Section Formula

EXERCISE – 11.1

Q1. Find the distance between the following pair of points :

(i) $(-5, 3), (3, 1)$

Sol. Distance $= \sqrt{[3-(-5)]^2 + (1-3)^2} = \sqrt{64+4} = \sqrt{68}$

$= 2\sqrt{17}$.

(ii) $(4, 5), (-3, 2)$

Distance $= \sqrt{(-3-4)^2 + (2-5)^2} = \sqrt{49+9} = \sqrt{58}$

(iii) $(-1, -4), (3, 5)$

Distance $= \sqrt{(3-(-1))^2 + (5-(-4))^2} = \sqrt{16+81} = \sqrt{97}$

Q2. Calculate the distance between A(7, 3) and B on the x-axis whose abscissa is 11.

Sol. $A(7, 3)$. $B(11, 0)$ as B lies on x-axis

$\therefore AB = \sqrt{(11-7)^2 + (0-3)^2} = \sqrt{25} = 5$

Q3. As a point on the y-axis whose ordinate is 5 and B is the point $(-3, 1)$. Calculate the length of AB.

Sol. A lies on y-axis, $x = 0$ and $y = 5$

$A(0, 5)$, $B(-3, 1)$

$AB = \sqrt{(-3-0)^2 + (1-5)^2} = \sqrt{9+16} = 5$.

Q4. Show that the point (4,4) is equidistant from the points A(1,0) and B(-1,4)

Sol. Distance between A(1,0) & (4,4) $= \sqrt{(4-1)^2+(4-0)^2} = \sqrt{25} = 5$

Distance between B(-1,4) & (4,4) $= \sqrt{(4-(-1))^2+(4-4)^2} = 5$

$\therefore$ Hence proved.

Q5. A is a point of y-axis whose ordinate is 4 and B is a point on x-axis whose abscissa is -3. find the length of the line segment AB.

Sol. A lies on y-axis, A (0,4)

B lies on x-axis, B(-3,0)

$AB = \sqrt{(-3-0)^2+(0-4)^2} = \sqrt{25} = 5$

Q6. The distance between A(1,3) and B(x,7) is 5. Calculate the possible values of x.

Sol. Distance = 5

$\Rightarrow \sqrt{(x-1)^2+(7-3)^2} = 5 \Rightarrow x^2-2x+1+16 = 25$

$\Rightarrow x^2-2x-8=0 \Rightarrow x^2-4x+2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$\Rightarrow x = 4, -2.$

Q7. point A(5,-1) on reflection in x-axis is mapped as A'. Also A on reflection in y-axis is mapped as A''. write the coordinates of A' and A''. also calculate the distance AA''.

Sol. coordinates of A' are (5,1)

coordinates of A'' are (-5,-1)

Q9. B and C have coordinates (3, 2) and (6, 3). find:

(i) The image B' of B under reflection in the x-axis.

(ii) The image C' of C under reflection in the line BB'.

(iii) The length of B'C'.

Sol. (i) The coordinates of B' reflected on x-axis will be (3, -2).

(ii) The coordinates of C', reflected on line BB' = (6, 3)

(iii) $B'C' = \sqrt{(6-3)^2 + (3-(-2))^2} = \sqrt{9+25} = \sqrt{34} = 5.83$.

Q10. what points on y-axis are at a distance of 10 units from the point (8, 8)?

Sol: let the coordinates of points be (x, y) which are at a distance of 10 units from the point (8, 8).

$$\sqrt{(8-x)^2 + (8-y)^2} = 10$$

$$\Rightarrow 64 + x^2 - 16x + 64 + y^2 - 16y = 100$$

$$\Rightarrow x^2 + y^2 - 16x - 16y + 28 = 0$$

$\therefore$ points are on y-axis, $x = 0$

Hence $y^2 - 16y + 28 = 0 \Rightarrow y^2 - 14y - 2y + 28 = 0$

$$\Rightarrow (y-14)(y-2) = 0$$

$$\Rightarrow y = 2, 14$$

$\therefore$ points will be (0, 14) and (0, 2).

Q11. find the points which are at a distance of $\sqrt{10}$ from the point (4, 3) given that the ordinate of the points is twice the abscissa.

Sol. let the abscissa of point = x

ordinate = 2x

$$\sqrt{(x-4)^2+(2x-3)^2}=\sqrt{10}$$

$\Rightarrow x^2-8x+16+4x^2+9-12x=10$

$\Rightarrow 5x^2-20x+15=0 \Rightarrow x^2-4x+3=0$

$\Rightarrow x^2-3x-x+3=0 \Rightarrow (x-3)(x-1)=0$

$\Rightarrow x=1,3$

$\therefore$ points will be $(1,2)$ and $(3,6)$.

Q12. $A(2,2)$, $B(-2,4)$ and $C(2,6)$ are the vertices of a triangle ABC. prove that ABC is an isosceles triangle.

Sol. $AB=\sqrt{(-2-2)^2+(4-2)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

$BC=\sqrt{(2-(-2))^2+(6-4)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

$AC=\sqrt{(2-2)^2+(6-2)^2}=\sqrt{16}=4.$

$\therefore AB=BC=2\sqrt{5}$

$\therefore \triangle ABC$ is an isosceles triangle.

Q13. Show that the points $(1,1)$ $(-1,-1)$ and $(-\sqrt{3},+\sqrt{3})$ form an equilateral triangle?

Sol. $AB=\sqrt{(1-(-1))^2+(1-(-1))^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

$BC=\sqrt{[-\sqrt{3}-(-1)]^2+[+\sqrt{3}-(-1)]^2}=\sqrt{(-\sqrt{3}+1)^2+(\sqrt{3}+1)^2}$

$=\sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}}=\sqrt{8}=2\sqrt{2}$

$AC=\sqrt{(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2}=\sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}}$
$=\sqrt{8}=2\sqrt{2}$

$\therefore AB=BC=AC=2\sqrt{2}$.

$\therefore \triangle ABC$ is an equilateral triangle.

Q14. Show that the points (3,3), (9,0) and (12,21) are the vertices of a right angled triangle.

Sol. let A(3,3), B(9,0), C(12,21)

$AB = \sqrt{(9-3)^2+(0-3)^2} = \sqrt{45} \Rightarrow AB^2 = 45$

$BC^2 = (12-9)^2+(21-0)^2 = 9+441 = 450$

$AC^2 = (12-3)^2+(21-3)^2 = 81+324 = 405$

$\therefore AB^2 + AC^2 = 405+45 = 450 = BC^2$

$\therefore \Delta ABC$ is a right angled triangle.

Q15. Show that the points (0,-1), (-2,3), (6,7) and (8,3) are the vertices of a rectangle.

Sol. let A(0,-1), B(-2,3), C(6,7) and D(8,3)

$AB = \sqrt{(-2-0)^2+(3-(-1))^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(-2-6)^2+(3-7)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$

$CD = \sqrt{(8-6)^2+(3-7)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$AD = \sqrt{(8-0)^2+(3-(-1))^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$

$\therefore AB = CD$ and $BC = AD$

$\therefore ABCD$ is a rectangle.

Q16. Show that the points (7,3), (3,0), (0,-4) and (4,-1) are the vertices of a rhombus. Also find the area of the rhombus.

Sol. let A(7,3), B(3,0), C(0,-4) and (4,-1)

$AB = \sqrt{(3-7)^2+(0-3)^2} = \sqrt{16+9} = 5$

$BC = \sqrt{(0-3)^2+(-4-0)^2} = \sqrt{9+16} = 5$

$$CD = \sqrt{(4-0)^2 + (-1+4)^2} = \sqrt{16+9} = 5$$

$$AD = \sqrt{(7-4)^2 + (3-(-1))^2} = \sqrt{9+16} = 5$$

$\therefore AB = BC = CD = DA$

$\therefore$ ABCD is a rhombus.

$$AC = \sqrt{(0-7)^2 + (-4-3)^2} = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2}$$

$$BD = \sqrt{(4-3)^2 + (-1+0)^2} = \sqrt{1+1} = \sqrt{2}$$

$\therefore$ Area of rhombus $= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 7\sqrt{2} \times \sqrt{2} = 7$ sq. units.

Q17. The points $A(0,3)$, $B(-2,a)$ and $C(-1,4)$ are the vertices of a right angled triangle at A, find the value of a.

Sol. $AB^2 = (-2-0)^2 + (a-3)^2 = 4 + (a-3)^2$

$AC^2 = (-1-0)^2 + (4-3)^2 = 1+1 = 2$

$BC^2 = (-1+2)^2 + (4-a)^2 = 1 + (4-a)^2$

$\therefore AB^2 + AC^2 = BC^2$ {$\because$ By pythagoras theorem}

$\Rightarrow 4 + (a-3)^2 + 2 = 1 + (4-a)^2$

$\Rightarrow 4 + a^2 - 6a + 9 + 2 = 1 + 16 + a^2 - 8a$

$\Rightarrow a^2 - 6a + 15 = a^2 - 8a + 17$

$\Rightarrow 2a = 2 \Rightarrow a = 1.$

Q18. Show by distance formula that the points $(-1,-1)$, $(2,3)$ and $(8,11)$ are collinear.

Sol. let $A(-1,-1)$, $B(2,3)$, $C(8,11)$

$$AB = \sqrt{(2-(-1))^2 + (3-(-1))^2} = \sqrt{9+16} = 5$$

$$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{36+64} = 10$$

$AC = \sqrt{(8-(-1))^2+(11-(-1))^2} = \sqrt{81+144} = 15$

$\therefore AB + BC = 5+10 = 15 = AC.$

$\therefore$ Hence A, B & C are collinear.

Q19. what point on the x-axis is equidistance from (7,6) and (-3,4) ?

Sol. let $P(x,0)$ be the equidistance from given points $A(7,6)$ and $B(-3,4)$, then

$$PA = \sqrt{(7-x)^2+(6-0)^2} = \sqrt{(7-x)^2+36}$$

$$PB = \sqrt{(-3-x)^2+(4-0)^2} = \sqrt{(-3-x)^2+16} = \sqrt{(3+x)^2+16}$$

$$\therefore PA = PB$$

$$\sqrt{(7-x)^2+36} = \sqrt{(3+x)^2+16}$$

Squaring on both sides,

$$\Rightarrow (7-x)^2+36 = (3+x)^2+16$$

$$\Rightarrow 49+x^2-14x+36 = 9+x^2+6x+16$$

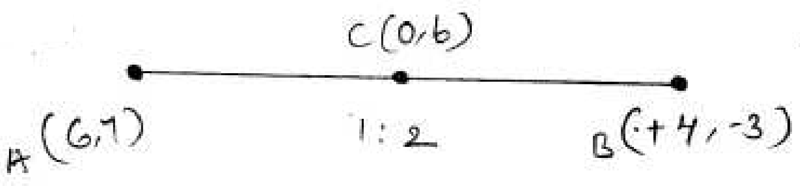
$$\Rightarrow x^2-14x+85 = x^2+6x+25$$

$$\Rightarrow -20x = -60 \Rightarrow x = 3.$$

$$\therefore P = (3,0).$$

Q20. Find the points on the y-axis whose distances from the points (6,7) and (4,-3) are in the ratio 1:2

Sol. let the point on y-axis is $(0,b)$.



$$AC = \sqrt{(0-6)^2 + (b-7)^2} = \sqrt{36+(b-7)^2}$$

$$BC = \sqrt{(4-0)^2 + (-3-b)^2} = \sqrt{16+(b+3)^2}$$

According to given, $\frac{AC}{BC} = \frac{1}{2}$

$\Rightarrow 2AC = BC$

$\Rightarrow 2\sqrt{36+(b-7)^2} = \sqrt{16+(b+3)^2}$

Squaring on bothsides,

$\Rightarrow 4(36+b^2+49-14b) = 16+b^2+9+6b$

$\Rightarrow 144+4b^2+196-56b = 16+b^2+9+6b$

$\Rightarrow 3b^2-62b+315 = 0$

$\Rightarrow 3b^2-27b-35b+315 = 0$

$\Rightarrow 3b(b-9)-35(b-9) = 0 \Rightarrow (b-9)(3b-35) = 0$

$\Rightarrow b = 9, \frac{35}{3}$

$\therefore$ Hence the point is $(0,9)$ or $\left(0, \frac{35}{3}\right)$

Q21. Find the abscissa of points whose ordinate is 4 and which are at a distance 5 units from $(5,0)$.

Sol. let abscissa of point $= x$

$\therefore$ coordinates of point $(x, 4)$

Distance between $(5,0)$ & $(x,4) = 5$

$\Rightarrow \sqrt{(x-5)^2+(4-0)^2} = 5$

$\Rightarrow x^2+25-10x+16 = 25$

$\Rightarrow x^2-10x+16 = 0$

$\Rightarrow x^2-8x-2x+16 = 0$

$\Rightarrow (x-8)(x-2) \Rightarrow x = 2$ or 8

Q22. Find the value of x such that $AB = BC$ where A, B and C are the points $(6,-1)$, $(1,3)$, $(x,8)$ respectively.

Sol. $A(6,-1)$, $B(1,3)$, $C(x,8)$

$$AB = \sqrt{(6-1)^2+(-1-3)^2} = \sqrt{25+16} = \sqrt{41}$$

$$BC = \sqrt{(x-1)^2+(8-3)^2} = \sqrt{(x-1)^2+25}$$

$\therefore AB = BC$ {given}

$$\sqrt{41} = \sqrt{(x-1)^2+25}$$

Squaring on bothsides

$$41 = (x-1)^2+25 \Rightarrow (x-1)^2 = 16$$

$$\Rightarrow x^2-2x+1 = 16 \Rightarrow x^2-2x-15 = 0$$

$$\Rightarrow x^2-5x+3x-15 = 0 \Rightarrow (x-5)(x+3) = 0$$

$$\Rightarrow x = 5, -3$$

Q23. If A, B and P are the points $(-4,3)$, $(0,-2)$ and (α,β) respectively and P is equidistant from A and B, Show that $8\alpha-10\beta+21 = 0$

Sol. $A(-4,3)$, $B(0,-2)$ and $P(\alpha,\beta)$

$$PA = \sqrt{(\alpha-(-4))^2+(\beta-3)^2} = \sqrt{(\alpha+4)^2+(\beta-3)^2}$$

$$PB = \sqrt{(\alpha-0)^2+(\beta-(-2))^2} = \sqrt{\alpha^2+(\beta+2)^2}$$

$$PA = PB$$

$$\sqrt{(\alpha+4)^2+(\beta-3)^2} = \sqrt{\alpha^2+(\beta+2)^2}$$

Squaring on bothsides

$$(\alpha+4)^2+(\beta-3)^2 = \alpha^2+(\beta+2)^2$$

$\Rightarrow \alpha^2 + 8\alpha + 16 + \beta^2 - 6\beta + 9 = \alpha^2 + \beta^2 + 4\beta + 4$

$\Rightarrow 8\alpha - 10\beta + 21 = 0$

$\therefore$ Hence proved.

Q24. The Centre of a Circle is $(2\alpha-1, 3\alpha+1)$ and it passes through the point $(-3,-1)$. find the values of α if a diameter of the circle is of length 20 units.

Sol. Centre $O(2\alpha-1, 3\alpha+1)$, $A(-3,-1)$

$AO = \sqrt{(2\alpha-1+3)^2 + (3\alpha+1+1)^2} = \sqrt{(2\alpha+2)^2 + (3\alpha+2)^2}$

$\therefore$ Diameter $= 20$

$2 \times AO = 20 \Rightarrow AO = 10$

$\Rightarrow \sqrt{(2\alpha+2)^2 + (3\alpha+2)^2} = 10$

$\Rightarrow 4\alpha^2 + 4 + 8\alpha + 9\alpha^2 + 4 + 12\alpha = 100$

$\Rightarrow 13\alpha^2 + 20\alpha - 92 = 0$

$\Rightarrow 13\alpha^2 + 46\alpha - 26\alpha - 92 = 0$

$\Rightarrow \alpha(13\alpha + 46) - 2(13\alpha + 46) = 0$

$\Rightarrow (13\alpha+46)(\alpha-2) = 0$

$\Rightarrow \alpha = 2, -46/13$

Q25. Find the centre of Circle passing through the points $(8,12)$, $(11,3)$, $(0,14)$. Also find its radius.

Sol. let $O(x,y)$ be the Centre of Circle

$\therefore AO = \sqrt{(x-8)^2 + (y-12)^2}$

$BO = \sqrt{(x-11)^2+(y-3)^2}$, $CO = \sqrt{(x-0)^2+(y-14)^2}$

$\therefore AO = BO = CO$ (radii of the same circle)

$\sqrt{(x-8)^2+(y-12)^2} = \sqrt{(x-11)^2+(y-3)^2}$

Squaring on bothsides

$\Rightarrow x^2-16x+64+y^2+144-24x = x^2-22x+121+y^2-6x+9$

$\Rightarrow 6x-18y+78=0$

$\Rightarrow x-3y=-13 \longleftarrow (i)$

Again $\sqrt{(x-11)^2+(y-3)^2} = \sqrt{x^2+(y-14)^2}$

Squaring on bothsides.

$\Rightarrow x^2-22x+121+y^2-6y+9 = x^2+y^2-28y+196$

$\Rightarrow -22x+22y=66 \Rightarrow x-y=-3 \longrightarrow (ii)$

Subtracting (i) & (ii) $\Rightarrow -2y=-10 \Rightarrow y=5$

substituting the values of y in (ii)

$x-5=-3 \Rightarrow x=2$

$\therefore x=2, \; y=5$

$OA = \sqrt{(x-8)^2+(y-12)^2} = \sqrt{(2-8)^2+(5-12)^2}$

$= \sqrt{36+49} = \sqrt{85}$ units.

EXERCISE - 11.2

Q1. Find the Coordinates of the mid points of the line segments joining the following pairs of points:

(i) $(2,-3)$, $(-6,7)$

Sol. $\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-6}{2}, \frac{-3+7}{2}\right)$

$= (-2, 2)$

(ii) $(5,-11)$, $(4,3)$

Sol. $\text{midpoint} = \left(\frac{5+4}{2}, \frac{-11+3}{2}\right) = \left(\frac{9}{2}, -4\right)$

(iii) $(a+3, 5b)$, $(2a-1, 3b+4)$

Sol. $\text{midpoint} = \left(\frac{a+3+2a-1}{2}, \frac{5b+3b+4}{2}\right) = \left(\frac{3a+2}{2}, (4b+2)\right)$

Q2. Given that the coordinates of points A $(-3,2)$ and B $(9,7)$ respectively, find : (i) The coordinates of midpoint of AB (ii) The distance between A and B.

Sol. $A(-3,2)$, $B=(9,7)$

(i) $\text{midpoint of } AB = \left(\frac{-3+9}{2}, \frac{2+7}{2}\right) = \left(3, \frac{9}{2}\right)$

(ii) $AB = \sqrt{(9-(-3))^2+(7-2)^2} = \sqrt{144+25} = \sqrt{169}$

$= 13$ units.

Q3. The coordinates of two points A and B are $(-3,3)$ and $(12,-7)$ respectively. P is a point on the linesegment AB such that $AP:PB = 2:3$

$A(-3,3)$, $B(12,-7)$

let $P(x,y)$ be the point which divides AB in the ratio of $m_1:m_2$ ie, $2:3$ then coordinates of P will be

$$x = \frac{m_1 x_2 : m_2 x_1}{m_1+m_2} = \frac{(2)(12)+3(-3)}{2+3} = \frac{15}{5} = 3$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1+m_2} = \frac{2(-7)+3(3)}{2+3} = \frac{-5}{5} = -1$$

$\therefore$ Coordinates of P are $(3,-1)$.

Q4. P divides the distance between $A(-2,1)$ and $B(1,4)$ in the ratio $2:1$. Calculate the coordinates of the point P.

Sol. $A(-2,1)$, $B(1,4)$ $\qquad m_1:m_2 = 2:1$

let $P(x,y)$

coordinates of P.

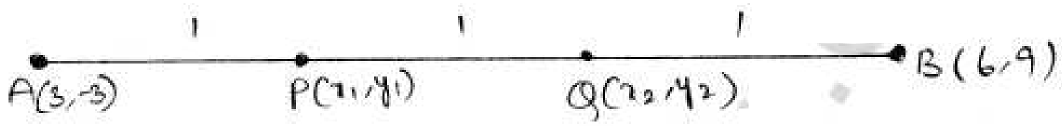
$$x = \frac{m_1 x_2 + m_2 x_1}{m_1+m_2} = \frac{2\times1 + 1(-2)}{2+1} = \frac{0}{3} = 0$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1+m_2} = \frac{2\times4 + 1\times1}{2+1} = \frac{9}{3} = 3$$

$\therefore P(x,y) = (0,3)$.

Q5. Find the coordinates of the points of trisection of the line segment joining the points (3, -3), (6, 9)

Sol. let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the points which trisects the line segment joining the points $A(3,-3)$, $B(6,9)$



$P(x_1, y_1)$ divides AB in the ratio of 1 : 2

$$x_1 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1\times6 + 2\times3}{1+2} = \frac{12}{3} = 4$$

$$y_1 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\times9 + 2(-3)}{1+2} = \frac{3}{3} = 1$$

$\therefore P = (4, 1)$

Again $Q(x_2, y_2)$ divides the line segment AB in the ratio of 2 : 1

$$x_2 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2\times6 + 1\times3}{2+1} = \frac{15}{3} = 5$$

$$y_2 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times9 + 1(-3)}{2+1} = \frac{15}{3} = 5$$

$\therefore$ coordinates of $Q = (5, 5)$.

Q6. Find the coordinates of the point which $3/4^{th}$ of the way from $A(3,1)$ to $B(-2,5)$

Sol. let $P = (x, y)$

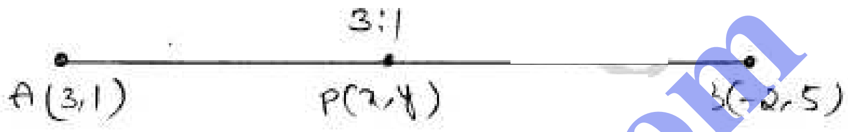
$$\frac{AP}{AB} = \frac{3}{4}$$

Applying section formula to the coordinate of $\frac{AP}{AB}$

$$\therefore \frac{AP}{PB} = \frac{AP}{AB - AP}$$

$$\Rightarrow \frac{PB}{AP} = \frac{AB - AP}{AP} = \frac{AB}{AP} - 1$$

$$\Rightarrow \frac{PB}{AP} = \frac{4}{3} - 1 = \frac{1}{3} \Rightarrow \frac{AP}{PB} = \frac{3}{1}$$



Now apply section formula.

$$x = \frac{3(-2) + 1(3)}{3+1} = \frac{-3}{4}, \quad y = \frac{3(5) + 1(1)}{3+1} = 4$$

$$\therefore P = (x, y) = \left(\frac{-3}{4}, 4\right)$$

Q7. Point $P(3, -5)$ is reflected to P' in the x-axis. Also P on reflection in the y-axis is mapped as P''.

(i) Find the coordinates of P' and P''.

(ii) Compute the distance $P'P''$.

(iii) Find the middle point of the line segment $P'P''$.

(iv) On which coordinate axis does the middle point of the line segment PP'' lie ?

Sol. (i) Coordinates of P = image of $P(3, -5)$ reflected on x-axis

$= (3, 5)$

Coordinates of P'' = image of $P(3,-5)$ reflected on y-axis
$= (3, -5)$

(ii) length of $PP'' = \sqrt{(-3-3)^2 + (-5-5)^2} = \sqrt{36+100} = \sqrt{136}$
$= 2\sqrt{34}$

(iii) let the coordinates of middle point M be (x, y)

$\therefore x = \frac{x_1+x_2}{2} = \frac{3-3}{2} = 0$, $y = \frac{y_1+y_2}{2} = \frac{-5+5}{2} = 0$

$\therefore$ middle point is $(0,0)$

(iv) middlepoint of PP'' be $N(x_1, y_1)$

$x_1 = \frac{3-3}{2} = 0$, $y_1 = \frac{-5-5}{2} = -5$

Coordinates of middle point of PP'' are $(0, -5)$

As $x=0$, this point lies on y-axis.

Q8. Use graph paper for this question. Take 1cm = 1unit on both axes. plot the points $A(3,0)$ and $B(0,4)$

(i) writedown the coordinates of A_1, the reflection of A in the y-axis.

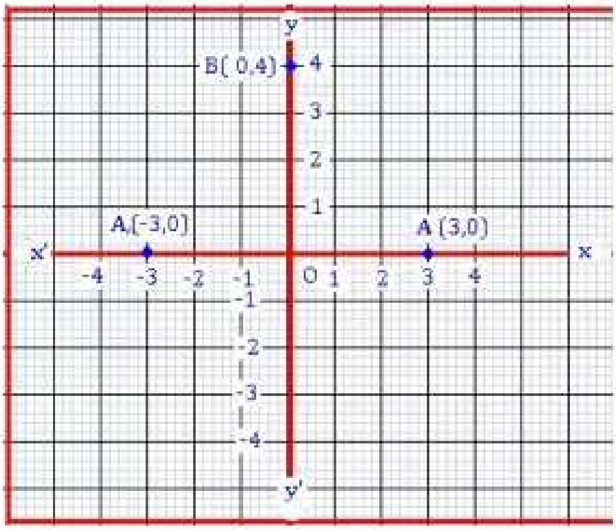
(ii) write down the coordinates of B_1, the reflection of B in the x-axis.

(iii) Assign the special name to quadrilateral ABA_1B_1

(iv) If C is the midpoint of AB, writedown the coordinates of C_1, the reflection of C in the origin.

(v) Assign the special name to quadrilateral ABC_1B_1

sd. (i)



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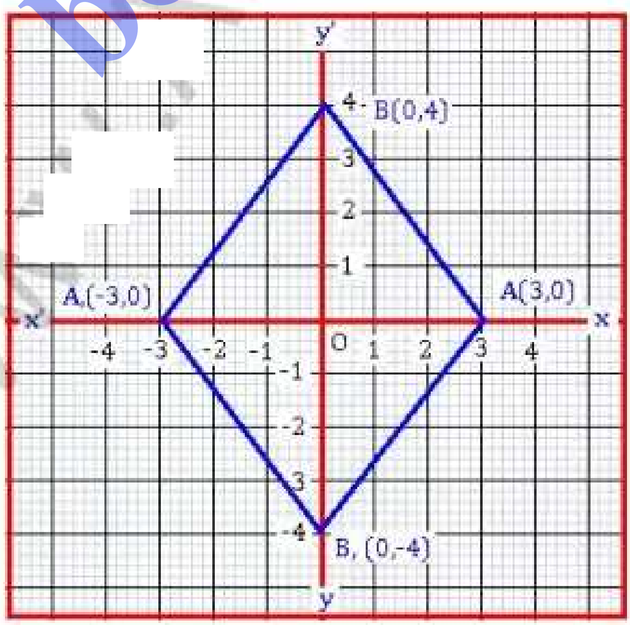
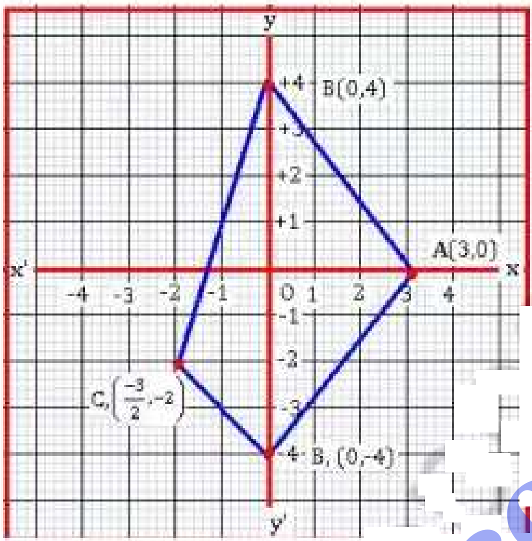
(iii)



As $AB = BA$, so the figure is rhombus.

(iv) As C is the midpoint of AB then coordinates of C are $\left(\frac{-3+0}{2}, \frac{0-4}{2}\right) = \left(\frac{-3}{2}, -2\right)$

(v)



The figure obtained after meeting ABCD is trapezium.

Q9. The line segment joining A(-3, 1) and B(5, -4) is a diameter of a circle whose centre is C. find the coordinates of the point C.

Sol. C is the midpoint of AB.

let $C = (x, y)$

$(x, y) = \left(\frac{-3+5}{2}, \frac{1-4}{2}\right) = \left(1, \frac{-3}{2}\right)$

$\therefore C = (x, y) = (1, -3/2)$

Q10. The midpoint of the line segment joining (2a, 4), (-2, 3b) is (1, 2a+1). find the values of a & b

Sol. $1 = \frac{2a-2}{2}$ and $2a+1 = \frac{4+3b}{2}$

$\Rightarrow a = 2$ $\qquad \Rightarrow 4a+2 = 4+3b$

$\Rightarrow 4a - 3b = 2$

$\Rightarrow 4(2) - 3b = 2 \Rightarrow 6 = 3b \Rightarrow b = 2$

$\therefore a = 2, \; b = 2$

Q11. The coordinates of the midpoint of the line segment PQ are $(1,-2)$. The coordinates of P are $(-3,2)$. Find the coordinates of Q.

Sol. Let the coordinates of Q be (x, y), $P(-3,2)$.

midpoint of PQ are $(1,-2)$.

then $1 = \frac{-3+x}{2} \Rightarrow -3+x = 2 \Rightarrow x = 5$

$-2 = \frac{2+y}{2} \Rightarrow 2+y = -4 \Rightarrow y = -6$

$\therefore$ Hence coordinates of Q are $(5,-6)$.

Q12. The centre O of a circle has the coordinates $(4,5)$ and one point on the circumference is $(8,10)$. Find the coordinates of the other end of the diameter of the circle through this point.

Sol. Centre $O(4,5)$, $A(8,10)$

let other end of diameter be (x, y) then

O is the midpoint of AB

$4 = \frac{8+x}{2} \Rightarrow 8+x = 8 \Rightarrow x = 0$

$5 = \frac{10+y}{2} \Rightarrow 10+y = 10 \Rightarrow y = 0$

$\therefore$ coordinates of other end are $(0,0)$.

Q13. Find the reflection (image) of the point (5, −3) in the point (−1, 3).

Sol. Let the coordinates of the image of the point A (5, −3) be $A_1(x, y)$ in the point (−1, 3) then the point (−1, 3) will be the midpoint of AA_1

$$-1 = \frac{5+x}{2} \Rightarrow 5+x = -2 \Rightarrow x = -7$$

$$3 = \frac{-3+y}{2} \Rightarrow -3+y = 6 \Rightarrow y = 9$$

∴ Coordinates of the image A will be (−7, 9).

Q14. The line segment joining $A(-1, \frac{5}{3})$ and B (a, 5) is divided in the ratio 1:3 at P, the point where the line segment AB intersects y-axis. Calculate (i) The value of a (ii) The co-ordinates of P.

Sol. Let P(x, y) divides the line segment joining the points A (−1, 5/3), B(a, 5) in the ratio 1:3

$$x = \frac{1 \times a + 3(-1)}{1+3} = \frac{a-3}{4}$$

$$y = \frac{1 \times 5 + 3\left(\frac{5}{3}\right)}{1+3} = \frac{10}{4} = \frac{5}{2}$$

(i) AB intersects y-axis at P, $\frac{a-3}{4} = 0 \Rightarrow a = 3$.

(ii) Coordinates of P are (0, 5/2).

Q15. The point P (−4, 1) divides the line segment joining the points A(2, −2) and B in the ratio of 3:5. Find the point B.

Sol. Let the coordinates of B be (x, y).

A (2, -2) and P(-4, 1) divides AB in the ratio of 3:5

$$-4 = \frac{3 \times x + 5(2)}{3+5} \Rightarrow -32 = 3x+10 \Rightarrow 3x = -42$$

$$\Rightarrow x = -14$$

$$1 = \frac{3 \times y + 5(-2)}{3+5} \Rightarrow 3y - 10 = 8 \Rightarrow y = 6$$

∴ Coordinates of B = (-14, 6)

Q16. (i) In what ratio does the point (5,4) divide the line segment joining the points (2,1) and (7,6) ?

Sol. (i) let the ratio be $m_1 : m_2$ that the point (5,4) divides the line segment joining the points (2,1), (7,6).

$$5 = \frac{m_1 \times 7 + m_2 \times 2}{m_1 + m_2} \Rightarrow 7m_1 + 2m_2 = 5m_1 + 5m_2$$

$$\Rightarrow 2m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{2} \Rightarrow m_1 : m_2 = 3:2$$

(ii) In what ratio does the point (-4, b) divide the line segment joining the points P(2, -2), Q (-14, 6). Hence find the value of b.

Sol. let the ratio be $m_1 : m_2$, the point (-4, b) divides the line segment joining the points P(2, -2) and Q(-14, 6)

$$-4 = \frac{m_1(-14) + m_2 \times 2}{m_1 + m_2} \Rightarrow -14m_1 + 2m_2 = -4m_1 - 4m_2$$

$$\Rightarrow 10m_1 = 6m_2 \Rightarrow \frac{m_1}{m_2} = \frac{6}{10} = \frac{3}{5}$$

Again,

$$b = \frac{m_1 \times 6 + m_2(-2)}{m_1 + m_2} = \frac{6m_1 - 2m_2}{m_1 + m_2}$$

$$\Rightarrow b = \frac{6 \times 3 - 2 \times 5}{3+5} = \frac{8}{8} = 1$$

$$\therefore b = 1$$

Q17. The line segment joining A(2, 3) and B(6, −5) is intersected by x-axis at a point K. Write down the ordinate of the point K. Hence, find the ratio in which K divides AB.

Sol. Let the coordinates of K be (x, 0) as it intersects x-axis.

Let point K divides the line segment joining the point A(2, 3) and B(6, −5) in the ratio $m_1 : m_2$.

$$0 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{m_1(-5) + m_2 \times 3}{m_1 + m_2}$$

$$\Rightarrow -5m_1 + 3m_2 = 0 \Rightarrow 5m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{5}$$

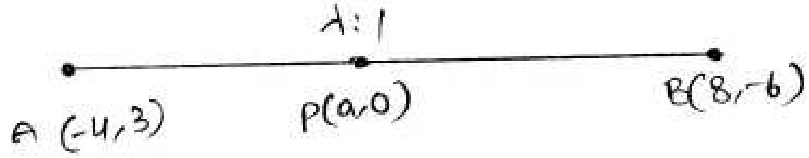
Ordinate of point K = 0.

Q18. If A = (−4, 3) and B = (8, −6), (i) find the length of AB (ii) In what ratio is the line joining AB, divided by the x-axis?

Sol. A = (−4, 3), B(8, −6)

(i) $AB = \sqrt{(8-(-4))^2 + (-6-3)^2} = \sqrt{144+81} = 15$

(ii) Let the ratio be $\lambda : 1$ and the point on the x-axis is P(a, 0)



By section formula, $0 = \frac{-6\lambda+3}{\lambda+1} \Rightarrow \lambda = \frac{1}{2}$

Hence the required ratio is 1 : 2

Q19. Calculate the ratio in which the line segment joining (3,4) and (-2,1) is divided by the y-axis.

Sol. let the point P divides the line segment joining the points A(3,4) and B(-2,4) in the ratio of $m_1 : m_2$ and let the coordinates of P be (0,y) as it intersects y-axis

$$0 = \frac{m_1x_2 + m_2x_1}{m_1+m_2} = \frac{m_1(-2) + m_2 \times 3}{m_1+m_2}$$

$$\Rightarrow -2m_1 + 3m_2 = 0 \Rightarrow 2m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{2}$$

Q20. (i) write down the coordinates of the point P that divides the line joining A(-4,1) and B(17,10) in the ratio 1:2

(ii) calculate the distance OP where O is the origin.

(iii) In what ratio does the y-axis divide the line AB?

Sol. (i) let coordinates of P be (x,y) which divides the line segment joining the points A(-4,1) and B(17,10) in the ratio of 1:2

$$x = \frac{m_1x_2 + m_2x_1}{m_1+m_2} = \frac{1 \times 17 + 2(-4)}{1+2} = \frac{9}{3} = 3$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 10 + 2 \times 1}{1+2} = \frac{12}{3} = 4$$

$\therefore$ coordinates of p are $(3, 4)$.

(ii) Distance of OP where O is origin i.e, $O = (0, 0)$

$$\text{Distance} = \sqrt{(3-0)^2 + (4-0)^2} = 5$$

(iii) let y-axis divides the AB in the ratio of $m_1 : m_2$ at P and let Co-ordinates of p be $(0, y)$

$$0 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{m_1 \times 17 + m_2(-4)}{m_1 + m_2}$$

$$\Rightarrow 17m_1 - 4m_2 = 0 \Rightarrow \frac{m_1}{m_2} = \frac{4}{17}$$

Q21. Calculate the length of the median through the vertex A of the triangle ABC with vertices A(7, -3), B(5, 3) and C(3, -1)

Sol. let D(x, y) be the median of ΔABC through A to BC.

$\therefore$ D will be the midpoint of BC.

coordinates of D will be,

$$x = \frac{5+3}{2} = 4, \quad y = \frac{3-1}{2} = 1$$

$\therefore D = (4, 1)$

$$\text{length of DA} = \sqrt{(7-4)^2 + (-3-1)^2}$$

$$= \sqrt{9+16}$$

$$= 5$$

Q22. prove by section formula that the points $(10,-6)$, $(2,-6)$, $(-4,-2)$ and $(4,-2)$ taken in this order, are the vertices of a parallelogram.

Sol. let $A(10,-6)$, $B(2,-6)$, $C(-4,-2)$, $D(4,-2)$

$$AB = \sqrt{(10-2)^2 + (-6+6)^2} = \sqrt{64} = 8$$

$$BC = \sqrt{(2+4)^2 + (-6+2)^2} = \sqrt{36+16} = \sqrt{52}$$

$$CD = \sqrt{(4+4)^2 + (-2+2)^2} = \sqrt{64} = 8$$

$$DA = \sqrt{(4-10)^2 + (-2+6)^2} = \sqrt{36+16} = \sqrt{52}$$

$\therefore$ $AB = CD$ and $BC = AD$

$\therefore$ Hence proved

Q23. Three consecutive vertices of a parallelogram ABCD are $A(1,2)$, $B(1,0)$, $C(4,0)$. find the fourth vertex D.

Sol. let O is the midpoint of AC the diagonal of ABCD

$\therefore$ Coordinates of O will be $\left(\frac{1+4}{2}, \frac{2+0}{2}\right) = \left(\frac{5}{2}, 1\right)$

$\therefore$ OA also the midpoint of second diagonal BD and let co-ordinates of D be (x,y)

$$\frac{5}{2} = \frac{1+x}{2} \Rightarrow x = 4.$$

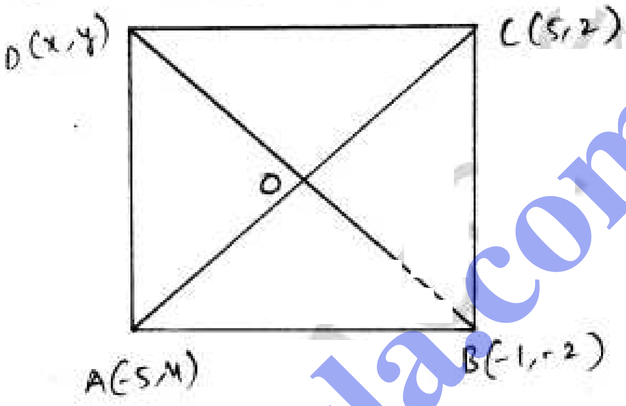
$$1 = \frac{0+y}{2} \Rightarrow y = 2$$

$\therefore$ Coordinates of D are $(4,2)$.

Q24. prove that the points $A(-5,4)$, $B(-1,-2)$ and $C(5,2)$ are the vertices of an isosceles right-angled triangle. find the co-ordinates of D so that ABCD is a square.

Sol. Given points $A(-5,4)$, $B(-1,2)$, $C(5,2)$

If they are vertices of an isoscles right angled triangle ABC then $AB = BC$.



$$AB = \sqrt{(-1-(-5))^2+(-2-4)^2} = \sqrt{16+36} = \sqrt{52}$$

$$BC = \sqrt{(5-(-1))^2+(2-(-2))^2} = \sqrt{36+16} = \sqrt{52}$$

$\therefore AB = BC$

ΔABC is an isosceles triangle.

$$AC = \sqrt{(-5-5)^2+(4-2)^2} = \sqrt{100+4} = \sqrt{104}$$

Now $AC^2 = AB^2 + BC^2$

$$(\sqrt{104})^2 = (\sqrt{52})^2 + (\sqrt{52})^2$$

$\Rightarrow 104 = 104$

$\therefore$ ΔABC is an isoscles right angled triangle.

let the coordinates of D be (x,y) and diagonals AC and BD of ABCD is a square, then diagonals bisect each other at O.

If ABCD is a Square, then diagonals bisects each other at O.

O is mid point of AC

$$O = \left(\frac{5-5}{1}, \frac{2+4}{2}\right) = (0, 3)$$

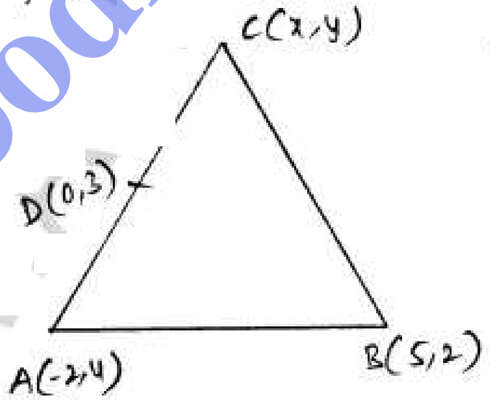
O is mid point of BD also.

$$0 = \frac{-1+x}{2} \Rightarrow x = 1, \quad 3 = \frac{-2+y}{2} \Rightarrow y = 8$$

$\therefore D = (1, 8)$

Q25. Find the third vertex of a triangle if its two vertices are $(-1, 4)$ and $(5, 2)$ and midpoint of one side is $(0, 3)$.

Sol. let $D(0,3)$ be the midpoint of AC and co-ordinates of C be (x, y).



$$0 = \frac{x-1}{2} \Rightarrow x = 1, \quad 3 = \frac{y+4}{2} \Rightarrow y = 2$$

D co-ordinates will be $(1, 2)$

If we take midpoint $D(0,3)$ of BC, then

$0 = \frac{5+x}{2} \Rightarrow x = -5$, $3 = \frac{2+y}{2} \Rightarrow y = 4$

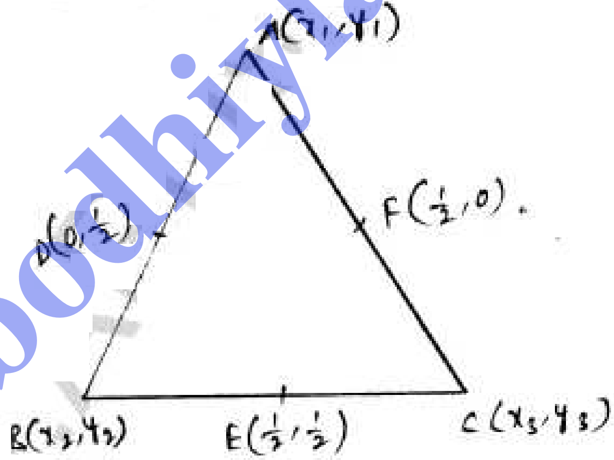
$\therefore$ Co-ordinates of C will be $(-5, 4)$.

The third vertex will be $(1, 2)$ or $(-5, 4)$.

Q26. Find the co-ordinates of the vertices of the triangle the middle points of whose sides are $(0, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, 0)$.

Sol. Let ABC be a triangle, in which $D(0, \frac{1}{2})$, $E(\frac{1}{2}, \frac{1}{2})$, $F(\frac{1}{2}, 0)$ are the midpoints of sides AB, BC and CA respectively.

Let co-ordinates of A be (x_1, y_1), $B(x_2, y_2)$ and $C(x_3, y_3)$



$0 = \frac{x_1+x_2}{2} \Rightarrow x_1 + x_2 = 0$ ——— (i)

$\frac{1}{2} = \frac{y_1+y_2}{2} \Rightarrow y + y_2 = 1$ ——— (ii)

Again $\frac{1}{2} = \frac{x_2+x_3}{2} \Rightarrow x_2 + x_3 = 1$ ——— (iii)

and $\frac{1}{2} = \frac{y_2+y_3}{2} \Rightarrow y_2 + y_3 = 1$ ⟶ (iv)

$\frac{1}{2} = \frac{x_3+x_1}{2} \Rightarrow x_3+x_1 = 1$ —— (v)

$0 = \frac{y_3+y_1}{2} \Rightarrow y_3+y_1 = 0$ —— (vi)

Adding (i), (iii) & (v)

$2(x_1+x_2+x_3) = 0+1+1 = 2$

$\Rightarrow x_1+x_2+x_3 = 1$

Now subtracting (iii), (v) and (i) respectively, we get

$x_1 = 0, \ x_2 = 0, \ x_3 = 1$

Again adding (ii), (iv) and (v)

$2(y_1+y_2+y_3) = 1+1+0 = 2$

$\Rightarrow y_1+y_2+y_3 = 1$

Now subtracting (iv), (v) and (ii) respectively, we get $y_1 = 0, \ y_2 = 1, \ y_3 = 0$

$\therefore A = (0,0), \ B(0,1), \ C(1,0)$.

Q27. Show by section formula that the points $(3,-2)$, $(5,2)$ and $(8,8)$ are collinear.

Sol. Let the point $(5,2)$ divides the line joining the points $(3,-2)$ and $(8,8)$ in the ratio of $m_1 : m_2$

$5 = \frac{m_1 \times 8 + m_2 \times 3}{m_1+m_2} \Rightarrow 8m_1 + 3m_2 = 5m_1 + 5m_2$.

$\Rightarrow 3m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$ —— (i)

Again, $2 = \frac{8m_1 - 2m_2}{m_1+m_2} \Rightarrow 8m_1 - 2m_2 = 2m_1 + 2m_2$

$\Rightarrow 6m_1 = 4m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$ —— (ii)

From (i) & (ii) it is clear that point (5,2) lies on the line joining the points (3,−2) and (8,8).

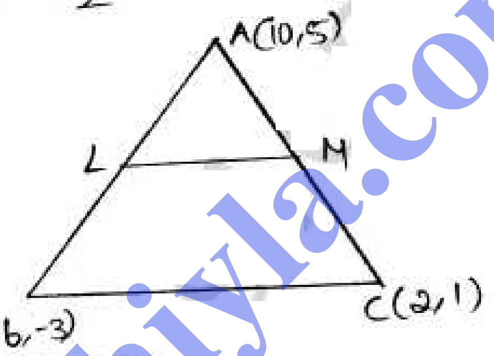
$\therefore$ Hence proved.

Q28. A(10,5), B(6,−3) and C(2,1) are the vertices of a triangle ABC. L is the midpoint of AB and M is the midpoint of AC. Write down the co-ordinates of L and M. Show that $LM = \frac{1}{2} BC$

Sol.

$$L = \left(\frac{10+6}{2}, \frac{5-3}{2}\right) = (8, 1)$$

$$M = \left(\frac{10+2}{2}, \frac{5+1}{2}\right) = (6, 3)$$



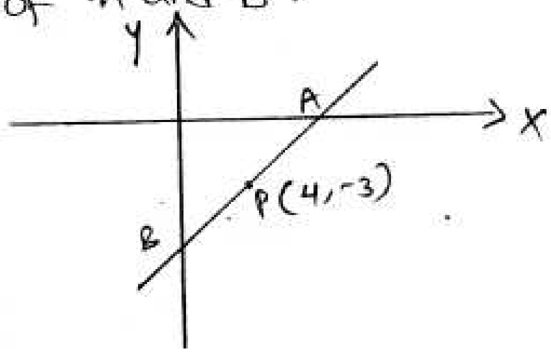
$$LM = \sqrt{(6-8)^2+(3-1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ —(i)}$$

$$BC = \sqrt{(2-6)^2+(1-(-3))^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ —(ii)}$$

From (i) & (ii), $LM = \frac{1}{2} BC$

Q29. The midpoint of the line segment AB shown in the adjoining diagram is (4,−3). Write down the co-ordinates of A and B.

Sol.



A lies on x-axis and B on y-axis

let co-ordinates of A be $(x,0)$ and B be $(0,y)$

$p(4,-3)$ is the third point of AB

$4 = \frac{x+0}{2} \Rightarrow x = 8$, $-3 = \frac{0+y}{2} \Rightarrow y = -6$

$\therefore$ co-ordinates of A will be $(8,0)$ and $B(0,-6)$

Q30. Find the co-ordinates of the centroid of a triangle whose vertices are : $A(-1,3)$, $B(1,-1)$ and $C(5,1)$

Sol. The co-ordinates of centroid $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\Rightarrow G = \left(\frac{-1+1+5}{3}, \frac{3-1+1}{3}\right) = \left(\frac{5}{3}, 1\right)$

Q31. Two vertices of a triangle are $(3,-5)$ and $(-7,4)$. Find the third vertex, given that the centroid is $(2,-1)$.

Sol. let the co-ordinates of third vertex is (x,y) and other two vertices are $(3,-5)$ and $(-7,4)$ and centroid $=(2,-1)$

$2 = \frac{3-7+x}{3} \Rightarrow \frac{x-4}{3} = 2 \Rightarrow x = 10$

$\Rightarrow -1 = \frac{-5+4+y}{3} \Rightarrow y-1 = -3 \Rightarrow y = -2$

$\therefore$ co-ordinates are $(10,-2)$.