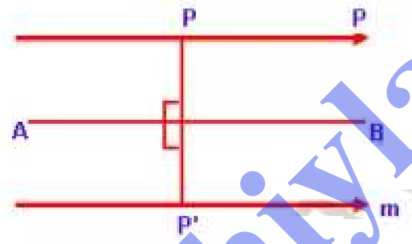
Locus

Exercise -14.1

1. A point moves such that its distance from a fixed line AB is always the same. what is the relation between AB and the path travelled by P ?

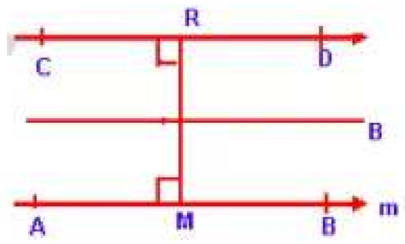
Sol. let point P moves in such a way that it is at a fixed distance from the fixed line AB.



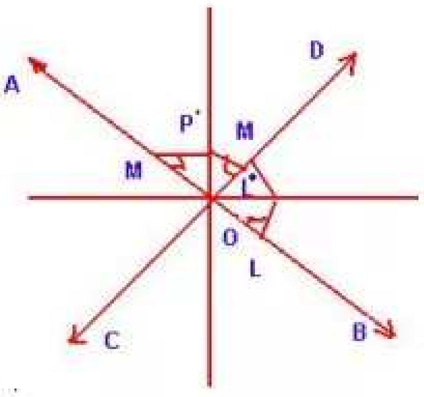
∴ It is a set of two lines parallel to AB drawn on either side of it at equal distance from it.

2. A point P moves so that it perpendicular distances from two given lines AB and CD are equal. State the locus of the point P.

Sol.



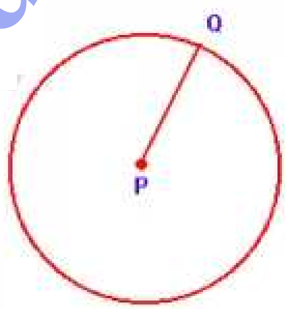
(i) when two Lines AB and CD are parallel, then the Locus of the point P which is equidistant from AB and CD is a Line in the midway of AB and CD and parallel to them.



(ii) If AB and CD are intersecting lines, then the Locus of the point P will be a pair of the straight lines which bisect the angles between the given lines AB & CD.

3. P is a fixed point and a point Q moves such that the distance PQ is Constant. what is the Locus of the path traced out by the point Q?

Sol.



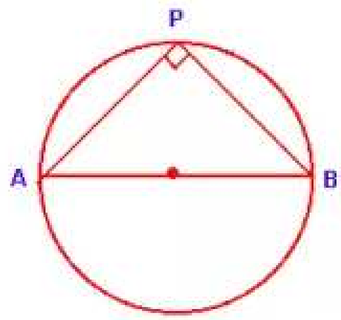
∴ P is the Centroid of the path of Q which is a Circle. The distance between P and Q is radius of the Circle.

∴ Hence Locus of the point Q is a Circle with P as Centre.

4. (i) AB is a fixed line. State the locus of the point P so that $\angle APB = 90^\circ$.

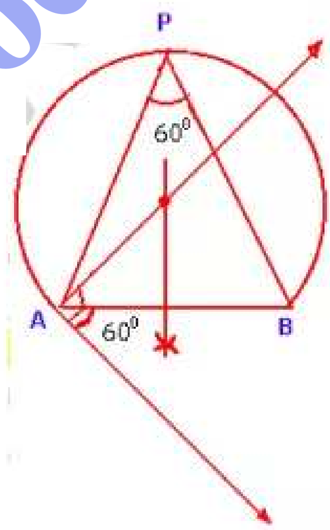
(ii) A, B are fixed points. State the locus of the point P so that $\angle APB = 60^\circ$.

Sol. (i) AB is a fixed line and P is a point such that $\angle APB = 90^\circ$.



The locus of P will be the circle whose diameter is AB. We know that angle in a semicircle is always equal to 90°. $\therefore \angle APB = 90^\circ$

(ii) AB is a fixed line and P is a point such that $\angle APB = 60^\circ$. Then locus of P will be a major segment of a circle whose AB is a chord.



5. Draw and describe the locus in each of the following cases :

(i) The locus of points at a distance 2.5cm from a fixed line.

(ii) The locus of Vertices of all isosceles triangles having a common base.

(iii) The locus of points inside a circle and equidistant from two fixed points on the circle.

(iv) The locus of centers of all circles passing through two fixed points.

(v) The locus of a point in rhombus ABCD which is equidistant from AB and AD

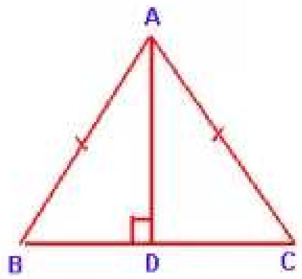
(vi) The locus of a point in the rhombus ABCD which is equidistant from points A and C.

Sol. (i)

1. Draw a given line AB.
2. Draw lines l and m parallel to AB at a distance of 2.5cm.

Lines l and m are the locus of point P which is at a distance of 2.5cm.

(ii)



ΔABC is an isosceles triangle in which $AB = AC$ from A, draw AD perpendicular to BC. AD is the locus of the point A the vertices of ΔABC.

In right ΔABD and ΔACD.

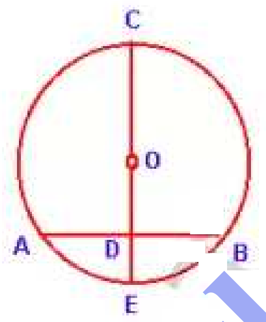
Side $AD = AD$ (common), Hypotenuse $AB = AC$ (given)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (R.H.S Axiom)

$BD = DC$ (C.P.C.T)

Hence the locus of vertices of isosceles triangles having common base is the perpendicular bisector of BC.

(iii)



a. Draw a circle with centre O.

b. Take points A and B on it and join them.

c. Draw the perpendicular bisector of AB which passes from O and meets the circle at C.

$\therefore$ CE the diameter which is the locus of a point inside the circle and equidistant from two points A and B at the circle.

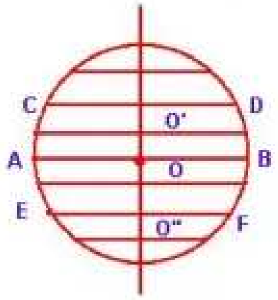
(iv) let $C_1, C_2, C_3 \ldots$ be centres of circle which pass through the two fixed points A and B.

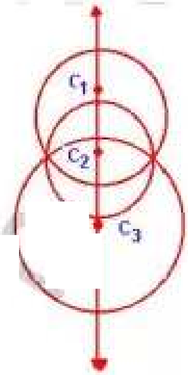
Draw a line xy passing through these centers C_1, C_2, C_3

$\therefore$ Hence locus of centers of circle passing through two points A and B is $\perp^{lar}$ bisector of line.

segment joining the two fixed points.

(v)



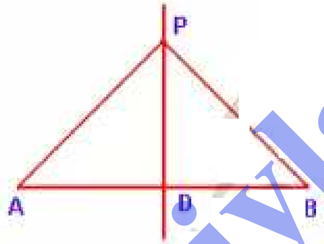


Draw a Circle. AB, CD and EF are the parallel chords of the Circle whose midpoints are O, O', O".

Join O, O', O" which form a diameter.

∴ Locus of midpoints of chords is a diameter which is perpendicular to these chords.

(vi)



a. Join the given points A and B.

b. from P draw perpendicular to AB which is the Locus of P.

c. Join PA and PB.

Now in right ΔPAD and ΔPBD

Hypotenuse $PA = PB$ (∵ P is equidistant from A and B)

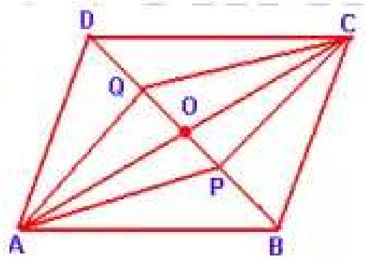
Side $PD = PD$ (common)

∴ $\Delta PAD \cong \Delta PBD$ (R.H.S Axiom of Congruency).

∴ $AD = BD$ (C.P.C.T)

∴ PD is the perpendicular bisector of AB. Hence locus of a point P is the perpendicular bisector of AB.

vii)



ABCD is a rhombus. join BD

BD is the locus of a point in the rhombus which is equidistant from A and C.

$\because$ Diagonal BD bisects the $\angle B$ and $\angle D$

any point on BD will be equidistant from A and C.

6. Describe completely the locus of points in each of the following cases:

(i) Mid point of radii of a circle.

(ii) Centre of a ball, rolling along as straight line on a level floor.

(iii) point in a plane equidistant from a given line.

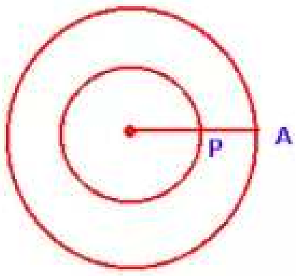
(iv) point in a plane at a constant distance of 5cm from a fixed point in the plane.

(v) centre of a circle varying radius and touching two arms of $\angle ABC$

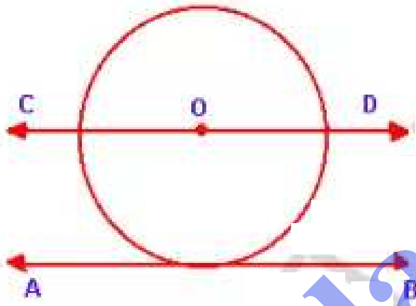
(vi) Centre of a circle varying radius and touching a fixed circle, centre O. at a fixed point A on it.

(vii) Centre of a circle of radius 2cm and touching a fixed circle of a radius 3cm with centre O.

Sol. (i) The locus of midpoint of the radii of a Circle is another concentric circle with radius is half of the radius of the given Circle.

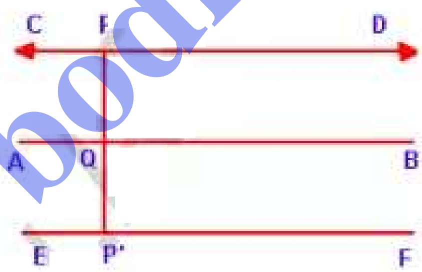


(ii) AB is the straight line on the ground and the ball is rolling on it.



∴ Locus of the centre of the ball is a line parallel to the given line AB.

(iii)

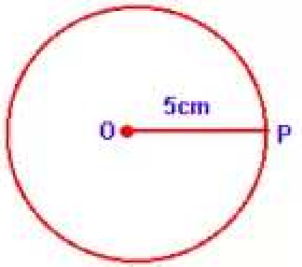


AB is the given line and P is a point in the plane

From P, draw a line CD and another line EF from P' parallel to AB.

Thus CD and EF are lines which are the locus of the point equidistant from AB.

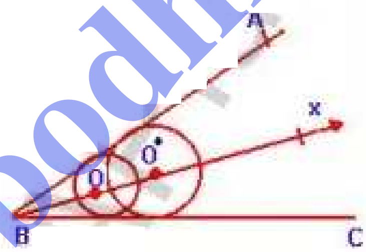
(iv)



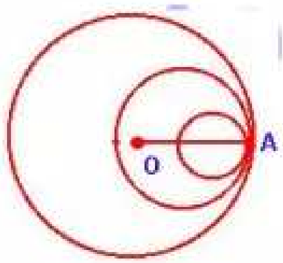
Take a point O and another point P such that OP = 5cm. With centre O and radius equal to OP, draw a circle. Thus this circle is the locus of point P which is at a distance of 5cm from O, the given point.

(V) Draw the bisector BX of ∠ABC. this bisector of angle is the locus of the centre of a circle with different radii.

Any point on BX, is equidistant from the arms BA and BC of the ∠ABC.

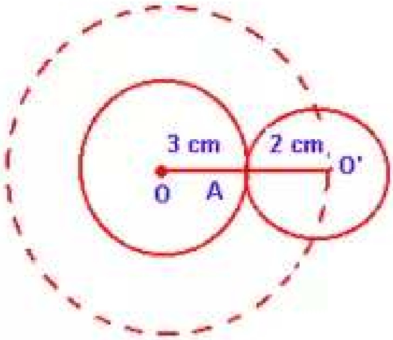


(vi) A circle with centre O is given and one point A on it.

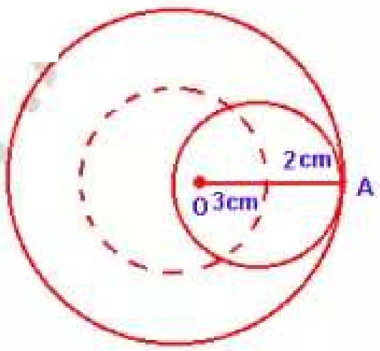


∴ The locus of the centre of a circle which touches the circle at the fixed point A on it is a line joining the points O and A.

(vii) If the circle with 2cm as radius touches the given circle externally then the locus of the centre of the circle will be a concentric circle with radius 3+2 = 5cm.

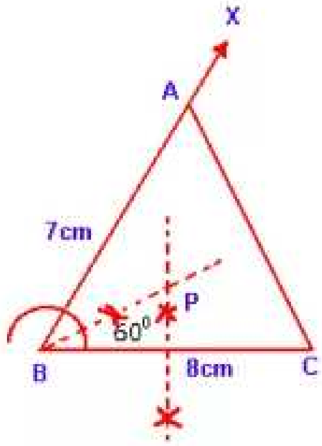


If the circle with 2cm as radius touches the given circle with 3cm as radius internally, then the locus of the centre of the circle will be a concentric circle with radius 3 - 2 = 1 cm.



7. Construct a triangle ABC, with AB = 7cm, BC = 8cm and $\angle ABC = 60^\circ$. Locate by construction the point P such that. (i) P is equidistant from B and C and
(ii) P is equidistant from AB and BC.
(iii) Measure and record the length of PB

Sol.



Steps of construction :

(i) Take BC = 8cm long line segment.
At B, draw a ray BX making an angle of 60° with BC. Cut off BA = 7cm, and joining AC.

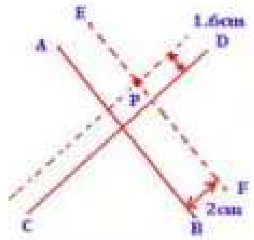
(ii) Draw the $\perp^{lar}$ bisector of BC.

(iii) Draw the angle bisector of $\angle B$ which intersect the $\perp^{lar}$ bisector of BC at P. P is the required point.

(iv) On measuring the length of BP = 4.6 cm (Approx.)

8. A straight line AB is 8cm Long. Locate by construction the locus of a point which is :
(i) Equidistant from A and B.
(ii) Always 4cm from the line AB.
(iii) Mark two points x and y, which are 4cm from AB and equidistant from A and B. Name the fig. AXBY.

sol.



Draw EF ∥ AB at a distance 2cm, and draw GH ∥ CD at a distance 1.6cm. point of intersection of EF and GH is the required point.

9. Ruler and compasses only be used in this question. All construction lines and arcs must be clearly shown, and be sufficient length and clarity to permit assessment.

(i) construct ΔABC, in which $BC = 8cm$, $AB = 5cm$ and $\angle ABC = 60^\circ$.

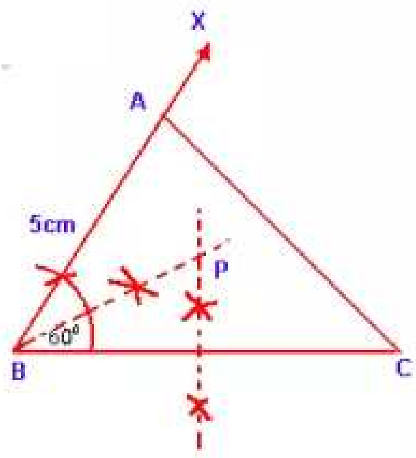
(ii) Construct the locus of points inside the triangle which are equidistant from BA and BC.

(iii) Construct the locus of points inside the triangle which are equidistant from B and C.

(iv) Mark as P the point which is equidistant from AB, BC and also equidistant from B and C.

(v) Measure and record the length of PB.

sol.



Steps of construction :

(i) Take BC = 8cm long

(ii) At B, draw a ray BX making an angle of 60° with BC.

(iii) Cut off AB = 5cm.

(iv) Join AC. ABC is the triangle.

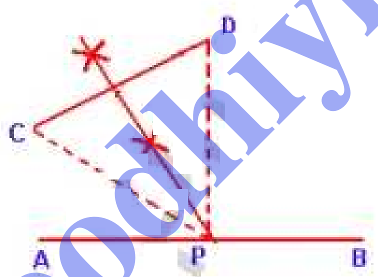
(V) Draw the angle bisector of $\angle B$.

(vi) Draw the $\perp^{lar}$ bisector of BC, which intersects the angle bisector of $\angle B$.

(vii) On measuring, the length BP = 4.6cm (Approx.)

10. In the adjoining diagram, AB is a fixed line and C, D are fixed points. Locate the point P on the line AB such that CP = PD.

Sol.



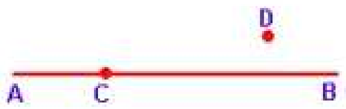
Steps of Construction :

(i) Join CD

(ii) Draw the $\perp^{lar}$ bisector of CD which meets AB at P.

(iii) P is the required point on AB which is equidistant from C and D. i.e., CP = PD.

Q11. In the adjoining diagram A, B, C are fixed collinear points ; D is a fixed point outside the line. locate



(i) The point P on AB such that CP = DP.

(ii) The point Q such that CQ = DQ = 3cm. how many such points are possible ?

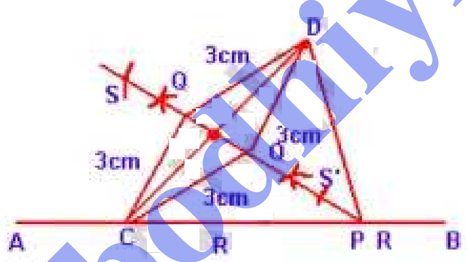
(iii) The point R on AB such that DR = 4cm. How many such points are possible ?

(iv) The point S such that CS = DS and S is 4cm away from the line CD. How many such points are possible ?

(v) Are the points P, Q and R collinear ?

(vi) Are the points P, Q and S collinear ?

Sol.



(i) Join CD

(ii) Draw the $\perp^{lar}$ bisector of CD which meets AB in P.

(iii) P is the required point such that CP = DP.

(iv) with Centres C and D, draw two arcs with 3cm radius which intersect each other at Q and Q'. Hence there are two points Q and Q' which are equidistant from C and D.

(V) with Centre D, and radius 4cm draw an arc which intersects ~~each other~~ at ~~Q and Q'~~.

~~Hence there are~~ AB at R and R'

∴ R and R' are the two points on AB.

(Vi) with Centres C and D, draw arcs with radius equal to 4cm which intersect each other in S and S'

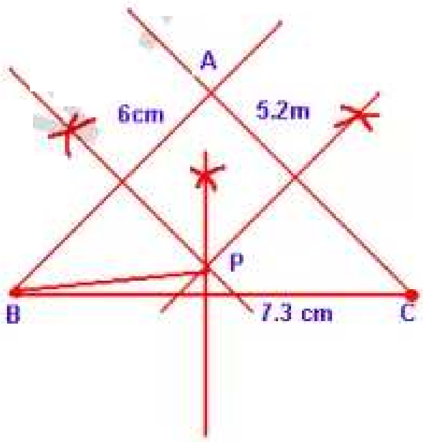
∴ There Can be two such points which are equidistant from C and D.

(Vii) No P, Q, R are not Collinear.

(Viii) Yes P, Q, S are Collinear.

Q12. points A, B and C represent position of three towers such that AB = 60m, BC - 73m and CA = 52m. taking a Scale of 10m ~~as~~ 1cm. make an accurate drawing of ΔABC. find by drawing, the location of a point which is equidistant from A, B and C. and its actual distance from any of the towers.

Sol. AB = 60m = 6.0cm, BC = 73m = 7.3cm and CA = 52m = 5.2cm



(i) Draw a line segment BC = 7.3cm.

(ii) with centre B and radius 6cm and with centre C and radius 5.2cm, draw two arcs intersecting each other at A.

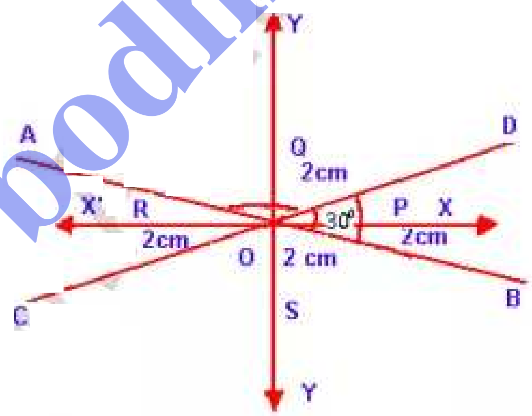
(iii) Joining AB and AC.

(iv) Draw the $\perp^{lar}$ bisector of AB, BC and CA respectively, which intersect each other at point P. P is equidistant from A, B and C on join PB. measuring PB = 3.7cm.

$\therefore$ Actual distance = 37 m.

13. Draw two intersecting lines to include an angle of 30°. Use ruler and compasses to locate points which are equidistant from these lines and also 2cm away from their point of intersection. How many such points exist?

Sol.



(i) Two lines AB and CD intersect eachother at O.

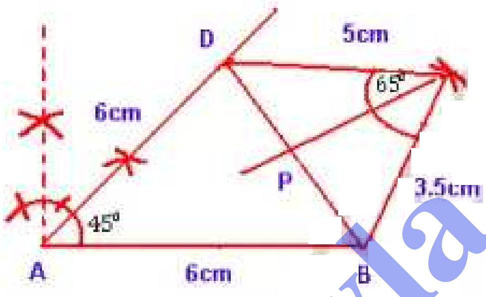
(ii) Draw the bisector of ∠BOD and ∠AOD.

(iii) with centre O and radius equal to 2cm. mark points on the bisector of angles at P, Q, R and S respectively.

Hence there are 4 points which are equidistant from AB and CD and 2cm from O, the point of intersection of AB and CD.

14. without using set square or protractor, construct the quadrilateral ABCD in which $\angle BAD = 45°$, AD = AB = 6cm, BC = 3.6cm and CD = 5cm
(i) Measure $\angle BCD$ (ii) locate the point P on BD which is equidistant from BC and CD.

Sol.



(i) Take AB = 6cm long.

(ii) At A, draw the angle of 45° and cutoff AD = 6cm.

(iii) with centre D and radius 5cm and with centre B, and radius 3.5cm draw two arcs intersecting each other at C.

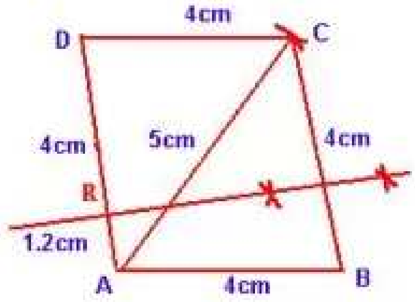
(iv) Join CD and CB and join BD.
ABCD is the required quadrilateral.

(V) on measuring $\angle BCD = 65°$.

(Vi) Draw the bisector of $\angle BCD$ which intersects BD at P which is the required point equidistant from CD and CB.

15. Without using set square or protractor, construct rhombus ABCD with sides of length 4cm and diagonal AC of length 5cm. Measure $\angle ABC$. Find the point R on AD such that RB = RC. Measure the length of AR.

Sol.



(i) Take AB = 4cm.

(ii) With centre A, draw an arc of 5cm radius and with B draw another arc of radius 4cm intersecting each other at C.

(iii) Join AC and BD.

(iv) Again with centre A and C, draw two arcs of radii 4cm intersecting each other on D.

(v) Join AD and CD.

ABCD is the required rhombus and on measuring the $\angle ABC$, it is 78°.

(vi) Draw $\perp^{lar}$ bisector of BC intersecting AD at R. On measuring, the length of AR it is equal to 1.2cm.

16. Without using set square or protractor construct:

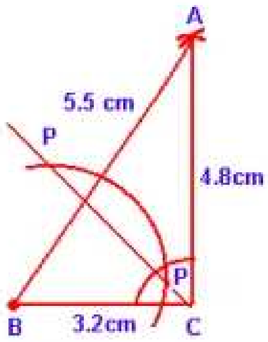
(i) ΔABC, in which AB = 5.5cm, BC = 3.2cm and CA = 4.8cm.

(ii) Draw the locus of a point moves so that it is always 2.5cm from B.

(iii) Draw the locus of a point moves so that it is equidistant from the sides BC and CA.

(iv) Mark the point of intersection of the loci with the letter P and measure PC.

Sol.



Steps of Construction:

(i) Take BC = 3.2 cm long.

(ii) With centre B and radius 5.5cm and with centre C and radius 4.8cm draw arcs intersecting each other at A.

(iii) Join ~~AB~~ and AC.

(iv) Draw the bisector of $\angle BCA$.

(v) With centre B and radius 2.5cm, draw an arc intersecting the angle bisector of $\angle BCA$ at P and P'.

$\therefore$ P and P' are two loci which satisfies the given condition.

On measuring CP and CP'

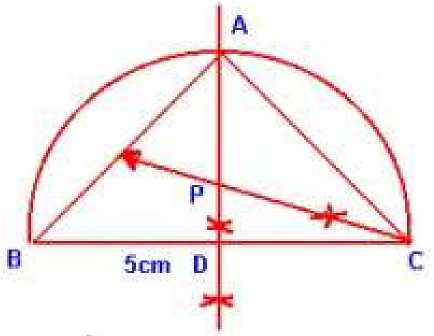
CP = 3.6cm, CP' = 1.1 cm.

17. By using ruler and compass only, construct an isosceles $\triangle ABC$ in which $BC = 5cm$, $AB = AC$ and $\angle BAC = 90°$. Locate the point P such that.

(i) P is equidistant from the sides BC and AC.

(ii) P is equidistant from the points B and C.

Sol.



Steps of construction :

(i) Take $BC = 5.0$ cm and bisect it at D.

(ii) Taking BC as diameter, draw a semicircle.

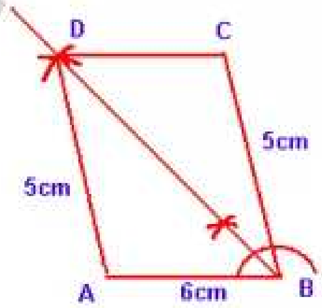
(iii) At D, draw a $\perp^{lar}$ intersecting the circle at A.

(iv) join AB and AC.

(v) Draw the angle bisector of C intersecting the $\perp^{lar}$ at P. P is the required point.

18. Using ruler and compasses only, construct a quadrilateral ABCD in which $AB = 6cm$, $BC = 5cm$, $\angle B = 60°$, $AD = 5cm$ and D is equidistant from AB and BC. Measure CD.

Sol.



Steps of construction:

(i) Take AB = 6cm

(ii) At B, draw angle of $60°$ and Cutoff BC = 5Cm.

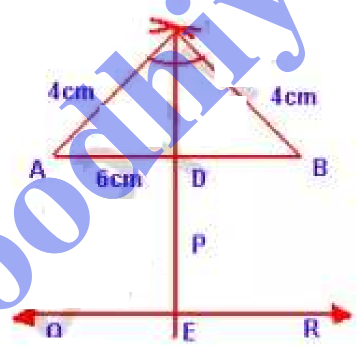
(iii) Draw the angle bisector of $\angle B$.

(iv) With centre A and radius 5cm. draw an arc which intersect the angle bisector of $\angle B$ at D

(v) Join AD and DC. ABCD is the required quadrilateral. On measuring CD it is 5.3Cm (approx.)

19. Construct an isosceles ΔABC such that AB = 6cm, BC = AC = 4cm. Bisect $\angle C$ internally and mark a point P on this bisector such that CP = 5cm. Find the points Q and R which are 5cm from P and also 5cm from the line AB.

Sol.



Steps of Construction:

(i) Take AB = 6cm

(ii) With centres A and B radius 4cm, draw two arcs intersecting each other at C.

(iii) Join CA and CB.

(iv) Draw the bisector of $\angle C$ and Cutoff CP = 5cm.

(v) Draw a line XY // AB at a distance of 5cm.

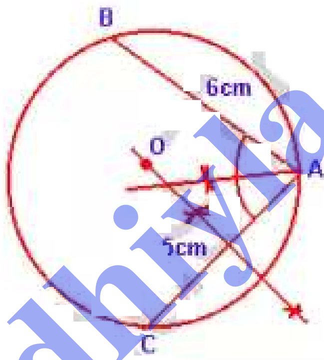
(vi) from P, draw arcs of radius 5cm each intersecting the line XY at Q and R.

∴ Hence Q and R are the required points.

20\. Use ruler and Compasses only for this question. draw a Circle of radius 4cm and mark two chords AB and AC of the circle of length 6cm and 5cm respectively.

(i) Construct the locus of points, inside the circle that are equidistant from A and C. prove your construction.

(ii) Construct the Locus of all points, inside ΔABC, which are equidistant from B and C.



Steps of Construction :

(i) with Centre O and radius 4cm draw a Circle.

(ii) Take a point A on this circle.

(iii) with Centre A and radius 6cm draw an arc cutting the circle at B.

(iv) Again with radius 5cm, draw another arc cutting the circle at C.

(v) join AB and AC.

(vi) Draw the $\perp^{lar}$ bisector of AC.

∴ Any point on it, will be equidistant from A and C:

(vii) Draw the angle bisector of $\angle A$ intersecting the $\perp^{lar}$ bisector of AC at P. P is the required locus.

21. Use ruler and compasses only in this question. All construction lines and arcs must be clearly shown, and be of sufficient length and clarity to permit assessment.

(i) Construct a $\triangle ABC$ in which $BC = 6cm$, $AB = 9cm$ and $\angle ABC = 60°$.

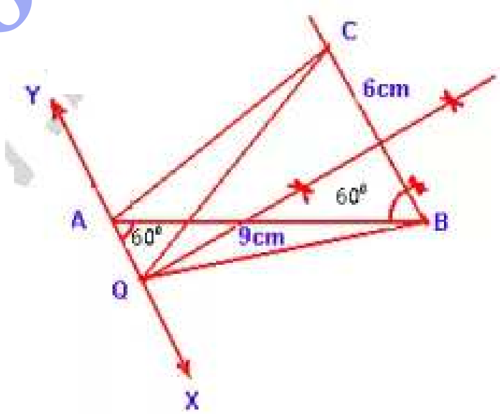
(ii) construct Locus of all points inside $\triangle ABC$, which are equidistant from B and C.

(iii) construct the locus of the vertices of the triangles with BC as base, which are equal in area to $\triangle ABC$.

(iv) Mark the point Q in your construction, which would make $\triangle QBC$ equal in area to $\triangle ABC$ and isosceles.

(v) Measure and record the length of CQ.

Sol.



Steps of Construction :

(i) Take $AB = 9cm$

(ii) At B draw an angle of $60°$ and cut off $BC = 6cm$.

(iii) join AC

(iv) Draw $\perp^{lar}$ bisector of BC. All points on it will be equidistant from B and C.

(V) From A, draw a line XY parallel to BC

(Vi) produce the $\perp^{lar}$ bisector of BC to meet XY in Q.

(Vii) join QC and QB.

ΔQBC will be the triangle equal in area to ΔABC because these are on the same base BC and between the same parallel lines. On measuring, the length of CQ is 8.2cm (Approx.)