Theorems on Area

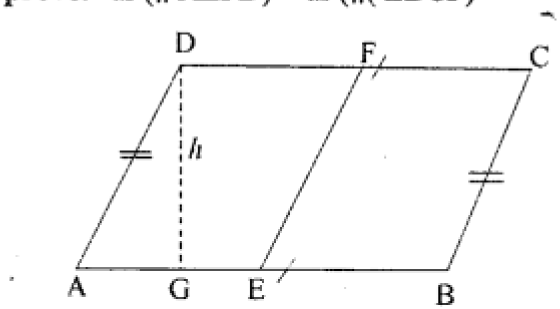
Question 1.

Prove that the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Solution:

Given. ABCD is a parallelogram in which E and F are mid-points of AB and CD respectively. Joining EF.

To prove. ar (|| AEFD) = ar (||(EBCF)



Construction. DG ⊥ AG and let DG = h i.e. h is the Altitude on side AB.

Proof. ar (|| ABCD) = AB × h

ar (|| AEFD) = AE × h = $\frac{1}{2}$ AB × h ..... (1)

(∵ E is the mid-point of AB)

ar (|| EBCF) = EF × h = $\frac{1}{2}$ AB × h ..... (2)

(∵ E is the mid-point of AB)

From (1) and (2)

ar (|| gm ABFD) = ar (|| EBCF)

Hence, the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms. **(Q.E.D.)**

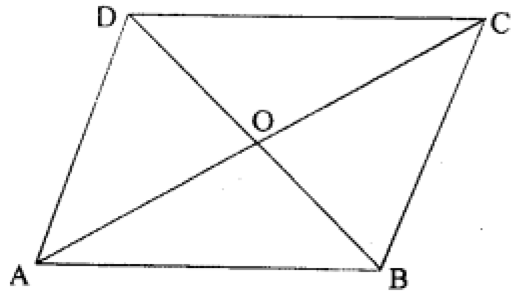
Question 2.

Prove that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution:

Given. In parallelogram ABCD the diagonals AC and BD are cut at point O.

To prove. $ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(\Delta AOD)$



Proof. We know that

In parallelogram ABCD the diagonals are bisecting each other.

$\therefore$ AO = OC

In ΔACD, O is the mid-point of AC.

$\therefore$ DO is median

$\therefore$ $ar(\Delta AOD) = ar(COD)$ ..... (1)

[Median of a triangle divides it into two triangles of equal areas]

Similarly, in ΔABC

$ar(\Delta AOB) = ar(\Delta COB)$ ..... (2)

In ΔADB

$ar(\Delta AOD) = ar(\Delta AOB)$ ..... (3)

and in ΔCDB

$ar(\Delta COD) = ar(\Delta COB)$ ..... (4)

From (1), (2), (3) and (4)

$ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(\Delta AOD)$

Hence, diagonals of a parallelogram divide it into four triangles of equal Area. **(Q.E.D.)**

Question 3.

(a) In the figure (1) given below, AD is median of ΔABC and P is any point on AD. Prove that

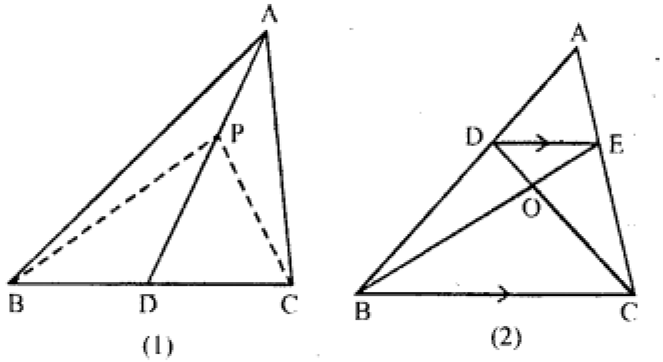
(i) area of ΔPBD = area of ΔPDC.

(ii) area of ΔABP = area of ΔACP.

(b) In the figure (2) given below, DE || BC. prove that (i) area of ΔACD = area of Δ

ABE.

(ii) area of $\triangle OBD$ = area of $\triangle OCE$.

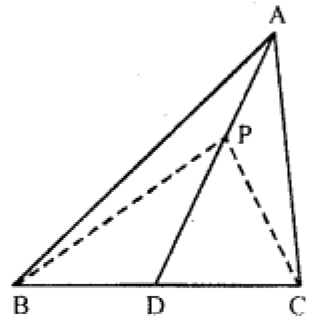


Solution:

(a) Given. A ΔABC in which AD is median. P is any point on AD. Join PB and PC.

To prove. (*i*) $ar(\Delta PBD) = ar(\Delta PDC)$

(*ii*) $ar(\Delta ABP) = ar(\Delta ACP)$



(1)

Proof. Since, AD is median of ΔABC

$\therefore \quad ar(\Delta ABD) = ar(\Delta ADC)$ ..... (1)

(Median of a triangle divides it into two triangles of equal areas)

Also PD is median of ΔBPD

Similarly ar $(\Delta PBD) = ar(\Delta PDC)$ ..... (2)

Subtracting (2) from (1),

$ar(\Delta ABD - ar(\Delta PBD) = ar(\Delta ADC) - ar(\Delta PDC)$

or, $ar(\Delta ABP) = ar(\Delta ACP)$ **(Q.E.D.)**

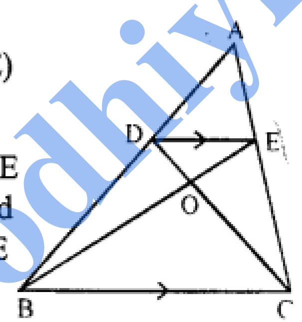
***(b)* Given.** ΔABC in which DE || BC.

To prove.

(*i*) $ar(\Delta ACD) = ar(\Delta ABE)$

(*ii*) $ar(\Delta OBD) = ar(\Delta OCE)$

Proof. (*i*) ΔDEC and ΔBDE are on the same base DE and between the same || line DE and BE.



(2)

$ar(\Delta DEC) = ar(\Delta BDE)$

Adding ar (ADE) to both side,

$ar(\Delta DEC) + ar(\Delta ADE) = ar(\Delta BDE) + ar(\Delta ADE)$

$\Rightarrow \quad ar(\Delta ACD) = ar(\Delta ABE)$ **(Q.E.D.)**

(ii) $ar(\Delta DEC) = ar(\Delta BDE)$

Subtracting ar (ΔDOE) from both side,

$ar(\Delta DEC) - ar(\Delta DOE) = ar(\Delta BDE) - ar(\Delta DOE)$

$\Rightarrow \quad ar(\Delta OBD) = ar(\Delta OCE)$ **(Q.E.D.)**

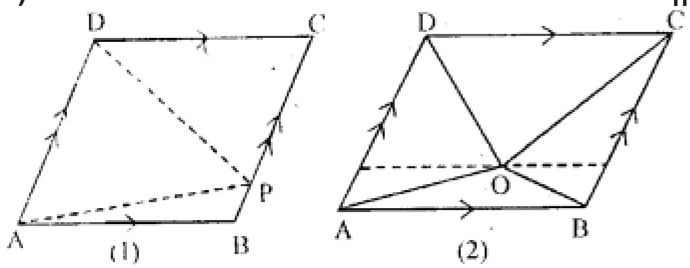
Question 4.

(a) In the figure (1) given below, ABCD is a parallelogram and P is any point in BC. Prove that: Area of $\triangle$ABP + area of $\triangle$DPC = Area of $\triangle$APD.

(b) In the figure (2) given below, O is any point inside a parallelogram ABCD. Prove that:

(i) area of $\triangle$OAB + area of $\triangle$OCD = $\frac{1}{2}$ area of || gm ABCD.

(ii) area of $\triangle$ OBC + area of $\triangle$ OAD = $\frac{1}{2}$ area of ||gmABCD

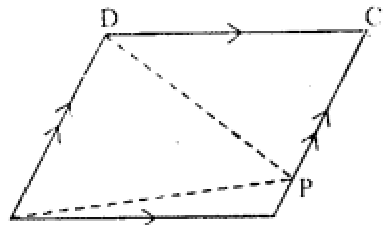


Solution:

(*a*) **Given.** ABCD is a parallelogram and P is any point in BC.

To prove.

$ar(\Delta ABP) + ar(\Delta DPC) = ar(\Delta APD)$



(*i*)

Proof. ΔAPD and $\|$ gm ABCD are on the same Base AD and between the same $\|$ lines AD and BC,

$$ar(\Delta APD) = \frac{1}{2} ar(\| \text{gm ABCD}) \quad \text{..... (1)}$$

In parallelogram ABCD

$ar(\| \text{gm ABCD}) = ar(\Delta ABP) + ar(\Delta APD) + ar(\Delta DPC)$

Dividing both sides by 2, we get

$$\frac{1}{2} ar(\| \text{gm ABCD}) = \frac{1}{2} ar(\Delta ABP) + \frac{1}{2} ar(\Delta APD) + \frac{1}{2} ar(\Delta DPC) \quad \text{..... (2)}$$

From (1) and (2)

$$ar(\Delta APD) = \frac{1}{2} ar(\Delta ABP) + \frac{1}{2} ar(\Delta APD) + \frac{1}{2} ar(\Delta DPC)$$

$$ar(\Delta APD) - \frac{1}{2} ar(\Delta APD) = \frac{1}{2} ar(\Delta ABP) + \frac{1}{2} ar(\Delta DPC)$$

$$\Rightarrow \quad \frac{1}{2} ar(\Delta APD) = \frac{1}{2} [ar(\Delta ABP) + ar(\Delta DPC)]$$

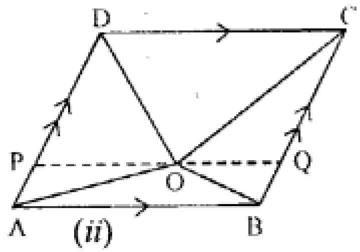
$$\Rightarrow \quad ar(\Delta APD) = ar(\Delta ABP) + ar(\Delta DPC)$$

$$\therefore \quad ar(\Delta ABP) + ar(\Delta DPC) = ar(\Delta APD)$$

(Q.E.D.)

(*b*) **Given.** || gm ABCD in which O is any point inside it.

To prove. (*i*) $ar(\Delta OAB) + ar(\Delta OCD) = \frac{1}{2} ar(\| \text{gm } ABCD)$



(*ii*)

(*ii*) $ar(\Delta OBC) + ar(\Delta OAD) = \frac{1}{2} ar(\| \text{gm } ABCD)$

Construction. Draw POQ || AB through O. It meets AD at P and BC at Q.

Proof. (*i*) AB || PQ and AP || BQ

∴ ABQP is a || gm

Similarly PQCD is a || gm

Now, ΔOAB and || gm ABQP are on same base AB and between same || lines AB and PQ.

$\therefore \quad ar(\Delta OAB) = \frac{1}{2} ar(\| \text{gm } ABQP) \qquad \dots (1)$

Similarly, $ar(\Delta OCD) = \frac{1}{2} ar(\| \text{gm } PQCD) \qquad \dots (2)$

Adding (1) and (2),

$ar(\Delta OAB) + ar(\Delta OCD)$

$= \frac{1}{2} ar(\| \text{gm } ABQP) + \frac{1}{2} ar(\| \text{gm } PQCD)$

$\Rightarrow \quad ar(\Delta OAB) + ar(\Delta OCD)$

$= \frac{1}{2} [ar(\| \text{gm } ABQP) + ar(\| \text{gm } PQCD)]$

$\Rightarrow \quad ar(\Delta OAB) + ar(\Delta OCD) = \frac{1}{2} ar(\| \text{gm } ABCD)$

(Q.E.D.)

(iii) $\because ar(\Delta OAB) + ar(\Delta OBC) + ar(\Delta OCD) + ar(\Delta OAD) = ar = (\| gm\ ABCD)$

$\Rightarrow [ar(\Delta OAB) + ar(\Delta OCD)] + [ar(\Delta OBC) + ar(\Delta OAD)] = ar(\| gm\ ABCD)$

$\Rightarrow \frac{1}{2} ar(\| gm\ ABCD) + ar(\Delta OBC) + ar(\Delta OAD) = ar(\| gm\ ABCD)$

$\Rightarrow ar(\Delta OBC) + ar(\Delta OAD) = ar(\| gm\ ABCD) - \frac{1}{2} ar(\| gm\ ABCD)$

$\Rightarrow ar(\Delta OBC) + ar(\Delta OAD) = \frac{1}{2} ar(\| gm\ ABCD)$

(Q.E.D.)

Question 5.

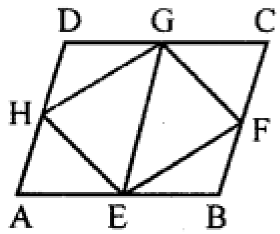
If E, F, G and H are mid-points of the sides AB, BC, CD and DA respectively of a parallelogram ABCD, prove that area of quad. EFGH = 1/2 area of || gm ABCD.

Solution:

Given : In parallelogram ABCD, E, F, G, H are the mid-points of its sides AB, BC, CD and DA respectively EF, FG, GH and HE are joined

To prove : Area of quad. EFGH = 1/2 area ||gm ABCD

Construction : Join EG



Proof : ∵ E and G are mid-points of AB and CD respectively

∴ EG || AD || BC

∴ AEGD and EBCG are parallelogram

Now ||gm AEGD and ΔEHG are on the same base and between the parallel lines

∴ area ΔEHG = $\frac{1}{2}$ area ||gm AEGD

Similarly,

area ΔEFG = $\frac{1}{2}$ area ||gm EBCG

Adding (*i*) and (*ii*),

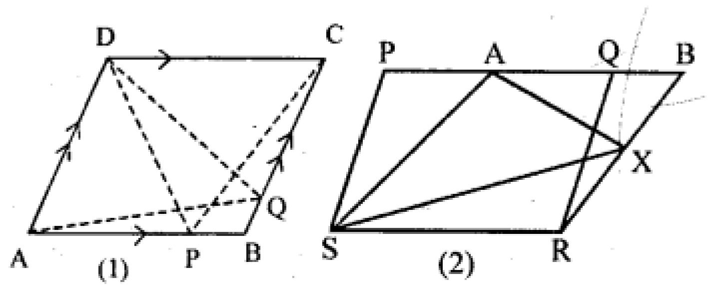
area ΔEHG + area ΔEFG = $\frac{1}{2}$ area ||gm AEGD + area ||gm EBCG

⇒ area quad. EFGH = $\frac{1}{2}$ area ||gm ABCD

Hence proved.

Question 6.

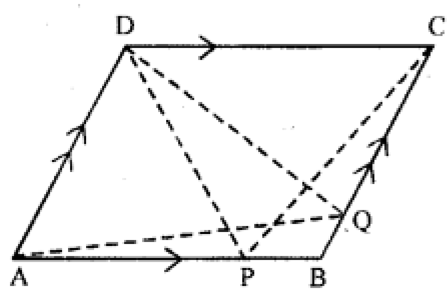
(a) In the figure (1) given below, ABCD is a parallelogram. P, Q are any two points on the sides AB and BC respectively. Prove that, area of △ CPD = area of △ AQD.



(b) In the figure (2) given below, PQRS and ABRS are parallelograms and X is any point on the side BR. Show that area of Δ AXS = $\frac{1}{2}$ area of ||gm PQRS

Solution:

(a) Given. || gm ABCD in which P is a point on AB and Q is a point on BC.



To prove. (i) ar (ΔCPD) = *ar* (AQD)

Proof. ΔCPD and || gm ABCD are on the same base CD and between the same parallels AB and CD.

$\therefore$ $ar(\Delta CPD) = \frac{1}{2} ar$ (|| gm ABCD) ..... (1)

ΔAQD and || gm ABCD are on the same base AD and between the same || lines AD and BC,

$\therefore$ $ar(\Delta AQD) = \frac{1}{2} ar$ (|| gm ABCD) ..... (2)

From (1) and (2),

ar (ΔCPD) = *ar* (ΔAQD)

Hence, area of ΔCPD = area of ΔAQD.

(Q.E.D.)

(b) Given : PQRS and ABRS are parallelogram on the same base SR. X is any point on BR. AX and SX are joined.

To prove : area ΔAXS = $\frac{1}{2}$ area ||gm PQRS

$\because$ ||gm PQRS and ABRS are on the same base SR and between the same parallels

$\therefore$ area ||gm PQRS = area ||gm ABRS ...(i)

$\because$ ΔAXS and ||gm ABRS are on the same base AS and between the same parallels

$\therefore$ area ΔAXS = $\frac{1}{2}$ area ||gm ABRS

= $\frac{1}{2}$ area ||gm PQRS [From (i)]

Question 7.

D,EandF are mid-point of the sides BC, CA and AB respectively of a ∆ ABC. Prove that

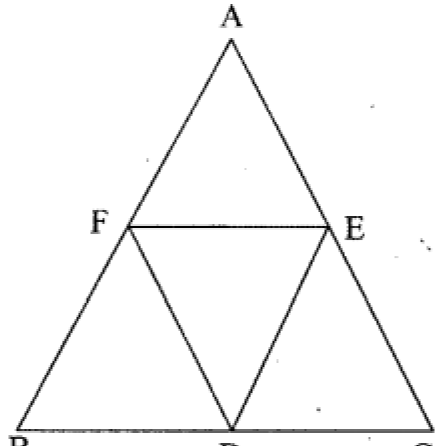
(i) FDCE is a parallelogram

(ii) area of ADEF = $\frac{1}{4}$ area of A ABC

(iii) area of || gm FDCE = $\frac{1}{2}$ area of ∆ ABC.

Solution:

Given. D, E, F are mid-points of the sides BC, CA, and AB respectively of a ΔABC.



To prove. (*i*) FDCE is a parallelogram.

(*ii*) Area of $\Delta DEF = \frac{1}{4}$ are of ΔABC.

(*iii*) Area of $\parallel$ gm FDCE $= \frac{1}{2}$ area of ΔABC.

Proof. $\because$ F and E are mid-points of AB and AC respectively.

$\therefore$ FE $\parallel$ BC and $FE = \frac{1}{2} BC$ ..... (1)

Also D is mid-point of BC

$CD = \frac{1}{2} BC$ ..... (2)

From (1) and (2)

FE $\parallel$ BC and FE = CD

i.e. FE $\parallel$ CD and FE = CD ..... (3)

Similarly D and F are mid-points of BC and AB respectively.

$\therefore$ DF $\parallel$ EC is a $\parallel$ gm

(Q.E.D.)

(ii) Since FDCE is a $\parallel$ gm

And DE is diagonal of $\parallel$ gm FDCE

$\therefore$ $ar(\Delta DEF) = ar(DEC)$ ..... (5)

[$\because$ A diagonal of a parallelogram divides, it into two triangle of equal areas]

Similarly, we show BDEF and DEAF are $\parallel$ gm and ar (Δ DEF) = ar (Δ BDF) = ar (Δ AFE) ..... (6)

From (5) and (6)

ar (Δ DEF) = ar (Δ BDF) = ar (Δ AFE) = ar (Δ DEC)

Now, ar (Δ ABC) = ar (Δ BDF) + ar (Δ DEF) + ar (Δ DEC) + ar (Δ AFE)

$\Rightarrow$ ar (Δ ABC) = ar (Δ DEF) + ar (Δ DEF) + ar (Δ DEF) + ar (Δ DEF)

$\Rightarrow$ ar (Δ ABC = 4 ar (Δ DEF)

$\Rightarrow$ ar (Δ DEF) = $\frac{1}{4}$ ar (Δ ABC)

$\therefore$ area of Δ DEF = $\frac{1}{4}$ area of Δ ABC ..... (7)

(Q.E.D.)

(iii) Area of || gm FDCE = ar (Δ DEF) + ar (Δ DEC)

= ar (Δ DEF) + ar (Δ DEF)

= 2ar (Δ DEF) [From (5)]

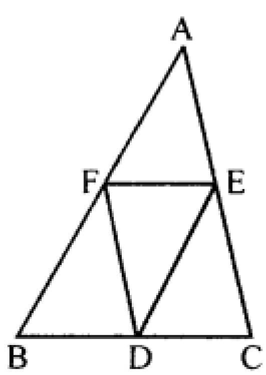
$= 2\left[\frac{1}{4} ar (\Delta ABC)\right]$ [From (7)]

$\therefore$ Area of || gm FDCE = $\frac{1}{2}$ area of Δ ABC

(Q.E.D.)

Question 8.

In the given figure, D, E and F are mid points of the sides BC, CA and AB respectively of AABC. Prove that BCEF is a trapezium and area of trap. BCEF = $\frac{3}{4}$ area of Δ ABC.



Solution:

Given : In ΔABC, D, E and F are the mid-points of the sides BC, CA and AB respectively and are joined in order.

To prove : Area trapezium BCEF = $\frac{3}{4}$ area ΔABC.

Proof : $\because$ D and E are the mid-points of BC and CA respectively.

$\therefore$ DE || AB and $\frac{1}{2}$ AB

Similarly, EF || BC and $\frac{1}{2}$ BC

and FD || AC and $\frac{1}{2}$ AC

$\therefore$ BDEF, CEFD and AFDE are parallelograms which are equal in area.

ED, DE and EF are the diagonals of these ||gms which divide correspondning parallelogram into two triangles equal in area.

Now, area of trapezium BCEF has the equal trianlges and ΔABC has 4 equal triangles.

$\therefore$ area of trap. BCEF = $\frac{3}{4}$ area (ΔABC)

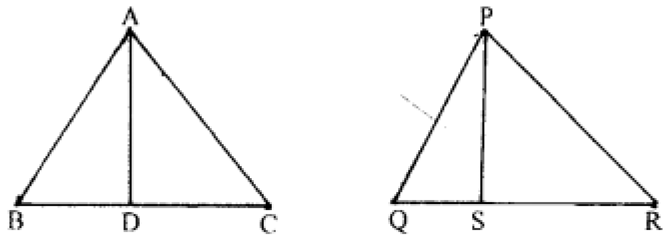
Question P.Q.

Prove that two triangles having equal areas and having one side of one of the triangles equal to one side of the other, have their corresponding altitudes equal.

Solution:

Given. Area of ΔABC = area of ΔPQR

Also BC = QR



To prove. AD = PS, where AD and PS are Altitudes of ΔABC and ΔPQR respectively.

Proof. Area of ΔABC = Area of ΔPQR ..... (1)

Now, area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times AD$ ..... (2)

Area of $\Delta PQR = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$\Rightarrow$ $ar(\Delta PQR) = \frac{1}{2} \times QR \times PS$ ..... (3)

From (1), (2) and (3),

$\frac{1}{2} \times BC \times AD = \frac{1}{2} \times QR \times PS$

or $BC \times AD = QR \times PS$

or $QR \times AD = QR \times PS$ [BC = QR (given)

or $AD = PS$

i.e. Altitude of ΔABC = Altitude of ΔPQR

(Q.E.D.)

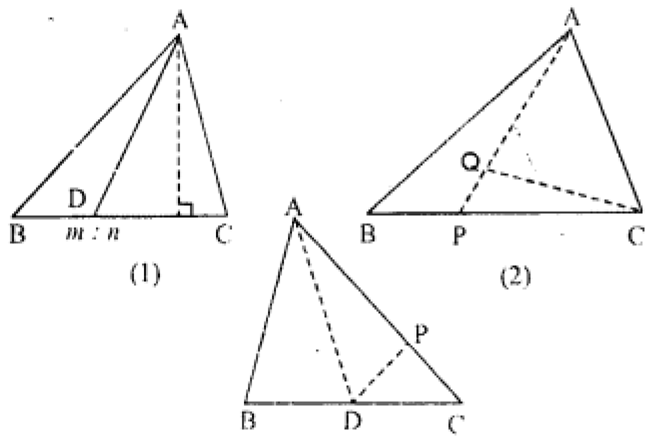
Question 9.

(a) In the figure (1) given below, the point D divides the side BC of ∆ABC in the ratio m : n. Prove that area of ∆ ABD: area of ∆ ADC = m : n.

(b) In the figure (2) given below, P is a point on the sidoBC of ∆ABC such that PC = 2BP, and Q is a point on AP such that QA = 5 PQ, find area of ∆AQC : area of ∆ABC.

(c) In the figure (3) given below, AD is a median of ∆ABC and P is a point in AC such that area of ∆ADP : area of AABD = 2:3. Find

(i) AP : PC (ii) area of ∆PDC : area of ∆ABC.



(3)

Solution:

Given. In Δ ABC, D divides the side BC in the ratio $m : n$ i.e BD : DC = $m : n$

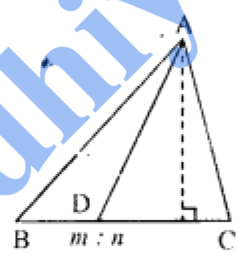
To prove. Area of Δ ABD : Area of Δ ADC = $m : n$

Proof. Area of $\Delta ABD = \frac{1}{2} \times \text{Base} \times \text{height}$

$\Rightarrow \quad ar(\Delta ABD) = \frac{1}{2} \times BD \times AE \qquad \text{..... (1)}$

Area of $(\Delta ACD) = \frac{1}{2} \times DC \times AE \qquad \text{..... (2)}$

Dividing (1) by (2)



(1)

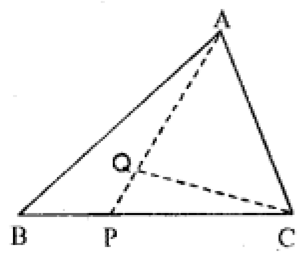
$$\frac{ar(\Delta ABD)}{ar(\Delta ACD)} = \frac{\frac{1}{2} \times BD \times AE}{\frac{1}{2} \times DC \times AE}$$

$$\Rightarrow \quad \frac{ar(\Delta ABD)}{ar(\Delta ACD)} = \frac{BD}{DC} = \frac{m}{n}$$

[given BD : DC $= m : n$]

(Q.E.D.)

(*b*) **Given.** In ΔABC, P is a point on side BC such that PC = 2BP and Q is a point on AP such that QA = 5PQ.



Required. Area of ΔAQC : Area of ΔABC

Sol. $PC = 2BP$ [given)

But $BC = BP + PC$

$$\Rightarrow \quad BC = \frac{PC}{2} + PC \qquad \left[\because \; BP = \frac{PC}{2}\right]$$

$$\Rightarrow \quad BC = \frac{PC + 2PC}{2} \Rightarrow BC = \frac{3PC}{2}$$

$$\Rightarrow \quad PC = \frac{3}{2} BC$$

$$\therefore \quad \text{Area of } \Delta APC = \frac{2}{3} \text{Area of } \Delta ABC \qquad ..(1)$$

$QA = 5PQ$ (given)

$$\Rightarrow \quad AQ = \frac{5}{6} AP \qquad [\because \; AQ = AQ + PQ]$$

$$\Rightarrow \quad \text{Area of } \Delta AQC = \frac{5}{6} \text{Area of } \Delta APC$$

$$= \frac{5}{6} \times \left(\frac{2}{3} \text{Area of } \Delta ABC\right) \qquad [\text{From } (1)]$$

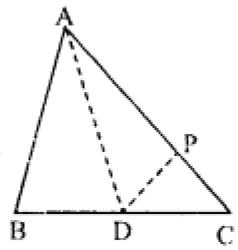
$$= \frac{5}{9} \text{ Area of } \Delta ABC$$

$$\therefore \quad \frac{\text{Area of } \Delta AQC}{\text{Area of } \Delta ABC} = \frac{5}{9}$$

Hence, Area of ΔAQC : Area of ΔABC = 5 : 9 **Ans.**

(c) **Given.** AD is a median of ΔABC. P is a point on AC such that

Area of ΔADP : area of ΔABD = 2 : 3



Required. (*i*) AP : PC

(*ii*) Area of ΔPDC : area of ΔABC

Sol. (*i*) Since AD is median of ΔABC

$\therefore$ Area of ΔABD = Area of ΔADC = $\frac{1}{2}$ area of ΔABC ..... (1)

[$\because$ Median divides a triangle into two triangle of equal area]

Given. Area of ΔADP : Area of $\Delta ABD = 2:3$ (given)

$\Rightarrow$ Area of ΔADP : Area of $\Delta ADC = 2:3$

$\Rightarrow$ $AP : AC = 2:3$

$$\Rightarrow \frac{AP}{AC} = \frac{2}{3} \Rightarrow AP = \frac{2}{3}AC$$

Now, $PC = AC - AP = AC - \frac{2}{3}AC = \frac{AC}{3}$ ..... (2)

$$\therefore \frac{AP}{PC} = \frac{\frac{2}{3}AC}{\frac{AC}{3}} = \frac{2}{1}$$

$\Rightarrow$ $AP : PC = 2:1$

(ii) From (2) $PC = \frac{AC}{3}$

$$\Rightarrow \frac{PC}{AC} = \frac{1}{3}$$

Since the base AC, of ΔPDC, ΔADC lie along the same line, and these triangles have equal heights, therefore,

$$\frac{\text{Area of } \Delta PDC}{\text{Area of } \Delta ADC} = \frac{PC}{AC}$$

$$\Rightarrow \frac{\text{Area of } \Delta PDC}{\text{Area of } \Delta ADC} = \frac{1}{3}$$

$$\Rightarrow \frac{\text{Area of } \Delta PDC}{\frac{1}{2}\text{area of } \Delta ABC} = \frac{1}{3} \quad \text{[From (1)]}$$

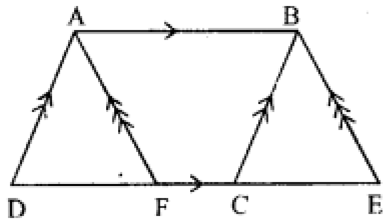
$$\Rightarrow \frac{\text{Area of } \Delta PDC}{\text{Area of } \Delta ABC} = \frac{1}{3} \times \frac{1}{2}$$

$$\Rightarrow \frac{\text{Area of } \Delta PDC}{\text{Area of } \Delta ABC} = \frac{1}{6}$$

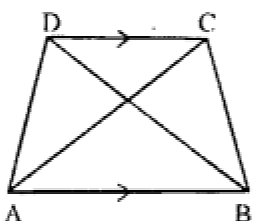
Hence, area of ΔPDC : area of $\Delta ABC = 1:6$

Question 10.

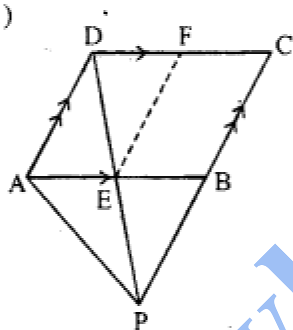
(a) In the figure (1) given below, area of parallelogram ABCD is 29 cm2. Calculate the height of parallelogram ABEF if AB = 5.8 cm
(b) In the figure (2) given below, area of ∆ABD is 24 sq. units. If AB = 8 units, find the height of ABC.
(c) In the figure (3) given below, E and F are mid points of sides AB and CD respectively of parallelogram ABCD. If the area of parallelogram ABCip is 36 cm2.
(i) State the area of ∆ APD.
(ii) Name the parallelogram whose area is equal to the area of ∆ APD.



(1)



(2)

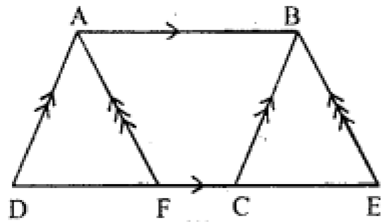


(3)

Solution:

Given. Area of || gm ABCD = 29 cm²

Required. Height of parallelogram ABEF if AB = 5.8 cm.



(1)

Sol. Since || gm ABCD and || gm ABEF with equal Bases and between the same parallels so that their area are same.

$\therefore$ *ar* (|| gm ABEF) = *ar* (|| ABCD)

$\Rightarrow$ *ar* (|| gm ABEF) = 29 cm² ..... (1)

[*ar* (|| gm ABCD = 29 cm² given)]

Also *ar* (|| gm ABEF = Base × height

$\Rightarrow$ 29 = AB × height [From (1)]

$\Rightarrow$ 29 = 5.8 × height [AB = 5.8 (given)]

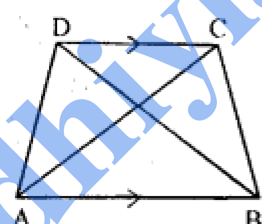
$$\Rightarrow \text{height} = \frac{29}{5.8} = \frac{29 \times 10}{58} = \frac{10}{2} = 5$$

$\therefore$ Height of parallelogram ABEF = 5cm

(*b*) Given. Area of ΔABD = 24 sq. units = AB = 8 units.

Required. Height of ΔABC.

Sol. Area of ΔABD = 24 sq. units ... (1)



(2)

$\therefore$ Area of ΔABD = Area of ΔABC ... (2)
($\because$ Triangles on the same base and between the same parallels are equal in area)
From (1) and (2),
Area of ΔABC = 24 sq. units.

$\Rightarrow \quad \frac{1}{2} \times AB \times \text{height} = 24$

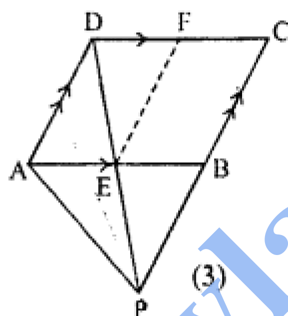
$\Rightarrow \quad \frac{1}{2} \times 8 \times \text{height} = 24 \quad \Rightarrow \quad \text{height} = \frac{24 \times 2}{8}$

$\Rightarrow \quad \text{height} = 3 \times 2 = 6$

Hence, height of ΔABC = 6 Units.

(*c*) **Given.** In || gm ABCD, E and F are mid-point of sides AB and CD respectively.
ar (|| gm ABCD) = 36 cm²

Required : (*i*) *ar* (ΔAPD)
(*ii*) name the || gm whose area is equal to the area of ΔAPD.



Sol. ΔAPD and || gm ABCD are on the same base AD and between the same || lines AD and BC.

$\therefore \quad ar(\Delta APD) = \frac{1}{2} ar(\| \text{ gm } ABCD)$ ..... (1)

But ar (|| gm ABCD) = 36 cm² .....(2)
From (1) and (2),

$ar(\Delta APD) = \frac{1}{2} \times 36 \text{ cm}^2$

$ar(\Delta APD) = 18 \text{ cm}^2$

(*ii*) E and F are mid-points of AB and CD

In ΔCPD, EF || PC.

Also EF bisect the || gm ABCD in two equal parts.

Now, EF || AD and AE || DF

$\therefore$ AEFD is a || gm.

$\therefore$ ar (|| gm AEFD $= \frac{1}{2} ar$ (|| gm ABCD) ...(3)

From (1) and (3),

$ar(\Delta APD) = ar$ (|| gm AEFD)

$\therefore$ AEFD is the required || gm which is equal to the area of ΔAPD. **(Q.E.D.)**

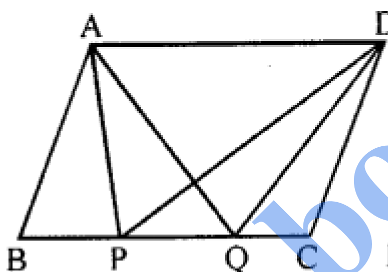
Question 11.

(a) In the figure (1) given below, ABCD is a parallelogram. Points P and Q on BC trisect BC into three equal parts. Prove that :

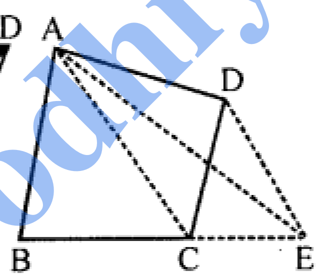
area of ΔAPQ = area of ΔDPQ = $\frac{1}{6}$ (area of ||gm ABCD)

(b) In the figure (2) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of ΔABE.

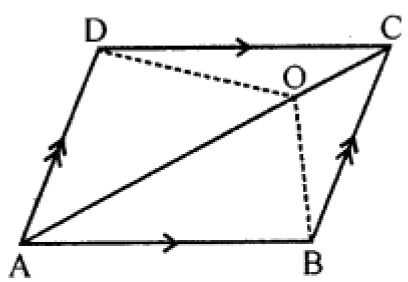
(c) In the figure (3) given below, ABCD is a parallelogram. O is any point on the diagonal AC of the parallelogram. Show that the area of ΔAOB is equal to the area of ΔAOD.



(1)



(2)



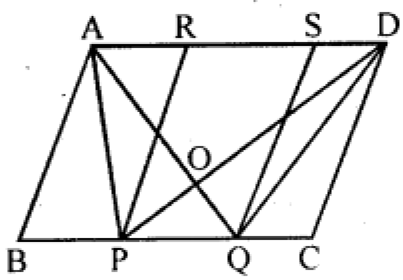
(3)

Solution:

(a) **Given :** In ||gm ABCD, points P and Q trisect BC into three equal parts.

To prove : area (ΔAPQ) = area (ΔDPQ) = $\frac{1}{6}$ area ||gm ABCD.

Construction : Through P and Q, draw PR and QS parallel to AB and CD.



Proof : Now, ΔAPD and ΔAQD lie on the same base AD and between the same parallel AD and BC.

$\therefore$ ar(ΔAPD) = ar(ΔAQD)

ar(ΔAPD) – ar(ΔAOD) = ar(ΔAQD) – ar(ΔAOD)

[on substracting ar(ΔAOD from both sides]

$\Rightarrow$ ar(ΔAPO = ar(ΔOQD) ...(*i*)

$\Rightarrow$ ar(ΔAPO) + ar(ΔOPQ) = ar(ΔOQD) + ar(ΔOPQ)

[on adding ar(ΔOPQ) on both sides]

ar(ΔAPQ) = ar(ΔDPQ) ...(*ii*)

Again, ΔAPQ and parallelogram PQSR are on the same base PQ and between same parallels PQ and AD.

$\therefore$ ar(ΔAPQ) = $\frac{1}{2}$ ar(||gm PQRS) ...(*iii*)

Now,

$$\frac{\text{ar(|| gm ABCD)}}{\text{ar(|| gm PQRS)}} = \frac{BC \times \text{height}}{PQ \times \text{height}} = \frac{3PQ \times \text{height}}{PQ \times \text{height}}$$

= 3

ar(||gm PQRS) = $\frac{1}{3}$ ar(||gm ABCD) ...(*iv*)

Using (*ii*), (*iii*) and (*iv*), we get

ar(ΔAPQ) = ar(ΔDPQ)

= $\frac{1}{2}$ ar(||gm PQRS)

= $\frac{1}{2} \times \frac{1}{3}$ ar(||gm ABCD)

$\Rightarrow$ ar(ΔAPQ) = ar(ΔDPQ) = $\frac{1}{6}$ ar(||gm ABCD)

Hence proved.

(b) **Given :** In the given figure, DE ∥ AC the diagonal of quadrilateral ABCD which meets at E on producing BC. AC, AE are joined.

To prove : Area of quadrilateral ABCD = area ΔABE.

Proof : ΔACE and ΔADE area on the same base AC and between the same parallelogram.

∴ area ΔACE = area ΔADC

Adding area ΔABC to both sides

area ΔACE + area ΔABC

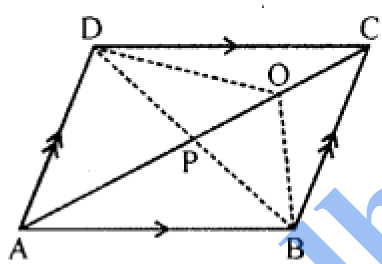
= area ΔADC + area ΔABC

⇒ area ΔABE = area quad. ABCD

(c) **Given :** In ∥gm ABCD, O is any point on diagonal AC.

To prove : area ΔAOB = area ΔAOD

Construction : Join BD which meets AC at P.



Proof : In ΔABD, AP is median

(∵ Diagonals of a ∥gm bisect each other)

∴ area ΔABP = area ΔADP ...(*i*)

Similarly, area ΔPBO = area ΔPDO ...(*ii*)

Adding, (*i*) and (*ii*), we get

area ΔABO = area ΔADO ...(*iii*)

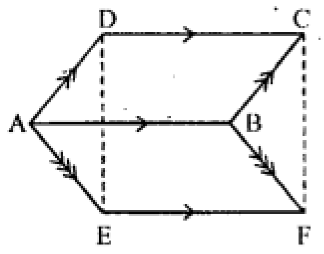
⇒ ΔAOB = area ΔAOD

Question P.Q.

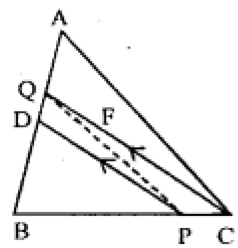
(a) In the figure (1) given below, two parallelograms ABCD and AEFB are drawn on opposite sides of AB, prove that: area of || gm ABCD + area of || gm AEFB = area of || gm EFCD.

(b) In the figure (2) given below, D is mid-point of the side AB of △ABC. P is any point on BC, CQ is drawn parallel to PD to meet AB in Q. Show that area of △BPQ = $\frac{1}{2}$ area of △ABC.

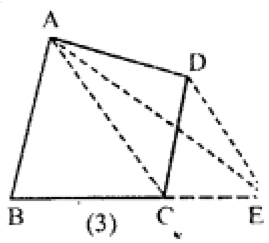
(c) In the figure (3) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of △ABE.



(1)



(2)

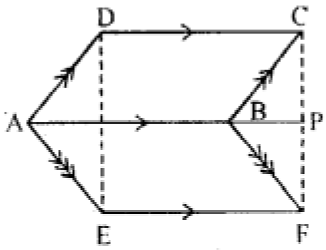


Solution:

(*a*) **Given.** ABCD and AEFB are two || gm on opposite sides of AB.

To prove. *ar* (|| gm ABCD) + *ar* (|| gm AEFB) + *ar* (|| gm EFCD)

Construction. Produced AB to meet CE at P.



Proof. || gm PQDC and || gm ABCD are on the same base CD and between same || lines.

$\therefore$ *ar* (|| gm PQDC) = *ar* (|| gm ABCD) ... (1)

Again || gm PQEF and || gm AEFB are on the same base EF and between same || lines

$\therefore$ *ar* (|| gm PQEF) = *ar* (|| gm AEFB) ... (2)

Adding (1) and (2)

ar (|| gm PQDC) + *ar* (|| gm PQEF) = *ar* (|| gm ABCD) + *ar* (|| gm AEFB)

$\Rightarrow$ *ar* (|| gm EFCD) = *ar* (|| gm ABCD) + *ar* (|| gm AEFB)

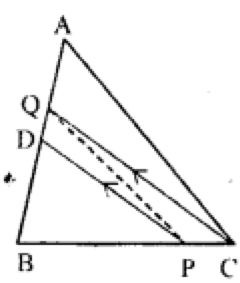
Hence, area of || gm ABCD + area of || gm AE FB = area of || gm EFCD. **(Q.E.D.)**

(*b*) **Given.** A ΔABC, in which D is mid-point of the side AB. P is any point on BC, CQ || PD to meet AB in Q.

To prove. $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$

Const. Join CD.

Proof.



$\therefore$ CD is median of ΔABC

$\therefore ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$

($\because$ Median divides a triangle into two triangle of equal area)

Now, ΔDPQ and ΔDPC are on the same Base DP and between the same parallel lines DP and QC.

$\therefore ar(\Delta DPQ) = ar(\Delta DPC)$ ..... (2)

From (1),

$ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$

or $ar(\Delta BPD) + ar(\Delta DPC) = \frac{1}{2} ar(\Delta ABC)$

or $ar(\Delta BPD) + ar(\Delta DPQ) = \frac{1}{2} ar(\Delta ABC)$

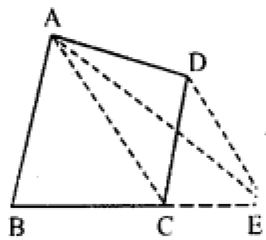
[From (2)]

or $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$

Hence, area of $\Delta BPQ = \frac{1}{2}$ area of ΔABC.

(Q.E.D.)

(*c*) **Given.** ABCD is a || gm. DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E.



To prove. Area of quad. ABCD = area of Δ ABE

Proof. DE || BC (given)

$\therefore$ ΔACE and ΔACD on the same base AC and between the same || lines AC and DE.

$\therefore$ area of ΔACE = area of ΔACD

Adding both sides area of ΔABC.

area of ΔACE + area of ΔABC = area of ΔACD + area of ΔABC.

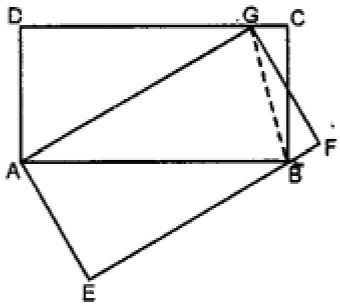
or area of ΔABE = area of quad. ABCD

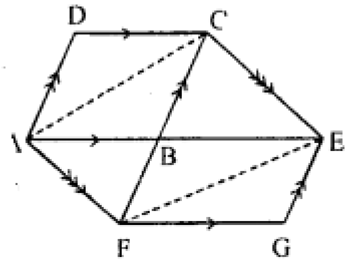
Hence, area of quad. ABCD = area of ΔABE.

(Q.E.D.)

Question 12.

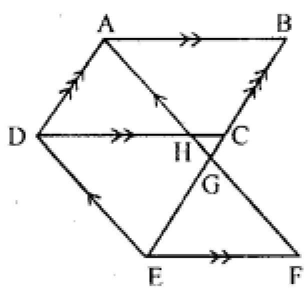
(a) In the figure given, ABCD and AEFG are two parallelograms.
Prove that area of || gm ABCD = area of || gm AEFG.
(b) In the fig. (2) given below, the side AB of the parallelogram ABCD is produced to E. A st. line At through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is Completed prove that area of || gm BFGE=Area of || gmABCD.





(2)

(c) In the figure (3) given below AB || DC || EF, AD || BEandDE || AF. Prove the area ofDEFH is equal to the area of ABCD.

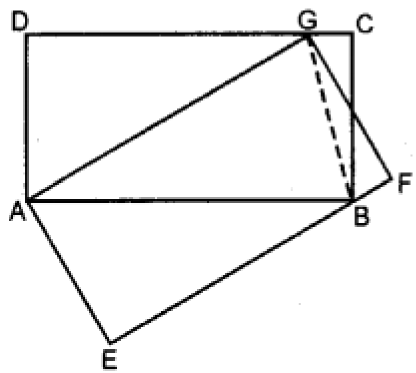


Solution:

(*a*) **Given.** ABCD and AEFG are two parallelograms as shown in the figure.

To prove. Area ABCD = area AEFG

Construction. Join BG.



Proof. $\because$ ΔABG and | | gm ABCD are on the same base AB and between the same parallels

$\therefore$ area $\Delta ABG = \frac{1}{2}$ (area | | gm ABCD) ...(*i*)

Similarly ΔABG and | | gm AEFG are on the same base AG and between the same parallels.

$\therefore$ area (ΔABG) $= \frac{1}{2}$ (area | | gm AEFG)

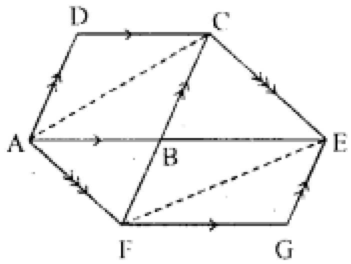
from (*i*) and (*ii*)

$\frac{1}{2}$ (area | | gm ABCD) $= \frac{1}{2}$ (area || gm AEFG)

$\therefore$ area || gm ABCD = area || gm AEFG.

Hence proved.

(*b*) **Given.** A || gm ABCD in which AB is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F, and || gm BFGE is completed.



(2)

To prove. area of || gm BFGE = area of || gm ABCD

Construction : Join AC and EF.

Proof. ∵ ΔAFC and ΔAFE are on the same base AF and between parallel lines AC and EF.

∴ ar (ΔAFC) = ar (ΔAFE) ...... (1)

subtracting both sides ar (ΔABF) ar (ΔAFC) – ar (ΔABF) = ar (ΔAFE) – ar (ΔABF)

or ar (ΔABC) = ar (ΔBEF)

Multiplying both sides by 2,

2 ar (ΔABC) = 2 ar (ΔBEF)

or ar (|| gm ABCD) = ar (|| gm BFGE)

(∵ diagonals of || gm divides it into two triangles of equal areas.)

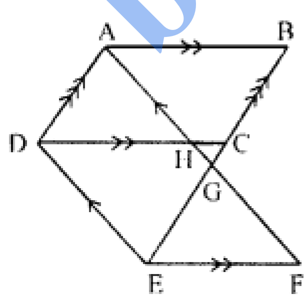
Hence, area of || gm BFGE = area of || gm ABCD.

(Q.E.D.)

(*c*) Given. DC || EF, AD || BE and DE || AF

To prove. *ar* (DEFH) = *ar* (ABCD)

Proof. DE || AF and AD || BE



(3)

∴ ADEG is a || gm. (given)

Now, || gm ABCD and || gm ADEG are on the same base AD and between the same || lines AD and BE.

∴ *ar* (|| gm ABCD) = *ar* ||(ADEG) ..... (1)

Again DEFG is a || gm

(∵ DE || AF and DC || EF (given))

∴ || gm DEFH and || gm ADEG are on the same base DE and between the same || lines DE and AF.

∴ *ar* (|| gm DEFH) = *ar* (|| gm ADEG) .....(2)

From (1) and (2),

ar (|| gm ABCD) = *ar* (|| gm DEFH)

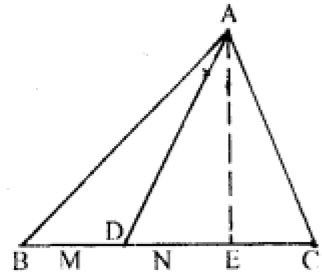
or *ar* (ABCD) = *ar* (DEFH) **(Q.E.D.)**

Question 13.

Any point D is taken on the side BC of, a △ ABC and AD is produced to E such that AD=DE, prove that area of △ BCE = area of △ ABC.

Solution:

Given. In ΔABC, D is taken on the side BC.
AD produced to E such that AD = DE.



To prove. Area of ΔBCE = area of ΔABC

Proof. In Δ ABE,

AD = DE (Given)

$\therefore$ BD is a median of ΔABE

$\Rightarrow$ $ar(\Delta ABD) = ar(\Delta BED)$ .....(1)

[median divides a triangle into two triangles of equal area]

Similarly, in ΔACE

CD is median of ΔACE.

$\Rightarrow$ $ar(\Delta ACD) = ar(\Delta CED)$ ..... (2)

Adding (1) and (2),

$ar(\Delta ABD) + ar(\Delta ACD) = ar(BED) + ar(\Delta CED)$

or, $ar(\Delta ABC) = ar(\Delta BCE)$

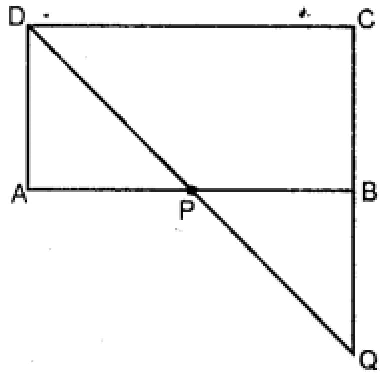
Hence, area of ΔBCE = area of Δ ABC. **(Q.E.D.)**

Question 14.

ABCD is a rectangle and P is mid-point of AB. DP is produced to meet CB at Q. Prove that area of rectangle ∆BCD = area of ∆ DQC.

Solution:

Given. ABCD is a rectangle P is mid-point of AB DP is joined and produced meeting CB produced at Q.



To prove. Area rectangle ABCD

$= \text{area} (\Delta DQC)$

Proof. In ΔAPD and ΔBQP,

$AP = BP$ ($\because$ D is mid-point of AB)

$\angle DAP = \angle QBP$ (each 90^o)

$\angle APD = \angle BPQ$ (vertically opposite angles)

$\therefore \quad \Delta APD \cong \Delta BQP$ (ASA postulate)

$\therefore \quad \text{area } \Delta APD = \text{area } \Delta BQP$

Now area ABCD = area ΔAPD + area PBCD

= area ΔBQP + area PBCD

= area ΔDQC

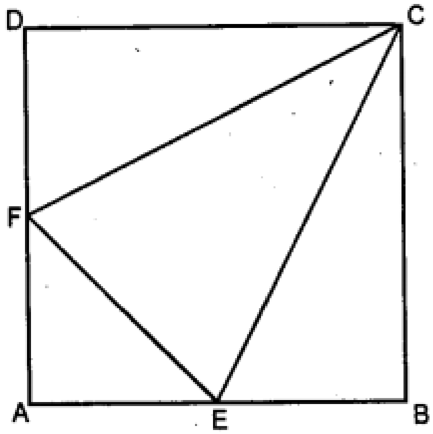
Hence proved

Question P.Q.

ABCD is a square, E and F are mid-points of the sides AB and AD respectively Prove that area of $\triangle CEF = \frac{3}{8}$(area of square ABCD).

Solution:

Given. ABCD is a square. E and F are the mid-points of sides AB and AD espectively EF, EC and FC are joined.



To prove. area $\Delta CEF = \frac{3}{8}$ (area of square ABCD)

Proof. Let side of square $= a$

then area of square ABCD $= a^2$

Now area of $\Delta AEF = \frac{1}{2}$ AE × AF

$$= \frac{1}{2} \times \frac{a}{2} \times \frac{a}{2} = \frac{a^2}{8}$$

$$\text{area of } \Delta EBC = \frac{1}{2} \times EB \times BC$$

$$= \frac{1}{2} \times \frac{a}{2} \times a = \frac{a^2}{4}$$

$$\text{and area of } \Delta CDF = \frac{1}{2} \times CD \times DF$$

$$= \frac{1}{2} a \times \frac{a}{2} = \frac{a^2}{4}$$

$\therefore$ Area of ΔCEF = area of sq. ABCD – (area of Δ AE) + area of Δ EBC + area of Δ CDF)

$$= a^2 - \left(\frac{a^2}{8} + \frac{a^2}{4} + \frac{a^2}{4}\right)$$

$$= a^2 - \left(\frac{a^2 + 2a^2 + 2a^2}{8}\right) = a^2 - \frac{5a^2}{8}$$

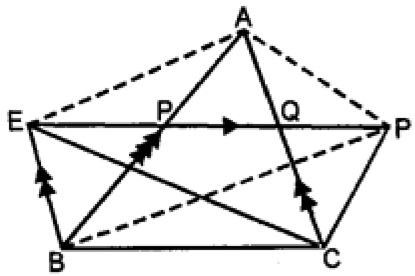
$$= \frac{3}{8}a^2 = \frac{3}{8} \text{ area of sq. ABCD.}$$

Question P.Q.

A line PQ is drawn parallel to the side BC of ∆ABC. BE is drawn parallel to CA to meet QP (produced) at E and CF is drawn parallel to BA to meet PQ (produced) at F. Prove that
area of ∆ABE=area of ∆ACF.

Solution:

Given. A line PQ is drawn parallel to side BC of ΔABC.



BE || CA and CF || BA drawn which meet PQ produced both sides at E and F respectively AE, AF and BF are joined.

To prove. area of ΔABE = area of ΔACF

Proof. ∵ ΔABE and ΔCBE are on the same base BE and between the same parallels

∴ area ΔABE = area ΔCBE ...(*i*)

Again ∵ ΔACF and ΔBCF are on the same base CF and between the same parallels

∴ area ΔACF = area ΔBCF ...(*ii*)

But ΔCBE and ΔCBF are on the same base BC and between the same parallels.

∴ area ΔCBE = area ΔBCF ...(*iii*)

∴ from (*i*), (*ii*) and (*iii*)

area ΔABE = area ΔACF

Hence proved.

Question 15.

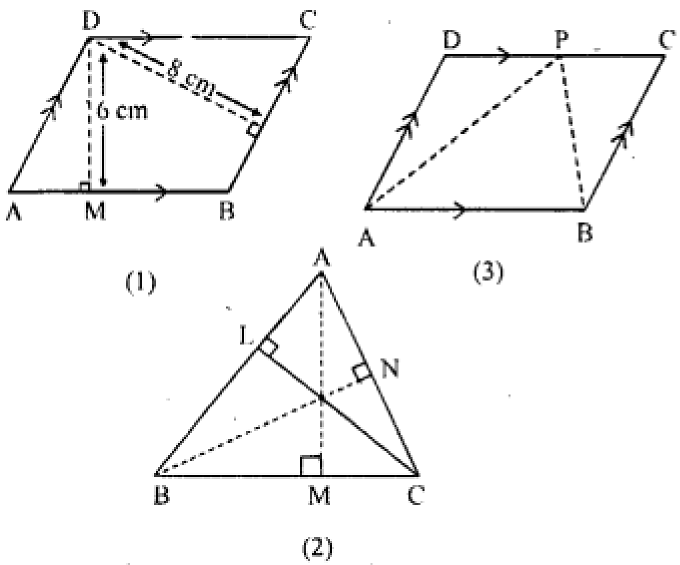
(a) In the figure (1) given below, the perimeter of parallelogram is 42 cm. Calculate the lengths of the sides of the parallelogram.

(b) In the figure (2) given below, the perimeter of △ ABC is 37 cm. If the lengths of the altitudes AM, BN and CL are 5x, 6x, and 4x respectively, Calculate the lengths of the sides of △ABC.

(c) In the fig. (3) given below, ABCD is a parallelogram. P is a point on DC such that area of △DAP = 25 cm² and area of △BCP = 15 cm². Find

(i) area of || gm ABCD

(ii) DP : PC.

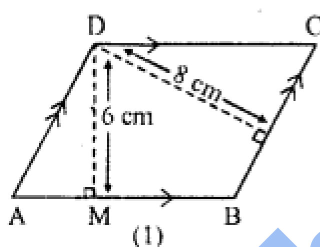


Solution:

(*a*) **Given.** Perimeter of || gm ABCD = 42 cm.

Required. Lengths of the sides of || gm ABCD.

Sol. Let AB = P



Then, perimeter of || gm = 2 (AB + BC)

$\Rightarrow \quad 42 = 2(P + BC)$

$\Rightarrow \quad \frac{42}{2} = P + BC$

$\Rightarrow \quad 21 = P + BC$

$\Rightarrow \quad BC = 21 - P$

Area of || gm ABCD = AB × DM

$= P \times 6 = 6P$ ....(1)

(Taking base AB and height DM)

Again, area of || gm ABCD = BC × DN

(Taking Base BC and height DN)

$= (21 - P) \times 8 = 8(21 - P)$ ..... (2)

From (1) and (2),

$6P = 8(21 - P) \Rightarrow 6P = 168 - 8P$

$\Rightarrow 6P + 8P = 168 \Rightarrow 14P = 168$

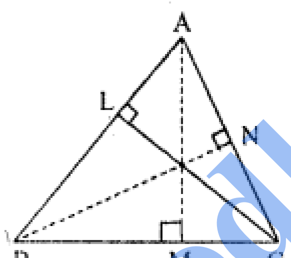
$\Rightarrow P = \frac{168}{14} = 12$

Hence, sides of || gm, AB = 12 cm and BC = (21 – 12) cm = 9 cm.

(*b*) **Given.** The perimeter of ΔABC = 37 cm. Length of the Altitudes AM, BN, and CL are $5x$, $6x$, and $4x$ respectively.

Required. Lengths of BC, CA, and AB.

Sol. Let BC = P and CA = Q



(2)

Then perimeter of ΔABC
$= AB + BC + CA$

$\Rightarrow \quad 37 = AB + P + Q$

$\Rightarrow \quad AB = 37 - P - Q$

Area of $\Delta ABC = \frac{1}{2} \times BC \times AM = \frac{1}{2} CA \times BN$

$= \frac{1}{2} \times AB \times CL$

i.e. $\frac{1}{2} \times P \times 5x = \frac{1}{2} \times Q \times 6x$

$= \frac{1}{2}(37 - P - Q) \times 4x \Rightarrow \frac{5P}{2} = 3Q = 2\ (37 - P - Q)$

Taking first two parts

$\frac{5P}{2} = 3Q \quad \Rightarrow \quad 5P = 6Q \quad \Rightarrow \quad 5P - 6Q = 0 \quad \text{..... (1)}$

Taking second and third parts

$3Q = 2\,(37 - P - Q) \quad \Rightarrow \quad 3Q = 74 - 2P - 2Q$

$\Rightarrow \quad 3Q + 2Q + 2P = 74 \quad \Rightarrow \quad 2P + 5Q = 74 \quad \text{..... (2)}$

Multiplying equation (1) by (5) & (2) by (6), we get

$$25P - 30Q = 0$$
$$12P + 30Q = 444$$

Adding, $\quad 37P = 444$

$\Rightarrow \quad P = \dfrac{444}{37} = 12$

Substituting the value of P in equation (1), we get

$5 \times 12 - 6Q = 0 \quad \Rightarrow \quad 60 - 6Q = 0 \quad \Rightarrow \quad 60 = 6Q$

$\Rightarrow \quad Q = \dfrac{60}{6} = 10$

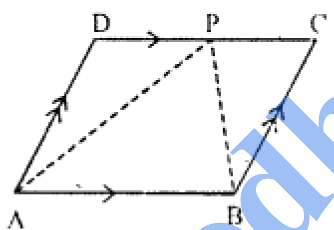
Hence, BC = P = 12 cm, CA = Q = 10 cm and AB = 37 – P – Q = 37 – 12 – 10 = 15 cm.

(*c*) Given. ABCD is a || gm. P is a point on DC such that ar (ΔDAP) = 25 cm² and ar (ΔBCP) = 15 cm²

Required. (*i*) *ar* (|| gm ABCD) (*ii*) DP : PC

Sol. (*i*) *ar* (ΔAPB) $= \dfrac{1}{2}$ *ar* (|| gm ABCD)

($\because$ Area of a triangle is half that of a || gm on the same base and between the same parallels)



(3)

Then $\dfrac{1}{2}$ *ar* (|| gm ABCD) = *ar* (ΔDAP) + *ar* (ΔBCP)

$= 25\text{cm}^2 + 15\text{cm}^2 = 40\text{cm}^2$

$\Rightarrow \quad$ *ar* (|| gm ABCD) = $2 \times 40 \text{ cm}^2 = 80 \text{ cm}^2$

(*ii*) Since ΔADP and ΔBCP are on the same base CD and between same || lines CD and AB.

$$\therefore \quad \frac{ar(\Delta DAP)}{ar(\Delta BCP)} = \frac{DP}{PC}$$

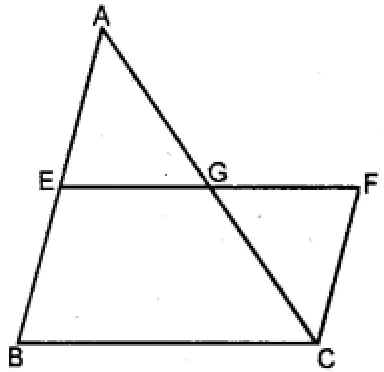
$$\Rightarrow \quad \frac{25}{15} = \frac{DP}{PC} \quad \Rightarrow \quad \frac{DP}{PC} = \frac{25}{15} = \frac{5}{3}$$

$\Rightarrow \quad DP : PC = 5 : 3$

Question 16.

In the adjoining figure, E is mid-point of the side AB of a triangle ABC and EBCF is a parallelogram. If the area of △ ABC is 25 sq. units, find the area of || gm EBCF.

Solution:



Let EF, side of || gm BCEF meets AC at G.

∵ E is mid point and EF || BC

∴ G is mid point of AC.

⇒ AG = GC

Now in ΔAEG and ΔCFG,

∠EAG, ∠GCF (Alternate angles)

∠EGA = ∠CGF

(vertically opposite angles)

AG = GC (proved)

∴ ΔAEG ≅ ΔCFG

⇒ area ΔAEG = area ΔCFG.

Now

area || gm EBCF = area BCGE + area Δ CFG

= area BCGE + area ΔAEG = area ΔABC

But area ΔABC = 25 sq. units.

∴ area || gm EBCF = 25 sq. units

Question 17.

(a) In the figure (1) given below, BC || AE and CD || BE. Prove that: area of △ABC= area of △EBD.

(b) In the llgure (2) given below, ABC is right angled triangle at A. AGFB is a square on the side AB and BCDE is a square on the hypotenuse BC. If AN ⊥ ED, prove that:

(i) △BCF ≅ △ ABE.

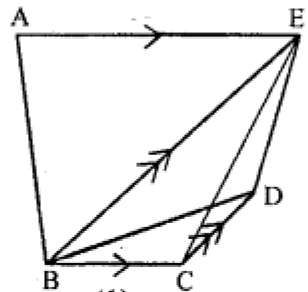
(ii)arca of square ABFG = area of rectangle BENM.

(a) **Given.** BC || AE and CD || BE

To prove. Area of ΔABC = area of ΔEBD.

Construction. Join CE

Proof. ΔABC and ΔEBC are on the same Base BC and between the same || lines AE and BC.



(1)

$\therefore$ *ar* (ΔABC) = *ar* (ΔEBC) ..... (1)

$\because$ ΔEBC and ΔEBD are on the same base BE and between same || lines BE and CD.

$\therefore$ ar (ΔEBC) = *ar* (ΔEBD) ..... (2)

From (1) and (2)

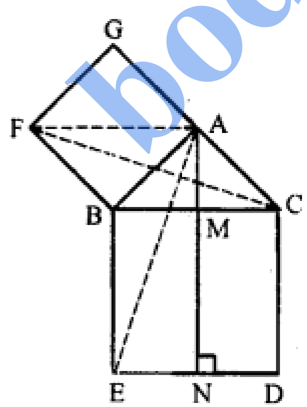
ar (ΔABC) = *ar* (ΔEBD)

Hence, area of ΔABC = area of ΔEBD **(Q.E.D.)**

(b) **Given.** A right angled Δ ABC in which $\angle A = 90°$. Squares AGFB and BCDE are drawn on the side AB and hypotenuse BC of ΔABC. AN $\perp$ ED meeting BC at M.

To Prove : (i) $\Delta BCF \cong \Delta ABE$

(ii) area of square ABFG = area of rectangle BENM



(2)

Solution:

Proof : (*i*) $\angle FBC = \angle FBA + \angle ABC$

$\Rightarrow \angle FBC = 90° + \angle ABC$ ..... (1)

$\angle ABE = \angle EAC + \angle ABC$

$\angle ABE = 90° + \angle ABC$ ..... (2)

From (1) and (2),

$\angle FBC = \angle ABE$ ..... (3)

Now, in ΔBCF and ΔABE

$BF = AB$

$\angle FBC = \angle ABE$ [From (3)]

$BC = BE$

$\therefore \Delta BCF \cong \Delta ABE$

(By S.A.S. axiom of congruency)

(*ii*) $\Delta BCF \cong \Delta ABE$ (Proved in part (*i*) above)

ar (ΔBCF) = *ar* (ΔABE) ..... (4)

$\angle BAG + \angle BAC = 90° + 90°$

$\Rightarrow \angle BAG + \angle BAC = 180°$

$\therefore$ GAC is a straight line.

Now, Δ BCF and square AGFB are on the same base BF and between the same || lines BF and GC.

$\therefore$ *ar* $(\Delta BCF) = \frac{1}{2}$ *ar* (square AGFB) .....(5)

Again, ΔABE and rectangle BENM are on the same base BE and between the same || lines BE and AN.

$\therefore$ *ar* $(\Delta ABE) = \frac{1}{2}$ *ar* (Rectangle BENM) .....(6)

From (4), (5) and (6)

$\frac{1}{2}$ *ar* (square AGFB) = $\frac{1}{2}$ *ar* (Rectangle BENM)

$\Rightarrow$ *ar* (square AGFB) = *ar* (Rectangle BENM)

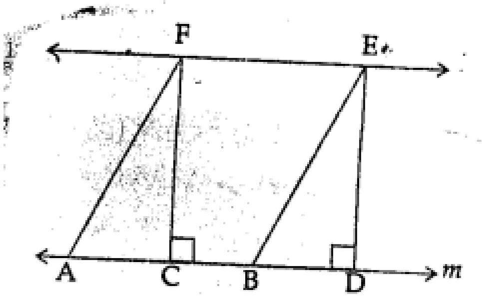
Hence, area of square AGFB = area of Rectangle BENM. **(Q.E.D.)**

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 8):

Question 1.

In the given figure, if l || m, AF || BE, FC ⊥ m and ED ⊥ m , then the correct statement is
(a) area of ||ABEF = area of rect. CDEF
(b) area of ||ABEF = area of quad. CBEF
(c) area of ||ABEF = 2 area of △ACF
(d) area of ||ABEF = 2 area of △EBD



Solution:
In the given figure,
l ||m, AF || BE, FC ⊥ m and ED ⊥ m
∵ ||gm ABEF and rectangle CDEF are on the same base EF and between the same parallel
∴ area ||gm ABEF = area rect. CDEF **(a)**

Question 2.
Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is
(a) 1 : 2
(b) 1 : 1
(c) 2 : 1
(d) 3 : 1
Solution:
A triangle and a parallelogram are on the same base and between same parallel, then
∴ They are equal in area
∴ Their ratio 1:1 **(b)**

Question 3.
If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of area of the triangle to the area of parallelogram is
(a) 1 : 3
(b) 1 : 2
(c) 3 : 1
(d) 1 : 4
Solution:
A triangle and a parallelogram are on the same base and between same parallel, then area of
triangle = $\frac{1}{2}$ area ||gm
∴ Their ratio 1 : 2 **(b)**

Question 4.

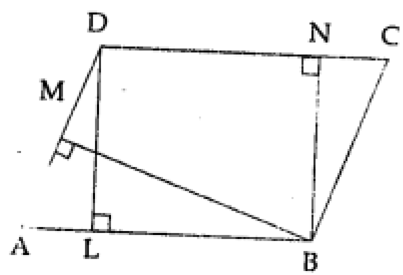
A median of a triangle divides it into two
(a) triangles of equal area
(b) congruent triangles
(c) right triangles
(d) isosceles triangles

Solution:

A median of a triangle divides it into two triangle equal in area. **(a)**

Question 5.

In the given figure, area of parallelogram ABCD is
(a) AB x BM
(b) BC x BN
(c) DC x DL
(d) AD x DL



Solution:

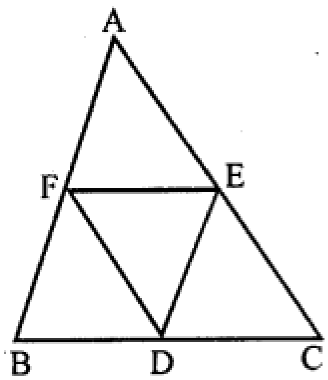
In the given figure,
Area of ||gm ABCD = AB x DL or DC x DL (∵ AB = DC) **(c)**

Question 6.

The mid-points of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to
(a) $\frac{1}{2}$ area of ΔABC
(b) $\frac{1}{3}$ area of ΔABC
(c) $\frac{1}{4}$ area of ΔABC
(d) area of ΔABC

Solution:

The mid-points of the sides of a triangle along with any of vertices as the fourth point makes a parallelogram of area equal to $\frac{1}{2}$ the area of ΔABC



i.e., area ||gm DEAF = $\frac{1}{2}$ area ΔABC (a)

Question 7.

In the given figure, ABCD is a trapezium with parallel sides AB = a cm and DC = b cm. E and F are mid-points of the non parallel sides. The ratio of area of ABEF and area of EFCD is

(a) a : b
(b) (3a + b) : (a + 3b)
(c) (a + 3b) : (3a + b)
(d) (2a + b) : (3a + b)

Solution:

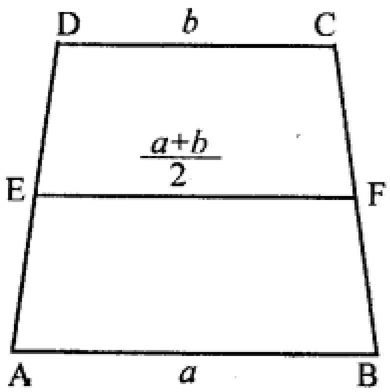
In the figure, ABCD is a trapezium in which AB || DC

AB = a, DC = b

E and F are mid points on DA and CB respectively

Let h be the height (∵ EF || AB || DC)

$$\therefore \text{EF} = \frac{1}{2}(a + b)$$



Area of trapezium ABFE

$$= \left[\frac{1}{2}\frac{(a+b)}{2} \times \frac{h}{2}\right]$$

$$= \frac{h}{4}\left(\frac{2a+a+b}{2}\right)$$

$$= \frac{h}{8}(3a+b)$$

and area of trap. EFCD

$$= \frac{1}{2}[\text{EF} + \text{DC}] \times \frac{h}{2}$$

$$= \frac{h}{4}\left[\frac{a+b}{2}+b\right] = \frac{h}{4}\left[\frac{a+b+2b}{2}\right]$$

$$= \frac{h}{4}[a+3b]$$

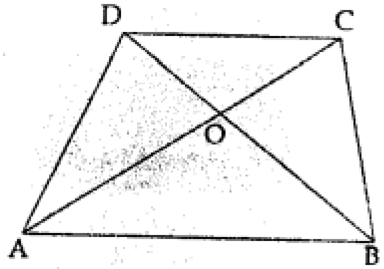
$$\therefore \text{ Ratio} = \frac{h}{8}(3a+b) : \frac{h}{8}(a+3b)$$

$$= (3a+b) : (a+3b)$$

(b)

Question 8.
In the given figure, AB || DC and AB ≠ DC. If the diagonals AC and BD of the trapezium ABCD intersect at O, then which of the following statements is not true?
(a) area of $\triangle$ABC = area of $\triangle$ABD

(b) area of △ACD = area of △BCD
(c) area of △OAB = area of △OCD
(d) area of △OAD = area of △OBC

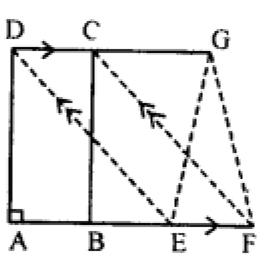


Solution:

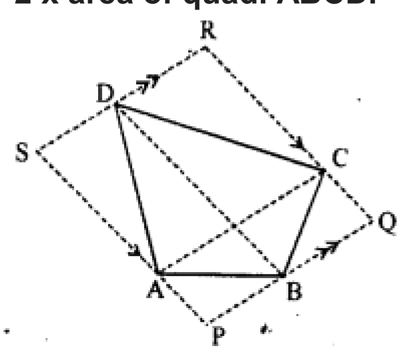
In the trapezium ABCD, AB || DC
AB ≠ DC
The diagonals BD and AC intersect each other at O
Only statement area of ΔOAB is not equal to area ΔCOD
Other all statements are true
Only (b) is not true. (b)

Chapter test

Question 1.
(a) In the figure (1) given below, ABCD is a rectangle (not drawn to scale) with side AB = 4 cm and AD = 6 cm. Find :
(i) the area of parallelogram DEFC
(ii) area of △EFG.
(b) In the figure (2) given below, PQRS is a parallelogram formed by drawing lines parallel to the diagonals of a quadrilateral ABCD through its corners. Prove that area of || gm PQRS = 2 x area of quad. ABCD.



(1)



(2)

Solution:

(a) Given. ABCD is a rectangle AB = 4 cm and AD = 6 cm.

Required. (*i*) The area of || gm DEFC.

(*ii*) area of ΔEFG

(*i*) Since AB = 4cm and AD = 6 cm (given)

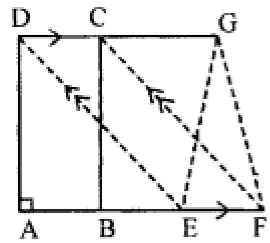
$\therefore$ Area of rectangle ABCD = AB × AD

$= 4 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$

Now, area of rectangle ABCD = area of || gm DEFC

($\because$ Both are on the same Base and between the same parallel lines)

$\Rightarrow$ Area of || gm DEFC = 24 cm^2



(1)

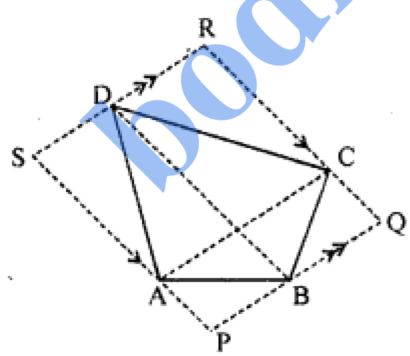
(*ii*) Area of ΔEFG = $\frac{1}{2}$(area of || gm DEFC)

($\therefore$ Both are on the same base and between the same parallel lines)

$\therefore$ Area of $\Delta EFG = \frac{1}{2} \times 24 \text{ cm}^2 = 12 \text{ cm}^2$ **Ans.**

(b) Given. PQRS is a || gm formed by drawing lines parallel to the diagonals of quadrilateral ABCD through its corners.

To prove. Area of || gm PQRS = 2. area of quad ABCD



(2)

Proof. $ar(\Delta ACD) = \frac{1}{2} ar$ (|| gm ACRS)

[∴ both are on same base AC and between the same || AC and SR]

$\Rightarrow$ ar (|| gm ACRS) $= 2ar(\Delta ACD)$ ..... (1)

Similarly,

$ar(\Delta ABC) = \frac{1}{2} ar$ (|| gm Δ APQC)

$\Rightarrow$ ar (|| gm APQC) $= 2ar(\Delta ABC)$ ..... (2)

Adding (1) from (2),

ar (|| gm ACRS) + ar (|| gm APQC) = $2ar(\Delta ACD) + 2ar(\Delta ABC)$

$\Rightarrow$ (|| gm PQRS) $= 2[ar(\Delta ACD) + ar(\Delta ABC)]$

$\Rightarrow$ ar (|| gm PQRS) $= 2ar$ (quad. ABCD)

Hence, area of || gm PQRS = 2.area of quad. ABCD. **(Q.E.D.)**

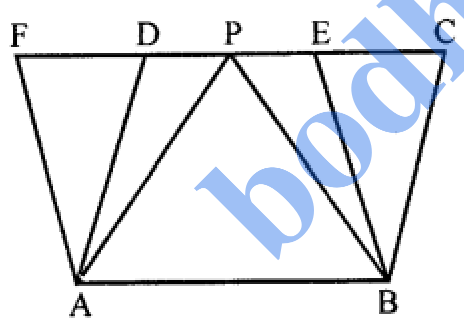
Question P.Q.

In the adjoining figure, ABCD and ABEF are parallelogram and P is any point on DC. If area of || gm ABCD = 90 cm2, find:

(i) area of || gm ABEF

(ii) area of △ABP.

(iii) area of △BEF.



Solution:

In the given figure,

ABCD and ABEF are parallelogram P is an point on DC

Area of ||gm ABCD = 90 cm^2

||gm ABCD and ABEF are on the same base AB are between the same parallels

(*i*) ∴ Area of ||gm ABEF = area of ||gm ABCD = 90 cm^2

(*ii*) ∵ ΔABP and ||gm ABCD are on the same base AB and between the same parallels

$$\therefore \text{ Area } \Delta ABP = \frac{1}{2} \text{ area } \|\text{gm } ABCD$$

$$= \frac{1}{2} \times 90 \text{ cm}^2 = 45 \text{ cm}^2$$

(*iii*) ∵ ΔBEF and ||gm ABEF are on the same base EF and between the same parallels

$$\therefore \text{ Area } \Delta BEF = \frac{1}{2} \text{ area } \|\text{gm } ABEF$$

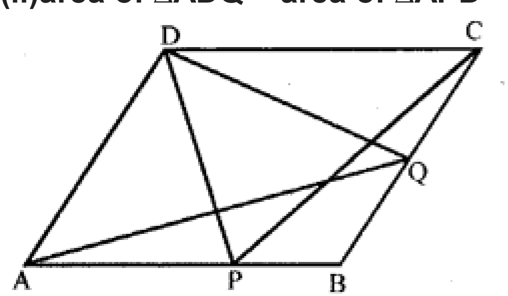
$$= \frac{1}{2} \times 90 = 45 \text{ cm}^2$$

Question 2.

In the parallelogram ABCD, P is a point on the side AB and Q is a point on the side BC. Prove that

(i) area of ΔCPD = area of ΔAQD

(ii)area of ΔADQ = area of ΔAPD + area of ΔCPB.



Solution:

Given. || gm ABCD in which P is a point on AB and Q is a point on BC.

To prove. (*i*) *ar* (ΔCPD) = *ar* (ΔAQD)

(*ii*) *ar* (ΔADQ) = *ar* (ΔAPD) + *ar* (ΔCPB)

Proof. ΔCPD and || gm ABCD are on the same base CD and between the same parallels lines AB and CD.

∴ $ar(\Delta CPD) = \frac{1}{2} ar(\| \text{gm } ABCD)$ ..... (1)

Δ ADQ and || gm ABCD are on the same base AD and between the same || lines AD and BC,

$ar(\Delta ADQ) = \frac{1}{2} (\| \text{gm } ABCD)$ ..... (2)

From (1) and (2),

ar (ΔCPD) = *ar* (ΔADQ)

or *ar* (ΔCPD) = *ar* (ΔADQ) **(Q.E.D.)**

(*ii*) $ar(\Delta ADQ) = \frac{1}{2} ar(\| \text{gm } ABCD)$

(Proved in part (*i*) above)

⇒ 2*ar* (ΔADQ) = *ar* (|| gm ABCD)

⇒ *ar* (ΔADQ) + *ar* (ΔADQ) = *ar* (|| gm ABCD) ..... (3)

But *ar* (ΔADQ) = *ar* (ΔCPD) ..... (4)

(Proved in part (*i*) above)

From (3) and (4),

ar (ΔADQ) + *ar* (ΔCPD) = *ar* (|| gm ABCD)

⇒ *ar* (ΔADQ) + *ar* (ΔCPD)

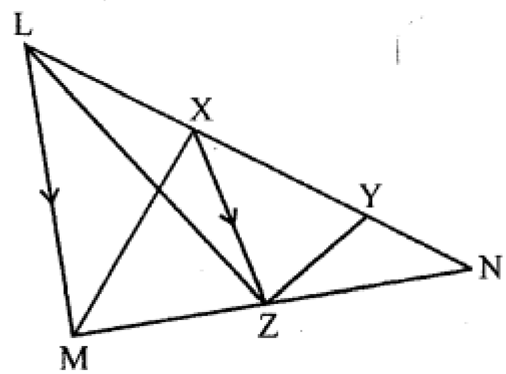
= *ar* (ΔAPD) + *ar* (ΔCPD) + *ar* (ΔCPB)

⇒ *ar* (ΔADQ) = *ar* (ΔAPD) + *ar* (ΔCPB)

(Q.E.D.)

Question 3.

In the adjoining figure, X and Y are points on the side LN of triangle LMN. Through X, a line is drawn parallel to LM to meet MN at Z. Prove that area of ΔLZY = area of quad. MZYX.



Solution:

Given : In the figure,

X and Y are points on side LN of ΔLMN. Through X, a line $XZ \parallel LM$ is drawn which meets MN at Z.

To prove : area of ΔLZY = area of quad. MZYX

Construction : Join MX, ZY and LZ

Proof : $\because LM \parallel XZ$

and ΔLZX and ΔMZX are on the same base XZ and between the same parallels

$\therefore$ area ΔLZX = area ΔMZX

Adding area ΔXZY to both sides

area ΔLZX + area ΔXZY

= area ΔMZX + area ΔXZY

$\Rightarrow$ area ΔLZY = area quadrilateral MZYX

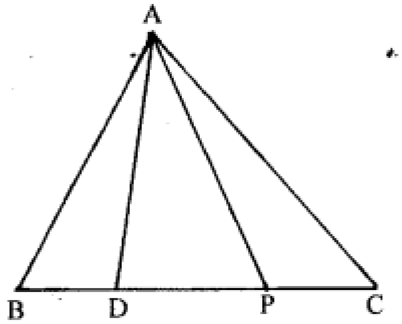
Question P.Q.

If D is a point on the base BC of a triangle ABC such that 2BD = DC, prove that area of $\triangle$ABD= $\frac{1}{3}$ area of $\triangle$ ABC.

Solution:

Given. ΔABC in which base BC. D is a point on BC such that 2BD = DC.

To prove. $ar(\Delta ABD) = \frac{1}{3} ar(ABC)$



Construction. Let P is the mid-point of DC join AD = DC

$\Rightarrow \quad BD = \frac{1}{2} DC$

i.e. BD = DP (P is mid-point of DC)

$\therefore$ D is mid-point of BP.

In ΔABP, AD is median of BP (D is mid-point of BP)

$\therefore \quad ar(\Delta ABD) = ar(\Delta ADP)$ ..... (1)

Again in ΔADC, AP is the median of DC. (P is mid-point of DC)

$\therefore \quad ar(\Delta ADP) = ar(\Delta APC)$ ..... (2)

From (1) and (2),

$\therefore \quad ar(\Delta ABD) = ar(\Delta ADP) = ar(\Delta APC)$

$\therefore \quad \Delta ABC$ is divided into three equal triangles and each Δ will be of $\frac{1}{3} \Delta ABC$.

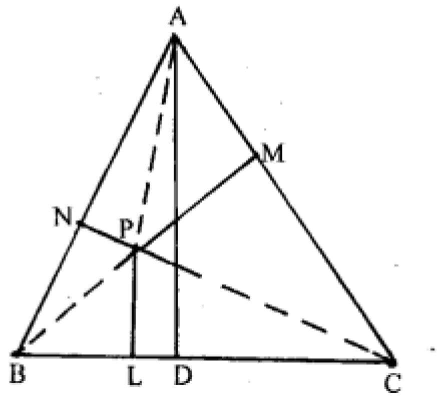
$\therefore \quad ar(\Delta ABD) = \frac{1}{3} ar(\Delta ABC)$ **(Q.E.D.)**

Question 4.

Perpendiculars are drawn from a point within an equilateral triangle to the three sides. Prove that the sum of the three perpendiculars is equal to the altitude of the triangle.

Solution:

ABC is an equilateral triangle. *i.e.* AB = BC = CA. P is any point within an equilateral triangle to the three sides.



PN, PM, and PL are perpendicular on side AB, AC and BC respectively. AD is any altitude from point A on side BC.

To prove. AD = NP + LP + MP

Construction. Join PA, PB and PC.

Proof. Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$ar(\Delta ABC) = \frac{1}{2} \times BC \times AD$ ..... (1)

Now, area of $\Delta APB = \frac{1}{2} \times AB \times NP$ ..... (2)

area of $\Delta APC = \frac{1}{2} \times AC \times MP$ ..... (3)

area of $\Delta BPC = \frac{1}{2} \times BC \times LP$ .....(4)

Adding (2), (3) and (4)

$ar(\Delta APB) + ar(\Delta APC) + ar(\Delta BPC)$

$= \frac{1}{2} \times AB \times NP + \frac{1}{2} \times AC \times MP + \frac{1}{2} \times BC \times LP$

$ar(\Delta ABC) = \frac{1}{2}[AB \times NP + AC \times MP \times BC \times LP]$

$= \frac{1}{2}[BC \times NP + BC \times MP \times BC \times LP]$

$(\because AB = AC = CA)$

$ar(\Delta ABC) = \frac{1}{2} \times BC\ [NP + MP + LP]$ ...(5)

From (4) and (5),

$\frac{1}{2} \times BC \times AD = \frac{1}{2} \times BC \times (NP + LP + MP)$

$\Rightarrow AD = NP + LP + MP$

$\Rightarrow NP + LP + MP = AD$

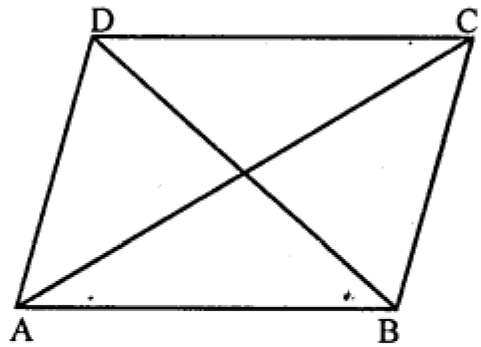
i.e.sum of three perpendiculars is equal to the altitude of the triangle.

Question 5.

If each diagonal of a quadrilateral' divides it into two triangles of equal areas, then prove that the quadrilateral is a parallelogram.

Solution:

Given : In quadrilateral ABCD, diagonal AC bisects the quadrilateral ABCD in two triangle of equal area i.e.



ar (Δ ABC) = ar (Δ ADC)

To prove : ABCD is a parallelogram.

Proof : Join BD.

Proof : $\because$ Diagonals of quad. ABCD divides the quad. into two triangles of equal area.

$\therefore$ ar(Δ ABC) = ar(Δ ABD)

$$= \frac{1}{2} \text{ ar (ABCD)}$$

But, these are on the same base AB

$\therefore$ Their heights are equal

$\therefore$ DC || AB ...(i)

Similarly, we can prove that :

ar (Δ ABC) = ar (Δ BDC)

$\therefore$ BC || AD ...(ii)

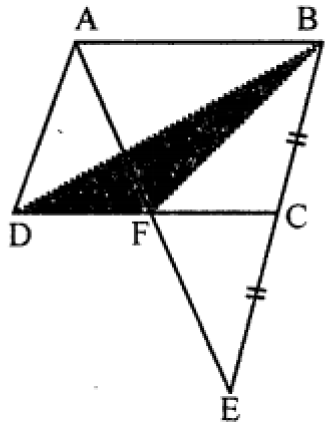
From (i) and (ii)

ABCD is a parallelogram.

Hence proved.

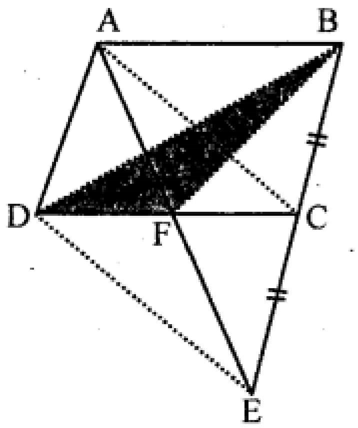
Question 6.

In the given figure, ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F. If area of ∆DFB = 3 cm², find the area of parallelogram ABCD.



Solution:

In the figure, ABCD is a parallelogram
BC is produced to E such that CE = BC



Join BD and AE

which intersects DC at F

Join BF, AC and DE

$\therefore$ Area of $\Delta DFB = 3\ cm^2$

Find the area of ||gm ABCD

Solution : $\because$ In ΔABE, C is mid-point of BE and CD || AB

$\therefore$ F is mid-point of AE and CD

$\therefore$ ABED is a ||gm

($\because$ Diagonals AE and CD bisect each other at F)

$\because$ BD is the diagonal of ||gm ABCD

$$\Delta BCD = \frac{1}{2} \text{ ||gm ABCD}$$

$\because$ F is mid-point of DC

$$\therefore \Delta DFB = \frac{1}{2} \Delta BCD$$

$$\Rightarrow \Delta DFB = \frac{1}{2} \times \frac{1}{2} (\text{||gm ABCD})$$

$$\Rightarrow \Delta DFB = \frac{1}{4} (\text{||gm ABCD})$$

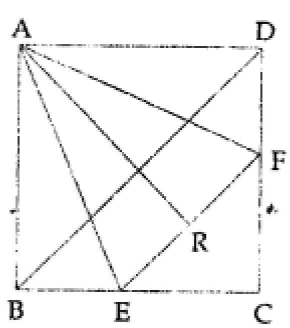
$\therefore$ area ||gm ABCD = 4 area ΔDFB

$= 4 \times 3 = 12\ cm^2$

Question 7.

In the given figure, ABCD is a square. E and F are mid-points of sides BC and CD respectively. If R is mid-point of EF, prove that: area of △AER = area of △AFR.

Solution:



Given : In square ABCD, BD is diagonals E and F are mid-point of BC and CD respectively. R is mid-point of EF.

To prove : area (ΔAER = area (ΔAFR)

Proof : In ΔABE and ΔADF

AB = AD (Sides of a square)

∠B = ∠D (Each 90°)

BE = CE (E is mid-point of BC)

∴ ΔABE ≅ ΔADF (SAS axiom)

∴ AE = AF (c.p.c.t.)

Again in ΔAER and ΔAFR

AE = AF (Produced)

AR = AR (Common)

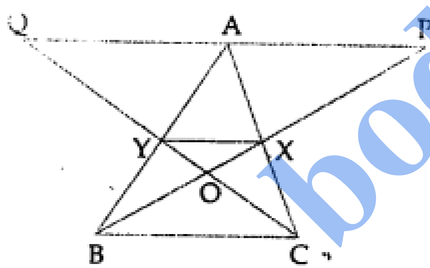
ER = FR (R is mid-point of EF)

∴ ΔAER ≅ ΔAFR (SSS axiom)

∴ area(ΔAER) = area (ΔAFR)

Question 8.

In the given figure, X and Y are mid-points of the sides AC and AB respectively of ΔABC. QP || BC and CYQ and BXP are straight lines. Prove that area of ΔABP = area of ΔACQ.



Solution:

Given : In the given figure,

X and Y are the mid-points of the sides AC and AB respectively of ΔABC

$QP \parallel BC$

CYQ and BXP are straight lines

To prove : area(ΔABP) = area(ΔACQ)

Proof : $\because$ X and Y are the mid-points of sides AC and AB respectively

$\therefore$ $YX \parallel BC$

But $QP \parallel BC$

$\therefore$ $QP \parallel BC \parallel YX$

In ΔBAP, Y is mid of AB and $YX \parallel QP$

$\therefore$ X is mid-point of BP

$\therefore$ $YX = \frac{1}{2} AP$ ...(*i*)

Similarly we can prove in ΔAQC

$YX = \frac{1}{2} QA$ ...(*ii*)

From (*i*) and (*ii*),

QA = AP

Now ΔABP and ΔACQ are on the equal base and between the same parallel lines

$\therefore$ area(ΔABP) = area(ΔACQ)