Chapter 3 Expansions

EXERCISE – 3.1

Solution- 1

(i) $(2x+7y)^2$

It is in the form of $(a+b)^2 = a^2+2ab+b^2$

$\therefore \; a = 2x \; ; \; b = 7y$

$\therefore \; (2x+7y)^2 = (2x)^2 + 2\cdot 2x \cdot 7y + (7y)^2$

$= 4x^2 + 28xy + 49y^2$.

(ii) $\left(\frac{1}{2}x + \frac{2}{3}y\right)^2$

$\Rightarrow \left(\frac{1}{2}x\right)^2 + \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot x\cdot\frac{2}{3}\cdot y + \left(\frac{2}{3}y\right)^2$

$\Rightarrow \frac{x^2}{4} + \frac{2xy}{3} + \frac{4}{9}y^2$

Solution – 2 :

(i) $\left(3x + \frac{1}{2x}\right)^2$

It is in the form of $(a+b)^2 = a^2+2ab+b^2$

$\therefore \; (3x)^2 + \cancel{2}\cdot 3\cancel{x}\cdot\frac{1}{\cancel{2}\cancel{x}} + \left(\frac{1}{2x}\right)^2$

$\Rightarrow \; 9x^2 + 3 + \frac{1}{4x^2}$

(ii) $(3x^2y + 5z)^2$

It is in the form of $(a+b)^2 = a^2 + 2ab + b^2$

Here $a = 3x^2y$; $b = 5z$.

$\Rightarrow (3x^2y)^2 + 2 \cdot 3x^2y \cdot 5z + (5z)^2$

$\Rightarrow 9x^4y^2 + 30x^2yz + 25z^2$.

Solution - 3

(i) $\left(3x - \frac{1}{2x}\right)^2$

It is in the form of $(a-b)^2 = a^2 - 2ab + b^2$

Here $a = 3x$; $b = \frac{1}{2x}$

$\Rightarrow (3x)^2 - \cancel{2} \cdot 3\cancel{x} \cdot \frac{1}{\cancel{2}\cancel{x}} + \left(\frac{1}{2x}\right)^2$.

$\Rightarrow 3^2 x^2 - 3 + \frac{1}{2^2 x^2}$

$\Rightarrow 9x^2 - 3 + \frac{1}{4x^2}$ //.

(ii) $\left(\frac{1}{2}x - \frac{3}{2}y\right)^2$

It is in the form of $(a-b)^2 = a^2 - 2ab + b^2$

here $a = \frac{1}{2}x$; $b = \frac{3}{2}y$

$\therefore \Rightarrow \left(\frac{1}{2}x\right)^2 - 2 \cdot \frac{1x}{2} \cdot \frac{3}{2}y + \left(\frac{3}{2}y\right)^2$

$\Rightarrow \frac{x^2}{4} - \frac{3}{2}xy + \frac{9y^2}{4}$

Solution-4:

(i) $(x+3)(x+5)$

$\Rightarrow x(x+5) + 3(x+5)$

$\Rightarrow x^2 + 5x + 3x + 15$

$\Rightarrow x^2 + 8x + 15$

(ii) $(x+3)(x-5)$

$\Rightarrow x(x-5) + 3(x-5)$

$\Rightarrow x \cdot x - x \cdot 5 + 3 \cdot x - 3 \cdot 5$

$\Rightarrow x^2 - 5x + 3x - 15$

$\Rightarrow x^2 - 2x - 15$

(iii) $(x-7)(x+9)$

$\Rightarrow x(x+9) - 7(x+9)$

$\Rightarrow x \cdot x + 9 \cdot x - 7 \cdot x - 7 \cdot 9$

$\Rightarrow x^2 + 9x - 7x - 63$

$\Rightarrow x^2 + 2x - 63$.

(iv) $(x-2y)(x-3y)$

$\Rightarrow x(x-3y) - 2y(x-3y)$

$\Rightarrow x \cdot x - x \cdot 3y - 2y \cdot x + 2y\, 3y$

$\Rightarrow x^2 - 3xy - 2xy + 6y^2$

$\Rightarrow x^2 - 5xy + 6y^2$

Solution - 5

(i) $(x-2y-z)^2$

it is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$.

Here $a = x$; $b = -2y$; $c = -z$.

$\therefore \Rightarrow x^2 + (-2y)^2 + (-z)^2 + 2\left(x(-2y) + (-2y)(-z) + (-z) \cdot x\right)$

$\Rightarrow x^2 + 4y^2 + z^2 + 2\left(-2xy + 2yz - zx\right)$

$\Rightarrow x^2 + 4y^2 + z^2 + 4yz - 4xy - 2zx$.

(ii) $(2x-3y+4z)^2$

It is in the form of $(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$.

Here $a=2x$; $b=-3y$; $c=4z$

$\therefore \Rightarrow (2x)^2+(-3y)^2+(4z)^2+2\left(2x\cdot(-3y)+(-3y)(4z)+4z\cdot 2x\right)$

$\Rightarrow 4x^2+9y^2+16z^2+2(-6xy-12yz+8xz)$

$\Rightarrow 4x^2+9y^2+16z^2-12xy-24yz+16xz$

Solution-6 :

(i) $\left(2x+\frac{3}{x}-1\right)^2$

It is in the form of $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Here $a=2x$; $b=\frac{3}{x}$; $c=-1$

$\therefore \Rightarrow (2x)^2+\left(\frac{3}{x}\right)^2+(-1)^2+2\left(2x\cdot\frac{3}{x}+\frac{3}{x}(-1)+(-1)\cdot 2x\right)$

$\Rightarrow 4x^2+\frac{9}{x^2}+1+2\left(6-\frac{3}{x}-2x\right)$

$\Rightarrow 4x^2+\frac{9}{x^2}+1+12-\frac{6}{x}-4x$

$\Rightarrow 4x^2+\frac{9}{x^2}-\frac{6}{x}-4x+13$

(ii) $\left(\frac{2}{3}x - \frac{3}{2x} - 1\right)^2$

It is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Here $a = \frac{2}{3}x$; $b = \frac{-3}{2x}$; $c = -1$

$\therefore \Rightarrow \left(\frac{2}{3}x\right)^2 + \left(\frac{-3}{2x}\right)^2 + (-1)^2 + 2\left[\frac{\cancel{2}}{\cancel{3}}\cancel{x}\left(\frac{-\cancel{3}}{\cancel{2x}}\right) + \left(\frac{-3}{2x}\right)\cdot(-1) + (-1)\left(\frac{2}{3}x\right)\right]$

$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + 1 + 2\left[-1 + \frac{3}{2x} - \frac{2}{3}x\right]$

$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + 1 - 2 + \frac{\overset{3}{\cancel{6}}}{\cancel{2}x} - \frac{4x}{3}$

$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + \frac{3}{x} - \frac{4x}{3} - 1$

Solution-7

(i) $(x+2)^3$

It is in the form of $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Here $a = x$; $b = 2$

$\therefore \Rightarrow x^3 + 3\cdot x^2\cdot 2 + 3\cdot x\cdot 2^2 + 2^3$

$\Rightarrow x^3 + 6x^2 + 3x\cdot 4 + 8$

$\Rightarrow x^3 + 6x^2 + 12x + 8$

(ii) $(2a+b)^3$

$\Rightarrow (2a)^3 + 3.(2a)^2.b + 3.2a.b^2 + b^3$

$\Rightarrow 8a^3 + 3\cdot4a^2\cdot b + 6ab^2 + b^3$

$\Rightarrow 8a^3 + 12a^2b + 6ab^2 + b^3.$

Solution - 8:

(i) $\left(3x + \frac{1}{x}\right)^3$

It is in the form of $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a = 3x$; $b = \frac{1}{x}$

$\therefore (3x)^3 + 3.(3x)^2.\frac{1}{x} + 3.3x\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$

$\Rightarrow 27x^3 + 3.9x^2.\frac{1}{x} + 9x.\frac{1}{x^2} + \frac{1}{x^3}$

$\Rightarrow 27x^3 + 27x + \frac{9}{x} + \frac{1}{x^3}$ //

(ii) $(2x-1)^3$

It is in the form of $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Here $a = 2x$; $b = 1$

$\therefore (2x)^3 - 3(2x)^2.1 + 3(2x)(1)^2 - (1)^3$

$\Rightarrow 8x^3 - 3.4x^2 + 6x - 1$

$\Rightarrow 8x^3 - 12x^2 + 6x - 1$ //

Solution-9:

(i) $(5x-3y)^3$

It is in the form of $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a = 5x;\quad b = 3y$

$\therefore (5x)^3 - 3\cdot(5x)^2\cdot 3y + 3\cdot 5x\cdot(3y)^2 - (3y)^3$

$\Rightarrow 125x^3 - 3\cdot 25x^2\cdot 3y + 3\cdot 5x\cdot 9y^2 - 27y^3$

$\Rightarrow 125x^3 - 225x^2y + 135y^2x - 27y^3$

(ii) $\left(2x - \frac{1}{3y}\right)^3$

$\Rightarrow (2x)^3 - 3\cdot(2x)^2\cdot\frac{1}{3y} + 3\cdot 2x\left(\frac{1}{3y}\right)^2 - \left(\frac{1}{3y}\right)^3$

$\Rightarrow 8x^3 - \cancel{3}\cdot 4x^2\cdot\frac{1}{\cancel{3}y} + \cancel{3}\cdot 2x\,\frac{1}{\cancel{9}_3 y^2} - \frac{1}{27y^3}$

$\Rightarrow 8x^3 - \frac{4x^2}{y} + \frac{2x}{3y^2} - \frac{1}{27y^3}$ //

Solution-10

(i) $(a+b)^2 + (a-b)^2$

$\Rightarrow a^2 + \cancel{2ab} + b^2 + a^2 - \cancel{2ab} + b^2$

$\Rightarrow 2a^2 + 2b^2$

$\Rightarrow 2(a^2+b^2)$

(ii) $(a+b)^2 - (a-b)^2$

$\Rightarrow (a^2+2ab+b^2) - (a^2-2ab+b^2)$

$\Rightarrow \cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}+2ab-\cancel{b^2}$

$\Rightarrow 2ab+2ab$

$\Rightarrow 4ab$

Solution - 11

(i) $\left(a+\frac{1}{a}\right)^2 + \left(a-\frac{1}{a}\right)^2$

$\Rightarrow \left(a^2+2\cdot\cancel{a}\cdot\frac{1}{\cancel{a}}+\frac{1}{a^2}\right) + \left(a^2-2\cdot\cancel{a}\cdot\frac{1}{\cancel{a}}+\frac{1}{a^2}\right)$

$\Rightarrow a^2+\cancel{2}+\frac{1}{a^2}+a^2-\cancel{2}+\frac{1}{a^2}$

$\Rightarrow 2a^2+\frac{2}{a^2}$

$\Rightarrow 2\left(a^2+\frac{1}{a^2}\right)$

(ii) $\left(a+\frac{1}{a}\right)^2 - \left(a-\frac{1}{a}\right)^2$

$\Rightarrow \left(a^2+2\cdot\cancel{a}\cdot\frac{1}{\cancel{a}}+\frac{1}{a^2}\right) - \left(a^2-2\cdot\cancel{a}\cdot\frac{1}{\cancel{a}}+\frac{1}{a^2}\right)$

$\Rightarrow \cancel{a^2}+2+\cancel{\frac{1}{a^2}}-\cancel{a^2}+2-\cancel{\frac{1}{a^2}}$

$\Rightarrow 2+2$

$\Rightarrow 4$

Solution - 12 :

(i) $(3x-1)^2 - (3x-2)(3x+1)$

$\Rightarrow (3x)^2 - 2\cdot 3x \cdot 1 + 1^2 - 3x(3x+1) + 2(3x+1)$

$\Rightarrow \cancel{9x^2} - \cancel{6x} + 1 - \cancel{9x^2} - 3x + \cancel{6x} + 2$

$\Rightarrow 3x + 3$

$\Rightarrow 3(x+1)$

(ii) $(4x+3y)^2 - (4x-3y)^2 - 48$

$\Rightarrow (4x)^2 + 2\cdot 3y \cdot 4x + (3y)^2 - \left((4x)^2 - 2\cdot 4x\cdot 3y + (3y)^2\right) - 48$

$\Rightarrow \cancel{16x^2} + 24xy + \cancel{9y^2} - \cancel{16x^2} + 24xy - \cancel{9y^2} - 48$

$\Rightarrow 48xy - 48$

$\Rightarrow 48(xy-1)$

Solution - 13 :

(i) $(7p+9q)(7p-9q)$

$\Rightarrow 7p(7p-9q) + 9q(7p-9q)$

$\Rightarrow 49p^2 - \cancel{63pq} + \cancel{63pq} - 81q^2$

$\Rightarrow 49p^2 - 81q^2$

(ii) $\left(2x-\frac{3}{x}\right)\left(2x+\frac{3}{x}\right)$

$\Rightarrow (2x)^2-\left(\frac{3}{x}\right)^2$

$\Rightarrow$ Since it is in the form of $(a+b)(a-b)=a^2-b^2$

$\therefore \Rightarrow 4x^2-\frac{9}{x^2}$

solution - 14 :

(i) $(2x-y+3)\ (2x-y-3)$

$\Rightarrow ((2x-y)+3)\ ((2x-y)-3)$

It is in the form of $(a+b)(a-b)=a^2-b^2$.

$\therefore (2x-y)^2-3^2$

$\Rightarrow (2x)^2-2\cdot 2x\cdot y+y^2-9$

$\Rightarrow 4x^2-4xy+y^2-9.$

(ii) $(3x+y-5)\ (3x-y-5)$

$\Rightarrow (3x+(y-5))\ (3x-(y+5))$

$\Rightarrow [(3x-5)+y]\ [(3x-5)-y]$

$\Rightarrow$ It is in the form of $(a+b)\ (a-b)=a^2-b^2$

$\therefore a=3x-5\ ;\ b=y.$

$\therefore (3x-5)^2-y^2$

$\Rightarrow (3x)^2-2\cdot 3x\cdot 5+5^2-y^2$

$\Rightarrow 9x^2-30x+25-y^2$ //

Solution – 15

(i) $\left(x+\frac{2}{x}-3\right)\left(x-\frac{2}{x}-3\right)$

$\Rightarrow \left((x-3)+\frac{2}{x}\right)\left((x-3)-\frac{2}{x}\right)$

It is in the form of $(a+b)(a-b) = a^2-b^2$

$\therefore a = x-3$; $b = \frac{2}{x}$

$\Rightarrow (x-3)^2 - \left(\frac{2}{x}\right)^2$

$\Rightarrow x^2 - 2 \cdot x \cdot 3 + 3^2 - \frac{4}{x^2}$

$\Rightarrow x^2 - 6x + 9 - \frac{4}{x^2}$

(ii) $(5-2x)(5+2x)(25+4x^2)$

It is in the form of $(a+b)(a-b) = a^2-b^2$

$\therefore (5^2-(2x)^2)(25+4x^2)$

$\Rightarrow (25-4x^2)(25+4x^2)$

$\Rightarrow (25)^2 - (4x^2)^2$

$\Rightarrow 625 - 16x^4$

Solution - 16 :

(i) $(x+2y+3)(2y+x+7)$

$\Rightarrow x(2y+x+7) + 2y(2y+x+7) + 3(2y+x+7)$

$\Rightarrow 2xy + x^2 + 7x + 4y^2 + 2xy + 14y + 6y + 3x + 21$

$\Rightarrow x^2 + 4y^2 + 4xy + 10x + 20y + 21$

(ii) $(2x+y+5)(2x+y-9)$

$\Rightarrow 2x(2x+y-9) + y(2x+y-9) + 5(2x+y-9)$

$\Rightarrow 4x^2 + 2xy - 18x + 2xy + y^2 - 9y + 10x + 5y - 45$

$\Rightarrow 4x^2 + y^2 + 4xy - 8x - 4y - 45$

(iii) $(x-2y-5)(x-2y+3)$

$\Rightarrow x(x-2y+3) - 2y(x-2y+3) - 5(x-2y+3)$

$\Rightarrow x^2 - 2xy + 3x - 2xy + 4y^2 - 6y - 5x + 10y - 15$

$\Rightarrow x^2 + 4y^2 - 4xy - 2x - 4y - 15$

(iv) $(3x-4y-2)(3x-4y-6)$

$\Rightarrow 3x(3x-4y-6) - 4y(3x-4y-6) - 2(3x-4y-6)$

$\Rightarrow 9x^2 - 12xy - 18x - 12xy + 16y^2 + 24y - 6x + 8y + 12$

$\Rightarrow 9x^2 + 16y^2 - 24xy \cancel{+6} - 24x + 32y + 12$ //

Solution – 17

(i) $(2p+3q)(4p^2-6pq+9q^2)$

$(2p+3q)\left((2p)^2-2p\cdot 3q+(3q)^2\right)$

It is in the form of $(a+b)(a^2-ab+b^2)$ is

a^3+b^3

$\therefore$ Here $a=2p$; $b=3q$

$\therefore (2p)^3+(3q)^3$

$\Rightarrow 8p^3+27q^3$

(ii) $\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$

It is in the form of $(a+b)(a^2-ab+b^2)$

is a^3+b^3

$\therefore$ Here $a=x$; $b=\frac{1}{x}$

$\therefore x^3+\frac{1}{x^3}$

Solution - 18 :

(i) $(3p-4q)\ (9p^2+12pq+16q^2)$

$(3p-4q)\ ((3p)^2+3p.4q+(4q)^2)$

It is in the form of $(a-b)\ (a^2+ab+b^2)$ is a^3-b^3

$\therefore$ Here $3p=a$; $b=4q$

$\therefore\ (3p)^3-(4q)^3$

$\Rightarrow\ 27p^3-64q^3$ //.

(ii) $\left(x-\frac{3}{x}\right)\left(x^2+3+\frac{9}{x^2}\right)$

$\therefore \left(x-\frac{3}{x}\right)\left(x^2+x\frac{3}{x}+\left(\frac{3}{x}\right)^2\right)$

It is in the form of $(a-b)\ (a^2+ab+b^2)$ is a^3-b^3.

$\therefore$ Here $a=x$; $b=\frac{3}{x}$

$\therefore\ x^3-\left(\frac{3}{x}\right)^3$

$\Rightarrow\ x^3-\frac{27}{x^3}$ //.

Solution-19

Given $(2x+3y+4z)(4x^2+9y^2+16z^2-6xy-12yz-8zx)$

$\Rightarrow (2x+3y+4z)((2x)^2+(3y)^2+(4z)^2-2x\cdot 3y-3y\cdot 4z - 4z\cdot 2x)$

$\therefore$ It is in the form of

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$

$\therefore$ Here $a = 2x$; $b = 3y$; $c = 4z$

$\therefore (2x)^3+(3y)^3+(4z)^3-3\cdot 2x\cdot 3y\cdot 4z$

$\Rightarrow 8x^3+27y^3+64z^3-72xyz$

Solution-20

(i) $(x+1)(x+2)(x+3)$

$[x(x+2)+1(x+2)](x+3)$.

$\Rightarrow (x^2+2x+x+2)(x+3)$

$\Rightarrow (x^2+3x+2)(x+3)$

$\Rightarrow (x^2+3x+2)(x)+(x^2+3x+2)3$

$\Rightarrow x^3+3x^2+2x+3x^2+9x+6$

$\Rightarrow x^3+6x^2+11x+6$

(ii) $(x-2)(x-3)(x+4)$

$\Rightarrow [x(x-3)-2(x-3)](x+4)$

$\Rightarrow (x^2-3x-2x+6)(x+4)$

$\Rightarrow (x^2-5x+6)(x+4)$

$\Rightarrow (x^2-5x+6)x+(x^2-5x+6)4$

$\Rightarrow x^3-5x^2+6x+4x^2-20x+24$

$\Rightarrow x^3-x^2-14x+24$

Solution - 21 :

Given $(x-3)(x+7)(x-4)$

$(x(x+7)-3(x+7))(x-4)$

$(x^2+7x-3x-21)(x-4)$

$(x^2+4x-21)(x-4)$

$(x^2+4x-21)x-4(x^2+4x-21)$

$x^3+\cancel{4x^2}-21x-\cancel{4x^2}-16x+84$

$x^3-37x+84$

$\therefore$ The coefficient of x^2 is $\underline{0}$

The coefficient of x is $\underline{-37}$

Solution- 22 :

Given $a^2+4a+x = (a+2)^2$

$\therefore \quad a^2+4a+x = a^2+2\cdot a\cdot 2+2^2$

$a^2+4a+x = a^2+4a+4$

$\therefore \quad x = a^2+4a+4-a^2-4a$

$\boxed{x = 4}$

Solution - 23 :

(i) $(101)^2$

$\Rightarrow \quad (100+1)^2$

$\Rightarrow \quad (100)^2 + 2\cdot 100\cdot 1 + 1^2$

$\Rightarrow \quad 10000 + 200 + 1$

$\Rightarrow \quad \underline{10201}$

(ii) $(1003)^2$

$\Rightarrow \quad (1000+3)^2$

$\Rightarrow \quad (1000)^2 + 2\cdot 1000\cdot 3 + 3^2$

$\Rightarrow \quad 1000000 + 6000 + 9$

$\Rightarrow \quad \underline{1006009}$

(iii) $(10.2)^2$

$(10 + 0.2)^2$

$(10)^2 + 2 \times 10 \times 0.2 + (0.2)^2$

$100 + 4 + 0.04$

104.04

<u>Solution - 24 :</u>

(i) 99^2

$\Rightarrow (100 - 1)^2$

$\Rightarrow (100)^2 - 2.100.1 + 1^2$

$\Rightarrow 10000 - 200 + 1$

$\Rightarrow 9801$

(ii) $(997)^2 \Rightarrow (1000 - 3)^2$

$\Rightarrow 1000^2 - 2\ 1000.3 + 3^2$

$\Rightarrow 1000000 - 6000 + 9$

$\Rightarrow 994009$

In this we used the $(a-b)^2$ formulae

ie., $a^2 - 2ab + b^2$.

(iii) $(9.8)^2$

$\Rightarrow (10-0.2)^2$

$\Rightarrow 10^2 - 2\times10\times0.2 + (0.2)^2$

$\Rightarrow 100 - 4 + 0.4$

$\Rightarrow 96.04$

<u>Solution – 25 :</u>

(i) $(103)^3$

$\Rightarrow (100+3)^3$

$\therefore$ It is in the form of

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore$ Here $a = 100$; $b = 3$

$\therefore \Rightarrow (100)^3 + 3\cdot(100)^2\cdot 3 + 3\cdot100\cdot 3^2 + 3^3$

$\Rightarrow 1000000 + 90000 + 2700 + 27$

$\Rightarrow 1092727$

(ii) 99^3

$\Rightarrow (100-1)^3$

$\Rightarrow 100^3 - 3\cdot100^2\cdot1 + 3\cdot100\cdot1^2 - 1^3$

$\Rightarrow 1000000 - 30000 + 300 - 1$

$\Rightarrow 970299.$

(iii) $(10.1)^3$

$\Rightarrow (10 + 0.1)^3$

$\Rightarrow 10^3 + 3 \cdot 10^2 \cdot (0.1) + 3 \cdot 10\, (0.1)^2 + (0.1)^3$

$\Rightarrow 1000 + 30 + 3 + 0.01$

$\Rightarrow 1030.301$

<u>Solution - 26 :</u>

Given $2a - b + c = 0$

$\Rightarrow (2a + c) = b$

Squaring on both sides

$\Rightarrow (2a + c)^2 = b^2$

$\Rightarrow (2a)^2 + 2 \cdot 2a \cdot c + c^2 = b^2$

$\Rightarrow 4a^2 + 4ac + c^2 = b^2$

$\Rightarrow 4a^2 - b^2 + c^2 + 4ac = 0$.

$\therefore$ Hence proved.

Solution- 27

Given $a+b+2c=0$

$a+b=-2c \quad \cdots (i)$

cubing on both sides

$(a+b)^3 = (-2c)^3$

$a^3+b^3+3a^2b+3ab^2 = -8c^3$

$a^3+b^3+3ab(a+b) = -8c^3 \quad (\because$ from (i))

$a^3+b^3+3ab(-2c) = -8c^3$

$a^3+b^3-6abc = -8c^3$

$a^3+b^3+8c^3 = 6abc$

Hence proved.

Solution- 28 :

Given $a+b+c=0$

$a+b=-c \quad \cdots (i)$

Cubing on both sides

$(a+b)^3 = (c)^3$

$\Rightarrow a^3+b^3+3a^2b+3ab^2 = -c^3$

$\Rightarrow a^3+b^3+3ab(a+b) = -c^3$

$$\Rightarrow a^3+b^3+3ab(-c) = -c^3$$

$$\Rightarrow a^3+b^3-3abc = -c^3$$

$$\Rightarrow a^3+b^3+c^3 = 3abc$$

$$\Rightarrow \frac{a^3+b^3+c^3}{abc} = 3$$

$$\Rightarrow \frac{a^3}{abc}+\frac{b^3}{abc}+\frac{c^3}{abc} = 3$$

$$\Rightarrow \therefore \frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab} = 3 //$$

<u>Solution - 29:</u>

(a) Given $x+y=4$

Cubing on both side:

$$(x+y)^3 = 4^3$$

$$\Rightarrow x^3+3x^2y+3xy^2+y^3 = 64$$

$$\Rightarrow x^3+3xy(x+y)+y^3 = 64$$

$$\Rightarrow x^3+3xy(4)+y^3 = 64$$

$$\Rightarrow x^3+12xy+y^3 = 64$$

$$\therefore x^3+y^3+12xy-64 = 0 //$$

Solution - 30 :

(i) $(27)^3 + (-17)^3 + (-10)^3$

$\therefore$ If $a+b+c=0$; then $a^3+b^3+c^3 = 3abc$

$\therefore$ Here $a = 27$

$b = -17$

$c = -10$

$\therefore$ $27-17-10 = 0$

$\therefore$ $a^3+b^3+c^3 = 3abc$

$= 3 \cdot 27 \cdot (-17)(-10)$

$= 13770$

(ii) $(-28)^3 + 15^3 + 13^3$

$\therefore$ If $a+b+c = 0$; then $a^3+b^3+c^3 = 3abc$.

$\Rightarrow$ $-28+15+13 = 0$

$\therefore$ $\Rightarrow a^3+b^3+c^3 = 3abc$

$= 3(-28)(15)(13)$

$= -16380$.

Solution – 31

Given $\dfrac{86\times 86\times 86 + 14\times 14\times 14}{86\times 86 - 86\times 14 + 14\times 14}$

$\therefore$ It is in the form of $\dfrac{a^3+b^3}{a^2-ab+b^2} = (a+b)$

$$\therefore \frac{(86)^3+(14)^3}{86^2-86\cdot 14+14^2} = 86+14$$

$$= 100$$

EXERCISE - 3.2

Solution-1 :

Given $x - y = 8$ ... (i)

$xy = 5$ ... (ii)

Squaring on both sides in equ (i)

$\therefore (x-y)^2 = 8^2$

$x^2 - 2xy + y^2 = 64$

$x^2 - 2(5) + y^2 = 64$ $\because$ from eq (ii)

$x^2 - 10 + y^2 = 64$

$x^2 + y^2 = 64 + 10$

$x^2 + y^2 = 74$ //

Solution-2 :

Given $x + y = 10$ ... (i)

$xy = 21$ ... (ii)

Squaring on both sides in equ(i)

$(x+y)^2 = 10^2$

$x^2 + 2xy + y^2 = 100$

$x^2 + 2(21) + y^2 = 100$

$x^2 + 42 + y^2 = 100$

$x^2 + y^2 = 100 - 42$

$x^2 + y^2 = 58$

$2(x^2 + y^2) = 2 \times 58$

$= 116$ //

Solution-3 :

Given $2a+3b=7$ — (i)

$ab=2$ — (ii)

Squaring on both sides in equ(i)

$$(2a+3b)^2 = 7^2$$

$$(2a)^2 + 2\cdot 2a\cdot 3b + (3b)^2 = 49$$

$$4a^2 + 12ab + 9b^2 = 49$$

$$4a^2 + 12(2) + 9b^2 = 49$$

$$4a^2 + 24 + 9b^2 = 49$$

$$4a^2 + 9b^2 = 49 - 24$$

$$4a^2 + 9b^2 = 25$$

Solution-4 :

Given $3x-4y=16$ — (i)

$xy=4$ — (ii)

Squaring on both sides in equ(i)

$$(3x-4y)^2 = 16^2$$

$$(3x)^2 - 2\cdot 3x\cdot 4y + (4y)^2 = 256$$

$$9x^2 - 24xy + 16y^2 = 256$$

$$9x^2 - 24(4) + 16y^2 = 256$$

$$9x^2 - 96 + 16y^2 = 256$$

$$9x^2 + 16y^2 = 256 + 96$$

$$9x^2 + 16y^2 = 352$$

Solution- 5:

Given $x + y = 8$ ---- (i)

$x - y = 2$ ---- (ii)

Since we know that

$$2(x)^2 + 2(y)^2 = (x+y)^2 + (x-y)^2$$

from -(i) → squaring on both sides.

$$(x+y)^2 = 8^2$$
$$= 64$$

from -(ii) → Squaring on both sides

$$(x-y)^2 = 2^2$$
$$= 4$$

$$\therefore \quad 2(x^2+y^2) = (x+y)^2 + (x-y)^2$$
$$= 64 + 4$$
$$= 68$$

Solution- 6

Given $a^2 + b^2 = 13$ — (i)

$ab = 6$ — (ii)

(i) $a+b$

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$= a^2 + b^2 + 2ab$$
$$= 13 + 2(6)$$
$$= 13 + 12$$

$$(a+b)^2 = 25$$

$$a+b = \sqrt{25}$$

$$a+b = 5 \, //.$$

(ii) <u>a-b</u>

$$\therefore (a-b)^2 = a^2 - 2ab + b^2$$

$$= a^2 + b^2 - 2ab$$

$$= 13 - 2(6)$$

$$= 13 - 12$$

$$(a-b)^2 = 1$$

$$(a-b) = \sqrt{1}$$

$$\therefore a-b = 1 \, //.$$

<u>Solution-7 :</u>

Given $a+b = 4$ ... i)

$ab = -12$ ..... (ii)

(i) <u>a-b</u>.

from (i) & (2)

$$ab = -12$$

$$a(4-a) = -12$$

$$4a - a^2 = -12$$

$$a^2 - 12 - 4a = 0$$

$$a^2 - 4a - 12 = 0.$$

$a^2 - 6a + 2a - 12 = 0$

$a(a-6) + 2(a-6) = 0$

$(a-6)(a+2) = 0$

$\therefore a - 6 = 0$; $a + 2 = 0$

$\therefore a = 6$; $a = -2$.

$\therefore a+b = 4$; $\therefore a+b = 4$

$6 + b = 4$; $-2 + b = 4$

$b = 4 - 6$; $b = 4 + 2$

$b = -2$. ; $b = 6$

$\therefore (a, b) = (6, -2)$

(i) $a - b \Rightarrow 6 - (-2) = 8$

(ii) $a^2 - b^2 = (a+b)(a-b)$

$= (4)(8)$

$= 32$.

Solution - 8

Given $\quad p - q = 9 \quad \text{--- (i)}$

$\quad pq = 36 \quad \text{--- (ii)}$

from (i) & (ii)

$$p = q + 9$$

$$\therefore \quad pq = 36$$

$$(q+9)\,q = 36$$

$$q^2 + 9q = 36$$

$$q^2 + 9q - 36 = 0$$

$$q^2 + 12q - 3q - 36 = 0$$

$$q(q+12) - 3(q+12) = 0$$

$$(q+12)(q-3) = 0$$

$$\therefore \quad q + 12 = 0 \qquad q - 3 = 0$$

$$q = -12 \qquad q = 3$$

$$\because \quad p - q = 9 \qquad p - q = 9$$

$$p - (-12) = 9 \qquad p - 3 = \cancel{q}\,9$$

$$p = 9 - 12 \qquad p = 9 + 3$$

$$p = -3. \qquad p = 12 \ \checkmark$$

$$\boxed{p = 12 \ ; \ q = 3}$$

(i) $p+q$

$\Rightarrow 12+3$

$= 15$

(ii) p^2-q^2

$\Rightarrow (p+q)(p-q)$

$\Rightarrow 15 \cdot 9$

$\Rightarrow 135$

Solution- 9:

Given $x+y=6$ --- (i)

$x-y=4$ --- (ii)

from (i) $\rightarrow$ squaring on both sides

$(x+y)^2 = 6^2$

$x^2+y^2+2xy = 36$

$x^2+y^2 = 36-2xy$ ---- (iii)

from -(ii) $\rightarrow$ squaring on both sides

$(x-y)^2 = 4^2$

$x^2-2xy+y^2 = 16$

$x^2+y^2 = 16+2xy$ ----(iv)

$\therefore$ equate equ (iii) & (iv)

$$36 - 2xy = 16 + 2xy$$

$$36 - 16 = 2xy + 2xy$$

$$20 = 4xy$$

$$4xy = 20$$

$$xy = \frac{20}{4}$$

$$xy = 5 \,//.$$

(i) $x^2 + y^2$

$\Rightarrow$ from equ (iii) $x^2 + y^2 = 36 - 2xy$

$$= 36 - 2(5)$$

$$= 36 - 10$$

$$= 26$$

(ii) $xy = 5 \,//.$

solution - 10 :

Given $x - 3 = \frac{1}{x}$.

$x - \frac{1}{x} = 3$.

squaring on both sides

$\left(x - \frac{1}{x}\right)^2 = 3^2$

$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 9$

$x^2 - 2 + \frac{1}{x^2} = 9$

$x^2 + \frac{1}{x^2} = 9 + 2$

$x^2 + \frac{1}{x^2} = 11$ //

solution - 11

Given $x + y = 8$ ---- (1)

$xy = 3\frac{3}{4} = \frac{15}{4}$ ---- (2)

from eq (1) $\Rightarrow$ $y = 8 - x$

from eq (2 $\Rightarrow$ $xy = \frac{15}{4}$

$x(8 - x) = \frac{15}{4}$

$8x - x^2 = \frac{15}{4}$

$4(8x - x^2) = 15$

$32x - 4x^2 = 15$

$4x^2 - 32x + 15 = 0$

$4x^2 - 32x + 15 = 0$ $\qquad$ $4 \times 15 = 60$

$\qquad\qquad\qquad\qquad\qquad\qquad 30 \quad 2$

$4x^2 - 30x - 2x + 15 = 0$

$2x(2x - 15) - 1(2x - 15) = 0$

$(2x - 15)(2x - 1) = 0$

$\therefore 2x - 15 = 0 \quad ; \quad 2x - 1 = 0$

$2x = 15 \quad\quad 2x = 1$

$x = \frac{15}{2} \quad\quad x = 1/2$

$\therefore x + y = 8$

$\frac{15}{2} + y = 8$

$y = 8 - \frac{15}{2}$

$y = \frac{16 - 15}{2}$

$y = \frac{1}{2}$

$x + y = 8$

$\frac{1}{2} + y = 8$

$y = 8 - \frac{1}{2}$

$y = \frac{16 - 1}{2}$

$y = \frac{15}{2}$

$\therefore x = \frac{15}{2} ; y = \frac{1}{2}$

(i) $x - y$

$\Rightarrow \frac{15}{2} - \frac{1}{2}$

$\Rightarrow \frac{15 - 1}{2}$

$\Rightarrow \frac{14}{2}$

$\Rightarrow 7$ //

(ii) $3(x^2+y^2)$

$\Rightarrow \quad 3\left[\left(\frac{15}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]$

$\Rightarrow \quad 3\left[\frac{225}{4}+\frac{1}{4}\right]$

$\Rightarrow \quad 3\left[\frac{225+1}{4}\right]$

$\Rightarrow \quad 3\left(\frac{\cancel{226}^{\,113}}{\cancel{4}_{\,2}}\right)$

$\Rightarrow \quad \frac{3\times 113}{2}$

$\Rightarrow \quad \frac{339}{2}$ //.

(iii) $5(x^2+y^2)+4(x-y)$

$5\left(\left(\frac{15}{2}\right)^2+\left(\frac{1}{2}\right)^2\right)+4(7)$

$\Rightarrow \quad 5\left(\frac{225}{4}+\frac{1}{4}\right)+4(7)$

$\Rightarrow \quad 5\left(\frac{113}{2}\right)+28$

$\Rightarrow \quad \frac{565}{2}+28$

$\Rightarrow \quad \frac{565+56}{2}$

$\Rightarrow \quad \frac{621}{2}$ //.

Solution - 12

Given $x^2+y^2=34$

$xy = 10\frac{1}{2} = \frac{21}{2}$

$\therefore \quad (x+y)^2 = x^2+y^2+2xy$

$= 34 + 2 \cdot \frac{21}{2}$

$= 34+21$

$= 55$

$\therefore \quad (x-y)^2 = x^2+y^2-2xy$

$= 34 - 2 \cdot \frac{21}{2}$

$= 34-21$

$= 13$

$\therefore \quad 2(x+y)^2 + (x-y)^2$

$\Rightarrow \quad 2(55) + 13$

$\Rightarrow \quad 110 + 13$

$\Rightarrow \quad 123$

Solution - 13

Given $a-b=3$ —— (i)

$ab = 4$ —— (ii)

$\therefore \quad a^3-b^3 = (a-b)(a^2+ab+b^2)$

from (i) squaring on both sides

$(a-b)^2 = 3^2$

$a^2+b^2 - 2ab = 9$

$a^2+b^2 - 2(4) = 9$

$a^2+b^2 - 8 = 9$

$a^2+b^2 = 9+8$

$a^2+b^2 = 17.$

$\therefore \; a^3 - b^3 = (a-b)\,(a^2+ab+b^2)$

$= 3\cdot(a^2+b^2+ab)$

$= 3\,(17+4)$

$= 3\,(21)$

$= 63$

<u>solution - 14 :</u>

Given $\quad 2a - 3b = 3 \quad$ —— (1)

$\qquad ab = 2 \quad$ —— (2)

from (1) squaring on both sides

$(2a-3b)^2 = 3^2$

$(2a)^2 - 2\cdot 2a\cdot 3b + (3b)^2 = 9$

$4a^2 - 12ab + 9b^2 = 9$

$4a^2 + 9b^2 - 12(2) = 9$

$4a^2 + 9b^2 - 24 = 9$

$4a^2 + 9b^2 = 9 + 24$

$= 33.$

$\therefore (2a)^3 - (3b)^3 \Rightarrow 8a^3 - 27b^3$

$\Rightarrow (2a-3b)\left((2a)^2 + 2a\cdot 3b + (3b)^2\right)$

$\Rightarrow 3\left(4a^2 + 6ab + 9b^2\right)$

$\Rightarrow 3\left(4a^2 + 9b^2 + 6ab\right)$

$\Rightarrow 3\left(33 + 6(2)\right)$

$\Rightarrow 3\left(33+12\right)$

$\Rightarrow 3\left(45\right)$

$\Rightarrow 135$ //.

Solution—15 :

Given $x + \frac{1}{x} = 4.$

(i) squaring on both sides

$\left(x+\frac{1}{x}\right)^2 = 4^2$

$x^2 + 2\cdot x\cdot\frac{1}{x} + \frac{1}{x^2} = 16$

$x^2 + 2 + \frac{1}{x^2} = 16$

$x^2 + \frac{1}{x^2} = 16 - 2$

$x^2 + \frac{1}{x^2} = 14.$

(ii) $x^4 + \frac{1}{x^4}$

$\because$ we know that $x^2 + \frac{1}{x^2} = 14$.

$\therefore$ squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 14^2$$

$$(x^2)^2 + 2 \cdot \cancel{x^2} \cdot \frac{1}{\cancel{x^2}} + \left(\frac{1}{x^2}\right)^2 = 196$$

$$x^4 + 2 + \frac{1}{x^4} = 196$$

$$x^4 + \frac{1}{x^4} = 196 - 2$$

$$x^4 + \frac{1}{x^4} = 194.$$

(iii) $x^3 + \frac{1}{x^3}$

$\because$ we know $x + \frac{1}{x} = 4$

cubing on both sides

$$\left(x + \frac{1}{x}\right)^3 = 4^3$$

$$x^3 + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$$

$$x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$$

$$x^3 + \frac{1}{x^3} + 3(4) = 64$$

$$x^3 + \frac{1}{x^3} = 64 - 12$$

$$= 52.$$

(iv) $x - \frac{1}{x}$

$$\left(x - \frac{1}{x}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$\because$ from (i) $x^2 + \frac{1}{x^2} = 14.$

$$= 14 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 12$$

$$x - \frac{1}{x} = \sqrt{12}$$

$$= \sqrt{4 \times 3}$$

$$= 2\sqrt{3}$$

Solution – 16 :

Given $x - \frac{1}{x} = 5$

Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = 5^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 25$$

$$x^2 + \frac{1}{x^2} = 25 + 2$$

$$x^2 + \frac{1}{x^2} = 23$$

Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 23^2$$

$$(x^2)^2 + 2\cdot\cancel{x^2}\cdot\frac{1}{\cancel{x^2}} + \left(\frac{1}{x^2}\right)^2 = 529$$

$$x^4 + 2 + \frac{1}{x^4} = 529$$

$$x^4 + \frac{1}{x^4} = 529 - 2$$

$$x^4 + \frac{1}{x^4} = 527 //$$

<u>Solution- 17</u>

Given $x - \frac{1}{x} = \sqrt{5}$

(i) squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = (\sqrt{5})^2$$

$$x^2 - 2\cdot\cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{x^2} = 5$$

$$x^2 - 2 + \frac{1}{x^2} = 5$$

$$x^2 + \frac{1}{x^2} = 5 + 2$$

$$x^2 + \frac{1}{x^2} = 7$$

(ii). $x + \frac{1}{x}$ //

$$\left(x + \frac{1}{x}\right)^2 = x^2 + 2\cdot\cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} + 2.$$ [∵ from (i)]

$$= 7 + 2$$

$$= 9.$$

$$\left(x+\frac{1}{x}\right)^2 = 9$$

$$x+\frac{1}{x} = \sqrt{9}$$

$$x+\frac{1}{x} = 3 \text{ //.}$$

(iii) $x^3+\frac{1}{x^3}$

$$\Rightarrow \left(x+\frac{1}{x}\right)\left(x^2 - x\cdot\frac{1}{x} + \frac{1}{x^2}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$$

$$\Rightarrow 3\cdot(7-1)$$

$$\Rightarrow 3\times 6$$

$$\Rightarrow 18 \text{ //.}$$

Solution - 18

Given $x+\frac{1}{x} = 6$.

Squaring on both sides

$$\left(x+\frac{1}{x}\right)^2 = 6^2$$

$$x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2} = 36$$

$$x^2 + 2 + \frac{1}{x^2} = 36$$

$$x^2 + \frac{1}{x^2} = 36 - 2$$

$$= 34 \text{ //.}$$

(i) $x - \frac{1}{x}$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$= 34 - 2$

$\left(x - \frac{1}{x}\right)^2 = 32$

$x - \frac{1}{x} = \sqrt{32}$

$x - \frac{1}{x} = \sqrt{16 \times 2}$

$x - \frac{1}{x} = 4\sqrt{2}$

(ii) $x^2 - \frac{1}{x^2}$

$\Rightarrow x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 6 \cdot 4\sqrt{2}$

$\Rightarrow 24\sqrt{2}$

Solution - 19

Given $x + \frac{1}{x} = 2$

Squaring on both sides

$\left(x + \frac{1}{x}\right)^2 = 2^2$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 4$$

$$x^2 + 2 + \frac{1}{x^2} = 4$$

$$x^2 + \frac{1}{x^2} = 4 - 2$$

$$x^2 + \frac{1}{x^2} = 2 \quad \text{. . . . (i)}$$

$$\therefore \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= 2 \cdot \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= 2\ (2 - 1)$$

$$= 2\ (1)$$

$$= 2 \quad \text{- - - - (ii)}$$

from (i) $\Rightarrow$ $x^2 + \frac{1}{x^2} = 2$

Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 2^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 4$$

$$x^4 + 2 + \frac{1}{x^4} = 4$$

$$x^4 + \frac{1}{x^4} = 4 - 2$$

$$= 2 \quad \text{- - - (iii)}$$

from equ's (i), (ii) & (iii)

$$x^2+\frac{1}{x^2} = 2$$

$$x^3+\frac{1}{x^3} = 2$$

$$x^4+\frac{1}{x^4} = 2$$

$$\therefore \quad x^2+\frac{1}{x^2} = x^3+\frac{1}{x^3} = x^4+\frac{1}{x^4}$$

Hence proved.

Solution- 20 :

Given $x-\frac{2}{x} = 3$.

cubing on both sides

$$\left(x-\frac{2}{x}\right)^3 = 3^3$$

$$x^3 - 3 \cdot x \cdot \frac{2}{x}\left(x-\frac{2}{x}\right) - \left(\frac{2}{x}\right)^3 = 27$$

$$x^3 - 6\left(x-\frac{2}{x}\right) - \frac{8}{x^3} = 27$$

$$x^3 - \frac{8}{x^3} - 6(3) = 27$$

$$x^3 - \frac{8}{x^3} = 27+18$$

$$= 45$$

Solution - 21

Given $a+2b = 5$

Cubing on both sides

$$(a+2b)^3 = 5^3$$

$$a^3 + 3a \cdot 2b(a+2b) + (2b)^3 = 125$$

$$a^3 + 6ab(5) + 8b^3 = 125$$

$$a^3 + 30ab + 8b^3 = 125$$

$$\therefore \quad a^3 + 8b^3 + 30ab = 125$$

Solution - 22 :

Given $a + \frac{1}{a} = p$

Cubing on both sides

$$\left(a+\frac{1}{a}\right)^3 = p^3$$

$$a^3 + 3a \cdot \frac{1}{a}\left(a+\frac{1}{a}\right) + \frac{1}{a^3} = p^3$$

$$a^3 + 3(p) + \frac{1}{a^3} = p^3$$

$$a^3 + \frac{1}{a^3} = p^3 - 3p$$

$$a^2 + \frac{1}{a^3} = p(p^2-3)$$

Solution-23 :

Given $x^2 + \frac{1}{x^2} = 27$

$\because \left(x - \frac{1}{x}\right)^2 = x^2 - 2\cancel{x} \cdot \frac{1}{\cancel{x}} + \frac{1}{x^2}$

$= x^2 - 2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$= 27 - 2$

$\left(x - \frac{1}{x}\right)^2 = 25$

$x - \frac{1}{x} = \sqrt{25}$

$x - \frac{1}{x} = 5$ //

Solution-24 :

Given $x^2 + \frac{1}{x^2} = 27$

Take $\left(x - \frac{1}{x}\right)^2 = x^2 - 2\cancel{x} \cdot \frac{1}{x} + \frac{1}{x^2}$

$= x^2 - 2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$= 27 - 2$

$\left(x - \frac{1}{x}\right)^2 = 25$

$x - \frac{1}{x} = \sqrt{25}$

$x - \frac{1}{x} = 5$

$\therefore \quad 3x^3 + 5x - \frac{3}{x^3} - \frac{5}{x}$

$\Rightarrow \quad 3x^3 - \frac{3}{x^3} + 5x - \frac{5}{x}$

$\Rightarrow \quad 3\left(x^3 - \frac{1}{x^3}\right) + 5\left(x - \frac{1}{x}\right)$

$\Rightarrow \quad 3\left(x - \frac{1}{x}\right)\left(x^2 + \cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{x^2}\right) + 5\left(x - \frac{1}{x}\right)$

$\Rightarrow \quad 3\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right) + 5\left(x - \frac{1}{x}\right)$

$\Rightarrow \quad 3(5)(27+1) + 5(5)$

$\Rightarrow \quad 15(28) + 25$

$\Rightarrow \quad 420 + 25$

$\Rightarrow \quad 445.$

Solution - 25 :

Given $\quad x^2 + \frac{1}{25x^2} = 8\frac{3}{5}$

$x^2 + \left(\frac{1}{5x}\right)^2 = \frac{43}{5}$

$\therefore$ let us consider

$\left(x + \frac{1}{5x}\right)^2 = x^2 + 2\cdot\cancel{x}\cdot\frac{1}{5\cancel{x}} + \left(\frac{1}{5x}\right)^2$

$= x^2 + \frac{2}{5} + \frac{1}{25x^2}$

$= x^2 + \frac{1}{25x^2} + \frac{2}{5}$

$= \frac{43}{5} + \frac{2}{5}$

$$\left(x+\frac{1}{5x}\right)^2 = \frac{43+2}{5}$$

$$\left(x+\frac{1}{5x}\right)^2 = \frac{47}{5}$$

$$x+\frac{1}{5x} = \sqrt{\frac{47}{5}}$$

Solution - 26 :

Given $x^2+\frac{1}{4x^2} = 8$

$$x^2+\left(\frac{1}{(2x)}\right)^2 = 8$$

let us consider

$$\left(x+\frac{1}{2x}\right)^2 = x^2+2\cdot x\cdot\frac{1}{2x}+\left(\frac{1}{2x}\right)^2$$

$$= x^2+1+\frac{1}{4x^2}$$

$$= x^2+\frac{1}{4x^2}+1$$

$$= 8+1$$

$$\left(x+\frac{1}{2x}\right)^2 = 9$$

$$x+\frac{1}{2x} = \sqrt{9}$$

$$x+\frac{1}{2x} = 3$$

$$x^3+\left(\frac{1}{2x}\right)^3$$

$$\Rightarrow x^3+\frac{1}{8x^3} = \left(x+\frac{1}{2x}\right)\left(x^2-x\cdot\frac{1}{2x}+\left(\frac{1}{2x}\right)^2\right)$$

$$x^3 + \frac{1}{8x^3} = \left(x + \frac{1}{2x}\right)\left(x^2 + \frac{1}{4x^2} - 1\right)$$

$$= 3\,(8-1)$$

$$= 3\,(7)$$

$$= \underline{\underline{21}}$$

Solution- 27 :

Given $a^2 - 3a + 1 = 0$

dividing each term by a, we get

$$\frac{a^2}{a} - \frac{3a}{a} + \frac{1}{a} = 0$$

$$a - 3 + \frac{1}{a} = 0$$

$$a + \frac{1}{a} = 3$$

Now

(i) $$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

$$a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$$

$$= 3^2 - 2$$

$$= 9 - 2$$

$$= 7 \,//$$

(ii) $a^3 + \frac{1}{a^3}$

$\Rightarrow \left(a + \frac{1}{a}\right)\left(a^2 - a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$

$\Rightarrow \left(a + \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)$

$\Rightarrow (3)(7-1)$

$\Rightarrow 3 \times 6$

$\Rightarrow 18$

Solution - 28

Given $a = \frac{1}{a-5}$

$a(a-5) = 1$

$a^2 - 5a = 1$

$a^2 - 5a - 1 = 0$

(i) Dividing each term by a, we get

$\frac{a^2}{a} - \frac{5a}{a} - \frac{1}{a} = 0$

$a - 5 - \frac{1}{a} = 0$

$\therefore a - \frac{1}{a} = 5$

(ii) Now $\left(a+\frac{1}{a}\right)$,

$\because \quad a-\frac{1}{a}=5$

$\therefore$ Squaring on both sides

$$\left(a-\frac{1}{a}\right)^2=5^2$$

$$a^2-2\cdot a\cdot\frac{1}{a}+\frac{1}{a^2}=25$$

$$a^2+\frac{1}{a^2}=25-2$$

$$a^2+\frac{1}{a^2}=23$$

$$\therefore \left(a+\frac{1}{a}\right)^2=a^2+2\cdot a\cdot\frac{1}{a}+\frac{1}{a^2}$$

$$=a^2+\frac{1}{a^2}+2$$

$$=23+2$$

$$=25 \,//$$

$$\therefore \left(a+\frac{1}{a}\right)^2=25$$

$$a+\frac{1}{a}=\sqrt{25}$$

$$=5 \,//$$

(iii) $a^2-\frac{1}{a^2}=\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$

$$=5\cdot 5$$

$$=25 \,//$$

Solution – 29

Given $\left(x+\frac{1}{x}\right)^{2} = 3$

$$x^{2} + 2\cdot\cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{x^{2}} = 3$$

$$x^{2} + \frac{1}{x^{2}} = 3-2$$

$$x^{2} + \frac{1}{x^{2}} = 1$$

$$\therefore \quad x^{3} + \frac{1}{x^{3}} = \left(x+\frac{1}{x}\right)\left(x^{2} - x\cdot\frac{1}{x} + \frac{1}{x^{2}}\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^{2}+\frac{1}{x^{2}}-1\right)$$

$$= \sqrt{3}\,(1-1)$$

$$= \sqrt{3}\,(0)$$

$$= 0 \,//$$

Solution – 30

Given $x = 5-2\sqrt{6}$

squaring on both sides

$$x^{2} = 5$$

Solution -31

Given $a+b+c=12$

Squaring on both sides

$$(a+b+c)^2 = 12^2$$

$$a^2+b^2+c^2+2(ab+bc+ca) = 144$$

$\because$ from given $ab+bc+ca = 22$

$$\therefore a^2+b^2+c^2 + 2(22) = 144$$

$$a^2+b^2+c^2 + 44 = 144$$

$$a^2+b^2+c^2 = 144-44$$

$$a^2+b^2+c^2 = 100$$

Solution- 32 :

Given $a+b+c=12$

squaring on both sides

$$(a+b+c)^2 = 12^2$$

$$a^2+b^2+c^2+2(ab+bc+ca) = 144$$

$$\because a^2+b^2+c^2 = 100$$

$$\therefore 100 + 2(ab+bc+ca) = 144$$

$$2(ab+bc+ca) = 144-100$$

$$2(ab+bc+ca) = 44$$

$$ab+bc+ca = \frac{44}{2} = 22.$$

Solution – 33

Given $a^2+b^2+c^2 = 125$

$ab+bc+ca = 50$

$\because (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$= 125+2(50)$

$= 125+100$

$= 225$

$\therefore (a+b+c)^2 = 225$

$a+b+c = \sqrt{225}$

$a+b+c = 15$

Solution – 34 :

Given $a+b-c = 5$

$a^2+b^2+c^2 = 29$

$(a+b-c)^2 = a^2+b^2+c^2+2(ab-bc-ca)$

$5^2 = 29+2(ab-bc-ca)$

$25 = 29+2(ab-bc-ca)$

$25-29 = 2(ab-bc-ca)$

$-4 = 2(ab-bc-ca)$

$ab-bc-ca = \frac{-4}{2} = -2.$

solution - 35

Given $a-b=7$

$a^2+b^2=85$

$(a-b)^2 = a^2+b^2-2ab$

$7^2 = 85-2ab$

$49 = 85-2ab$

$2ab = 85-49$

$2ab = 36$

$ab = \frac{36}{2}$

$ab = 18.$

$\therefore \quad a^3-b^3 = (a-b)(a^2+ab+b^2)$

$= 7(a^2+b^2+ab)$

$= 7(85+18)$

$= 7(103)$

$= 721$

Solution – 36

Given, the number is x.

$\therefore \quad x = y - 3$

$\therefore \quad x - y = -3$

$y - x = 3$

and $\quad x^2 + y^2 = 29$

$\therefore \quad y - x = 3$

squaring on both sides

$(y-x)^2 = 3^2$

$y^2 + x^2 - 2xy = 9$

$29 - 2xy = 9$

$29 - 9 = 2xy$

$2xy = 20$

$xy = \frac{20}{2}$

$xy = 10.$

Solution - 37 :

Given, sum of two numbers = 8

product of two numbers = 15

let, numbers be x and y.

$\therefore$ $x+y=8$

$xy=15$

$\therefore$ $x+y=8$

Squaring on both sides

$(x+y)^2=8^2$

$x^2+y^2+2xy=64$

$x^2+y^2+2(15)=64$

$x^2+y^2=64-30$

$x^2+y^2=34$

$\therefore$ ~~at~~ $\underline{x^3+y^3}$

$\therefore \Rightarrow (x+y)(x^2-xy+y^2)$

$\Rightarrow 8(x^2+y^2-xy)$

$\Rightarrow 8(34-8)$

$\Rightarrow 8(26)$

$\Rightarrow 208$