OMC$, $OC^2 = OM^2 + MC^2$

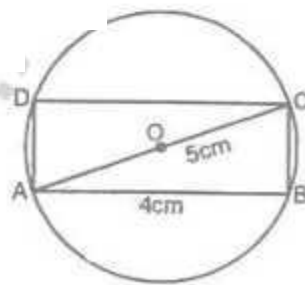
$$\Rightarrow (13)^2 = (5)^2 + MC^2 \Rightarrow MC^2 = 13^2 - 5^2 = 144 \Rightarrow MC = 12\text{cm}$$

$\therefore M$ is midpoint of CD.

$$\therefore CD = 2 \times MC = 2 \times 12 = 24\text{cm}.$$

13. A rectangle with one side of length 4cm is inscribed in a circle of diameter 5cm . Find the area of the rectangle.

Sol.



ABCD is a rectangle inscribed in a circle with centre O and diameter 5cm.

$$AB = 4\text{cm}, AC = 5\text{cm}.$$

$$\text{In right } \triangle ABC, AC^2 = AB^2 + BC^2$$

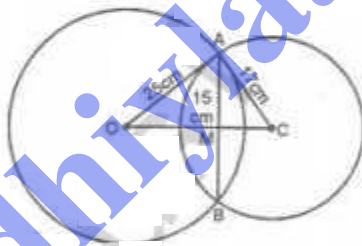
$$\Rightarrow (5)^2 = (4)^2 + BC^2 \Rightarrow BC^2 = 25 - 16 = 9$$

$$\Rightarrow BC = 3\text{cm}.$$

$$\text{Area of rectangle } ABCD = AB \times BC = 4 \times 3 = 12\text{cm}^2.$$

14. The length of the common chord of two intersecting circles is 30cm. If the radii of the two circles are 25cm and 17cm, find the distance between their centers.

sol.



AB is the common chord of two circles with centre O and C.

Join OA, CA and OC.

$$AB = 30\text{cm}, OA = 25\text{cm}, AC = 17\text{cm}.$$

$\therefore$ OC is the $\perp$ bisector of AB at M.

$$\therefore AM = MB = 15\text{cm}.$$

$$\text{In right } \triangle OAM, OA^2 = OM^2 + AM^2.$$

$$\Rightarrow (25)^2 = OM^2 + (15)^2$$

$$\Rightarrow OH^2 = 625 - 225 = 400 = (20)^2$$

$$\Rightarrow OM = 20\text{cm.}$$

Again in $\triangle AMC$, $AC^2 = AM^2 + MC^2$

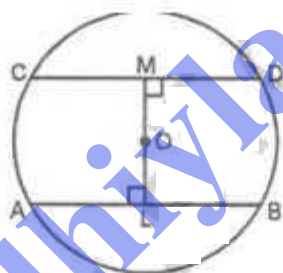
$$\Rightarrow 17^2 = 15^2 + MC^2 \Rightarrow MC^2 = 289 - 225 = 64$$

$$\Rightarrow MC = 8\text{cm.}$$

$$\text{Now, } OC = OM + MC = 20 + 8 = 28\text{cm.}$$

15. The line joining the midpoints of two chords of a circle passes through its centre. prove that the chords are parallel.

Sol.



Given: Two chords AB and CD where L and M are midpoints of AB and CD respectively.

LM passes through O, the centre.

To prove $AB \parallel CD$.

proof: L is the midpoint of AB.

$$OL \perp AB.$$

$$\therefore \angle OLA = 90^\circ \text{ — (i)}$$

Again M is midpoint of CD. $OM \perp CD$

$$\therefore \angle OMD = 90^\circ \text{ — (ii)}$$

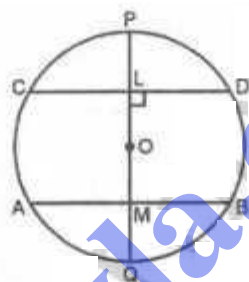
From (i) & (ii), $\angle OLA = \angle OMD$

But these are alternate angles

$$\therefore AB \parallel CD.$$

16. If a diameter of a circle is $\perp$ to one of two parallel chords of the circle, prove that it is $\perp$ to the other and bisects it.

Sol.



Given: Chord $AB \parallel CD$ and diameter $PQ \perp AB$.

To prove: $PQ \perp CD$.

proof: Diameter of $PQ \perp AB$.

$$\therefore \angle AMO = 90^\circ$$

$\therefore PQ$ bisects AB .

$\therefore AB \parallel CD$ (given)

$$\therefore \angle OLD = 90^\circ \text{ (Alternate angles)}$$

$\therefore OL$ or PQ is perpendicular to CD .

Hence PQ bisects CD .

17. In an equilateral Δ^e , prove that the centroid and the circumcenter of the triangle coincide.

sd. Given : ΔABC in which $AB = BC = CA$.

To prove : The centroid and the circumcenter coincide each other.

Construction: Draw $\perp$ bisectors of AB and BC intersecting each other at O .

Join AD , OB and OC .

proof : O lies on the $\perp$ bisectors of AB and BC .

$$OA = OB = OC.$$

O is the circumcentre of ΔABC .

D is midpoint of BC .

AD is the median of ΔABC .

Now in ΔABD and ΔACD ,

$$AB = AC \quad (\text{given})$$

$$AD = AD \quad (\text{common})$$

$$BD = DC \quad (D \text{ is midpoint of } BC)$$

$$\therefore \Delta ABD \cong \Delta ACD \quad (\text{SSS axiom of congruency})$$

$$\therefore \angle ADB = \angle ADC \quad (\text{C.P.C.T.})$$

$$\text{But } \angle ADB + \angle ADC = 180^\circ \quad (\text{linear pair})$$

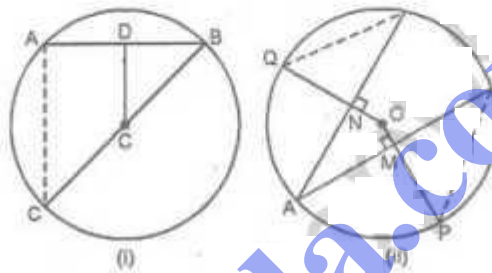
$$\therefore \angle ADB = \angle ADC = 90^\circ.$$

AD is perpendicular on BC which passes through O .

Hence centroid and circumcentre of ΔABC coincide each other.

18. (a) In the fig.(i) given below, OD is perpendicular to the chord AB of a circle whose centre is O . If BC is a diameter, show that $CA = 2 \cdot OD$.
- (b) In the fig.(ii) given below, O is the centre of a circle. If AB and AC are chords of the circle such that $AB = AC$ and $OP \perp AB$, $OQ \perp AC$, prove that $PB = QC$.

Sol.



(a) Given: $OD \perp$ to chord AB of the circle and BOC is the diameter. CA is joined.

To prove: $CA = 2 OD$.

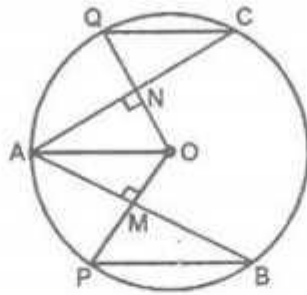
Proof: $OD \perp AB$.

$\therefore D$ is the midpoint of AB and O is the midpoint of BC .

In $\triangle BAC$, $OD \parallel CA$ and $OD = \frac{1}{2} CA$.

$\therefore CA = 2 OD$.

(b) Given : AB and AC are chords of a circle with Centre O and $AB = AC$, $OP \perp AB$ and $OQ \perp AC$. BP and QC are joined.



To prove : $PB = QC$

Proof : $OP \perp AB$ (given)

$\therefore M$ is the midpoint of AB .

$$\therefore AM = MB \Rightarrow MB = \frac{1}{2} AB$$

Similarly, $OQ \perp AC \therefore AN = NC$.

$$\Rightarrow NC = \frac{1}{2} AC$$

But $AB = AC \therefore MB = NC$

$\therefore$ Chord $AB =$ Chord AC .

$OM = ON$ but $OP = OQ$ (radii of the same circle)

$$\therefore MP = NQ$$

Now in $\triangle MPB$ and $\triangle NQC$,

$MB = NC$ (proved), $MP = NQ$ (proved)

$$\angle PMB = \angle QNC \text{ (each } 90^\circ)$$

$\therefore \triangle MPB \cong \triangle NQC$ (SAS axiom of congruency)

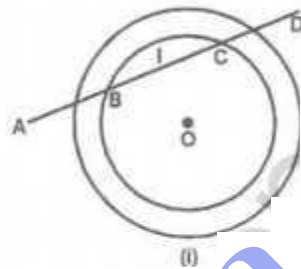
$$\therefore PB = QC \quad (C.P.C.T)$$

19. (a) In the fig(i) given below, a line l intersects two concentric circles at the points A, B, C and D.

prove that $AB = CD$.

(b) In the fig(ii) given below, chords AB and CD of a circle with centre O intersect at E. If OE bisects $\angle AED$, prove that $AB = CD$.

Sol.



(a) Given: A line l intersects two circles with centre O

To prove: $AB = CD$

Construction: Draw $OM \perp l$

proof: $OM \perp BC$ — (i)

Again $OM \perp AD$. $\therefore AM = MD$ — (ii)

Subtracting (i) from (ii)

$$AM - BM = MD - MC \Rightarrow AB = CD.$$

(b) Given: Two chords AB and CD intersect each other at E inside the circle with centre O.

OE bisects $\angle AED$ i.e., $\angle OEA = \angle OED$

To prove: $AB = CD$.

Construction : From O, draw $OM \perp AB$ and $ON \perp CD$.

Proof : In $\triangle OME$ and $\triangle ONE$

$$\angle M = \angle N \text{ (each } 90^\circ) \quad \cdot \quad OE = OE \text{ (common)}$$

$$\angle OEM = \angle OEN \text{ (given)}$$

$$\therefore \triangle OME \cong \triangle ONE \text{ (ASS axiom of Congruency)}$$

$$\therefore OM = ON.$$

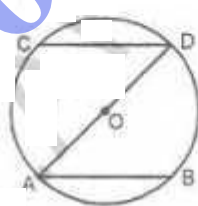
$\therefore AB = CD$ (chords which are equidistant from the centre are equal)

20. (a) In the fig (i) given below, AD is a diameter of a circle with centre O. If $AB \parallel CD$, prove that $AB = CD$.

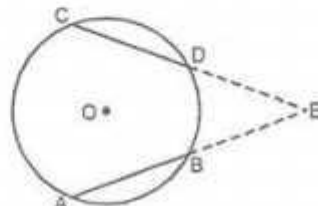
(b) In the fig (ii) given below, AB and CD are equal chords with centre O.

If AB and CD meet at E (outside the circle).

prove that : (i) $AE = CE$ (ii) $BE = DE$.

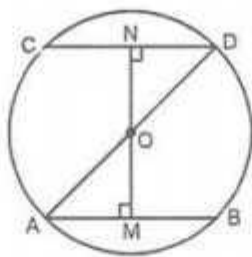


(i)



(ii)

Sol.



(a) Given: AD is the diameter of a circle with centre O and chords AB and CD are parallel.

To prove: $AB = CD$.

Construction: From O, draw $OM \perp AB$ and $ON \perp CD$.

proof: In $\triangle OMA$ and $\triangle OND$

$\angle AOM = \angle DON$ (Vertically opposite angles)

$OA = OD$ (radii of the same circle)

and $\angle M = \angle N$ (each 90°)

$\therefore \triangle OMA \cong \triangle OND$ (AAS axiom of congruency)

$\therefore OM = ON$ (C.P.C.T)

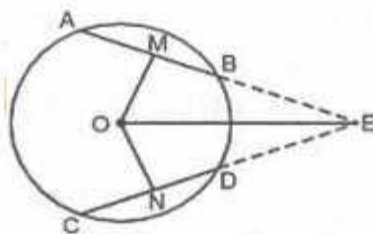
But $OM \perp AB$ and $ON \perp CD$.

$\therefore AB = CD$ (chords which are equidistant from the centre are equal).

(b) Given: chord $AB =$ chord CD of circle with centre O and meet at E on producing them.

To prove: (i) $AE = CE$

(ii) $BE = DE$.



Construction: From O, draw $OM \perp AB$
and $ON \perp CD$. Join OE.

In right $\triangle OME$ and $\triangle ONE$.

Hypotenuse $OE = OE$ (Common)

Side $OM = ON$.

(Equal chords are equidistant from the centre)

$\therefore \triangle OME \cong \triangle ONE$ (RHS axiom of Congruency)

$\therefore ME = NE$ (C.P.C.T) — (i)

$\therefore OM \perp AB$ and $ON \perp CD$.

$\therefore M$ is the midpoint of AB and N is a midpoint of CD .

$\therefore MB = \frac{1}{2} AB$ and $ND = \frac{1}{2} CD$.

But $AB = CD$ — (ii)

$MB = ND$

Subtracting (i) from (ii), $ME - MB = NE - ND$

$\Rightarrow BE = DE$ but $AB = CD$ (given)

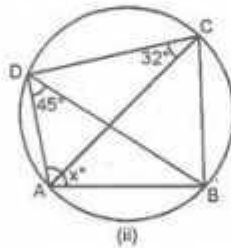
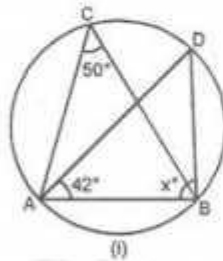
Adding, we get

$AB + BE = CD + DE$

$\Rightarrow AE = CE$ (Hence proved)

EXERCISE - 16.2

Q1. Using the given information, find the value of x in each of the following figures:



Sol. (i) $\angle ADB$ and $\angle ACB$ are in the same segment

$$\therefore \angle ADB = \angle ACB = 50^\circ$$

Now in $\triangle ADB$, $\angle DAB + \angle ABD + \angle ADB = 180^\circ$ (Angles of a triangle)

$$\Rightarrow 42^\circ + x + 50^\circ = 180^\circ \Rightarrow x = 88^\circ$$

(ii) $\angle ABD = \angle ACD$ (Angles in the same segment)

$$\text{But } \angle ACD = 32^\circ \therefore \angle ABD = 32^\circ$$

Now in $\triangle ABD$, $\angle ABD + \angle ADB + \angle DAB = 180^\circ$ (Angles in a triangle)

$$\Rightarrow 32^\circ + 45^\circ + x = 180^\circ \Rightarrow x = 103^\circ$$

(iii) $\angle BAD = \angle BCD$ (Angles in the same segment)

$$\text{But } \angle BAD = 20^\circ \therefore \angle BCD = 20^\circ$$

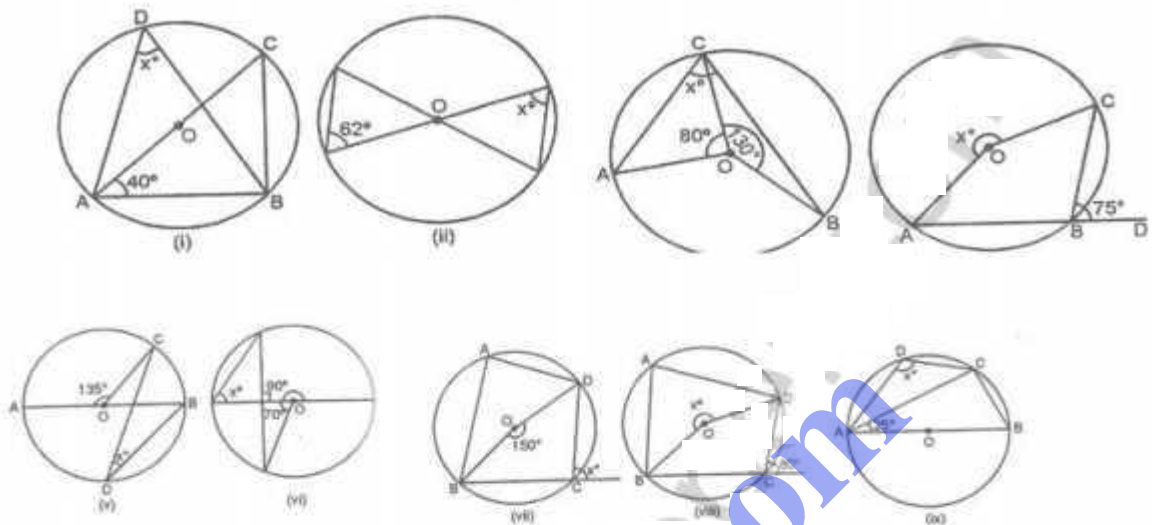
$$\Rightarrow \therefore \angle CEA = 90^\circ, \angle CED = 90^\circ$$

Now in $\triangle CED$, $\angle CED + \angle BCD + \angle CDE = 180^\circ$

$$\Rightarrow 90^\circ + 20^\circ + x = 180^\circ$$

$$\Rightarrow x = 70^\circ$$

Q2. If O is the centre of the circle, find the value of x in each of the following figures (using the given information):



Sol. (i) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)

$$\text{But } \angle ADB = x^\circ \Rightarrow \angle ACB = x^\circ$$

Now in $\triangle ABC$, $\angle CAB + \angle ABC + \angle ACB = 180^\circ$

$$\Rightarrow 40^\circ + 90^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 50^\circ$$

(ii) $\angle ACD = \angle ABD$ $\therefore \angle ACB = x^\circ \Rightarrow \angle ABD = x^\circ$

Now in $\triangle OAC$, $OA = OC$ (radii of the same circle)

$\therefore \angle ACO = \angle OAC$ (Opp. angles of equal sides)

$$\therefore x^\circ = 62^\circ$$

(iii) $\angle AOB + \angle AOC + \angle BOC = 360^\circ$ (angle at a point)

$$\Rightarrow \angle AOB + 80^\circ + 130^\circ = 360^\circ$$

$$\Rightarrow \angle AOB = 150^\circ$$

Now AB subtends $\angle AOB$ at the centre $\angle ACB$ at the remaining part of the circle.

$$\therefore \angle AOB = 2 \angle ACB$$

$$\Rightarrow \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 150^\circ = 75^\circ$$

$$(iv) \angle ABC + \angle CBD = 180^\circ \text{ (linear pair)}$$

$$\Rightarrow \angle ABC + 75^\circ = 180^\circ \Rightarrow \angle ABC = 105^\circ$$

Now Arc AC subtends reflex $\angle AOC$ at the Centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \text{Reflex } \angle AOC = 2 \angle ABC = 2 \times 105^\circ = 210^\circ$$

$$(v) \angle AOC + \angle COB = 180^\circ \text{ (linear pair)}$$

$$\Rightarrow 135^\circ + \angle COB = 180^\circ \Rightarrow \angle COB = 45^\circ$$

Now arc BC subtends $\angle COB$ at the Centre and $\angle CDB$ at the remaining part of the circle.

$$\therefore \angle COB = 2 \angle CDB \Rightarrow \angle CDB = \frac{1}{2} \angle COB = \frac{1}{2} \times 45^\circ = 22\frac{1}{2}^\circ$$

(vi) Arc AD subtends $\angle AOD$ at the Centre and $\angle ACD$ at the remaining part of the circle.

$$\therefore \angle AOD = 2 \angle ACB \Rightarrow \angle ACB = \frac{1}{2} \angle AOD = \frac{1}{2} \times 70^\circ = 35^\circ$$

$$\therefore \angle CMO = 90^\circ \Rightarrow \therefore \angle AMC = 90^\circ \text{ (} \angle AMC + \angle CMO = 180^\circ \text{)}$$

Now in $\triangle ACM$, $\angle ACM + \angle AMC + \angle CAM = 180^\circ$ (Angles in a triangle)

$$\Rightarrow 35^\circ + 90^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 55^\circ$$

(vii) ABCD is a cyclic quadrilateral, Ext. $\angle DCE = \angle BAD$

$$\therefore \angle BAD = x^\circ$$

Now arc BD subtends $\angle BOD$ at the Centre and $\angle BAD$ at the remaining part of the circle.

$$\therefore \angle BOD = 2 \angle BAD = 2x \Rightarrow 2x = 150^\circ \Rightarrow x = 75^\circ$$

$$(viii) \angle BCD + \angle DCE = 180^\circ \Rightarrow \angle BCD + 80^\circ = 180^\circ \Rightarrow \angle BCD = 100^\circ$$

Arc BAD subtends reflex $\angle BOD$ at the Centre and $\angle BCD$ at the remaining part of the circle.

$$\therefore \text{Reflex } \angle BOD = 2 \angle BCD \Rightarrow x^\circ = 2 \times 100^\circ = 200^\circ$$

$$(ix) \text{ In } \triangle ACB, \angle CAB + \angle ABC + \angle ACB = 180^\circ$$

$$\text{But } \angle ACB = 90^\circ$$

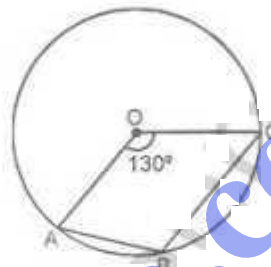
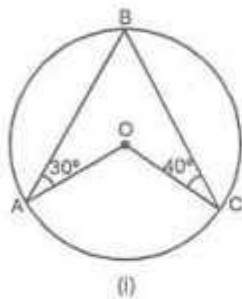
$$\therefore 25^\circ + 90^\circ + \angle ABC = 180^\circ \Rightarrow \angle ABC = 65^\circ$$

$\therefore ABCD$ is a cyclic quadrilateral.

$\angle ABC + \angle ADC = 180^\circ$ (opp. angles of a cyclic quadrilateral)

$$\Rightarrow 65^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 115^\circ$$

3. (a) In the fig(i) given below, O is the Centre of the circle. $\angle OAB$ and $\angle OCB$ are 30° and 40° respectively. find $\angle AOC$.
- (b) In the fig(ii) given below, it is given that O is the Centre of the circle and $\angle AOC = 130^\circ$, find $\angle ABC$.



Sol. (a) Join OB .

In $\triangle OAB$, $OA = OB$ (radii of the same circle)

$$\therefore \angle OBA = \angle OAB = 30^\circ \quad \text{--- (i)}$$

Similarly in $\triangle OBC$, $OC = OB$ --- (ii)

$$\therefore \angle OBC = \angle OCB = 40^\circ$$

Adding (i) and (ii), $\angle OBA + \angle OBC = 30^\circ + 40^\circ$

$$\Rightarrow \angle ABC = 70^\circ$$

Now arc AC subtends $\angle AOC$ at the Centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \angle AOC = 2 \angle ABC = 2 \times 70^\circ = 140^\circ$$

(b) $\angle AOC + \text{reflex } \angle AOC = 360^\circ$

$$\Rightarrow 130^\circ + \text{reflex } \angle AOC = 360^\circ$$

$$\Rightarrow \text{reflex } \angle AOC = 230^\circ$$

Now arc AC subtends reflex $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \text{Reflex } \angle AOC = 2\angle ABC \Rightarrow \angle ABC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 230^\circ = 115^\circ$$

84. (a) In the fig(i) given below, AC is a diameter of the given circle and $\angle BCD = 75^\circ$. Calculate the size of (i) $\angle ABC$ (ii) $\angle EAF$

(b) In the fig(ii) given below, O is the centre of the circle.

If $\angle AOB = 140^\circ$ and $\angle OAC = 50^\circ$, find: (i) $\angle ACB$ (ii) $\angle OBC$

(iii) $\angle OAB$ (iv) $\angle CBA$

Sol.

(a) (i) AC is the diameter of the circle.

$$\therefore \angle ABC = 90^\circ \quad (\text{Angles in a semi circle})$$

(ii) ABCD is a cyclic quadrilateral

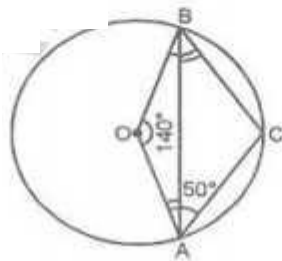
$$\angle BAD + \angle BCD = 180^\circ \Rightarrow \angle BAD + 75^\circ = 180^\circ \Rightarrow \angle BAD = 105^\circ$$

$$\text{But } \angle EAF = 105^\circ$$

(b) (i) $\angle AOB + \text{reflex } \angle AOB = 360^\circ$ (angle at a point)

$$\Rightarrow 140^\circ + \text{reflex } \angle AOB = 360^\circ$$

$$\Rightarrow \text{reflex } \angle AOB = 220^\circ$$



Now major arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ is at the remaining part of the circle.

$$\text{Reflex } \angle AOB = 2 \angle ACB \Rightarrow \angle ACB = \frac{1}{2} \text{ Reflex } \angle AOB.$$

$$\Rightarrow \angle ACB = \frac{1}{2} \times 220^\circ = 110^\circ$$

$$(ii) \text{ In quad } OACB, \angle OAC + \angle ACB + \angle AOB + \angle OBC = 360^\circ$$

$$\Rightarrow 50^\circ + 110^\circ + 140^\circ + \angle OBC = 360^\circ \Rightarrow \angle OBC = 60^\circ$$

$$(iii) \text{ In } \triangle OAB, OA = OB \text{ (radii of the same circle)}$$

$$\therefore \angle OAB = \angle OBA$$

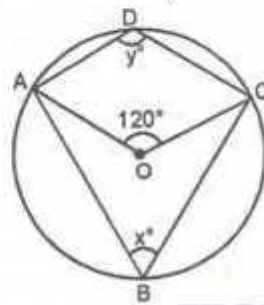
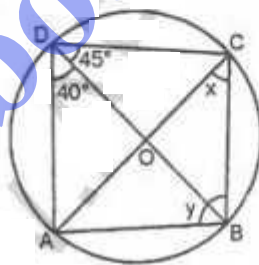
$$\text{But } \angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\Rightarrow \angle OAB + \angle OAB + 140^\circ = 180^\circ \Rightarrow 2 \angle OAB = 40^\circ \Rightarrow \angle OAB = 20^\circ$$

$$(iv) \text{ But } \angle OBC = 60^\circ$$

$$\therefore \angle CBA = \angle OBC - \angle OBA = 60^\circ - 20^\circ = 40^\circ$$

- 5 (a) In the fig(i) given below, calculate the value of x and y .
 (b) In the fig(ii) given below, O is the Centre of the circle. Calculate the values of x and y .



Sol. (a) ABCD is a cyclic quadrilateral

$$\therefore \angle B + \angle D = 180^\circ \Rightarrow y + 40^\circ + 45^\circ = 180^\circ \Rightarrow y = 95^\circ$$

$$\therefore \angle ACB = \angle ADB \quad (\text{Angles in the same segment})$$

$$\therefore x^\circ = 40^\circ$$

(b) Arc ADC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \angle AOC = 2\angle ABC \Rightarrow \angle ABC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 120^\circ = 60^\circ$$

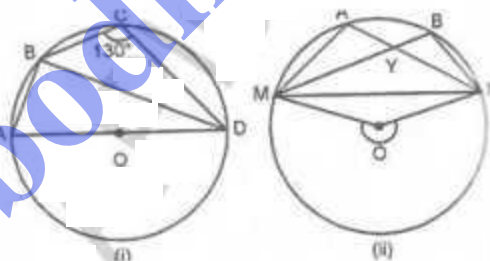
$$\therefore x^\circ = 60^\circ$$

Again ABCD is a cyclic quadrilateral

$$\therefore \angle B + \angle D = 180^\circ \Rightarrow 60^\circ + y^\circ = 180^\circ \Rightarrow y^\circ = 120^\circ$$

6. (a) In the fig (i) given below, if $\angle BCD = 130^\circ$ and AD is a diameter of the circle, calculate (i) $\angle DAB$ (ii) $\angle ADB$.

(b) In the fig (ii) given below, MABN are points on a circle O. AN and MB cut at Y if $\angle NYB = 50^\circ$ and $\angle YNB = 20^\circ$, find $\angle MAN$ and the reflex angle MON.



Sol. (a) AD is the diameter of the circle and $\angle BCD = 130^\circ$

(i) ABCD is a cyclic quadrilateral

$$\therefore \angle DAB + \angle BCD = 180^\circ$$

$$\Rightarrow \angle DAB + 130^\circ = 180^\circ$$

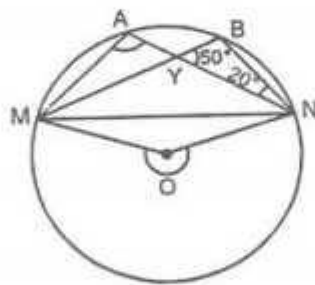
$$\Rightarrow \angle DAB = 50^\circ$$

(ii) In $\triangle ABD \Rightarrow \angle ABD = 90^\circ$ (angles in a semicircle)

and $\angle DAB + \angle ABD + \angle ADB = 180^\circ$ (Angles in a triangle)

$$\Rightarrow 50^\circ + 90^\circ + \angle ADB = 180^\circ \Rightarrow \angle ADB = 40^\circ$$

(b) $\angle NYB = 50^\circ$, $\angle YNB = 20^\circ$



In $\triangle YNB$, $\angle NYB + \angle YNB + \angle YBN = 180^\circ$

$$\Rightarrow 50^\circ + 20^\circ + \angle YBN = 180^\circ$$

$$\Rightarrow \angle YBN = 110^\circ$$

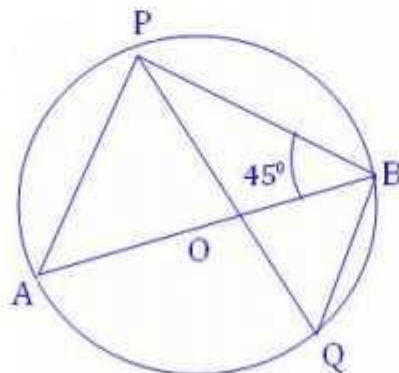
But $\angle MAN = \angle YBN$ (angles in the same segment)

$$\therefore \angle MAN = 110^\circ$$

Major arc MN subtends reflex $\angle MON$ at the centre and $\angle MAN$ at the remaining part of the circle.

$$\therefore \text{Reflex } \angle MON = 2 \angle MAN = 2 \times 110^\circ = 220^\circ$$

7. In the given fig, O is the centre of the circle and $\angle PBA = 45^\circ$. Calculate the value of $\angle PSB$.



Sol. Since angle in a semi circle is a right angle.

$$\therefore \angle APB = 90^\circ$$

In $\triangle APB$ have, $\angle PAB + \angle PBA + \angle APB = 180^\circ$

$$\Rightarrow \angle PAB + 45^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow \angle PAB = 45^\circ$$

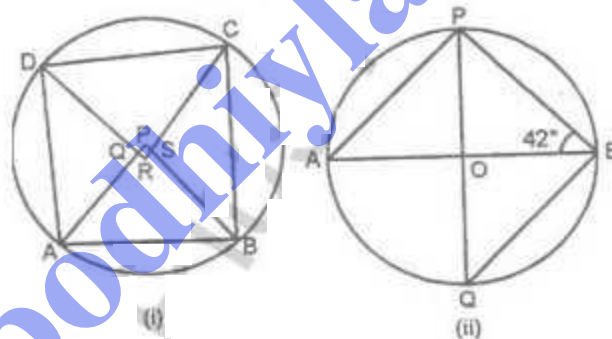
Consider arc BP we find that $\angle PAB$ and $\angle PQB$ are angle in the same segment of a circle.

$$\therefore \angle PQB = \angle PAB \Rightarrow \angle PQB = 45^\circ$$

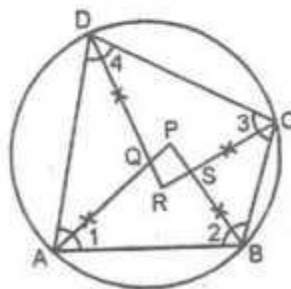
Q8. (a) In the fig(i) given below, ABCD is a cyclic quadrilateral. prove that the quadrilateral PQRS formed by angle bisectors of ABCD is also a cyclic.

(b) In the fig(ii) given below, O is the centre and $\angle PBA = 42^\circ$. Calculate the value of $\angle PQB$.

Sol.



Given: ABCD is a cyclic quadrilateral and quad PQRS is a formed by the angle bisectors of ABCD.



To prove: PQRS is a cyclic.

proof: $\because$ AP is the bisector of $\angle A$

$$\therefore \angle 1 = \frac{1}{2} \angle A \quad \text{similarly, } \angle 2 = \frac{1}{2} \angle B$$

$$\Rightarrow \angle 3 = \frac{1}{2} \angle C \quad \text{and } \angle 4 = \frac{1}{2} \angle D$$

Now in $\triangle APB$ and $\triangle CRD$.

$$\angle 1 + \angle 2 + \angle P + \angle 3 + \angle 4 + \angle R = 180^\circ + 180^\circ$$

$$\Rightarrow \angle P + \angle R + \frac{1}{2} \angle A + \frac{1}{2} \angle B + \frac{1}{2} \angle C + \frac{1}{2} \angle D = 360^\circ$$

$$\Rightarrow \angle P + \angle R + \frac{1}{2} [\angle A + \angle B + \angle C + \angle D] = 360^\circ$$

$$\Rightarrow \angle P + \angle R + \frac{1}{2} \times 360^\circ = 360^\circ$$

$$\Rightarrow \angle P + \angle R = 180^\circ$$

But these are the opp. angles of the quadrilateral PQRS.

$\therefore$ PQRS is a quadrilateral

(b) In $\triangle APB$, $\angle APB = 90^\circ$ (angle of a semicircle)

But $\angle A + \angle APB + \angle ABP = 180^\circ$ (angle of a triangle)

$$\Rightarrow \angle A + 90^\circ + 42^\circ = 180^\circ \Rightarrow \angle A = 48^\circ$$

$$\therefore \text{But } \angle A = \angle PQB = 48^\circ$$

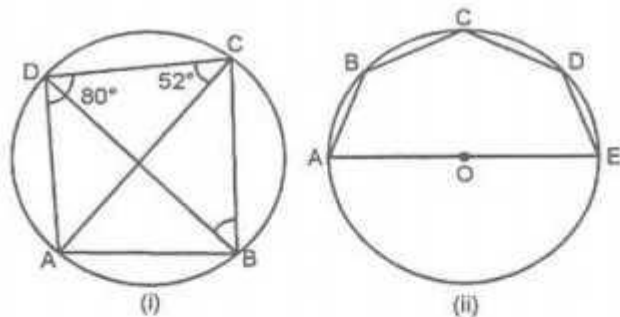
Q9.

(a) In the fig(i) given below, ABCD is a cyclic quadrilateral.

If $\angle ADC = 80^\circ$ and $\angle ACD = 52^\circ$, find the values of $\angle CBD$ and $\angle ABC$.

(b) In the fig(ii) given below, AE is diameter of the circle.

Find $\angle ABC + \angle CDE$. Give reason for your answer.



Sol. (a) In $\triangle ADC$, $\angle ADC + \angle CAD + \angle ACD = 180^\circ$ (angles of a triangle)

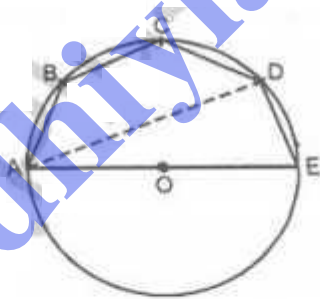
$$\Rightarrow 80^\circ + 52^\circ + \angle CAD = 180^\circ \Rightarrow \angle CAD = 48^\circ$$

$$\therefore \angle CAD = \angle CBD = 48^\circ$$

ABCD is a cyclic quadrilateral.

$$\therefore \angle ADC + \angle ABC = 180^\circ \Rightarrow 80^\circ + \angle ABC = 180^\circ \Rightarrow \angle ABC = 100^\circ$$

(b) AE is the diameter of the circle with centre O. Join AD.



AE is the diameter of the circle.

$$\therefore \angle ADE = 90^\circ \text{ (angle in a semi-circle)} \quad \text{--- (i)}$$

$$\therefore \text{ABCD is a cyclic quadrilateral} \therefore \angle ABC + \angle ADC = 180^\circ \quad \text{--- (ii)}$$

Adding (i) and (ii)

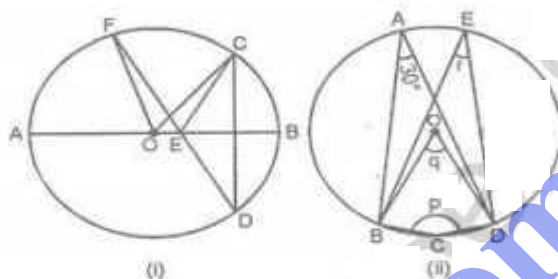
$$\angle ABC + \angle ADC + \angle ADE = 180^\circ + 90^\circ$$

$$\Rightarrow \angle ABC + \angle CDE = 270^\circ$$

Q10 (a) In the fig.(i) given below, AB is a diameter of the circle whose Centre is O. Given that $\angle ECD = \angle EDC = 32^\circ$, Calculate:

(i) $\angle CEF$ (ii) $\angle COF$

(b) In the fig.(ii) given below, O is the Centre of the Circle. If $\angle BAD = 30^\circ$, find the value of P, q and r.

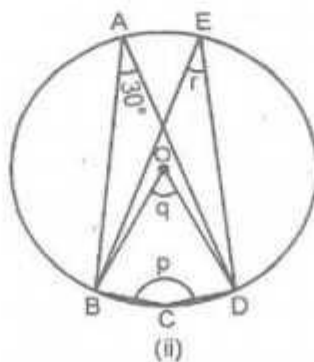


Sol (a) (i) In $\triangle EDC$, Ext. $\angle CEF = \angle ECD + \angle EDC = 32^\circ + 32^\circ = 64^\circ$

(ii) arc CF subtends $\angle COF$ at the Centre and $\angle CDF$ at the remaining part of the circle.

$$\therefore \angle COF = 2\angle CDF = 2\angle CDE = 2 \times 32^\circ = 64^\circ$$

(b) (i) ABCD is a cyclic quadrilateral.



$$\therefore \angle A + \angle C = 180^\circ \Rightarrow 30^\circ + \hat{P} = 180^\circ \Rightarrow \hat{P} = 150^\circ$$

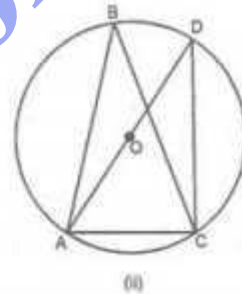
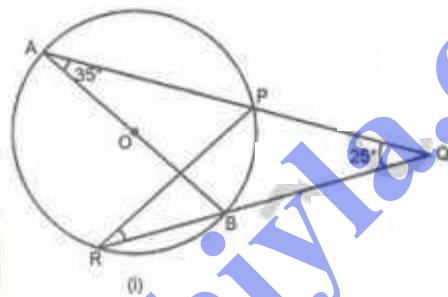
(ii) Arc BD subtends $\angle BOD$ at the Centre. and $\angle BAD$ at the remaining part of the Circle.

$$\therefore \angle BOD = 2 \angle BAD \Rightarrow \hat{r} = 2 \times 30^\circ = 60^\circ$$

(iii) $\angle BAD$ and $\angle BED$ are in the same segment of a circle.

$$\angle BAD = \angle BED \Rightarrow 30^\circ = \hat{r} \Rightarrow \hat{r} = 30^\circ$$

Q11. (a) In the fig.(i) given below, AB is a diameter of the Circle APBR. APQ and RBQ are straight lines. $\angle A = 35^\circ$, $\angle Q = 25^\circ$. find (i) $\angle PRB$ (ii) $\angle PBR$ (iii) $\angle BPR$



(b) In the fig.(ii) given below, it is given that $\angle ABC = 40^\circ$ and AD is a diameter of the circle. calculate $\angle DAC$.

sol. (a) (i) $\angle PRB = \angle BAP$

$$\therefore \angle PRB = 35^\circ \quad (\because \angle PAB = 35^\circ \text{ given})$$

$$(ii) \text{ In } \triangle PRQ, \text{ Ext } \angle APR = \angle PRQ + \angle PQR = \angle PRB + \angle Q \\ = 35^\circ + 25^\circ = 60^\circ$$

But $\angle APB = 90^\circ$ (angle in a semicircle)

$$\therefore \angle BPR = \angle APB - \angle APR = 90^\circ - 60^\circ = 30^\circ$$

$$(iii) \angle APR = \angle ABR \Rightarrow 60^\circ = \angle ABR$$

In $\triangle PBQ$, Ext. $\angle PBR = \angle Q + \angle BPQ = 25^\circ + 90^\circ = 115^\circ$

(b) $\angle B = \angle D \quad \therefore \angle D = 40^\circ$

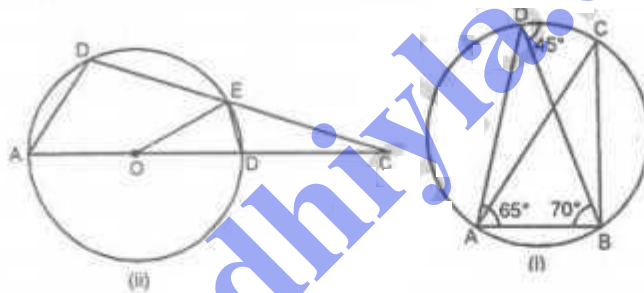
$\angle ACD = 90^\circ$ (angle in the semicircle)

Now in $\triangle ADC$, $\angle ACD + \angle D + \angle DAC = 180^\circ$ (Angles in a $\triangle$),

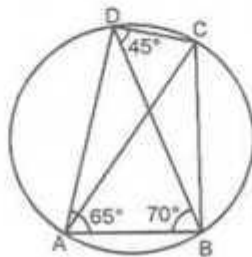
$\Rightarrow 90^\circ + 40^\circ + \angle DAC = 180^\circ \Rightarrow \angle DAC = 50^\circ$

Q12. (a) In the fig(i) given below, if $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$ and $\angle BDC = 45^\circ$ calculate (i) $\angle BCD$ (ii) $\angle ADB$ and show that AC is a diameter.

(b) In the fig.(ii) given below, O is the centre of the circle. $\angle AOE = 150^\circ$, $\angle DAO = 51^\circ$. Calculate the sizes of the angles CEB and CBE.



Sol. (a) In $\triangle ABD$, $\angle ABD + \angle BAD + \angle ADB = 180^\circ$ (angles of a $\triangle$)



$$\Rightarrow 70^\circ + 65^\circ + \angle ADB = 180^\circ \Rightarrow \angle ADB = 45^\circ$$

$$\therefore \angle ADB = \angle ACB = 45^\circ$$

$$\text{Similarly, } \angle ACD = \angle ABD = 70^\circ$$

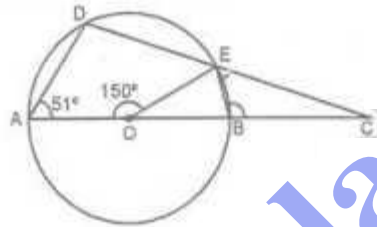
$$\therefore \angle BCD = \angle ACB + \angle ACD = 45^\circ + 70^\circ = 115^\circ$$

(b) (i) ABED is cyclic quadrilateral

$$\therefore \text{Ext. } \angle CEB = \angle A = 51^\circ$$

(ii) In $\triangle OBE$, $\text{Ext. } \angle AOE = \angle OEB + \angle OBE$

$$\text{But } \angle OEB = \angle OBE \quad (\because OE = OB \text{ radii of the same circle})$$



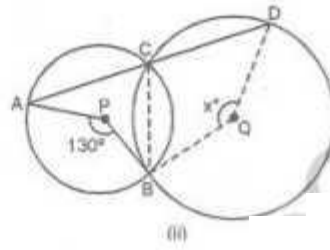
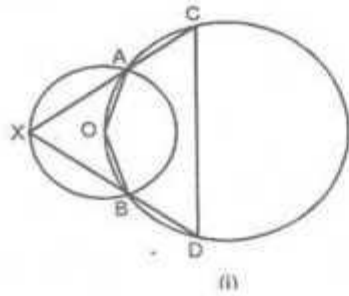
$$\therefore \angle OBE = \frac{1}{2} \angle AOE = \frac{1}{2} \times 150^\circ = 75^\circ$$

$$\text{But } \angle OBE + \angle CBE = 180^\circ \quad (\text{Linear pair})$$

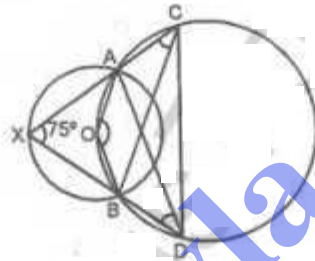
$$75^\circ + \angle CBE = 180^\circ \Rightarrow \angle CBE = 105^\circ$$

- Q13. (a) The fig(i) given below shows two circles, which intersect at A and B. The centre of the smaller circle O and lies on the circumference of the larger circle. XAC and XBD are st. lines. Given $\angle AXB = 75^\circ$, calculate the value of (i) obtuse $\angle AOB$ (ii) $\angle ACB$ (iii) $\angle ADB$. Give clear reason for your answer.

- (b) In the fig(ii) given below, P and Q are centres of two circles intersecting at B and C. ACD is a st. line. Calculate the numerical value of α .



sol. (a) (i) arc AB subtends $\angle AOB$ at the Centre and $\angle AXB$ at the remaining part of the circle.



$$\therefore \angle AOB = 2 \angle AXB = 2 \times 75^\circ = 150^\circ$$

(ii) In cyclic quadrilateral AOB D

$$\angle AOB + \angle ADB = 180^\circ \Rightarrow 150^\circ + \angle ADB = 180^\circ$$

$$\Rightarrow \angle ADB = 30^\circ$$

Again in cyclic quadrilateral AOB C,

$$\angle AOB + \angle ACB = 180^\circ$$

$$\Rightarrow 150^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 30^\circ$$

(b) arc AB subtends $\angle APB$ at the Centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore \angle ACB = \frac{1}{2} \angle APB = \frac{1}{2} \times 130^\circ = 65^\circ$$

But $\angle ACB + \angle BCD = 180^\circ$ (Linear pair)

$$\Rightarrow 65^\circ + \angle BCD = 180^\circ \Rightarrow \angle BCD = 115^\circ$$

Major arc BD subtends reflex $\angle BOD$ at the Centre and $\angle BCD$ at the remaining part of the circle.

$$\therefore \text{Reflex } \angle BOD = 2 \angle BCD = 2 \times 115^\circ = 230^\circ$$

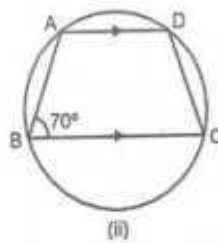
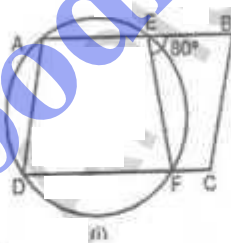
But reflex $\angle BOD + x = 360^\circ$ (Angles at a point)

$$\therefore 230^\circ + x = 360^\circ \Rightarrow x = 130^\circ$$

Q14 (a) In the fig(i) given below, ABCD is a parallelogram. A circle passes through A and D and cuts AB to E and DC at F.

Given that $\angle BEF = 80^\circ$, find $\angle ABC$.

(b) In the fig(ii) given below, ABCD is a cyclic trapezium in which AD is parallel to BC and $\angle B = 70^\circ$, find: (i) $\angle BAD$ (ii) $\angle BCD$



Sol. (a) APFE is a cyclic quadrilateral.

$$\therefore \text{Ext. } \angle FEB = \angle ADF = 80^\circ$$

$\therefore$ ABCD is a parallelogram.

$$\therefore \angle B = \angle D = \angle ADF = 80^\circ \text{ (or) } \angle ABC = 80^\circ$$

(b) In trapezium ABCD, $AD \parallel BC$

$$(i) \quad \angle B + \angle A = 180^\circ \Rightarrow 70^\circ + \angle A = 180^\circ \Rightarrow \angle A = 110^\circ$$

$$\therefore \angle BAD = 110^\circ$$

(ii) ABCD is a cyclic quadrilateral

$$\therefore \angle A + \angle C = 180^\circ$$

$$\Rightarrow 110^\circ + \angle C = 180^\circ$$

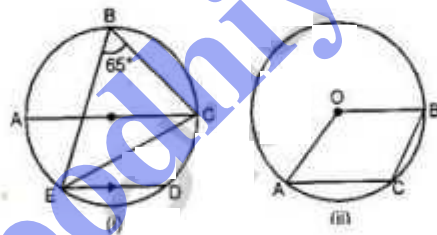
$$\Rightarrow \angle C = 70^\circ$$

$$\therefore \angle BCD = 70^\circ$$

Q15. (a) In the fig(i) given below, chord ED is parallel to the diameter AC of the circle given $\angle CBE = 65^\circ$, calculate $\angle DEC$.

(b) In the fig(ii) given below, C is a point on the minor arc AB of the circle with centre O. given $\angle ACB = p^\circ$, $\angle AOB = q^\circ$. Express q in terms of p . Calculate p if OACB is a parallelogram.

Sol.



(a) $\angle CBE = \angle CAE$ (Angle in the same segment)

$\therefore \angle CAE = 65^\circ$, $\angle AEC = 90^\circ$ (Angle in a semi-circle)

Now $\triangle AEC$,

$$\angle AEC + \angle CAE + \angle ACE = 180^\circ$$

$$\Rightarrow 90^\circ + 65^\circ + \angle ACE = 180^\circ$$

$$\Rightarrow \angle ACE = 25^\circ$$

$\therefore AC \parallel ED$ (given)

$\therefore \angle ACE = \angle DEC$ (alternate angles)

$$\therefore \angle DEC = 25^\circ$$

(b) Major arc AB subtends reflex $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore \text{reflex } \angle AOB = 2\angle ACB = 2p \quad \text{--- (i)}$$

$$\text{But reflex } \angle AOB + \gamma = 360^\circ$$

$$\Rightarrow \text{Reflex } \angle AOB = 360^\circ - \gamma \quad \text{--- (ii)}$$

from (i) and (ii),

$$2p = 360^\circ - \gamma \Rightarrow \gamma = 2(180^\circ - p)$$

If OACB is a parallelogram, then $p = \gamma$

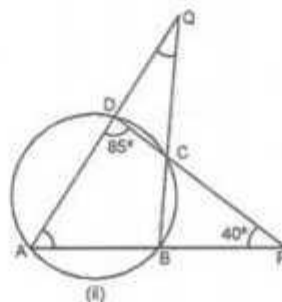
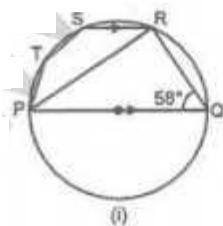
$$\Rightarrow p = 360 - 2p \Rightarrow 3p = 360 \Rightarrow p = 120^\circ$$

$$\therefore \gamma = p = 120^\circ$$

16. (a) In the fig(i) given below, PQ is a diameter chord SR is parallel to PQ. Given $\angle PQR = 58^\circ$, calculate (i) $\angle RPQ$ (ii) $\angle STP$

(T is a point on the minor arc SP)

(b) In the given fig(ii), if $\angle ACE = 43^\circ$ and $\angle CAF = 62^\circ$ find the value of a, b and c.



sol. (a) In $\triangle PQR$, $\angle PRQ = 90^\circ$ (Angle in a Semi-circle)
and $\angle PQR = 58^\circ$

$$\therefore \angle RPQ = 90^\circ - \angle PQR = 90^\circ - 58^\circ = 32^\circ$$

$$\therefore SR \parallel PQ \text{ (given)} \quad \therefore \angle SRP = \angle RPQ = 32^\circ$$

Now $PRST$ is a cyclic quadrilateral.

$$\therefore \angle STP + \angle SRP = 180^\circ \Rightarrow \angle STP + 32^\circ = 180^\circ$$

$$\Rightarrow \angle STP = 148^\circ$$

(b) In $\triangle ADP$, $\angle ADP + \angle BPC + \angle BAD = 180^\circ$

$$\Rightarrow 85^\circ + 40^\circ + \angle BAD = 180^\circ \Rightarrow \angle BAD = 55^\circ$$

$ABCD$ is a cyclic quadrilateral.

$$\angle ABC + \angle ADC = 180^\circ \Rightarrow \angle ABC + 85^\circ = 180^\circ \Rightarrow \angle ABC = 95^\circ$$

Now in $\triangle ABQ$, $\angle A + \angle B + \angle Q = 180^\circ$

$$\Rightarrow \angle BAD + \angle ABC + \angle Q = 180^\circ$$

$$\Rightarrow 55^\circ + 95^\circ + \angle Q = 180^\circ \Rightarrow \angle Q = 30^\circ$$

In $\triangle AEC$ we have $\angle CAE + \angle AEC + \angle ECA = 180^\circ$

$$\Rightarrow 62^\circ + \angle AEC + 43^\circ = 180^\circ \Rightarrow \angle AEC = 75^\circ$$

In the cyclic quadrilateral $AEDB$ we have,

$$a + \angle AED = 180^\circ \Rightarrow a = 180^\circ - \angle AED = 180^\circ - 75^\circ = 105^\circ$$

$$\text{and } 62^\circ + \angle EDB = 180^\circ \Rightarrow \angle EDB = 180^\circ - 62^\circ = 118^\circ$$

$$\text{also } \angle EDB + c = 180^\circ \Rightarrow 118^\circ + c = 180^\circ \Rightarrow c = 62^\circ$$

In $\triangle ABF$ we have,

$$\angle FBA + \angle BAF + \angle AFB = 180^\circ \text{ (Sum of angles of a } \triangle = 180^\circ)$$

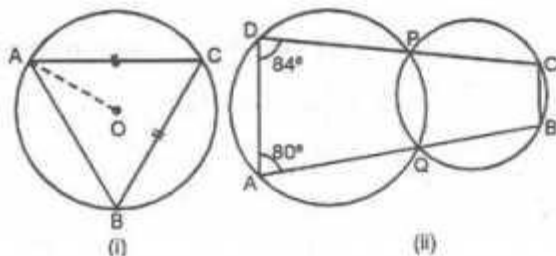
$$\Rightarrow a + 62^\circ + b = 180^\circ$$

$$\Rightarrow b = 180^\circ - 62^\circ - a = 118^\circ - 105^\circ = 13^\circ$$

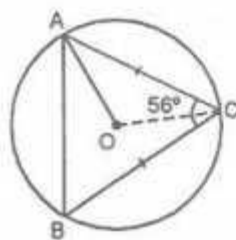
$$\text{Hence } a = 105^\circ, b = 13^\circ, c = 62^\circ$$

Q17. (a) In the fig(i) given below, O is the circumcentre of a ΔABC in which $AC = BC$. Given that $\angle ACB = 56^\circ$, calculate (i) $\angle CAB$ (ii) $\angle OAC$

(b) In the fig(ii) given below, two circles intersect at point P and Q. If $\angle A = 80^\circ$ and $\angle D = 84^\circ$, calculate (i) $\angle QBC$ (ii) $\angle BCP$



Sol.



(a) Join OC.

In ΔABC , $AC = BC \therefore \angle A = \angle B$

But $\angle A + \angle B + \angle C = 180^\circ \Rightarrow \angle A + \angle A + 56^\circ = 180^\circ$

$\Rightarrow 2\angle A = 124^\circ \Rightarrow \angle A = 62^\circ$ (or) $\angle CAB = 62^\circ$

$\therefore OC$ is the radius of the circle.

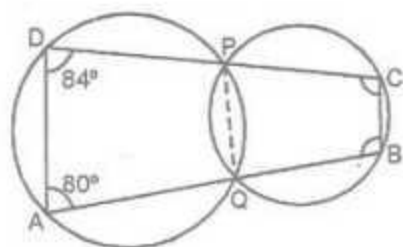
OC bisects $\angle ACB$

$\therefore \angle OCA = \frac{1}{2} \angle ACB = \frac{1}{2} \times 56^\circ = 28^\circ$

Now in ΔOAC , $OA = OC$ (radii of same circle)

$\therefore \angle OAC = \angle OCA = 28^\circ$

(b) join PQ



$\triangle ASPD$ is a cyclic quadrilateral.

$$\angle A + \angle QPD = 180^\circ \Rightarrow \angle QPD = 100^\circ$$

$$\text{and } \angle D + \angle AQP \Rightarrow 84^\circ + \angle AQP = 180^\circ \Rightarrow \angle AQP = 96^\circ$$

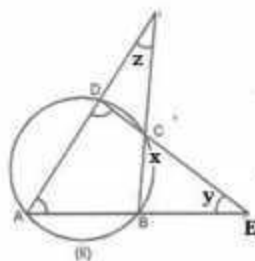
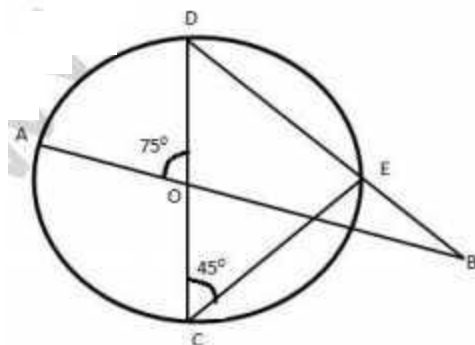
Now $PQBC$ is a cyclic quadrilateral

$$\text{Ext } \angle QPD = \angle QBC = 100^\circ$$

$$\text{and Ext. } \angle AQP = \angle BCP = 96^\circ$$

Q18 (a) In the fig(i) given below, st. lines AB and CD pass through the Centre O of a circle. If $\angle OCE = 40^\circ$ and $\angle AOX = 75^\circ$, find the number of degrees in (i) $\angle CDE$ (ii) $\angle OBE$.

(b) In the fig(ii) given below, sides AB and DC of a cyclic quadrilateral ABCD are produced to meet at E, the sides AD and BC are produced to meet at F. If $x:y:z = 3:4:5$ find the values of x, y and z .



(a) (i) $\angle CED = 90^\circ$ (Angle in Semi-Circle)

In $\triangle CED$, $\angle CED + \angle CDE + \angle DCE = 180^\circ$

$$\Rightarrow 90^\circ + \angle CDE + 40^\circ = 180^\circ \Rightarrow \angle CDE = 50^\circ$$

(ii) In $\triangle OBD$,

$$\text{Ext. } \angle AOD = \angle OBE + \angle ODB = \angle OBE + \angle CDE$$

$$\Rightarrow 75^\circ = \angle OBE + 50^\circ \Rightarrow \angle OBE = 25^\circ$$

(b) $\angle DCF = \angle BCE$ (Vertically opp. angles)

$$\text{Now in } \triangle DCF, \text{ Ext. } \angle 2 = x + z \quad \text{--- (i)}$$

$$\text{and in } \triangle CBE, \text{ Ext. } \angle 1 = x + y \quad \text{--- (ii)}$$

Adding (i) and (ii)

$$x + y + x + z = \angle 1 + \angle 2$$

$$\Rightarrow 2x + y + z = 180^\circ \quad (\text{ABCD is a cyclic quad.}) \quad \text{--- (iii)}$$

$$\text{But } x : y : z = 3 : 4 : 5$$

$$\frac{x}{y} = \frac{3}{4} \Rightarrow y = \frac{4}{3}x, \quad \frac{x}{z} = \frac{3}{5} \Rightarrow z = \frac{5}{3}x$$

Sub. the value of y and z in (iii)

$$\Rightarrow 2x + \frac{4}{3}x + \frac{5}{3}x = 180^\circ$$

$$\Rightarrow 6x + 4x + 5x = 180^\circ \times 3$$

$$\Rightarrow 15x = 180^\circ \times 3$$

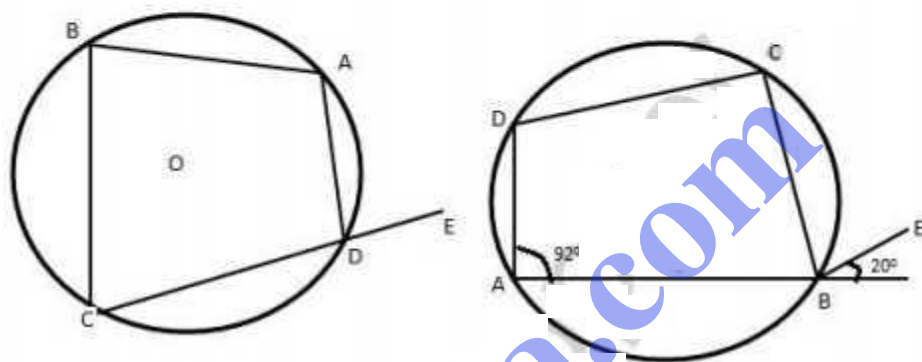
$$\Rightarrow x = 36^\circ$$

$$\therefore y = \frac{4}{3}x = \frac{4}{3} \times 36^\circ = 48^\circ$$

$$z = \frac{5}{3}x = \frac{5}{3} \times 36^\circ = 60^\circ$$

$$\text{Hence } x = 36^\circ, y = 48^\circ, z = 60^\circ$$

19. (a) In the fig (i) given below, ABCD is a quadrilateral inscribed in a circle with centre O. CD is produced to E. If $\angle ADE = 70^\circ$ and $\angle OBA = 45^\circ$, calculate (i) $\angle OCA$ (ii) $\angle BAC$
- (b) In the fig (ii) given below, ABF is a st. line and $BE \parallel DC$. If $\angle DAB = 92^\circ$, find (i) $\angle BCD$ (ii) $\angle ADC$.



Sol. (a) Join OA.

(i) ABCD is a cyclic quad.

$$\text{Ext. } \angle ADE = \angle ABC = 70^\circ$$

$$\angle ABO + \angle OBC = 70^\circ \Rightarrow 45^\circ + \angle OBC = 70^\circ \Rightarrow \angle OBC = 25^\circ$$

Now in $\triangle BOC$, $OB = OC$

$$\angle OBC = \angle OCB = 25^\circ$$

$$\angle BOC = 180^\circ - \angle OBC - \angle OCB = 180^\circ - 25^\circ - 25^\circ = 130^\circ$$

Now arc BC subtends $\angle BOC$ at the centre and $\angle BAC$ at the remaining part of the circle.

$$\therefore \angle BOC = 2 \angle BAC \Rightarrow \angle BAC = \frac{1}{2} \angle BOC$$

$$\Rightarrow \angle BAC = \frac{1}{2} \times 130^\circ = 65^\circ$$

(ii) In $\triangle OAB$, $OA = OB$ (radii of the same circle)

$$\therefore \angle OAB = \angle OBA = 45^\circ$$

$$\therefore \angle OAC = 65^\circ - 45^\circ = 20^\circ$$

Now in $\triangle OAC$, $OA = OC$

$$\therefore \angle OCA = \angle OAC = 20^\circ$$

(b) ABCD is a cyclic quadrilateral

$$\therefore \angle BCD + \angle DAB = 180^\circ \Rightarrow \angle BCD + 92^\circ = 180^\circ \Rightarrow \angle BCD = 88^\circ$$

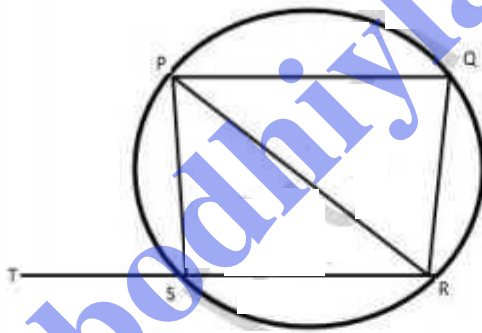
Again Ext. $\angle CBF = \angle ADC$

$$\angle ADC = \angle CBF = \angle CBE + \angle EBF$$

$$\Rightarrow \angle ADC = \angle BCD + \angle EBF \quad (BC \parallel BE \text{ and alternate angles are equal})$$

$$\angle ADC = 88^\circ + 20^\circ = 108^\circ$$

Q20(a) In the given fig below, PQRS is a cyclic quadrilateral in which PQ and QR and RS is produced to T. If $\angle QPR = 52^\circ$, calculate $\angle PST$.



Sol.

(a) PQRS is a cyclic quadrilateral in which $PQ = QR$

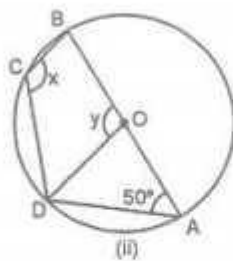
$$\text{In } \triangle PQR, \angle QPR = \angle QRP = 52^\circ$$

$$\Rightarrow \angle PQR = 180^\circ - (\angle QPR + \angle QRP) = 180^\circ - (52^\circ + 52^\circ) = 76^\circ$$

Now in cyclic quadrilateral PQRS,

$$\text{Ext. } \angle PST = \angle PQR = 76^\circ$$

- (b) In the fig(ii) given below, O is the Centre of the Circle.
If $\angle OAD = 50^\circ$, find the values of x and y .



- sol. (b) ABCD is a cyclic quadrilateral

$$\therefore \angle OAD + \angle BCD = 180^\circ$$

$$\Rightarrow 50^\circ + x = 180^\circ \Rightarrow x = 130^\circ$$

In $\triangle OAD$, $OA = OD$ (Radii of same circle)

$$\therefore \angle ODA = \angle OAD = 50^\circ$$

$$\text{Now Ext. } \angle BOD = \angle ODA + \angle OAD \Rightarrow y = 50^\circ + 50^\circ = 100^\circ$$

- Q21. (a) In the fig(i) given below, O is the Centre of the Circle.
If $\angle COD = 40^\circ$ and $\angle CBE = 100^\circ$, calculate the size in degrees of:

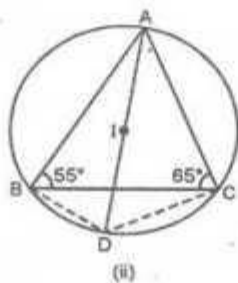
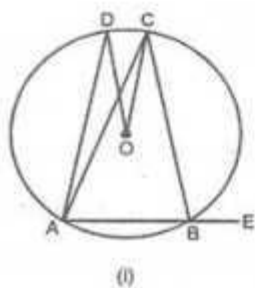
(i) $\angle ADC$ (ii) $\angle DAC$ (iii) $\angle ODA$ (iv) $\angle OCA$.

- (b) In the fig(ii) given below, I is the Centre of $\triangle ABC$.

AI produced meets the Circumcircle of $\triangle ABC$ at D. given that

$\angle ABC = 55^\circ$ and $\angle ACB = 65^\circ$, calculate (i) $\angle BCD$ (ii) $\angle CBD$.

(iii) $\angle DCI$ (iv) $\angle BIC$

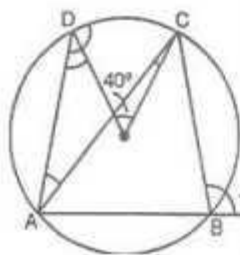


Sol. (a) (i). ABCD is a cyclic quadrilateral

$$\therefore \text{Ext } \angle CBE = \angle ADC = 100^\circ$$

(ii) Arc CD subtends $\angle COD$ at the centre and $\angle CAD$ at the remaining part of the circle.

$$\therefore \angle COD = 2 \angle CAD \Rightarrow \angle CAD = \frac{1}{2} \angle COD = \frac{1}{2} \times 40^\circ = 20^\circ$$



(iii) In $\triangle COD$, $OC = OD$ (radii of same circle)

$$\therefore \angle CDO = \angle DCO$$

$$\text{But } \angle CDO + \angle DCO + \angle COD = 180^\circ$$

$$\Rightarrow 2 \angle CDO + 40^\circ = 180^\circ \quad (\because \angle CDO = \angle DCO)$$

$$\Rightarrow 2 \angle CDO = 140^\circ$$

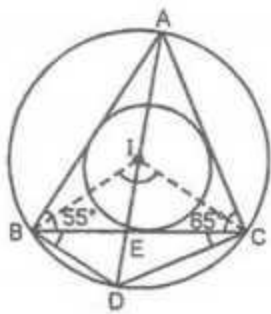
$$\Rightarrow \angle CDO = 70^\circ$$

Now $\angle ODA = \angle ADC - \angle CDO = 100^\circ - 70^\circ = 30^\circ$

(iv) $\angle OCA = \angle OCD - \angle ACD = 70^\circ - (180^\circ - \angle ADC - \angle CAD)$
 $= 70^\circ - (180^\circ - 100^\circ - 20^\circ) = 10^\circ$

(b) Join BI and CI

In $\triangle ABC$, $\angle BAC + \angle ABC + \angle ACB = 180^\circ$ (Angles of a $\triangle$)



$\angle BAC + 55^\circ + 65^\circ = 180^\circ \Rightarrow \angle BAC = 60^\circ$

$\therefore I$ is incentre.

$\therefore I$ lies on the bisectors of angle of the $\triangle ABC$.

$\therefore \angle BAD = \angle CAD = \frac{60^\circ}{2} = 30^\circ$

But $\angle BCD = \angle BAD = 30^\circ$ (Angles in the same segment)

Similarly $\angle CBD = \angle CAD = 30^\circ$

and $\angle IBC = \frac{55^\circ}{2} = 27\frac{1}{2}^\circ$

and $\angle ICB = \frac{65^\circ}{2} = 32\frac{1}{2}^\circ$

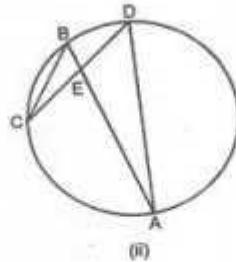
$\therefore \angle BIC = 180^\circ - (27\frac{1}{2}^\circ + 32\frac{1}{2}^\circ) = 180^\circ - 60^\circ = 120^\circ$

Now $\angle ICD = \angle ICB + \angle BCD$

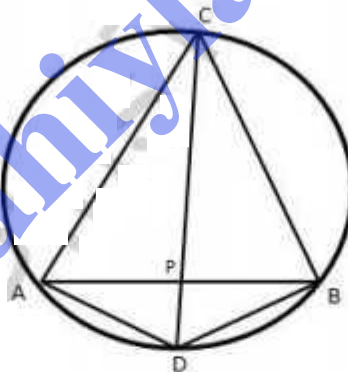
$= 32\frac{1}{2}^\circ + 30^\circ$

$\therefore \angle ICD = 62\frac{1}{2}^\circ$

- Q22. (a) In the fig(i) given below, chords AB and CD of a circle intersect at E. (i) prove that triangles ADE and CBE are similar.
 (ii) Given $DC = 12\text{cm}$, $DE = 4\text{cm}$ and $AE = 16\text{cm}$, calculate the length of BE.



- (b) In the fig(ii) given below, AB and CD are two intersecting chords of a circle. Name two triangles which are similar. Hence calculate CP given that $AP = 6\text{cm}$, $PB = 4\text{cm}$ and $CD = 14\text{cm}$ ($PC > PD$).



sol. To prove: (i) $\triangle ADE \sim \triangle CBE$

(ii) If $DC = 12\text{cm}$, $DE = 4\text{cm}$ and $AE = 16\text{cm}$, calculate the length of BE.

proof: (i) In $\triangle ADE$ and $\triangle CBE$

$$\therefore \angle D = \angle B, \angle A = \angle C \text{ (angles in the same segment)}$$

$$\therefore \triangle ADE \sim \triangle CBE \text{ (AA axiom of similarity)}$$

(ii) $DC = 12\text{cm}$, $DE = 4\text{cm}$

$\therefore EC = 12 - 4 = 8\text{cm}$

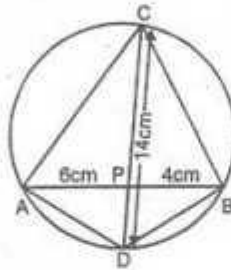
$\Rightarrow AE = 16\text{cm}$

$\therefore$ chords $AB = CD$ intersect each other at E .

$\therefore AE \times EB = CE \times ED$

$\Rightarrow 16 \times EB = 8 \times 4 \Rightarrow EB = 2\text{cm}$

(b) let $CP = x$ then $PD = 14 - x$



Now in $\triangle APD$ and $\triangle CPB$

$\therefore \angle DAB = \angle DCB$ $\angle CDA = \angle CBA$

$\therefore \triangle APD \sim \triangle CPB$ (AA axiom of similarity)

$\frac{AP}{PD} = \frac{CP}{PB}$

$\Rightarrow AP \times PB = CP \times PD \Rightarrow 6 \times 4 = x \times (14 - x)$

$\Rightarrow 24 = 14x - x^2 \Rightarrow x^2 - 14x + 24 = 0$

$\Rightarrow (x - 12)(x - 2) = 0$

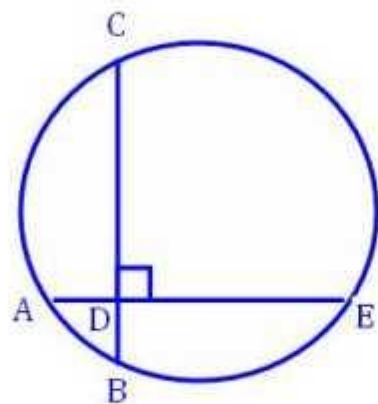
Either $x - 12 = 0 \Rightarrow x = 12$ or $x - 2 = 0 \Rightarrow x = 2$

$\therefore CP = 12\text{cm}$ or 2cm

But $CP > PD$ (given)

$\therefore CP = 12\text{cm}$

Q.8. In the given fig. AE and BC intersect each other at point D.
if $\angle CDE = 90^\circ$, $AB = 5\text{cm}$, $BD = 4\text{cm}$, $CD = 9\text{cm}$, find DE.



Sol. Here $AE \perp CB \Rightarrow \angle ADB = 90^\circ$

By pythagoras theorem, we have

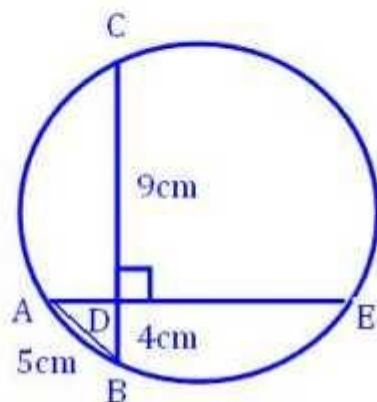
$$AD^2 = AB^2 - BD^2 = 5^2 - 4^2 = 9 \Rightarrow AD = 3\text{cm}.$$

Now AE and BC are two chords of a circle intersecting at D.
inside the circle.

$$AD \times DE = BD \times DC$$

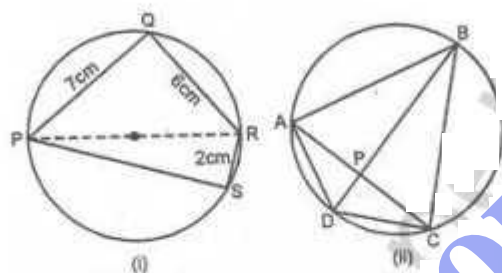
$$\Rightarrow 3 \times DE = 4 \times 9 \Rightarrow DE = 12\text{cm}.$$

Hence the required length of DE is 12cm.



Q24. (a) In the fig(i) given below, PR is a diameter of the circle, PQ = 7cm, QR = 6cm and RS = 2cm. Calculate the perimeter of the cyclic quadrilateral PQRS.

(b) In the fig(ii) given below, the diagonals of a cyclic quadrilateral ABCD intersect in P and the area of the $\triangle APB$ is 24 cm^2 . If AB = 8cm and CD = 5cm, calculate the area of $\triangle CPD$.



Sol. (a) PR is the diameter of the circle.

$$PQ = 7\text{cm}, \quad QR = 6\text{cm}, \quad RS = 2\text{cm}.$$

In $\triangle PQR$, $\angle Q = 90^\circ$ (Angle in a Semi-circle)

$$\therefore PR^2 = PQ^2 + QR^2 = (7)^2 + (6)^2 = 85.$$

Again in $\triangle PSR$, $\angle S = 90^\circ$ (Angle in a Semi-circle)

$$\therefore PR^2 = PS^2 + RS^2 \Rightarrow 85 = PS^2 + (2)^2 \Rightarrow PS^2 = 81$$

$$\Rightarrow PS = 9\text{cm}.$$

Now the perimeter of PQRS = $7 + 6 + 2 + 9 = 24 \text{ cm}$.

(b) In $\triangle APB$ and $\triangle CPD$.

$$\therefore \angle APB = \angle CPD \quad (\text{Vertically opp. angles})$$

$$\therefore \angle ABP = \angle DCP \quad (\text{Angles in the same segment})$$

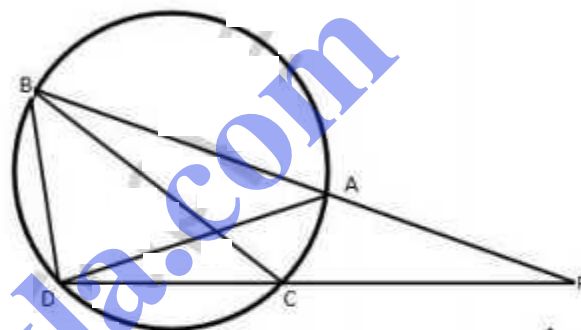
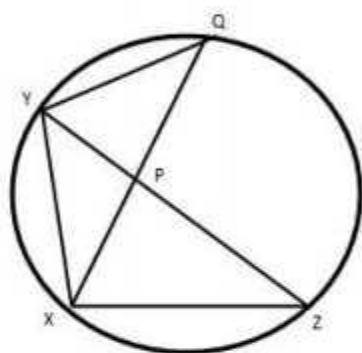
$$\therefore \triangle APB \sim \triangle CPD \quad (\text{AA axiom of similarity})$$

$$\therefore \frac{\text{Area of } \triangle APB}{\text{Area of } \triangle CPD} = \frac{AB^2}{CD^2}$$

$$\Rightarrow \frac{24}{\text{area of } \triangle CPD} = \frac{(8)^2}{(5)^2} \Rightarrow \frac{24}{\text{area of } \triangle CPD} = \frac{64}{25}$$

$$\Rightarrow \text{Area of } \triangle CPD = 24 \times \frac{25}{64} = \frac{75}{8} = 9\frac{3}{8} \text{ cm}^2$$

- Q25. (a) In the fig(i) given below, QP is the bisector of $\angle YXZ$ of the $\triangle XYZ$. prove that $XY : XQ = XP : XZ$ (Hint: $\triangle XYQ \sim \triangle XPZ$)
- (b) In the fig(ii) given below, chords BA and DC of a circle meet at P. prove that : (i) $\angle PAD = \angle PCB$ (ii) $PA \cdot PB = PC \cdot PD$



Sol. Given: $\triangle XYZ$ is inscribed in a circle bisector of $\angle YXZ$ meets the circle at Q. QY is joined.

To prove: $XY : XQ = XP : XZ$.

Proof: In $\triangle XYQ$ and $\triangle XPZ$

$\angle Q = \angle Z$ (Angles in the same segment)

$\angle YXQ = \angle P X Z$ ($\because$ XQ is the bisector of $\angle YXZ$)

$\therefore \triangle XYQ \sim \triangle XPZ$

$$\frac{XY}{XP} = \frac{XQ}{XZ} \Rightarrow \frac{XY}{XQ} = \frac{XP}{XZ} \Rightarrow XY : XQ = XP : XZ$$

(b) Given: Two chords BA and DC meet each other at P outside the circle. AD and BC are joined.

To prove : (i) $\angle PAD = \angle PCB$

(ii) $PA \cdot PB = PC \cdot PD$

proof : $\angle PAD + \angle DAB = \angle PCB + \angle BCD$ (each 180°)

But $\angle DAB = \angle BCD$ (angle in the same segment)

$\therefore \angle PAD = \angle PCB$

Now in $\triangle PBC$ and $\triangle PAD$

$\angle PCB = \angle PAD$ (proved)

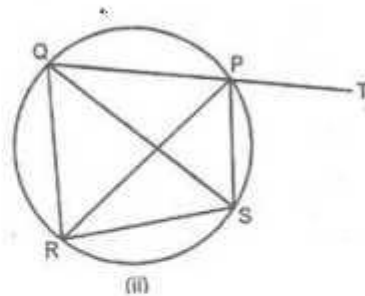
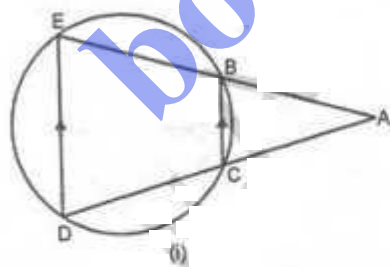
$\angle P = \angle P$ (common)

$\therefore \triangle PBC \sim \triangle PAD$ (AA axiom of Similarity)

$$\frac{PC}{PA} = \frac{PB}{PD} \Rightarrow PA \cdot PB = PC \cdot PD$$

Q26. (a) In the fig(i) given below, ED and BC are two parallel chords of the circle and ABE , ACD are two st. lines. prove that $\triangle AED$ is an isosceles triangle.

(b) In the fig(ii) given below, SP is the bisector of $\angle RPT$ and $PQRS$ is a cyclic quadrilateral. prove that $SQ = RS$.



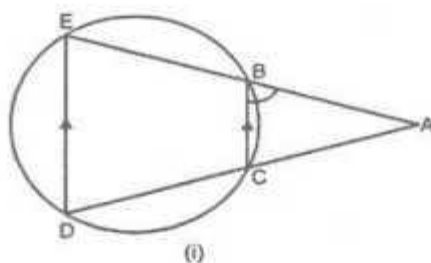
(a) Given : chord $BC \parallel ED$, ABE and ACD are straight lines.

To prove : $\triangle AED$ is an isosceles triangle.

proof : $BCDE$ is a cyclic quadrilateral

$\therefore \text{Ext. } \angle ABC = \angle D$ — (i)

But $BC \parallel ED$ (given)



$$\therefore \angle ABC = \angle E \quad (\text{Corresponding angles}) \quad \text{--- (ii)}$$

$$\text{from (i) and (ii), } \angle D = \angle E$$

$$\text{In } \triangle AED, \angle D = \angle E \quad (\text{proved})$$

$$\therefore AE = AD \quad (\text{Sides opp to equal angle})$$

Hence $\triangle AED$ is an isosceles $\triangle$

(b) given: In a circle, PQRS is a cyclic quadrilateral. QP is produced to T such that QS is the bisector of $\angle RPT$

To prove $SQ = RS$

proof: $\because$ PQRS is a cyclic quadrilateral.

$$\therefore \text{Ext. } \angle TPS = \angle SRQ \quad \text{--- (i)}$$

$$\text{and } \angle RPS = \angle RQS \quad (\text{Angle in the Same Segment}) \quad \text{--- (ii)}$$

$$\text{But } \angle TPS = \angle RPS \quad (\text{QS is the bisector of } \angle TPR)$$

$$\text{from (i) and (ii), } \angle SRQ = \angle RQS$$

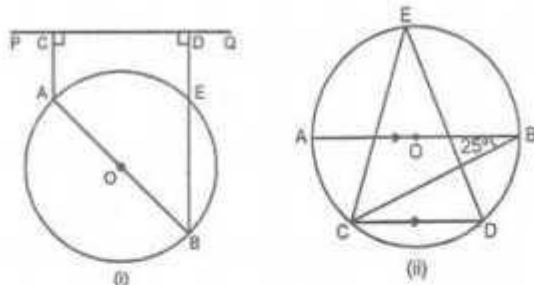
$$\text{Now in } \triangle SQR, \angle SRQ = \angle RQS \quad \text{proved}$$

$$\therefore SQ = RS \quad (\text{Sides opp to equal angles})$$

Q27. (a) In the fig. (i) given below, AB is a diameter of a circle with centre O. AC and BD are $\perp$ on a line PQ. BD meets the circle at E. prove that $AC = ED$.

(b) In the fig. (ii) given below, O is the Centre of the circle. chord CD is parallel to the diameter AB.

If $\angle ABC = 25^\circ$, calculate $\angle CED$.



Sol. Given: AB is the diameter of a circle with centre O . AC and BD are $\perp$ on a line PQ , such that BD meets the circle at E .

To prove: $AC = ED$

Construction: Join AE .

Proof: $\angle AEB = 90^\circ$ (angle in a semi-circle)

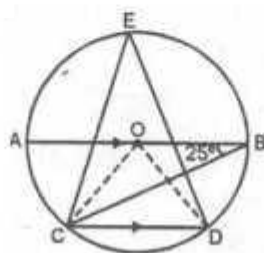
But $ED \perp PQ$ (given) $\therefore AE \parallel PQ$

But BD and AC are $\perp$ to PQ

$\therefore AEDC$ is a rectangle

Hence $AC = ED$ (Opp. sides of a rectangle)

(b) $AB \parallel CD$ and O is the centre of the circle where AB is diameter, $\angle ABC = 25^\circ$. Join OC and OD .



Arc AC subtends $\angle AOC$ at the Centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \angle AOC = 2 \angle ABC = 2 \times 25^\circ = 50^\circ$$

$$\text{But } \angle OCD = \angle AOC = 50^\circ \text{ (Alternate angles)}$$

$$\text{But } \angle ODC = \angle OCD = 50^\circ \text{ (OC = OD)}$$

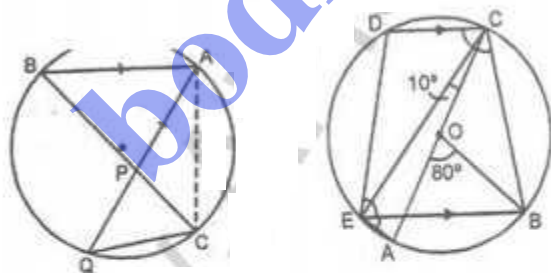
$$\text{In } \triangle OCD, \angle COD + \angle OCD + \angle ODC = 180^\circ \text{ (Angles of a } \triangle)$$

$$\Rightarrow \angle COD + 50^\circ + 50^\circ = 180^\circ \Rightarrow \angle COD = 80^\circ$$

Now arc CD subtends $\angle COD$ at the Centre and $\angle CED$ at the remaining part of the circle.

$$\therefore \angle COD = 2 \angle CED \Rightarrow 80^\circ = 2 \angle CED \Rightarrow \angle CED = 40^\circ$$

- Q28. (a) In the fig(i) given below, P is the point of intersection of the chords BC and AQ such that $AB = AP$. Prove that $CP = CQ$.
 (b) In the fig(ii) given below, AC is a diameter of the circle with Centre O. If $CD \parallel BE$, $\angle ACB = 80^\circ$ and $\angle ACE = 10^\circ$, find:
 (i) $\angle BEC$ (ii) $\angle BCD$ (iii) $\angle CED$.



- Sol. (a) Given: Two chords AQ and BC intersect each other at P inside the circle.

AB and CQ are joined and $AB = AP$.

To prove: $CP = CQ$.

Construction: Join AC.

proof: In $\triangle ABP$ and $\triangle CQP$.

$$\therefore \angle B = \angle Q \quad (\text{Angles in the same segment})$$

$$\angle BAP = \angle PCQ \quad (\text{Angles in the same segment})$$

$$\angle BPA = \angle CPQ \quad (\text{Vertically opp. angles})$$

$$\therefore \triangle ABP \sim \triangle CQP \quad (\text{AAA axiom of similarity})$$

$$\therefore \frac{AB}{CQ} = \frac{AP}{CP}$$

$$\text{But } AB = AP \quad (\text{given})$$

$$\therefore CQ = CP$$

(b) AC is the diameter of the circle with centre O. $CD \parallel BE$,

$$\angle AOB = 80^\circ, \angle ACE = 10^\circ. \text{ Join AE}$$

$$\angle AEC = 90^\circ \quad (\text{Angles in a semi circle})$$

Arc subtends $\angle AOB$ at the centre and $\angle AEB$ at the remaining part of the circle.

$$\therefore \angle AOB = 2 \angle AEB$$

$$\Rightarrow \angle AEB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 80^\circ = 40^\circ$$

$$\angle AEB + \angle BEC = \angle AEC \Rightarrow 40^\circ + \angle BEC = 90^\circ \Rightarrow \angle BEC = 50^\circ$$

$$\text{But } \angle AEB = \angle ACB \quad (\text{Angles in the same segment})$$

$$\therefore \angle ACB = \angle AEB = 40^\circ$$

$$\angle BCE = \angle BEC \quad (\text{Alternate angles})$$

$$\therefore \angle DCE = 50^\circ$$

$$\text{Now } \angle BCD = \angle DCE + \angle ECA + \angle ACB = 50^\circ + 10^\circ + 40^\circ = 100^\circ$$

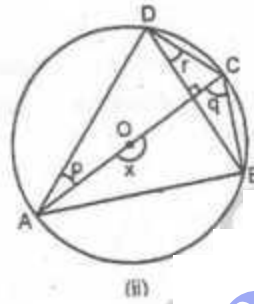
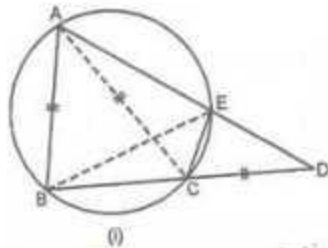
$$\text{In cyclic quad. BCDE, } \angle BCD + \angle BED = 180^\circ$$

$$\Rightarrow 100^\circ + \angle BED = 180^\circ \Rightarrow \angle BED = 80^\circ$$

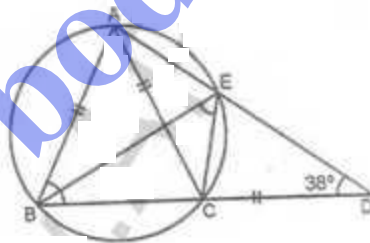
$$\therefore \angle CED = \angle BED - \angle BEC = 80^\circ - 50^\circ = 30^\circ$$

Q29. (a) In the fig(i) given below, $AB = AC = CD$, $\angle ADC = 38^\circ$.
calculate: (i) $\angle ABC$ (ii) $\angle BEC$.

(b) In the fig(ii) given below, AC is diameter of circle with Centre O . chord BD is $\perp$ to AC . write down the angles p, q, r in terms of x .



Sol. $AB = AC = CD$, $\angle ADC = 38^\circ$
(a) In $\triangle ACD$, $AC = CD$ (given)
 $\therefore \angle CAD = \angle ADC = 38^\circ$



and Ext. $\angle ACB = \angle CAD + \angle ADC = 38^\circ + 38^\circ = 76^\circ$

In $\triangle ABC$, $AB = AC$.

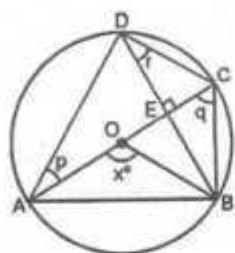
$\therefore \angle ABC = \angle ACB = 76^\circ$

and $\angle BAC = 180^\circ - (\angle ABC + \angle ACB) = 180^\circ - (76^\circ + 76^\circ) = 28^\circ$

But $\angle BEC = \angle BAC$ (angle in the same segment)

$$\therefore \angle BEC = 28^\circ$$

(b) arc AB subtends $\angle AOB$ at the Centre and $\angle ADB$ at the remaining part of the circle.



$$\therefore \angle ADB = \frac{1}{2} \angle AOB = \frac{x^\circ}{2}$$

But $\angle ADC = 90^\circ$ (angle in a semi-circle)

$$\therefore y + \frac{x}{2} = 90^\circ \Rightarrow y = 90^\circ - \frac{x}{2}$$

In $\triangle ADE$, $\angle DEA = 90^\circ$ ($DB \perp AC$)

$$p + \angle ADE = 90^\circ$$

$$\Rightarrow p + \angle ADB = 90^\circ \Rightarrow p = 90^\circ - \angle ADB = 90^\circ - \frac{x}{2}$$

$\angle ACB = \angle ADB$ (angles in the same segment)

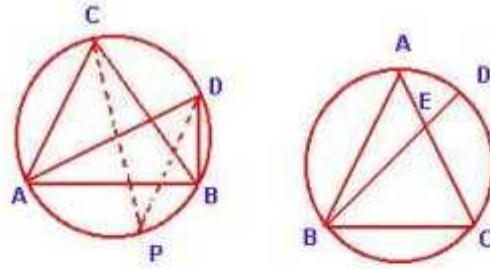
$$q = \frac{x^\circ}{2}$$

$$\text{Hence } p = 90^\circ - \frac{x}{2}, \quad q = \frac{x}{2} \quad \text{and } r = 90^\circ - \frac{x}{2}$$

Q30 (a) In the fig(i) given below, CP bisects $\angle ACB$. prove that DP bisects $\angle ADB$.

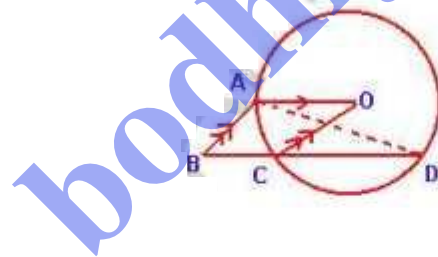
(b) In the fig(ii) given below, BD bisects $\angle ABC$.

$$\text{prove that } \frac{AB}{BD} = \frac{BE}{BC}.$$



Sol. $\angle ACB = \angle ADB$ (angle in the same segment) $\rightarrow (1)$
 $\angle ACP = \angle ADP$ (angle in the same segment) $\rightarrow (2)$
 -from (1) and (2), DP bisects $\angle ADB$

Q31. In the adjoining fig, O is the Centre of the given circle and OABC is a parallelogram. BC is produced to meet the circle at D. prove that $\angle ABC = \angle OAD$.



Sol. $\angle AOC = 2 \angle ADC$
 (angle at Centre = Double the angle at the remaining part of the circle.)
 $\angle ABC = \angle AOC$ (Opp. angle of a parallelogram)
 and $\angle OAD = \angle ADC$ (alternate angles)

Q32 (a) In the fig(i) given below, chords AB and CD of a circle meet internally at P. Given that AP = 3cm, PB = 6cm and CP = 2.4cm.

(i) prove that $\triangle ACP$ and $\triangle DBP$ are similar. (ii) find PD.

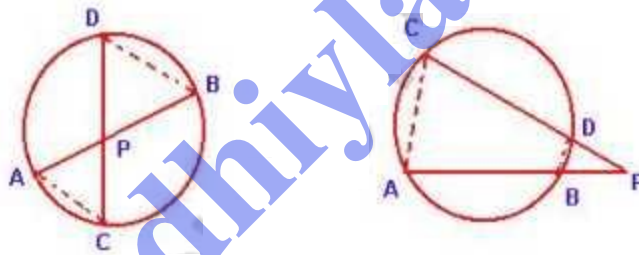
(iii) find $\frac{\text{area of } \triangle ACP}{\text{area of } \triangle DBP}$.

(b) In the fig(ii) given below, chords AB and CD of a circle meet externally at P. given that BP = 4cm, CD = 15cm and DP = 5cm.

(i) prove that $\triangle CAP$ and $\triangle BDP$ are similar

(ii) find AB.

(iii) find $\frac{\text{area of quad. CABD}}{\text{area of } \triangle CAP}$.



Sol. $\angle APC = \angle DPB$ (vertically opp. angles)

$\angle PAC = \angle PDB$

$\angle ACB = \angle PBD$ (angle in the same segment)

Hence $\triangle ACP$ and $\triangle DBP$ are similar.

(ii) $AP \times PB = PC \times PD \Rightarrow 3 \times 6 = 2.4 \times x \Rightarrow x = 7.5$

(iii) $\therefore \triangle ACP \sim \triangle BPD$

$$\therefore \frac{\text{Area of } \triangle ACP}{\text{Area of } \triangle BPD} = \frac{CP^2}{PB^2} = \frac{(2.4)^2}{(6)^2} = \frac{5.76}{36} = \frac{4}{25}$$

(b) (i) $\angle P = \angle P$ (Common)

$$\angle C = \angle DBP$$

$\therefore \triangle CAP \sim \triangle DBP$ (AA axiom)

$$(ii) CP = CD + DP = 15 + 5 = 20 \text{ cm.}$$

$\therefore \triangle CAP \sim \triangle DPB$ (proved)

$$\frac{CP}{BP} = \frac{AP}{DP} \Rightarrow \frac{20}{4} = \frac{AB+BP}{5} \Rightarrow 5 = \frac{AB+4}{1} \Rightarrow AB = 21 \text{ cm.}$$

$\therefore \triangle CAP \sim \triangle DPB$

$$\therefore \frac{\text{area of } \triangle CAP}{\text{area of } \triangle DPB} = \frac{CP^2}{BP^2} = \frac{(20)^2}{(4)^2} = 25.$$

$$\Rightarrow \text{area of } \triangle CAP = 25 \times \text{area of } \triangle DPB$$

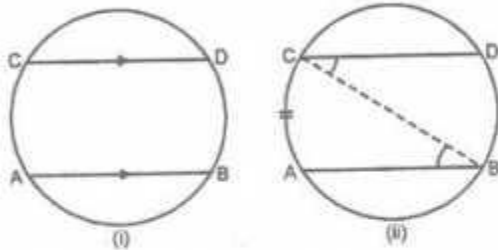
$$\begin{aligned} \text{area of quad. CABD} &= \text{area } \triangle CAP - \text{area } \triangle DPB \\ &= \text{area } \triangle CAP - \frac{1}{25} \text{ area } \triangle CAP \end{aligned}$$

$$\Rightarrow \text{area of quad. CABD} = \frac{24}{25} (\text{area of } \triangle CAP)$$

$$\therefore \frac{\text{area of quad. CABD}}{\text{area of } \triangle CAP} = \frac{24}{25}$$

EXERCISE - 16.3

- Q1. (a) In the fig.(i) given below, chords AB and CD of a circle are parallel. prove that $\text{arc } AC = \text{arc } BD$.
- (b) In the fig.(ii) given below, $\text{arc } AC = \text{arc } BD$. prove that chords AB and CD are parallel.

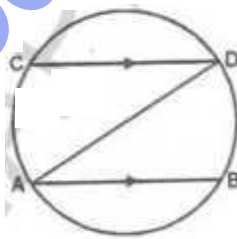


Sol. (a) Given : In a circle with centre O, chord $AB \parallel CD$.

To prove : $\text{arc } AC = \text{arc } BD$

Construction : Join AD.

proof : $AB \parallel CD$.

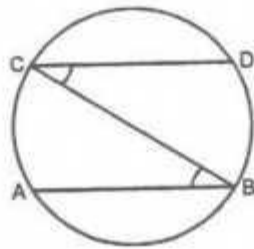


$\therefore \angle ADC = \angle BAD$ (alternate angles)

$\text{arc } AC = \text{arc } BD$

(arcs subtend equal angles at the circumference are equal)

(b) Given : In a circle, arc AC = arc BD.



AB and CD are chords.

To prove : $AB \parallel CD$

Construction : Join BC.

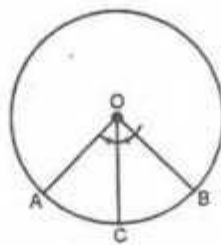
proof : arc AC = arc BD.

$$\angle ABC = \angle BCD$$

But these are alternate angles. $\therefore AB \parallel CD$.

Q2. A and B are points on a circle with centre O. C is a point on the circle such that OC bisects $\angle AOB$. prove that OC bisects the arc AB.

Sol. Given : In a given circle with centre O, A and B are two points on the circle C is another point on the circle. such that $\angle AOC = \angle BOC$.



To prove: arc AC = arc BD

proof: OC is the bisector of $\angle AOB$ or $\angle AOC = \angle BOC$

But these are the angle subtended by the arc AC and BD.

$\therefore$ arc AC = arc BD.

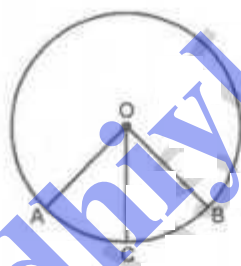
Q3. prove that the angle subtended at the centre of a circle is bisected by the radius passing through the midpoint of the arc.

Sol. Given: AB is the arc of the circle with centre O and C is the midpoint of arc AB.

To prove: OC bisects the $\angle AOB$ i.e. $\angle AOC = \angle BOC$

proof: C is the midpoint of arc AB.

$\therefore$ arc AC = arc BC.



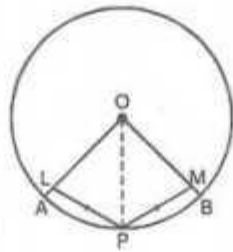
But arc AC and arc BC subtend $\angle AOC$ and $\angle BOC$ at the centre.

$\therefore \angle AOC = \angle BOC$

Hence OC bisects the $\angle AOB$.

Q4. P is a point on a circle with centre O. If P is equidistant from two radii OA and OB, prove that arc AP = arc BP.

Sol. Given: A and B are two points on the circle with centre O.



P is the point on arc AB such that $\angle PL = \angle PM$

To prove : arc AP = arc BP

Construction : Join OP

proof : In right $\triangle OLP$ and $\triangle OMP$,

Hyp. $OP = OP$ (Common) , Side $PL = PM$ (given)

$\therefore \triangle OLP \cong \triangle OMP$ (RHS axiom of congruency)

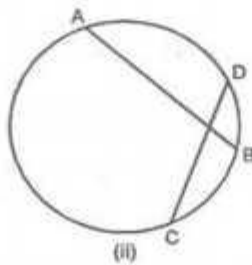
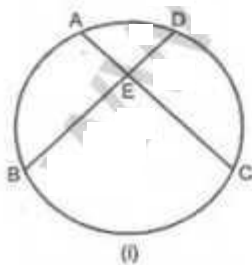
$\therefore \angle POL = \angle POM$ or $\angle POA = \angle POB$

But these are the angles subtended by arc AP and BP respectively

$\therefore$ arc AP = arc BP

Q5. (a) In the fig(i) given below, two chords AC and BD of a circle intersect at E. if arc AB = arc CD, prove that BE = EC and AE = ED.

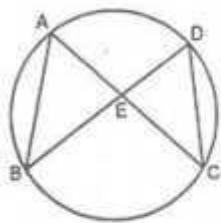
(b) In the fig(ii) given below, two chords AB = CD of a circle intersect at P. if AB = CD, prove that arc AD = arc CB.



Sol. Given: Two chords AC and BD of a circle intersect each other at E and $\text{arc } AB = \text{arc } CD$.

To prove: $BE = EC$ and $AE = ED$.

Construction: Join AB and CD.



proof: $\text{arc } AB = \text{arc } CD$

$$AB = CD$$

Now in $\triangle AEB$ and $\triangle CED$,

$$AB = CD \text{ (proved)}$$

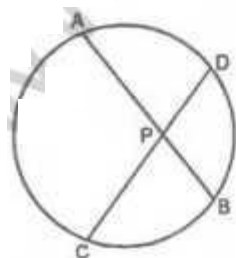
$$\angle A = \angle D \text{ (angles in the same segment)}$$

$$\angle B = \angle C$$

$$\therefore \triangle AEB \cong \triangle CED \text{ (ASA axiom of congruency)}$$

$$BE = EC \text{ and } AE = ED.$$

(b) Given: Two chords AB and CD of a circle intersect at P and $AB = CD$.



To prove : arc AD = arc CB

proof : $AB = CD$ (given)

minor arc AB = minor arc CD

Subtracting arc BD from both sides.

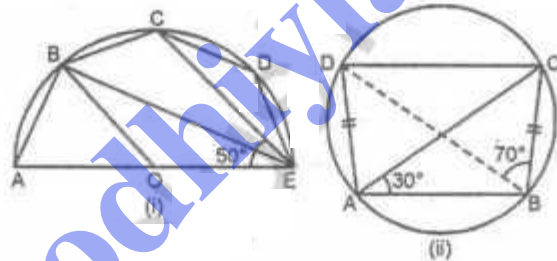
$$\text{arc AB} - \text{arc BD} = \text{arc CD} - \text{arc BD}$$

$$\Rightarrow \text{arc AD} = \text{arc CB}$$

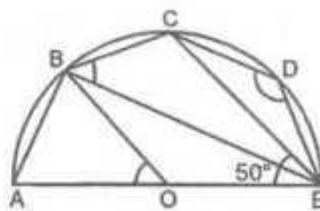
- Q6. (a) In the fig(i) given below, O is the Centre and AOE is the diameter of the Semicircle and AOE is the diameter of the Semicircle ABCDE. if $AB = BC$ and $\angle OEC = 50^\circ$, find
(i) $\angle CBE$ (ii) $\angle CDE$ (iii) $\angle AOB$.

prove that $OB \parallel EC$.

- (b) In the fig(ii) given below, $AD = BC$, $\angle BAC = 30^\circ$ and $\angle CBD = 70^\circ$, find (i) $\angle BCD$ (ii) $\angle BCA$ (iii) $\angle ABC$ (iv) $\angle ADB$.



sol. $AB = BC$, $\angle AEB = \angle BEC = \frac{1}{2} \angle OEC = \frac{1}{2} \times 50^\circ = 25^\circ$



In $\triangle OBE$, $OB = OE$ (radii of the same circle)

$$\therefore \angle OEB = \angle OBE = 25^\circ$$

(i) Now in cyclic quad. $ABCE$, $\angle AEC + \angle ABC = 180^\circ$

$$\Rightarrow 50^\circ + \angle ABC = 180^\circ \Rightarrow \angle ABC = 130^\circ$$

But $\angle ABE = 90^\circ$ (angle in a semi circle)

$$\angle ECB = \angle ABC - \angle ABE = 130^\circ - 90^\circ = 40^\circ$$

In cyclic quad. $EBCD$, $\angle ECB + \angle CDE = 180^\circ$

$$\Rightarrow 40^\circ + \angle CDE = 180^\circ \Rightarrow \angle CDE = 140^\circ$$

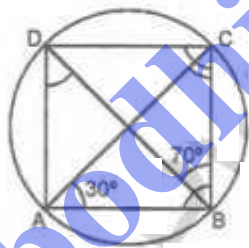
In $\triangle OBE$, Ext. $\angle AOB = \angle OBE + \angle OEB = 25^\circ + 25^\circ = 50^\circ$

$$\therefore \angle AOB = \angle OEC = 50^\circ$$

But these are the corresponding angles.

$$\therefore OB \parallel EC$$

(b) $AD = BC$, $\angle BAC = 30^\circ$ and $\angle CBD = 70^\circ$



(i) $\angle DAC = \angle DBC = 70^\circ$

$$\angle DAB = 100^\circ$$

$ABCD$ is a cyclic quad.

$$\angle BCD + \angle DAB = 180^\circ$$

$$\Rightarrow \angle BCD + 100^\circ = 180^\circ$$

$$\Rightarrow \angle BCD = 80^\circ$$

(ii) chord $AD = BC$

$$\angle DCA = \angle BAC = 30^\circ$$

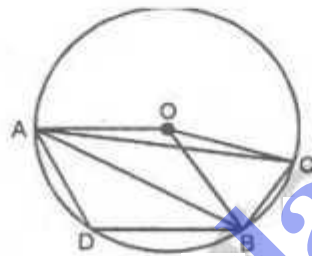
$$\therefore \angle BCA = \angle BCD - \angle DCA = 80^\circ - 30^\circ = 50^\circ$$

$$(iii) \angle ADB = \angle ACB = 50^\circ \text{ (angle in the same segment)}$$

$$(iv) \angle ACD = \angle ABD = 30^\circ \text{ (angle in the same segment)}$$

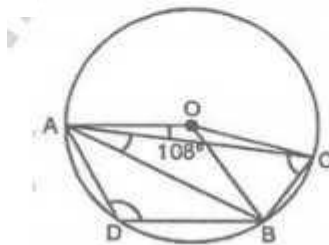
$$\angle ABC = \angle ABD + \angle DBC = 30^\circ + 70^\circ = 100^\circ$$

- Q7. In the adjoining fig, A, D, B, C are 4 points on the circumference of a circle with centre O. arc AB = 2 arc BC and $\angle AOB = 108^\circ$. Calculate in degrees of (i) $\angle ACB$ (ii) $\angle CAB$ (iii) $\angle ADB$. Justify your calculations.



- Sol. Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 108^\circ = 54^\circ$$

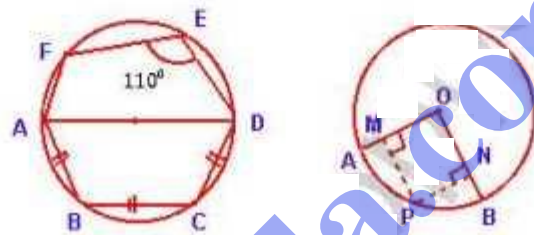


(ii) $\text{arc } AB = 2 \cdot \text{arc } BC$

$$\angle ACB = 2 \angle CAB \Rightarrow \angle CAB = \frac{1}{2} \angle ACB = \frac{1}{2} \times 54^\circ = 27^\circ$$

Q8. (a) In the fig(i) given below, AD is diameter of the circle and chords AB, BC and CD are equal in length. Given $\angle DEF = 110^\circ$, calculate (i) $\angle AEF$ (ii) $\angle BAF$.

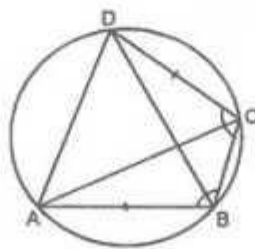
(b) In the fig(ii) given below, O is the centre of the given circle, P is a point on the circle, $PM \perp OA$ and $PN \perp OB$. If $PM = PN$, prove that P is the mid-point of the arc AB.



Sol. (a) Join AE, BE and CE. $\angle AEB = \angle BEC = \angle CED$ and $\angle AED = 90^\circ$

(b) Join OP. $\triangle OMP \cong \triangle ONP$, so $\angle MOP = \angle NOP$.

Q9. ABCD is a cyclic quadrilateral. If $AB = DC$, prove that $\angle B = \angle C$.



Sol. Given: ABCD is a cyclic quadrilateral and $AB = CD$.

To prove: $\angle B = \angle C$

Construction: Join AC and BD.

proof: $\because$ chord $AB =$ chord CD .

$$\angle ACB = \angle DBC$$

But $\angle ACD = \angle ABD$ (angles in the same segment)

$$\text{Adding } \angle ACB + \angle ACD = \angle DBC + \angle ABD$$

$$\Rightarrow \angle BCD = \angle ABC \text{ or } \angle C = \angle B$$

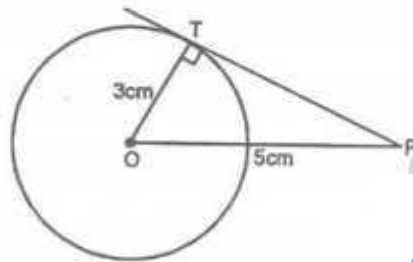
Hence proved.

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EXERCISE - 16.4

Q1. Find the length of the tangent drawn to a circle of radius 3cm, from a point distant 5cm from the centre.

sol. In a circle with centre O and radius 3cm and P is at a distance of 5cm. i.e. $OT = 3\text{cm}$, $OP = 5\text{cm}$.



$\therefore$ OT is the radius of the circle. $OT \perp PT$

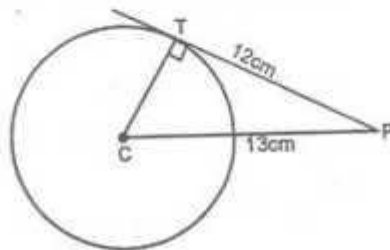
Now in right $\triangle OTP$, by pythagoras theorem,

$$OP^2 = OT^2 + PT^2 \Rightarrow (5)^2 = (3)^2 + PT^2$$

$$\Rightarrow PT = 4\text{cm}$$

Q2. A point P is at a distance 13cm from the centre C of a circle. PT is the tangent to the given circle. If $PT = 12\text{cm}$, find the radius of the circle.

sol.



CT is the radius

CP = 13cm and tangent PT = 12cm.

$\therefore$ CT is the radius and TP is the tangent

$\therefore$ CT $\perp$ TP

Now in right $\triangle CPT$,

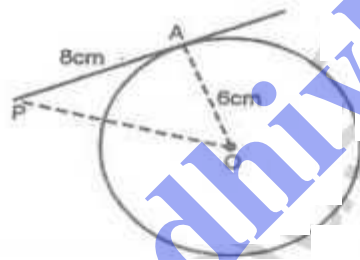
$$CP^2 = CT^2 + PT^2 \quad (\text{Pythagoras axiom})$$

$$\Rightarrow (13)^2 = (CT)^2 + (12)^2 \Rightarrow CT^2 = 25 \Rightarrow CT = 5\text{cm}$$

Hence radius of the circle = 5cm.

- Q3. The tangent to a circle of radius 6cm from an external point P is of the length 8cm. Calculate the distance of P from the nearest point of the circle.

Sol.



Radius of the circle = 6cm

length of tangent = 8cm.

let OP be the distance i.e. OA = 6cm, AP = 8cm.

$\therefore$ OA is the radius $\Rightarrow$ OA $\perp$ AP.

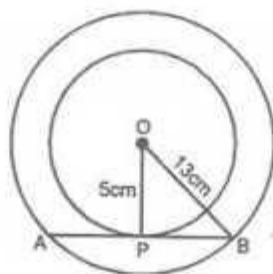
Now in right $\triangle OAP$, $OP^2 = OA^2 + AP^2$

$$OP^2 = (6)^2 + (8)^2 = 36 + 64 = 100$$

$$\Rightarrow OP = 10\text{cm}.$$

Q4. Two concentric circles are of the radii 13cm and 5cm. find the length of the chord of the outer circle which touches the inner circle.

Sol. Two concentric circles with centre O and OP and OB are the radii of the circles respectively, then
 $OP = 5\text{cm}$, $OB = 13\text{cm}$.



AB is the chord of the outer circle which touches the inner circle at P.

$\therefore$ OP is the radius and APB is the tangent to the inner circle.

$\therefore$ In right $\triangle OPB$, by Pythagoras theorem,

$$(OB)^2 = (OP)^2 + (PB)^2 \Rightarrow (13)^2 = (5)^2 + (PB)^2$$

$$\Rightarrow (PB)^2 = 169 - 25 = 144 \Rightarrow PB = 12\text{cm}.$$

But P is the midpoint of AB.

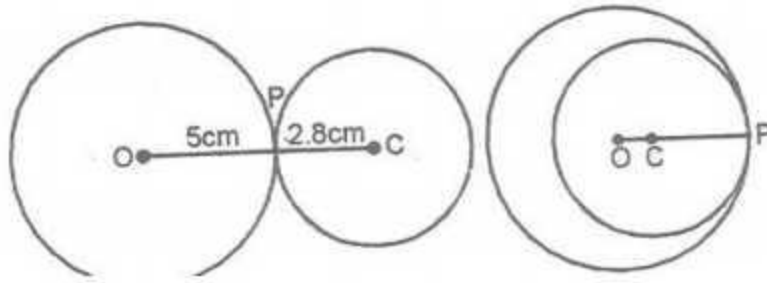
$$\therefore AB = 2 \times PB = 2 \times 12 = 24\text{cm}.$$

Q5. Two circles of radii 5cm and 2.8cm touch each other. find the distance between their centre if they touch:

(i) Externally (ii) Internally.

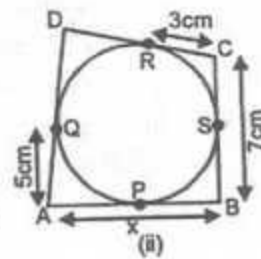
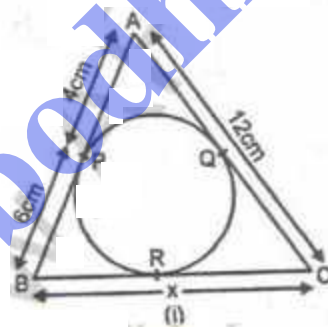
Sol. Radii of the circles are 5cm and 2.8cm.

i.e. $OP = 5\text{cm}$ and $CP = 2.8\text{cm}$.

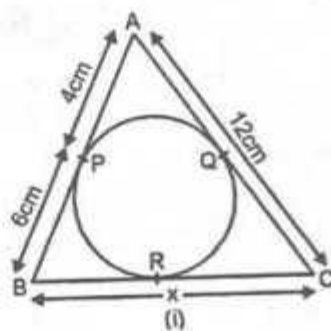


- (i) when the circles touch externally, then the distance between their centres $= OC = 5 + 2.8 = 7.8 \text{ cm}$.
- (ii) when the circles touch internally, then the distance between their centres $= OC = 5 - 2.8 = 2.2$.

- Q6. (a) In fig(i) given below, triangle ABC is circumscribed, find x .
- (b) In fig(ii) given below, quadrilateral ABCD is circumscribed, find x .



- Sol. (a) From A, AP and AQ are the tangents to the circle.
 $\therefore AQ = AP = 4\text{cm}$.



But $AC = 12\text{cm}$, $CQ = 12 - 4 = 8\text{cm}$.

From B, BP and BR are the tangents to the circle.

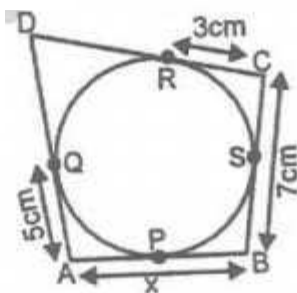
$$\therefore BR = BP = 6\text{cm}.$$

Similarly from C, CQ and CR are the tangents

$$\therefore CR = CQ = 8\text{cm}.$$

$$\Rightarrow x = BC = BR + CR = 6 + 8 = 14\text{cm}.$$

- (b) From C, CR and CS are the tangents to the circle.



$$\therefore CS = CR = 3 \text{ cm}$$

$$\text{But } BC = 7 \text{ cm.}$$

$$\therefore BS = BC - SC = 7 - 3 = 4 \text{ cm.}$$

Now from B, BP and BS are tangents to the circle.

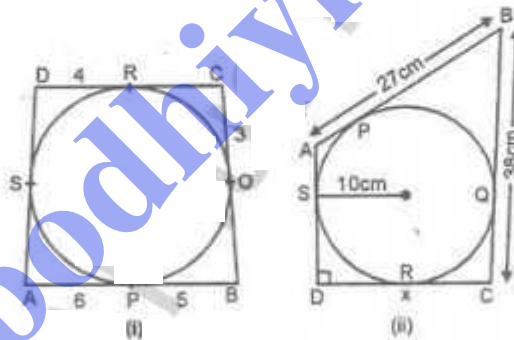
$$\therefore BP = BS = 4 \text{ cm.}$$

and from A, AP and AQ are the tangents to the circle.

$$\therefore AP = AQ = 5 \text{ cm.}$$

$$x = AB = AP + BP = 5 + 4 = 9 \text{ cm.}$$

- Q7. (a) In fig(i) given below, quad. ABCD is circumscribed; find the perimeter of quad. ABCD.
 (b) In fig(ii) given below, quad. ABCD is circumscribed and $AD + DC$, find x if radius of incircle is 10 cm.



Sol. From A, AP and AS are the tangents to the circle.

$$\therefore AS = AP = 6$$

from B, BP and BQ are the tangents. $\therefore BQ = BP = 5 \text{ cm.}$

from C, CQ and CR are the tangents. $\therefore CR = CQ = 3 \text{ cm.}$

and from D, DS and DR are the tangents. $\therefore DS = DR = 4 \text{ cm.}$

$\therefore$ perimeter of the quad. ABCD = $6+5+3+3+4+4+6+5 = 36\text{cm}$.

(b) In the circle with centre O, radius = 10cm , $\angle ADC = 90^\circ$

$PB = 27\text{cm}$, $BC = 38\text{cm}$.

$\therefore OS$ is radius and AD is the tangent

$\therefore OS \perp AD \Rightarrow SD = OS = 10\text{cm}$.

Now from D, DR and DS are the tangents to the circle.

$\therefore DR = DS = 10\text{cm}$.

From B, BP and BQ are tangents to the circle.

$BQ = BP = 27$

$CQ = CB - BQ = 38 - 27 = 11\text{cm}$.

Now from C, CQ and CR are the tangents to the circle.

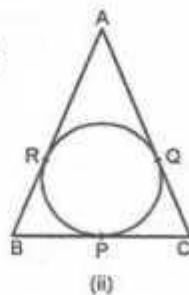
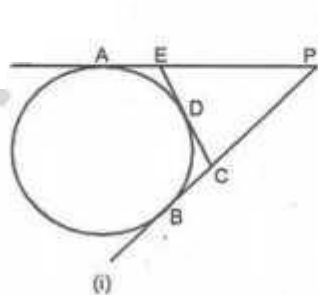
$CR = CQ = 11\text{cm}$.

$DC = x = DR + CR = 10 + 11 = 21\text{cm}$.

Q8.

(a) In the fig.(i) given below, from an external point P tangents PA and PB are drawn to a circle. CE is a tangent to the circle at D. If $AP = 15\text{cm}$, find the perimeter of the $\triangle PEC$.

(b) In the fig.(ii) given below, the incircle of $\triangle ABC$ touches the sides BC , CA and AB at point P, Q and R respectively. prove that $AR + BP + CQ = RB + PC + QA$.



Sol (a) From E, EA and ED are the tangents

$$\therefore EA = ED \quad \text{--- (i)}$$

Similarly, from C CD and CB are the tangents

$$\therefore CD = CB \quad \text{--- (ii)}$$

and from P, PA and PB are the tangents

$$\therefore PA = PB = 15 \text{ cm.}$$

$$\begin{aligned} \text{Now perimeter of } \triangle PEC &= PE + EC + CP = PE + ED + DC + PC \\ &= PE + EA + PC + CB = PA + PB = 15 + 15 = 30 \text{ cm.} \end{aligned}$$

(b) Given: In circle of $\triangle ABC$ touches the sides of the triangle BC, CA and AB at points P, Q, R respectively

To prove: $AR + BP + CQ = RB + PC + QA$

proof: $\therefore AR$ and QA are the tangents to the circle.

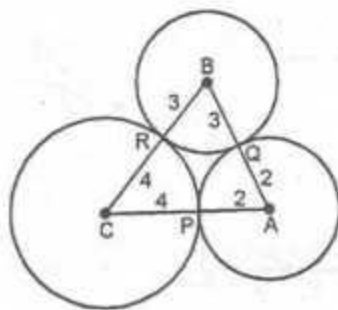
$$\therefore AR = QA$$

Similarly, $BP = RB$ and $CQ = PC$

$$\text{Adding, we get } AR + BP + CQ = QA + RB + PC = RB + PC + QA.$$

Q7. 3 circles of radii 2cm, 3cm and 4cm touch each other Externally. Find the perimeter of the triangle obtained on joining the centres of these circle.

Sol. Three circles with centres A, B and C touch each other externally at P, Q and R respectively and the radii of these circles are 2cm, 3cm, 4cm.



By joining their Centres $\triangle ABC$ is formed in which

$$AB = 2+3 = 5\text{cm.}$$

$$BC = 3+4 = 7\text{cm.}$$

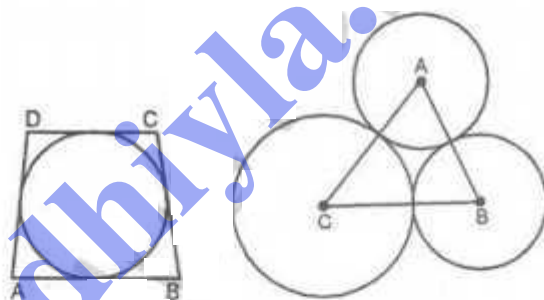
$$CA = 4+2 = 6\text{cm.}$$

$$\therefore \text{perimeter of the } \triangle ABC, AB+BC+CA = 5+7+6 = 18\text{cm.}$$

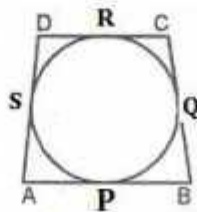
Q10. (a) In the fig(i) given below, the sides of the quadrilateral touch the circle. prove that $AB+CD = BC+DA$.

(b) In the fig(ii) given below, 3 circles with centres A, B and C touch each other externally. If $AB = 5\text{cm}$, $BC = 7\text{cm}$ and $CA = 6\text{cm}$, find the radii of the 3 circles.

(Hint: let radii of 3 circles be x, y and z then $x+y = 5$, $y+z = 7$ and $z+x = 6$)



Sol. (a) Given: Sides of quad. ABCD touches the circle at P, Q, R and S respectively.



To prove: $AB + CD = BC + DA$

proof: Tangents from A, AP and AS are to the Circle.

$$\therefore AP = AS$$

Similarly, $PB = BQ$, $CR = CQ$ and $DR = DS$

Adding then, we get $AP + PB + CR + RD = SA + BQ + CQ + DS$

$$\Rightarrow AP + PB + CR + RD = BQ + CQ + AS + SD$$

$$\Rightarrow AB + CD = BC + DA$$

(b) 3 circles with centres A, B and C are drawn in such a way that they touch in each other Externally and $AB = 5\text{cm}$, $BC = 7\text{cm}$, $CA = 6\text{cm}$.

Let the radii of these circles be x, y and z

$$\therefore x + y = 5\text{cm} \quad \text{--- (i)}$$

$$y + z = 7\text{cm} \quad \text{--- (ii)}$$

$$z + x = 6\text{cm} \quad \text{--- (iii)}$$

adding, we get $2(x + y + z) = 18 \Rightarrow x + y + z = 9\text{cm} \quad \text{--- (iv)}$

Now Subtracting (i) from (iv)

$$z = 9 - 5 = 4\text{cm}$$

Similarly Subtracting (ii) and (iii) respectively from (iv)

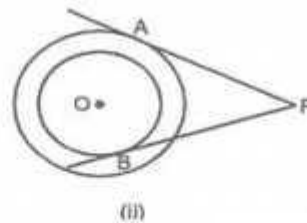
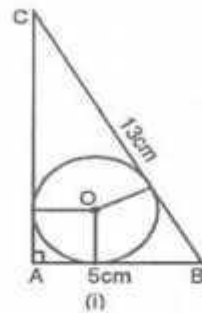
$$x = 9 - 7 = 2\text{cm}$$

$$y = 9 - 6 = 3\text{cm}$$

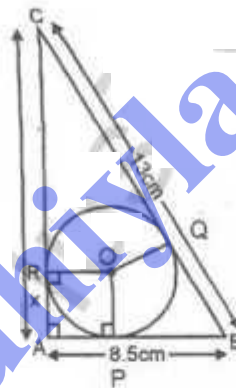
$\therefore$ Radii of the circle are 2cm, 3cm and 4cm.

Q11: (a) In the fig (a) given below, ABC is a right angled triangle at A with sides $AB = 5\text{cm}$ and $BC = 13\text{cm}$. A circle with centre O has been inscribed in the $\triangle ABC$. Calculate the radius of the incircle.

(b) In the fig (b) given below, two concentric circles with centre O are of radii 5cm and 3cm. From an external point P, tangent PA and PB are drawn to these circles. If $AP = 12\text{cm}$, find BP.



- Sol. (a) $\triangle ABC$ is a right triangle right angle at A and $AB = 5\text{cm}$, $BC = 13\text{cm}$.
 $\therefore BC^2 = AC^2 + AB^2$ (By Pythagoras axiom)
 $\Rightarrow (13)^2 = AC^2 + (5)^2 \Rightarrow AC^2 = 169 - 25 = 144 \Rightarrow AC = 12\text{cm}$.



O is the Orthocentre of the incircle of the $\triangle ABC$ and the circle touches the sides AB, BC and CA at P, Q and R respectively.

$\therefore OP, OQ$ and OR are joined.

$\therefore OP$ and OR are radius on the tangents AB and AC respectively.

$\therefore OP \perp PB$ and $OR \perp AC$.

$\therefore APOR$ is a Square.

$\therefore AP = AR = OP = OR$.

Now in right $\triangle ABC$, $BC^2 = AB^2 + AC^2$

$$\Rightarrow (13)^2 = (5)^2 + AC^2 \Rightarrow AC = 12 \text{ cm.}$$

let $AP = AR = x$ (Side of the square APOR)

$\therefore BP = 5 - x$ $BO = BQ$ (Tangents from B to the circle)

$$\therefore BQ = 5 - x$$

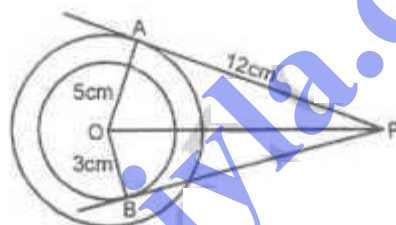
again $CR = AC - x = 12 - x$ and $CR = CQ$ (tangents from C to the circle)

$$\therefore CQ = 12 - x$$

$$\text{Now } BC = BQ + CQ \Rightarrow 13 = 5 - x + 12 - x \Rightarrow 2x = 4 \Rightarrow x = 2 \text{ cm.}$$

Hence radius of the circle = 2 cm.

(b) radius of outer circle = 5 cm and radius of inner circle = 3 cm.



$$\therefore OA = 5 \text{ cm, } OB = 3 \text{ cm and } AP = 12 \text{ cm.}$$

$\therefore OA$ is radius and AP is the tangent.

$$OA \perp AP.$$

$$\text{In right } \triangle OAP, OP^2 = OA^2 + AP^2 = 5^2 + 12^2 = 169 \Rightarrow OP = 13 \text{ cm.}$$

Similarly, in right $\triangle OBP$,

$$OP^2 = OB^2 + BP^2$$

$$\Rightarrow (13)^2 = (3)^2 + BP^2$$

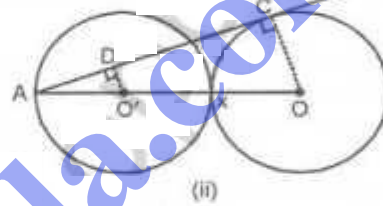
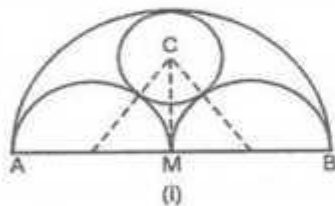
$$\Rightarrow BP^2 = 169 - 9 = 160$$

$$\Rightarrow BP = \sqrt{160} = \sqrt{16 \times 10} = 4\sqrt{10} \text{ cm.}$$

$$\therefore BP = 4\sqrt{10} \text{ cm.}$$

Q12. (a) In the fig (i) given below, $AB = 8\text{cm}$ and M is the Mid point of AB . Semi-circles are drawn on AB , AM and MB as diameter. A circle with Centre C touches all three Semi-circles as shown, find its radius.

(b) In the fig (ii) given below, equal circle with Centre O and O' touch each other at X . OO' is produced to meet a Circle O' at A . AC is tangent to the circle whose Centre is O . $O'D$ is perpendicular to AC . find the value of: (i) $\frac{AO'}{AO}$
(ii) $\frac{\text{area of } \triangle ADO'}{\text{area of } \triangle ACO}$



Sol. (a) let x be the radius of the circle with Centre C and radii of each circle equal semi circles = $\frac{4}{2} = 2\text{cm}$.

$$CP = x + 2 \text{ and } CM = 4 - x.$$

$$\text{In right } \triangle PCM, (PC)^2 = (PM)^2 + (CM)^2$$

$$\Rightarrow (x+2)^2 = (2)^2 + (4-x)^2 \Rightarrow x^2 + 4x + 4 = 4 + 16 - 8x + x^2$$

$$\Rightarrow 12x = 16 \Rightarrow x = \frac{4}{3} = 1\frac{1}{3}\text{cm}.$$

$$\therefore \text{radius} = 1\frac{1}{3}\text{cm}.$$

(b) Two equal circles with Centre O and O' touch each other externally at X . OO' is joined and produced to meet the Circle at A . AC is the tangent to the circle with Centre O . $O'D$ is perpendicular AC . join OC .

$\therefore$ OC is radius and AC is tangent, then $OC \perp AC$.

let radius of each equal circle = r

In $\triangle ADO'$ and $\triangle ACO$, $\angle A = \angle A$ (common), $\angle D = \angle C$ (each 90°)

$\therefore \triangle ADO' \sim \triangle ACO$ (AA axiom of similarity)

$$(i) \frac{AO'}{AO} = \frac{r}{3r} = \frac{1}{3}$$

(ii) $\triangle ADO' \sim \triangle ACO$

$$\frac{\text{Area of } \triangle ADO'}{\text{Area of } \triangle ACO} = \left(\frac{AO'}{AO}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

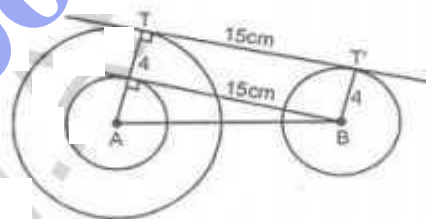
Q13. The length of the direct common tangent to two circles of radii 12cm and 4cm is 15cm. Calculate the distance b/w their centres.

Sol. Let r be the radius of the circles with centre A and B respectively and let TT' be their common tangent.

$$(TT')^2 = (AB)^2 - (R-r)^2 \Rightarrow (15)^2 = (AB)^2 - (12-4)^2$$

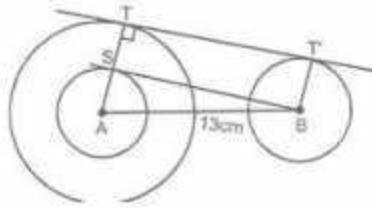
$$\Rightarrow 225 = (AB)^2 - 64 \Rightarrow AB^2 = 289 \Rightarrow AB = 17\text{cm.}$$

Hence distance between their centres 17cm.



14. Calculate the length of a direct common tangent to two circles of radii 3cm and 8cm with their centres 13cm apart.

Sol. Let A and B be the centres of the circles whose radii are 8cm and 3cm and TT' length of their common tangent and $AB = 13\text{cm}$.

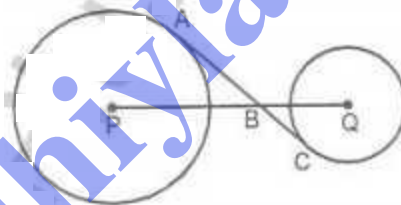


Now $(TT')^2 = (AB)^2 - (R-r)^2 = (13)^2 - (8-3)^2 = 169 - 25$

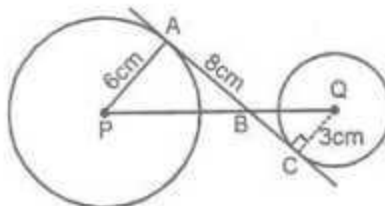
$(TT')^2 = 144 \Rightarrow TT' = 12\text{cm.}$

Hence length of common tangent = 12cm.

- Q15. In the given figure, AC is a transverse common tangent to two circles with centres P and Q and of radii 6cm and 3cm respectively. Given that AB = 8cm, calculate PQ.



- Sol. AC is a transverse common tangent to the two circles with centres P and Q and of radii 6cm and 3cm respectively. AB = 8cm. Join AP and CQ.



In $\triangle PAB$, $PB^2 = PA^2 + AB^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow PB = 10\text{cm}$

Now in $\triangle PAB$ and $\triangle BCQ$,

$\angle A = \angle C$ (each 90°), $\angle ABP = \angle CBQ$ (vertically opp. angles)

$\therefore \triangle PAB \sim \triangle BCQ$ (AA axiom of similarity)

$\therefore \frac{AP}{CQ} = \frac{PB}{BQ} \Rightarrow \frac{6}{3} = \frac{10}{BQ} \Rightarrow BQ = 5\text{cm}$

$\therefore PQ = PB + BQ = 10 + 5 = 15\text{cm}$

Q16. Two circles with Centres A, B of radii 6cm and 3cm respectively. If $AB = 15\text{cm}$, find the length of a transverse common tangent to these circles.

Sol.

$AB = 15\text{cm}$

Radius of the circle with Centre A = 6cm.

and radius of circle with Centre B = 3cm.



Let $AP = x$, then $PB = 15 - x$.

In $\triangle ATP$ and $\triangle SBP$

$\angle T = \angle S$ (each 90°), $\angle APT = \angle BPS$ (vertically opp. angles)

$\therefore \triangle ATP \sim \triangle SBP$ (AA axiom of similarity)

$\frac{AT}{BS} = \frac{AP}{PB} \Rightarrow \frac{6}{3} = \frac{x}{15-x} \Rightarrow 30 - 2x = x \Rightarrow 30 = 3x$

$\Rightarrow x = 10\text{cm}$

$\therefore AP = 10\text{cm}$, $PB = 15 - 10 = 5\text{cm}$

Now in right $\triangle ATP$, $AP^2 = AT^2 + TP^2$

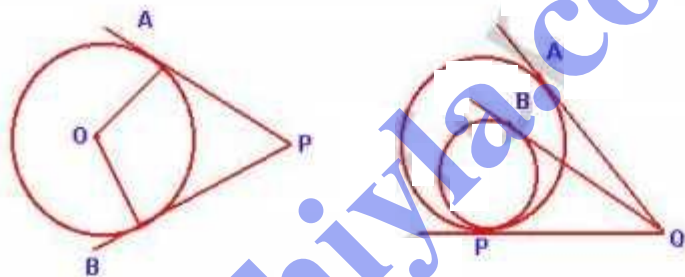
$$\Rightarrow 10^2 = 6^2 + TP^2 \Rightarrow TP = 8 \text{ cm.}$$

Similarly, in right $\triangle PSB$, $PB^2 = BS^2 + PS^2$

$$\Rightarrow 5^2 = 3^2 + (PS)^2 \Rightarrow PS = 4 \text{ cm.}$$

$$\text{Hence } TS = TP + PS = 8 + 4 = 12 \text{ cm.}$$

- Q17. (a) In the fig(i) given below, PA and PB are tangents to a circle with centre O. prove that $\angle APB$ and $\angle AOB$ are supplementary.
- (b) In the fig(ii) given below, two circles touch internally at P. from an external point Q on the common tangent at P, two tangents QA and QB are drawn to the two circles. prove that $QA = QB$.



Sol.

$$\text{In } \triangle BOP, \angle POB + \angle OPB = 90^\circ \quad \text{--- (1)}$$

$$\text{In } \triangle OAP, \angle AOP + \angle APO = 90^\circ \quad \text{--- (2)}$$

By adding (1) and (2),

$\angle APB$ and $\angle AOB$ are supplementary.

(ii) $PQ = BQ$ (tangents drawn from an external point to the circle are equal)

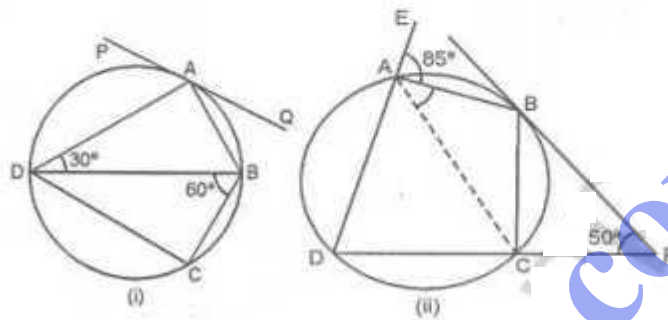
Similarly $PQ = AQ$

Hence $BQ = AQ$.

Q18. (a) In the fig(i) given below, PQ is a tangent to the circle at A, BB is a diameter, $\angle ADB = 30^\circ$ and $\angle CBD = 60^\circ$, calculate

(i) $\angle QAB$ (ii) $\angle PAD$ (iii) $\angle CDB$

(b) In the fig(ii) given below, ABCD is a cyclic quadrilateral. the tangent to the circle at B meets DC produced at F. if $\angle EAB = 85^\circ$ and $\angle BFC = 50^\circ$, find $\angle CAB$.



sol. (a) PQ is tangent and AD is chord

(i) $\angle QAB = \angle BDA = 30^\circ$ (Angles in the alternate segment)

(ii) In $\triangle ADB$, $\angle DAB = 90^\circ$ (angle in a semi-circle)

and $\angle ADB = 30^\circ$

$$\therefore \angle ABD = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

But $\angle PAD = \angle ABD$ (Angles in the alternate segment)
 $= 60^\circ$

(iii) In right $\triangle BCD$, $\angle BCD = 90^\circ$ (angle in a semi-circle)

$\angle CBD = 60^\circ$ (given)

$$\therefore \angle CDB = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

(b) ABCD is a cyclic quadrilateral

$$\text{Ext. } \angle EAB = \angle BCD = 85^\circ$$

But $\angle BCD + \angle BCF = 180^\circ$ (linear pair)

$$\Rightarrow 85^\circ + \angle BCF = 180^\circ \Rightarrow \angle BCF = 95^\circ$$

Now in $\triangle BCF$, $\angle BCF + \angle BFC + \angle CBF = 180^\circ$

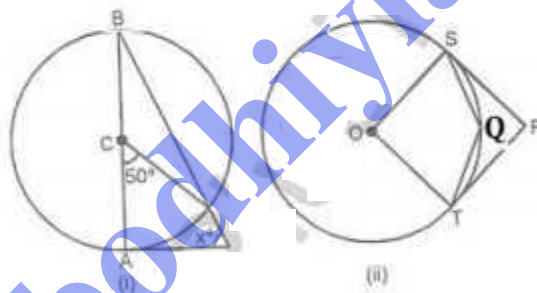
$$\Rightarrow 95^\circ + 50^\circ + \angle CBF = 180^\circ \Rightarrow \angle CBF = 35^\circ$$

$\therefore$ BF is a tangent and BC is the chord.

$$\therefore \angle CAB = \angle CBF = 35^\circ$$

Q9. (a) In the fig(i) given below, AB is a diameter of the circle, with centre O and AT is a tangent. Calculate the numerical value of x .

(b) In the fig(ii) given below, O is the centre of the circle. PS and PT are tangents and $\angle SPT = 84^\circ$. Calculate the sizes of the angles TOS and TQS.



Sol. (a) arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore \angle ABC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 50^\circ = 25^\circ \text{ or } \angle ABT = 25^\circ$$

OA is radius and AT is the tangent. $\therefore OA \perp AT$

$$\text{or } \angle OAT \text{ or } \angle BAT = 90^\circ$$

Now in $\triangle ABT$, $\angle BAT + \angle ABT + \angle ATB = 180^\circ$

$$\Rightarrow 90^\circ + 25^\circ + x = 180^\circ \Rightarrow x = 65^\circ$$

(b) TP and SP are the tangents and OS and OT are the radii

$\therefore OS \perp SP$ and $OT \perp TP$

$$\therefore \angle OSP = \angle OTP = 90^\circ \quad \therefore \angle SPT = 84^\circ$$

$$\text{But } \angle SOT + \angle OTP + \angle TPS + \angle OSP = 360^\circ$$

$$\Rightarrow \angle SOT + 264^\circ = 360^\circ \Rightarrow \angle SOT = 96^\circ$$

$$\text{and reflex } \angle SOT = 360^\circ - 96^\circ = 264^\circ$$

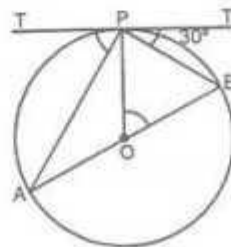
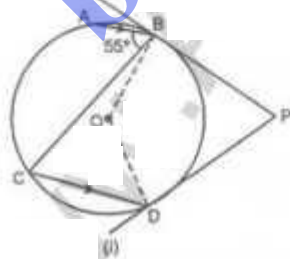
Now major arc ST subtends reflex $\angle SOT$ at the Centre and $\angle TQS$ at the remaining part of the circle.

$$\therefore \angle TQS = \frac{1}{2} \text{ reflex } \angle SOT = \frac{1}{2} \times 264^\circ = 132^\circ$$

$$\text{Hence } \angle TOS = 96^\circ \text{ and } \angle TQS = 132^\circ$$

Q20. (a) In the fig(i) given below, O is the Centre of the Circle. The tangent at B and D meet at P. If $AB \parallel CD$ and $\angle ABC = 55^\circ$, find (i) $\angle BOD$ (ii) $\angle BPD$.

(b) In the fig(ii) given below, O is the Centre of the Circle. AB is a diameter, TPT' is a tangent to the circle at P. If $\angle BPT' = 30^\circ$, calculate (i) $\angle APT$ (ii) $\angle BOP$.



Sol.

(a) $AB \parallel CD$

(i) $\angle ABC = \angle BCD$ (Alternate angles)

$$\therefore \angle BCD = 55^\circ$$

Now arc BD subtends $\angle BOD$ at the Centre and $\angle BCD$ at the remaining part of the circle.

$$\therefore \angle BOD = 2 \angle BCD = 2 \times 55^\circ = 110^\circ$$

OB is radius and BP is tangent

$\therefore OB \perp BP$. Similarly $OD \perp DP$

Now in quadrilateral OBPD,

$$\angle BOD + \angle ODP + \angle OBP + \angle BPD = 360^\circ \text{ (angles of quadrilateral)}$$

$$\Rightarrow 110^\circ + 90^\circ + 90^\circ + \angle BPD = 360^\circ$$

$$\Rightarrow \angle BPD = 70^\circ$$

(b) TPT' is the tangent

(i) AB is diameter $\therefore \angle APB = 90^\circ$ (angle in a semi-circle)

and $\angle BPT' = 30^\circ$

$$\therefore \angle APT' = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

(ii) TPT' is tangent and PB is the chord

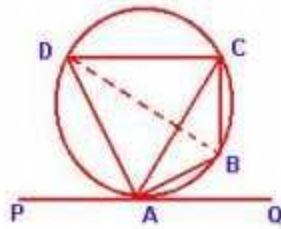
$$\therefore \angle BAP = \angle BPT' = 30^\circ \text{ (angle in the alternate segment)}$$

Now arc BP subtends $\angle BOP$ at the Centre and $\angle BAP$ at the remaining part of the circle.

$$\therefore \angle BOP = 2 \angle BAP = 2 \times 30^\circ = 60^\circ$$

Q21.

In the adjoining fig, ABCD is a cyclic quadrilateral. The line PQ is the tangent to the circle at A. If $\angle CAQ : \angle CAP = 1 : 2$, AB bisects $\angle CAQ$ and AD bisects $\angle CAP$, then find the measure of the angles of the cyclic quadrilateral. Also prove that BD is a diameter of the circle.



Sol. ABCD is a cyclic quadrilateral. PAQ is the tangent to the circle at A.
 $\angle CAD : \angle CAP = 1:2$, AB and AD are the bisectors of $\angle CAQ$ and $\angle CAP$ respectively.

To prove: (i) BD is the diameter of the circle.

(ii) find the measure of the angles of the cyclic quadrilateral.

proof: (i) AB and AD are the bisectors of $\angle CAQ$ and $\angle CAP$ respectively and $\angle CAQ + \angle CAB = 180^\circ$ (linear pair)

$$\therefore \angle CAB + \angle CAD = 90^\circ$$

$$\Rightarrow \angle BAD = 90^\circ$$

Hence BD is at diameter of the circle.

$$(ii) \angle CAQ : \angle CAP = 1:2$$

$$\text{let } \angle CAQ = x \text{ and } \angle CAP = 2x$$

$$\therefore x + 2x = 180^\circ \Rightarrow 3x = 180^\circ \Rightarrow x = 60^\circ$$

$$\therefore \angle CAQ = 60^\circ \text{ and } \angle CAP = 120^\circ$$

$\therefore$ PAQ is the tangent and AC is the chord of the circle

$$\therefore \angle ADC = \angle CAQ = 60^\circ$$

$$\text{Similarly, } \angle ABC = \angle CAP = 120^\circ$$

$$\text{Hence } \angle A = 90^\circ, \angle B = 120^\circ$$

$$\angle C = 90^\circ \text{ and } \angle D = 60^\circ$$

Q22. In a $\triangle ABC$, the incircle (centre O) touches BC, CA and AB at P, Q and R respectively. Calculate (i) $\angle QOR$ (ii) $\angle QPR$.

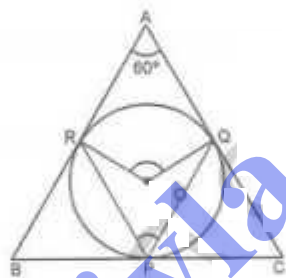
Given that $\angle A = 60^\circ$

Sol. OQ and OR are the radii and AC and AB are tangents
 $\therefore OQ \perp AC$ and $OR \perp AB$

Now in quadrilateral AROQ,

$$\angle A = 60^\circ, \angle ORA = 90^\circ \text{ and } \angle OQA = 90^\circ$$

$$\begin{aligned} \angle ROQ &= 360^\circ - (\angle A + \angle ORA + \angle OQA) \\ &= 360^\circ - (60^\circ + 90^\circ + 90^\circ) = 120^\circ \end{aligned}$$



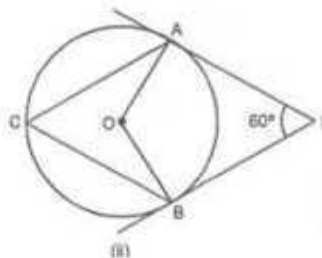
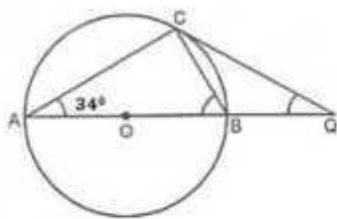
Now arc RQ subtends $\angle ROQ$ at the Centre and $\angle RPQ$ at the remaining part of the circle.

$$\therefore \angle ROQ = 2 \angle RPQ \Rightarrow \angle RPQ = \frac{1}{2} \angle ROQ = \frac{1}{2} \times 120^\circ = 60^\circ$$

Q23. (a) In the fig(i) given below, AB is a diameter. the tangent at C meets AB produced at Q, $\angle CAB = 34^\circ$, find (i) $\angle CBA$ (ii) $\angle CQA$.

(b) In the fig(ii) given below, AP and BP are tangents to the circle with centre O. Given $\angle APB = 60^\circ$ calculate:

(i) $\angle AOB$ (ii) $\angle OAB$ (iii) $\angle ACB$.



Sol. (a) AB is the diameter.

$$\angle ACB = 90^\circ \text{ and } \angle CAB = 30^\circ.$$

$$\text{In } \triangle ABC, \angle ACB + \angle CAB + \angle CBA = 180^\circ$$

$$\Rightarrow 90^\circ + 30^\circ + \angle CBA = 180^\circ \Rightarrow \angle CBA = 60^\circ$$

$$\text{and ext. } \angle CBQ = 180^\circ - 60^\circ = 120^\circ$$

$\therefore$ CQ is tangent and CB is chord

$$\therefore \angle BCQ = \angle CAB = 30^\circ \text{ (Angles in the alternate segment)}$$

Now in $\triangle BCQ$,

$$\angle BCQ + \angle CBQ + \angle CQB = 180^\circ \text{ (Angles of a } \triangle)$$

$$\Rightarrow 30^\circ + 120^\circ + \angle CQB = 180^\circ$$

$$\Rightarrow \angle CQB = 30^\circ$$

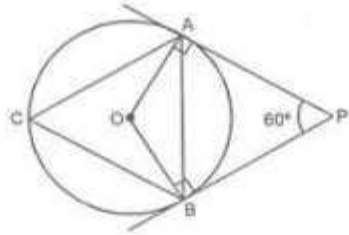
(b) (i) AP and BP are the tangents to the circle and OA and OB are radii on it.

$$\therefore OA \perp AP \text{ and } OB \perp BP$$

$$\therefore \angle AOB = 180^\circ - 60^\circ = 120^\circ$$

Now in $\triangle OAB$, $OA = OB$ (Radii of the same circle)

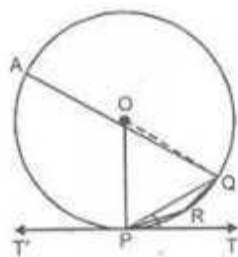
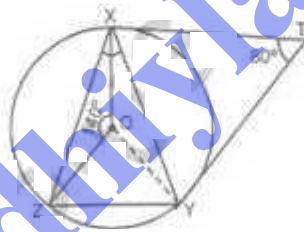
$$\therefore \angle OAB = \angle OBA = \frac{1}{2}(180^\circ - 120^\circ) = \frac{1}{2} \times 60^\circ = 30^\circ$$



(iii) arc AB subtends $\angle AOB$ at the Centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore \angle AOB = 2 \angle ACB \Rightarrow \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 120^\circ = 60^\circ.$$

Q24 (a) In the fig(1) given below, O is the Centre of the Circumcircle of $\triangle XYZ$. tangents at X and Y intersect at T. Given $\angle XTY = 80^\circ$ and $\angle XOZ = 140^\circ$, calculate the value of $\angle ZXY$.



Join OY , OX and OY is radius of the circle.

$\therefore OX \perp XT$ and $OY \perp YT$ and $\angle XTY = 80^\circ$

$$\therefore \angle XOY = 180^\circ - \angle XTY = 180^\circ - 80^\circ = 100^\circ$$

$$\text{Now } \angle ZOY = 360^\circ - (\angle XOZ + \angle XOY) = 360^\circ - (140^\circ + 100^\circ) \\ = 120^\circ$$

But arc ZY subtends $\angle ZOY$ at the centre and $\angle ZXY$ at the remaining part of the circle.

$$\therefore \angle ZXY = \frac{1}{2} \angle ZOY = \frac{1}{2} \times 120^\circ = 60^\circ$$

(b) In the fig(ii) given below, O is the centre of the circle and PT is the tangent to the circle at P . Given $\angle QPT = 30^\circ$, calculate: (i) $\angle PRQ$ (ii) $\angle POQ$.

Sol. (i) PT is tangent and OP is radius, then $OP \perp PT$.

$$\therefore \angle OPT = 90^\circ \text{ but } \angle QPT = 30^\circ$$

$$\therefore \angle OPQ = 90^\circ - 30^\circ = 60^\circ$$

$$\therefore OP = OQ \text{ (radii of the same circle)}$$

$$\therefore \angle QOP = \angle OPQ = 60^\circ$$

$$\text{Hence in } \triangle POQ, \angle POQ = 60^\circ$$

(ii) Take a point A on the circumference and join AP and AQ .
Now arc PQ subtends $\angle POQ$ at the centre and $\angle PAQ$ on the circumference of the circle.

$$\therefore \angle PAQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 60^\circ = 30^\circ$$

Now $APRQ$ is a cyclic quadrilateral

$$\therefore \angle PAQ + \angle PRQ = 180^\circ$$

$$\Rightarrow 30^\circ + \angle PRQ = 180^\circ$$

$$\Rightarrow \angle PRQ = 150^\circ$$

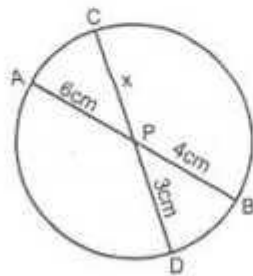
Q25. Two chords AB, CD of a circle intersect internally at a point P. if

(i) $AB = 6\text{cm}$, $PB = 4\text{cm}$ and $PD = 3\text{cm}$, find PC.

(ii) $AB = 12\text{cm}$, $AP = 2\text{cm}$ and $PC = 5\text{cm}$, find PD.

(iii) $AP = 5\text{cm}$, $PB = 6\text{cm}$ and $CD = 13\text{cm}$, find CP.

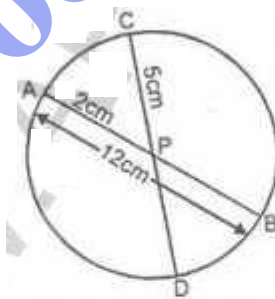
Sol. In a circle, two chords AB and CD intersect each other at P internally. $\therefore AP \cdot PB = CP \cdot PD$.



(i), when $AP = 6\text{cm}$, $PB = 4\text{cm}$, $PD = 3\text{cm}$, then

$$\Rightarrow 6 \times 4 = CP \times 3 \Rightarrow CP = 8\text{cm}.$$

(ii) when $AB = 12\text{cm}$, $AP = 2\text{cm}$, $PC = 5\text{cm}$.



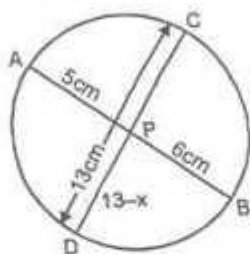
$$PB = AB - AP = 12 - 2 = 10\text{cm}.$$

$$2 \times 10 = 5 \times PD$$

$$\Rightarrow PD = \frac{2 \times 10}{5} = 4\text{cm}.$$

(iii) when $AP = 5\text{cm}$, $PB = 6\text{cm}$, $CD = 13\text{cm}$.

let $CP = x$, then $PD = CD - CP = 13 - x$.



$$\therefore AP \times PB = CP \times PD$$

$$5 \times 6 = x(13 - x) \Rightarrow 30 = 13x - x^2 \Rightarrow x^2 - 13x + 30 = 0$$

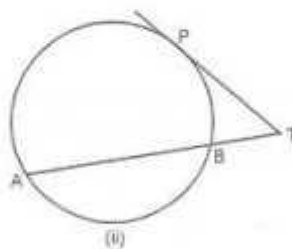
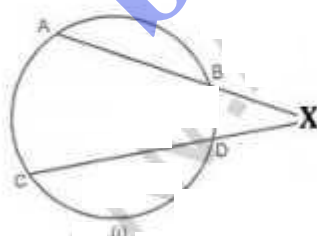
$$\Rightarrow (x - 10)(x - 3) = 0$$

Either $x - 10 = 0$, then $x = 10$ or $x - 3 = 0$, then $x = 3$.

$\therefore CP = 10\text{cm}$ or 3cm .

Q26 (a) In the fig(i) given below, chords AB and CD when extended meet at X. Given $AB = 4\text{cm}$, $BX = 6\text{cm}$, $XD = 5\text{cm}$, calculate the length of D.

(b) In the fig(ii) given below, PT is a tangent to the circle. find if TP if $AT = 16\text{cm}$ and $AB = 12\text{cm}$.



Sol. (a) AB and CD are chords of a circle which intersect at X outside the circle.

$$\therefore AX \cdot XB = CX \cdot XD.$$

$$\text{Now } AB = 4 \text{ cm, } BX = 6 \text{ cm, } XD = 5 \text{ cm.}$$

$$\therefore AX = AB + BX = 4 + 6 = 10 \text{ cm.}$$

$$\therefore 10 \times 6 = CX \times 5 \Rightarrow CX = \frac{10 \times 6}{5} = 12 \text{ cm.}$$

$$\therefore CD = CX - DX = 12 - 5 = 7 \text{ cm.}$$

(b) PT is the tangent to the circle and AT is a secant.

$$\therefore PT^2 = TA \times TB$$

$$\text{Now } TA = 16 \text{ cm, } AB = 12 \text{ cm.}$$

$$TB = AT - AB = 16 - 12 = 4 \text{ cm.}$$

$$PT^2 = 16 \times 4 = 64 \Rightarrow PT = 8 \text{ cm.}$$

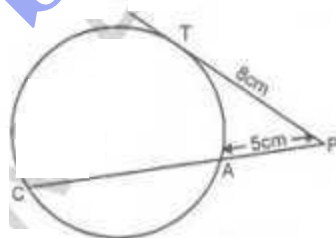
Q27. PAB is a secant and PT is tangent to a circle. if

(i) $PT = 8 \text{ cm}$ and $PA = 5 \text{ cm}$, find the length of AB.

(ii) $PA = 4.5 \text{ cm}$ and $AB = 13.5 \text{ cm}$, find the length of PT.

Sol.

$\therefore$ PT is the tangent and PAB is the secant of the circle.



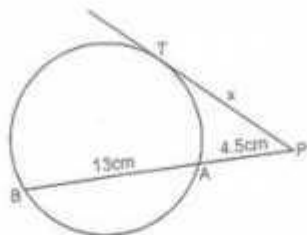
$$\therefore PT^2 = PA \cdot PB.$$

(i) $PT = 8\text{cm}$, $PA = 5\text{cm}$

$$(8)^2 = 5 \times PB \Rightarrow PB = \frac{64}{5} = 12.8\text{cm}$$

$$\therefore AB = PB - PA = 12.8 - 5.0 = 7.8\text{cm}$$

(ii) $PT^2 = PA \times PB$



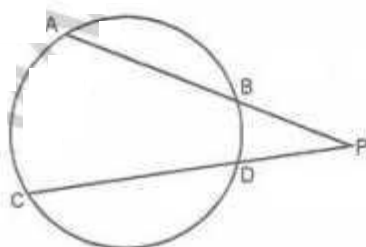
But $PA = 4.5\text{cm}$, $AB = 13.5\text{cm}$

$$PB = PA + AB = 4.5 + 13.5 = 18\text{cm}$$

$$\text{Now } PT^2 = 4.5 \times 18 = 81 \Rightarrow (9)^2 \Rightarrow PT = 9\text{cm}$$

Q28 Two chords AB, CD of a circle intersect externally at a point P.
if $PA = PC$, prove that $AB = CD$.

sol.



Given: Two chords AB and CD intersect each other at P outside the circle.
 $PA = PC$.

To prove: $AB = CD$.

Proof: Chords AB and CD intersect each other at P outside the circle.

$$\therefore PA \times PB = PC \times PD.$$

$$\text{But } PA = PC \text{ (given)} \quad \text{--- (i)}$$

$$\therefore PB = PD \quad \text{--- (ii)}$$

Subtracting (ii) from (i)

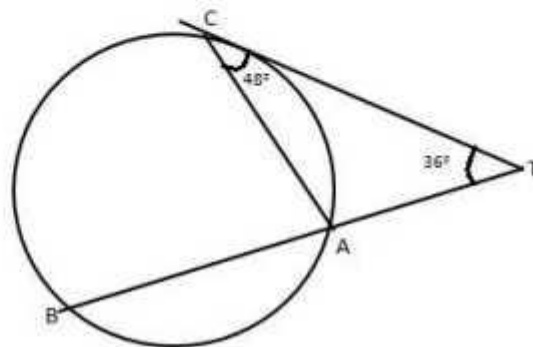
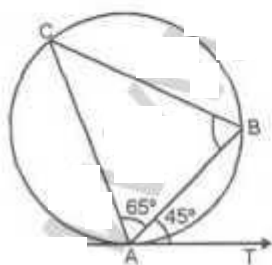
$$PA - PB = PC - PD$$

$$\Rightarrow AB = CD.$$

Q29 (a) In the fig(i) given below, AT is tangent to a circle at A.

If $\angle BAT = 45^\circ$ and $\angle BAC = 65^\circ$, find $\angle ABC$.

(b) In the fig(ii) given below, A, B and C are 3 points on a circle, the tangent at C meets BA produced at T. Given that $\angle ATC = 36^\circ$ and $\angle ACT = 48^\circ$, calculate the angle subtended by AB at the centre of the circle.



Sol. (a) AT is the tangent to the circle at A and AB is the chord of the circle.

$$\therefore \angle ACB = \angle BAT \quad (\text{Angles in the alternate segment})$$

$$= 45^\circ$$

Now in $\triangle ABC$, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$$\Rightarrow \angle ABC + 65^\circ + 60^\circ = 180^\circ \Rightarrow \angle ABC = 55^\circ$$

(b) join OA, OB and CB.

In $\triangle AOC$, Ext. $\angle CAB = \angle AOC + \angle OCA = 36^\circ + 48^\circ = 84^\circ$

But $\angle OCA = \angle ABC$ (angle in the alternate segment)

$$\therefore \angle ABC = 48^\circ$$

But in $\triangle ABC$, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

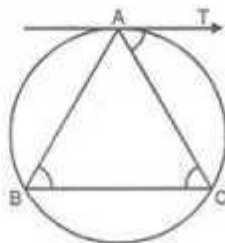
$$\Rightarrow 48^\circ + 84^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 48^\circ$$

$\therefore$ Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore \angle AOB = 2\angle ACB = 2 \times 48^\circ = 96^\circ$$

Q30 In the fig (i) given below, $\triangle ABC$ is isosceles with $AB = AC$. prove that the tangent at A to the circumcircle of $\triangle ABC$ is parallel to BC.



Sol. Given: $\triangle ABC$ is an isosceles triangle with $AB = AC$.
AT is the tangent to the circumcircle at A.

To prove: $AT \parallel BC$.

Proof: In $\triangle ABC$, $AB = AC$ (given)

$\therefore \angle C = \angle B$ (Angles opp. to equal sides)

But AT is the tangent and AC is the chord

$\therefore \angle TAC = \angle B$ (Angles in the alternate segment)

But $\angle B = \angle C$ (given)

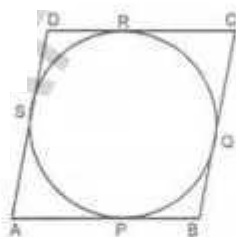
$\therefore \angle TAC = \angle C$

But these are alternate angles

$\therefore AT \parallel BC$.

Q31. If all the sides of a parallelogram touch a circle, prove that the parallelogram is a rhombus.

Sol. Given: ABCD is a parallelogram and a circle touches the sides of parallelogram at P, Q, R and S respectively.



To prove: ABCD is a rhombus.

Proof: tangents from a point to a circle are equal in length.

$\therefore AP = AS$

Similarly, $BP = BQ$, $CR = CQ$ and $DR = DS$.

Adding, we get

$$AP + BP + CR + DR = AS + BQ + CQ + RS$$

$$\Rightarrow AB + CD = AS + DS + BC$$

$$\Rightarrow AB + CD = AD + BC$$

$\therefore ABCD$ is a parallelogram.

$$\therefore AB = CD \text{ and } AD = BC$$

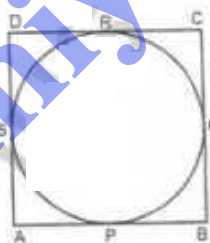
$$\Rightarrow 2AB = 2BC \Rightarrow AB = BC$$

$$\therefore AB = BC = CD = DA$$

$\therefore$ Hence $ABCD$ is a rhombus.

Q32. If the sides of a rectangle touch a circle, prove that the rectangle is a square.

Sol. Given: A circle touches the sides AB , BC , CD and DA of a rectangle $ABCD$ at P , Q , R and S respectively.



To prove: $ABCD$ is a square.

Proof: Tangents from a point to the circle are equal.

$$\therefore AP = PS$$

Similarly, $BP = BQ$, $CR = CQ$ and $DR = DS$

Adding, we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow AB + CD = AD + BC$$

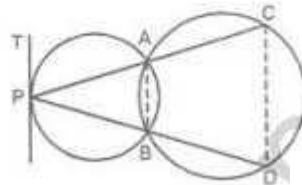
But $AB = CD$ and $AD = BC$ (opp. sides of a rectangle)

$$\therefore AB + AB = BC + BC$$

$$\Rightarrow 2AB = 2BC \Rightarrow AB = BC$$

$$\therefore AB = BC = CD = DA.$$

- Q33. (a) In the figure given below, two circles intersect at A, B. from a point P on one of these circles, two line segments PAC and PBD are drawn, intersecting the other circle at C and D respectively. prove that CD is parallel to the tangent at P.



Sol. Given: Two circles intersect each other at A and B. from a point P on one circle PAC and PBD are drawn. from P, PT is a tangent drawn. CD is joined.

To prove: $PT \parallel CD$.

Construction: Join AB.

proof: PT is tangent and PA is chord.

$$\therefore \angle APT = \angle ABP \quad \text{--- (i)}$$

But BDCA is a cyclic quadrilateral

$$\therefore \text{Ext. } \angle ABP = \angle ACD \quad \text{--- (ii)}$$

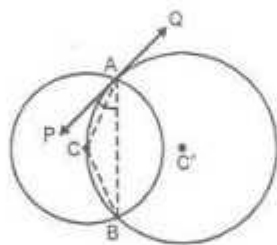
From (i) and (ii),

$$\angle APT = \angle ACD$$

But these are alternate angles.

$$\therefore CD \parallel PT$$

(b) In the fig given below, two circles with centre C & C' intersect at A, B and the point C lies on the circle with centre C' . PQ is a tangent to the circle with centre C' at A . prove that AC bisects $\angle PAB$.



Sol. Given: Two circles with centres C and C' intersect each other at A and B . PQ is a tangent to the circle with centre C' at A .

To prove: AC bisects $\angle PAB$.

Construction: join AB and CP .

proof: In $\triangle ACB$, $AC = BC$ (radii of the same circle)

$$\therefore \angle BAC = \angle ABC \quad \text{--- (i)}$$

PAQ is tangent and AC is the chord of the circle.

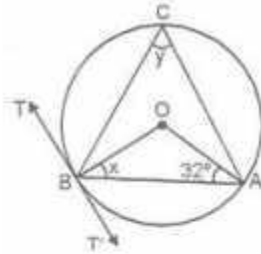
$$\therefore \angle PAC = \angle ABC \quad \text{--- (ii)}$$

From (i) and (ii),

$$\angle BAC = \angle PAC$$

$\therefore$ Hence AC is the bisector of $\angle PAB$.

Q34 (a) In the fig(i) given below, AB is a chord of the circle with centre O . BT is tangent to the circle. If $\angle OAB = 32^\circ$, find the values of x and y .



Sol (a) AB is a chord of circle with centre O. BT is a tangent to the circle and $\angle OAB = 32^\circ$.

In $\triangle OAB$, $OA = OB$ (radii of the same circle)

$$\therefore \angle OAB = \angle OBA$$

$$\therefore x = 32^\circ$$

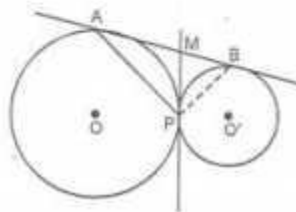
$$\text{and } \angle AOB = 180^\circ - (x + 32^\circ) = 180^\circ - (32^\circ + 32^\circ) = 116^\circ$$

Now arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle

$$\therefore \angle AOB = 2 \angle ACB \Rightarrow \angle ACB = \frac{1}{2} \angle AOB$$

$$\Rightarrow \angle ACB = \frac{1}{2} \times 116^\circ = 58^\circ$$

(b) In the fig. given below, O and O' are centres of two circles touching each other externally at the point P. The common tangent at P meets a direct common tangent AB at M. prove that (i) M bisects AB (ii) $\angle APB = 90^\circ$



Sol.

Given: Two circles with centre O^* and O' touch each other at P externally. From P , a common tangent is drawn and meets the common direct tangent AB at M .

To prove: (i) M bisects AB . i.e. $AM = MB$.

$$(ii) \angle APB = 90^\circ$$

Proof: (i) from M , MA and MP are the tangents

$$\therefore MA = MP \text{ --- (i)}$$

$$\text{Similarly, } MB = MP \text{ --- (ii)}$$

$$\text{from (i) and (ii), } MA = MB$$

or M is the midpoint of AB .

$$(ii) MA = MP$$

$$\therefore \angle MAP = \angle MPA \text{ --- (i)}$$

$$\text{Similarly, } MP = MB \therefore \angle MPB = \angle MBP \text{ --- (ii)}$$

Adding (i) and (ii),

$$\angle MAP + \angle MPB = \angle MPA + \angle MBP$$

$$\Rightarrow \angle APB = \angle MAP + \angle MBP$$

$$\text{But } \angle APB + \angle MAP + \angle MBP = 180^\circ \text{ (Angles of a triangle)}$$

$$\Rightarrow \angle APB + \angle APB = 180^\circ$$

$$\Rightarrow 2 \angle APB = 180^\circ$$

$$\Rightarrow \angle APB = 90^\circ$$