

Chapter 1. Rational and Irrational Numbers

Exercise 1.1

Solution (1)

Given rational numbers are $\frac{2}{9}$ and $\frac{3}{8}$

LCM of denominators 9 and 8 is 72

Equivalent fractions of ' $\frac{2}{9}$ ' and ' $\frac{3}{8}$ ' with denominator 72

$$\frac{2}{9} = \frac{2 \times 8}{9 \times 8} = \frac{16}{72}$$

$$\frac{3}{8} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72}$$

Since, $16 < 27$, $\frac{16}{72} < \frac{27}{72}$

So, $\frac{2}{9} < \frac{3}{8}$

A rational number between $\frac{2}{9}$ and $\frac{3}{8}$ is

$$\begin{aligned} & \frac{\frac{2}{9} + \frac{3}{8}}{2} \\ &= \frac{\frac{2 \times 8 + 3 \times 9}{72}}{2} \end{aligned}$$

$$= \frac{16 + 27}{72 \times 2}$$

$$= \frac{43}{144}$$

Descending order of the numbers is $\frac{3}{8}, \frac{43}{144}, \frac{2}{9}$

$$\frac{2}{9} < \frac{43}{144} < \frac{3}{8}$$

Solution (2):

Method II:

L.C.M of 3 and 4 is 12.

Rational number between $\frac{1}{3}$ and $\frac{1}{4}$ is $\frac{\frac{1}{3} + \frac{1}{4}}{2}$

$$= \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12 \times 2}$$

$$= \frac{7}{24}$$

$$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12} ;$$

$$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12} .$$

$$\text{Since, } 4 > 3 \therefore \frac{4}{12} > \frac{3}{12}$$

$$\frac{1}{3} > \frac{1}{4}$$

' $\frac{7}{24}$ ' is in between $\frac{1}{3}$ and $\frac{1}{4}$. So, $\frac{1}{3} > \frac{7}{24} > \frac{1}{4} \rightarrow \textcircled{1}$

Rational number between $\frac{1}{3}$ and $\frac{7}{24}$ is $\frac{\frac{1}{3} + \frac{7}{24}}{2}$

$$= \frac{\frac{1 \times 8 + 7 \times 1}{24}}{2}$$

$$= \frac{15}{2 \times 24}$$

$$= \frac{15}{48}$$

$\frac{15}{48}$ is in between $\frac{1}{3}$ and $\frac{7}{24}$. So, $\frac{1}{3} > \frac{15}{48} > \frac{7}{24} \rightarrow \textcircled{2}$

From $\textcircled{1}$ and $\textcircled{2}$, ascending order of numbers (increasing order)

$$\text{is } \frac{1}{4} < \frac{7}{24} < \frac{15}{48} < \frac{1}{3} .$$

Solution ③

Given rational numbers are $-\frac{1}{3}$ and $-\frac{1}{2}$

LCM of 3 and 2 is 6.

$$-\frac{1}{3} = \frac{-1 \times 2}{3 \times 2} = \frac{-2}{6} ; \quad -\frac{1}{2} = \frac{-1 \times 3}{2 \times 3} = \frac{-3}{6}$$

Since, $2 < 3$

$$-2 > -3$$

$$-\frac{2}{6} > \frac{-3}{6}$$

$$\text{So, } -\frac{1}{3} > -\frac{1}{2}$$

Rational number between $-\frac{1}{3}$ and $-\frac{1}{2}$ is $\frac{-\frac{1}{3} + (-\frac{1}{2})}{2}$

$$= \frac{-1 \times 2 + (-1) \times 3}{6}$$

$$= \frac{-2-3}{6 \times 2}$$

$$= \frac{-5}{12}$$

$$\therefore -\frac{1}{3} > -\frac{5}{12} > -\frac{1}{2} \rightarrow \textcircled{1}$$

Rational number between $-\frac{1}{3}$ and $-\frac{5}{12}$ is $\frac{-\frac{1}{3} + (-\frac{5}{12})}{2}$

$$= \frac{-1 \times 4 + (-5) \times 1}{12}$$

$$= \frac{-4-5}{12 \times 2}$$

$$= \frac{-9}{24}$$

$$\therefore -\frac{1}{3} > -\frac{9}{24} > -\frac{5}{12} \rightarrow \textcircled{2}$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2}, \quad -\frac{1}{3} > -\frac{9}{24} > -\frac{5}{12} > -\frac{1}{2}$$

Ascending order (increasing order) of rational numbers

$$-\frac{1}{2}, -\frac{5}{12}, -\frac{9}{24}, -\frac{1}{3}$$

Solution (4):

LCM of 3 and 5 is 15.

$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15} \quad ; \quad \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

Since, $5 < 12$

$$\text{So, } \frac{5}{15} < \frac{12}{15}$$

$$\frac{1}{3} < \frac{4}{5}$$



Rational number between $\frac{1}{3}$ and $\frac{4}{5}$ is $\frac{\frac{1}{3} + \frac{4}{5}}{2}$

$$= \frac{\frac{1 \times 5 + 4 \times 3}{3 \times 5}}{2}$$

$$= \frac{\frac{5 + 12}{15}}{2}$$

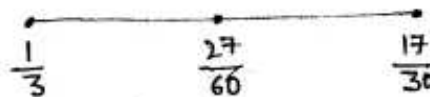
$$= \frac{17}{30}$$

Rational number between $\frac{1}{3}$ and $\frac{17}{30}$ is

$$\frac{\frac{1}{3} + \frac{17}{30}}{2}$$

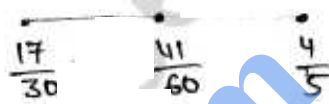
$$= \frac{\frac{1 \times 10 + 17}{30}}{2}$$

$$= \frac{27}{60}$$

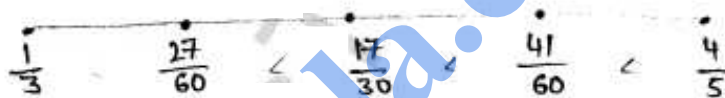


Rational number between $\frac{17}{30}$ and $\frac{4}{5}$ is

$$\begin{aligned} & \frac{\frac{17}{30} + \frac{4}{5}}{2} \\ &= \frac{\frac{17 + 4 \times 6}{30}}{2} \\ &= \frac{41}{60} \end{aligned}$$



→ Rational numbers between $\frac{1}{3}$ and $\frac{4}{5}$ are



Descending order (decreasing order) of numbers are

$$\frac{4}{5}, \frac{41}{60}, \frac{17}{30}, \frac{27}{60}, \frac{1}{3}$$

Solution ⑤:

A rational number between 4 and 4.5 is $\frac{4+4.5}{2} = \frac{8.5}{2}$
 $= 4.25$

A rational number between 4 and 4.25 is $\frac{4+4.25}{2} = \frac{8.25}{2}$
 $= 4.125$

A rational number between 4 and 4.125 is $\frac{4+4.125}{2} = \frac{8.125}{2}$
 $= 4.0625$

Three rational numbers between 4 and 4.5 are
4.0625, 4.125, 4.25,,

Solution ⑥:

We need to insert six rational numbers between 3 and 4. So, we multiply both numerator and denominator of rational numbers with $6+1$ i.e. 7.

$$\text{So, } \frac{3}{1} = \frac{3 \times 7}{1 \times 7} = \frac{21}{7}$$

$$\frac{4}{1} = \frac{4 \times 7}{1 \times 7} = \frac{28}{7}$$

We have, $21 < 22 < 23 < 24 < 25 < 26 < 27$

$$\Rightarrow \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$$

Therefore, six rational numbers between 3 and 4

$$\text{are } \frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}.$$

Solution ⑦:

We need to insert five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. So, we multiply both numerator and denominator with $5+1$ i.e. 6.

$$\text{So, } \frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

We have, $18 < 19 < 20 < 21 < 22 < 23 < 24$

$$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

Therefore, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}.$$

Solution (8):

LCM of 5 and 7 is 35.

$$\frac{-2}{5} = \frac{-2 \times 7}{5 \times 7} = \frac{-14}{35} \quad ; \quad \frac{1}{7} = \frac{1 \times 5}{7 \times 5} = \frac{5}{35}$$

We need to insert ten rational numbers between $\frac{-2}{5} (= \frac{-14}{35})$ and $\frac{1}{7} (= \frac{5}{35})$. So, we can select any ten numbers between -14 and 5 as numerators and '35' as denominator.

$$\therefore \frac{-13}{35}, \frac{-12}{35}, \frac{-11}{35}, \frac{-10}{35}, \frac{-9}{35}, \frac{-8}{35}, \frac{-7}{35}, \frac{-6}{35},$$

$\frac{-5}{35}, \frac{-4}{35}$ are ten rational numbers which are in between $\frac{-2}{5}$ and $\frac{1}{7}$.

Solution (9)

LCM of 2 and 3 is 6.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \quad , \quad \frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

We need to insert six rational numbers. So, multiply both numerator and denominator by 6+1 i.e., 7.

$$\frac{3}{6} = \frac{3 \times 7}{6 \times 7} = \frac{21}{42} \quad ;$$

$$\frac{4}{6} = \frac{4 \times 7}{6 \times 7} = \frac{28}{42}$$

Since, $21 < 22 < 23 < 24 < 25 < 26 < 27 < 28$

$$\frac{21}{42} < \frac{22}{42} < \frac{23}{42} < \frac{24}{42} < \frac{25}{42} < \frac{26}{42} < \frac{27}{42} < \frac{28}{42}$$

Therefore, Six numbers between $\frac{1}{2}$ and $\frac{2}{3}$

$$\text{are } \frac{22}{42}, \frac{23}{42}, \frac{24}{42}, \frac{25}{42}, \frac{26}{42}, \frac{27}{42}$$

Exercise - 1.2

Solution 1:

Let $\sqrt{5}$ be a rational number, then

$\sqrt{5} = \frac{p}{q}$, where p, q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$\Rightarrow 5 = \frac{p^2}{q^2}$$

$$p^2 = 5 \cdot q^2 \longrightarrow (i)$$

As '5' divides $5q^2$, so '5' divides p^2 and '5' is prime

$\Rightarrow 5$ divides p .

Let $p = 5 \cdot m$, where m is an integer

Substituting the value of 'p' in (i)

$$(5m)^2 = 5 \cdot q^2$$

$$25 \cdot m^2 = 5 \cdot q^2$$

$$\Rightarrow q^2 = 5 \cdot m^2$$

As 5 divides $5m^2$, so 5 divides q^2 but 5 is prime

$\Rightarrow 5$ divides q .

Thus, p and q have a common factor 5. This contradicts that 'p' and 'q' have no common factors (except 1).

Hence, $\sqrt{5}$ is not rational number.

So, we conclude $\sqrt{5}$ is an irrational number.

Solution 2:

Let $\sqrt{7}$ be a rational number.

$\sqrt{7} = \frac{p}{q}$, where p, q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$\Rightarrow 7 = \frac{p^2}{q^2}$$

$$p^2 = 7 \cdot q^2 \rightarrow (i)$$

As 7 divides $7q^2$, so 7 divides p^2 and 7 is a prime

$\Rightarrow$ 7 divides p

Let $p = 7m$, where 'm' is an integer

Substitute this value of 'p' in $\therefore$ we have.

$$(7m)^2 = 7 \cdot q^2$$

$$49 \cdot m^2 = 7 \cdot q^2$$

$$7m^2 = q^2$$

As 7 divides $7m^2$, so 7 divides q^2 but 7 is a prime number.

$\Rightarrow$ 7 divides q

Thus, 'p' and 'q' have a common factor 7. This contradicts that p and q have no common factor (except 1).

Hence, $\sqrt{7}$ is not a rational number.

So, we conclude $\sqrt{7}$ is irrational number.

Solution 3:

Let $\sqrt{6}$ be a rational number.

$\sqrt{6} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$\Rightarrow 6 = \frac{p^2}{q^2}$$

$$p^2 = 6 \cdot q^2 \rightarrow (i)$$

As '2' divides $6q^2$, so 2 divides p^2 but 2 is prime

$\Rightarrow$ 2 divides p .

Let $p = 2 \cdot m$, where 'm' is an integer.

Substitute this value of 'p' in (i)

$$(2m)^2 = 6 \cdot q^2$$

$$4 \cdot m^2 = 6 \cdot q^2$$

$$2 \cdot m^2 = 3 \cdot q^2$$

'2' divides ' $2m^2$ ', so 2 divides ' $3q^2$ '

2 should either divide '3' or divide q^2 .

But 2 should does not divide 3.

Therefore, 2 divides q^2 and 2 is a prime

2 divides q .

Thus, p and q have a common factor 2. This contradicts that ' p ' and ' q ' have no common factors (except 1).

Hence, $\sqrt{6}$ is not rational number.

So, we conclude $\sqrt{6}$ is irrational number.

Solution 4 :-

Let $\frac{1}{\sqrt{11}}$ be a rational number.

$\frac{1}{\sqrt{11}} = \frac{p}{q}$, where 'p', 'q' are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$\Rightarrow \frac{1}{11} = \frac{p^2}{q^2}$$

$$\Rightarrow q^2 = 11 \cdot p^2 \rightarrow \text{ii}$$

As 11 divides $11p^2$, so 11 divides q^2 but 11 is a prime.

$$\Rightarrow 11 \text{ divides } q.$$

$q = 11m$, where 'm' is an integer.

$$(11m)^2 = 11p^2$$

$$\Rightarrow 121 \cdot m^2 = 11 \cdot p^2$$

$$\Rightarrow p^2 = 11 \cdot m^2$$

As 11 divides $11m^2$, so 11 divides p^2 but 11 is a prime.

$$\Rightarrow 11 \text{ divides } p.$$

Thus, p and q have a common factor 11. This contradicts the fact that p and q has no common factors (except 1).

Hence, $\frac{1}{\sqrt{11}}$ is not rational number.

So, we conclude $\frac{1}{\sqrt{11}}$ is irrational number.

Solution 5:

Let $\sqrt{2}$ is a rational number.

$\sqrt{2} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$\Rightarrow 2 = \frac{p^2}{q^2}$$

$$p^2 = 2 \cdot q^2 \rightarrow (i)$$

As '2' divides $2q^2$, so 2 divides p^2 but 2 is prime.

$\Rightarrow$ 2 divides p .

Let $p = 2m$, where 'm' is an integer.

Substitute this value of p in (i).

$$(2m)^2 = 2q^2$$

$$4m^2 = 2q^2$$

$$\Rightarrow q^2 = 2m^2$$

As 2 divides $2m^2$, so 2 divides q^2 but 2 is prime.

2 divides q

Thus, p and q have a common factor 2. This contradicts the fact that p and q has no common factor (except 1).

Hence, $\sqrt{2}$ is not rational number.

So, we conclude $\sqrt{2}$ is irrational number.

Let us assume $3 - \sqrt{2}$ is rational number, say r .

$$\text{Thus, } 3 - \sqrt{2} = r \Rightarrow 3 - r = \sqrt{2}$$

As 'r' is rational, $3 - r$ is rational $\Rightarrow \sqrt{2}$ is rational

this contradicts the fact that $\sqrt{2}$ is irrational

Hence, our assumption is wrong. Therefore, $3 - \sqrt{2}$ is an irrational number.

Solution 6:

Let $\sqrt{3}$ is a rational number.

$\sqrt{3} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$3 = \frac{p^2}{q^2}$$

$$\Rightarrow p^2 = 3q^2 \longrightarrow (i)$$

As '3' divides $3q^2$, so 3 divides p^2 but 3 is a prime.

$$\Rightarrow 3 \text{ divides } p$$

Let $p = 3m$, where 'm' is an integer

substituting this value of p in (i).

$$(3m)^2 = 3q^2$$

$$9m^2 = 3q^2$$

$$\Rightarrow q^2 = 3m^2$$

As '3' divides $3m^2$, so '3' divides q^2 but 3 is not prime.

$$\Rightarrow 3 \text{ divides } q$$

Thus, p and q have a common factor 3. This contradicts the fact that p and q has no common factor (except 1).

Hence, $\sqrt{3}$ is not rational number.

So, we conclude $\sqrt{3}$ is irrational number.

Let us assume $\frac{2}{5}\sqrt{3}$ is a rational number, say r .

$$\text{Thus } \frac{2}{5}\sqrt{3} = r \Rightarrow \sqrt{3} = \frac{5}{2} \cdot r$$

As r is rational, $\frac{5}{2}r$ is rational $\Rightarrow \sqrt{3}$ is rational

This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, our assumption is wrong. Therefore, $\frac{2}{5}\sqrt{3}$ is irrational number.

Solution 7 :

Let $\sqrt{5}$ is a rational number.

$\sqrt{5} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$5 = \frac{p^2}{q^2}$$

$$\Rightarrow p^2 = 5 \cdot q^2 \rightarrow (i)$$

As 5 divides $5q^2$, so 5 divides p^2 but 5 is a prime.

$\Rightarrow$ 5 divides p .

Let $p = 5m$ where m is an integer

Substitute this value of m in (i)

$$(5m)^2 = 5q^2$$

$$25m^2 = 5q^2$$

$$q^2 = 5m^2$$

As 5 divides $5m^2$, 5 divides q^2 but 5 is a prime.

5 divides q .

Thus, p and q have a common factor 5. This contradicts the fact that p and q has no common factors (except 1).

Hence, $\sqrt{5}$ is not rational number.

So, we conclude $\sqrt{5}$ is irrational number.

Let us assume $-3+2\sqrt{5}$ is a rational number, say r .

$$\text{Thus, } -3+2\sqrt{5} = r \Rightarrow -3-r = 2\sqrt{5}$$

$$\Rightarrow \sqrt{5} = \frac{-(3+r)}{2}$$

As ' r ' is rational, $-\frac{(3+r)}{2}$ is rational $\Rightarrow \sqrt{5}$ is rational

This contradict the fact that $\sqrt{5}$ is irrational.

Hence, our assumption is wrong, therefore, $-3+2\sqrt{5}$ is irrational number.

Solution (8) :

(i) Let ' $5+\sqrt{2}$ ' is rational number, say r .

$$5+\sqrt{2}=r \Rightarrow \sqrt{2}=r-5$$

As ' r ' is rational, $r-5$ is rational $\Rightarrow \sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is wrong. Therefore, $5+\sqrt{2}$ is an irrational number.

(ii)

Let $3-5\sqrt{3}$ is rational number, say r .

$$3-5\sqrt{3}=r \Rightarrow 5\sqrt{3}=3-r$$

$$\Rightarrow \sqrt{3}=\left(\frac{3-r}{5}\right)$$

As r is rational, $\left(\frac{3-r}{5}\right)$ is rational $\Rightarrow \sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, our assumption is wrong. Therefore, $3-5\sqrt{3}$ is an irrational number.

(iii)

Let $2\sqrt{3}-7$ is a rational number, say r .

$$2\sqrt{3}-7=r \Rightarrow 2\sqrt{3}=r+7$$

$$\sqrt{3}=\frac{r+7}{2}$$

As ' r ' is rational, $\left(\frac{r+7}{2}\right)$ is rational $\Rightarrow \sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, our assumption is wrong. Therefore, $2\sqrt{3}-7$ is an irrational number.

Solution 8:

(iv) Let $\sqrt{2} + \sqrt{5}$ is a rational number, say r .

$$\sqrt{2} + \sqrt{5} = r$$

$$\sqrt{5} = r - \sqrt{2}$$

$$(\sqrt{5})^2 = (r - \sqrt{2})^2 \quad (\text{on squaring both sides})$$

$$5 = r^2 + (\sqrt{2})^2 - 2 \times r \times \sqrt{2} \quad \left[\because (a-b)^2 = a^2 + b^2 - 2ab \right]$$

$$5 = r^2 + 2 - 2\sqrt{2} \cdot r$$

$$2\sqrt{2} \cdot r = r^2 - 3$$

$$\sqrt{2} = \frac{r^2 - 3}{2 \cdot r}$$

As r is rational, $r^2 - 3$ is rational, $\left(\frac{r^2 - 3}{2r}\right)$ is rational

$\Rightarrow \sqrt{2}$ is rational

this contradicts the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is wrong. Therefore,

$(\sqrt{2} + \sqrt{5})$ is an irrational number.

Exercise 1.3

Solution 1:



PQ is a number line.

We have two right angle triangles. They are

$\triangle OAB$ and $\triangle OCD$.

In a right angled triangle,

$$(\text{hypotenuse})^2 = (\text{side 1})^2 + (\text{side 2})^2$$

$$\therefore OB^2 = OA^2 + AB^2$$

$$OB^2 = 3^2 + 1^2 \quad (\because OA = 3, AB = 1)$$

$$OB^2 = 9 + 1$$

$$OB = \sqrt{10}$$

Similarly, in $\triangle OCD$,

$$OD^2 = OC^2 + CD^2$$

$$OD^2 = 4^2 + 1^2$$

$$(\because OC = 4, CD = 1)$$

$$OD^2 = 16 + 1$$

$$OD = \sqrt{17}$$

Solution 2:-

(i) $\frac{36}{100}$

	0.036
100	360
	300
	600
	600
	0

Remainder becomes zero.

Decimal expansion of $\frac{36}{100} (=0.36)$ is terminating.

(ii) $4\frac{1}{8}$

$$= \frac{4 \times 8 + 1}{8}$$

$$= \frac{33}{8}$$

	4.125
8	33
	32
	10
	8
	20
	16
	40
	40
	0

Remainder becomes zero

Decimal expansion of $4\frac{1}{8} (=4.125)$ is terminating.

(iii) $\frac{2}{9}$

9	0.22
20	18
20	18
2	

← Remainder is repeating

In the above decimal expansion remainder is repeating. So, it is a non-terminating decimal.

So, $\frac{2}{9} = 0.222\ldots = 0.\dot{2} = 0.\bar{2}$

(iv) $\frac{2}{11}$

11	0.1818
20	11
90	88
20	11
90	88
2	

← Remainder '2' is repeating.

Decimal expansion of $\frac{2}{11} = 0.1818\ldots$

Here, remainder is repeating. So, it is a non-terminating repeating decimal.

$\therefore \frac{2}{11} = 0.\overline{18}$

(v) $\frac{3}{13}$

	0.230769
13	30
	26
	40
	39
	100
	91
	90
	78
	120
	117
	3

← Remainder '3' is repeating.

Decimal Expansion of $\frac{3}{13}$ is 0.230769.....

Here, remainder is repeating. So, it a non-terminating repeating decimal.

$\therefore \frac{3}{13} = 0.\overline{230769}$

(vi) $\frac{329}{400}$

	0.8225
100	3290
	3200
	900
	800
	1000
	800
	2000
	2000
	0

← Remainder is zero

Decimal Expansion of $\frac{329}{400} = 0.8225$.

∴ It is a ~~repet~~ terminating decimal Expansion.

Solution (3) :-

(i) $\frac{13}{3125}$

Prime factorization of denominator 3125.

$$3125 = 5 \times 5 \times 5 \times 5 \times 5 \times 1$$

$$= 5^5 \times 1$$

$$= 1 \times 5^5$$

$$\therefore 3125 = 2^0 \times 5^5 \quad (\because 2^0 = 1)$$

$$\begin{array}{r|l} 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Since, denominator is in the form of $2^0 \times 5^5$,
the decimal Expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

Prime factorization of denominator 8.

$$8 = 2 \times 2 \times 2$$

$$8 = 2^3 \times 1$$

$$8 = 2^3 \times 5^0$$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Since denominator is in the form of $2^3 \times 5^0$,
decimal Expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{23}{75}$

Prime factorization of 75.

$$75 = 3 \times 5 \times 5 \times 1$$

$$75 = 3 \times 5^2 \times 1$$

$$75 = 3 \times 2^0 \times 5^2 \quad (\because 2^0 = 1)$$

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Since, denominator contains prime factor 3 other than 2 or 5.

Decimal Expansion of $\frac{23}{75}$ is non-terminating.

(iv) $\frac{6}{15}$

Both numerator and denominator contains common factor 3.

$$\frac{6}{15} = \frac{3 \times 2}{3 \times 5} = \frac{2}{5}$$

$$\therefore \frac{6}{15} = \frac{2}{5}$$

Since, denominator is in the form $2^0 \times 5^1$.

Decimal Expansion of $\frac{6}{15} (= \frac{2}{5})$ is terminating.

(v) $\frac{1258}{625}$

Prime factorization of denominator 625.

$$625 = 5 \times 5 \times 5 \times 5 \times 1$$

$$625 = 5^4 \times 2^0$$

$$\begin{array}{r|l} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Since, denominator is in the form $2^0 \times 5^4$, decimal Expansion of $\frac{1258}{625}$ is terminating.

(vi) $\frac{77}{210}$

Both numerator and denominator contains common factor 7.

$$\frac{77}{210} = \frac{7 \times 11}{7 \times 30} = \frac{11}{30}$$

$$\therefore \frac{77}{210} = \frac{11}{30}$$

Prime factorization of denominator 30.

$$30 = 2 \times 3 \times 5 \times 1$$

$$30 = 3 \times 2 \times 5$$

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Since, denominator contains prime factor 3 other 2 or 5

Decimal Expansion of $\frac{77}{210}$ is non-terminating.

Solution (4) :-

Expressing both numerator and denominator of fraction

$\frac{987}{10500}$ as product of prime numbers by prime factorization method

$$\begin{array}{r|l} 3 & 987 \\ \hline 7 & 329 \\ \hline 47 & 47 \\ \hline & 1 \end{array}$$

$$\therefore 987 = 3 \times 7 \times 47$$

$$\begin{array}{r|l} 2 & 10500 \\ \hline 2 & 5250 \\ \hline 3 & 2625 \\ \hline 5 & 875 \\ \hline 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$$\therefore 10500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7$$

$$\frac{987}{10500} = \frac{\cancel{3} \times \cancel{7} \times 47}{2 \times 2 \times \cancel{3} \times 5 \times 5 \times 5 \times \cancel{7}}$$

$$= \frac{47}{2^2 \times 5^3}$$

Since, denominator is in the form $2^2 \times 5^3$, decimal expansion of $\frac{987}{10500}$ is terminating

Solution (5) :-

(i) $\frac{17}{8}$

Prime factorization of denominator 8

$$8 = 2 \times 2 \times 2 \times 1$$

$$8 = 2^3 \times 5^0 \quad (\because 1 = 5^0)$$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\frac{17}{8} = \frac{17}{2^3}$$

$$= \frac{17 \times 5^3}{2^3 \times 5^3} \quad (\text{By multiplying both numerator and denominator with } 5^3).$$

$$= \frac{17 \times 125}{(2 \times 5)^3}$$

$$= \frac{2125}{10^3}$$

$$= 2.125$$

(Since, denominator is in the form 10^3 , decimal expansion is obtained by moving decimal point to three digits from right).

$$\begin{array}{r} 2.125 \\ \times 17 \\ \hline 875 \\ 1250 \times \\ \hline 2125 \end{array}$$

$$\therefore \frac{17}{8} = 2.125 //$$

$$(ii) \frac{13}{3125}$$

Prime factorization of 3125.

$$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$$

$$\begin{array}{r|l} 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\frac{13}{3125} = \frac{13}{5^5}$$

$$= \frac{13 \times 2^5}{5^5 \times 2^5} \quad (\text{Multiplying numerator and denominator by } 2^5)$$

$$= \frac{13 \times 32}{(2 \times 5)^5} \quad \left(\begin{array}{l} 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32 \\ \& a^m \times b^m = (a \times b)^m \end{array} \right)$$

$$= \frac{416}{10^5}$$

$$= 0.00416$$

$$\therefore \frac{13}{3125} = 0.00416$$

$$\begin{array}{r} 32 \\ \times 13 \\ \hline 96 \\ 32 \times \\ \hline 416 \end{array}$$

$$(iii) \frac{7}{80}$$

Prime factorization of 80.

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

$$80 = 2^4 \times 5^1$$

$$\frac{7}{80} = \frac{7}{2^4 \times 5^1}$$

$$\begin{array}{r|l} 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

(Multiplying numerator and denominator by 5^3)

$$= \frac{7 \times 5^3}{2^4 \times 5^1 \times 5^3}$$

$$= \frac{7 \times 125}{2^4 \times 5^4}$$

$$= \frac{7 \times 125}{(2 \times 5)^4}$$

$$= \frac{875}{10^4}$$

$$= 0.0875$$

$$\begin{array}{r} 125 \\ \times 7 \\ \hline 875 \end{array}$$

(Since, denominator is 10^4 , decimal expansion can be obtained by moving decimal point of numerator to four digits from right.)

$$\therefore \frac{7}{80} = 0.0875$$

$$(iv) \frac{6}{15}$$

Prime factorization of 6 and 15

$$\begin{array}{r} 2 \overline{) 6} \\ 3 \overline{) 3} \\ 1 \end{array}$$

$$\begin{array}{r} 3 \overline{) 15} \\ 5 \overline{) 5} \\ 1 \end{array}$$

$$\therefore 6 = 2 \times 3$$

$$\therefore 15 = 3 \times 5$$

$$\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$$

$$= \frac{2 \times 2}{5 \times 2} \quad \left(\begin{array}{l} \text{By multiplying both numerator and} \\ \text{denominator by 2} \end{array} \right)$$

$$= \frac{4}{10}$$

$$= 0.4$$

$$(v) \frac{2^3 \times 7}{5^4}$$

$$= \frac{2^3 \times 7 \times 2^4}{5^4 \times 2^4}$$

(By multiplying both numerator and denominator by 2^4)

$$= \frac{4 \times 7 \times 16}{(2 \times 5)^4}$$

$$= \frac{28 \times 16}{10^4}$$

$$= \frac{448}{10^4}$$

$$= 0.0448$$

$$\therefore \frac{2^3 \times 7}{5^4} = 0.0448$$

$$\begin{array}{r} 4 \\ 28 \\ \times 16 \\ \hline 168 \\ 28 \times \\ \hline 448 \end{array}$$

$$(vi) \frac{237}{1500}$$

Prime factorization of 237 and 1500.

$$\begin{array}{r} 3 \overline{) 237} \\ 79 \overline{) 79} \\ \hline 1 \end{array}$$

$$\begin{array}{r} 2 \overline{) 1500} \\ 2 \overline{) 750} \\ 3 \overline{) 375} \\ 5 \overline{) 125} \\ 5 \overline{) 25} \\ 5 \overline{) 5} \\ \hline 1 \end{array}$$

$$\therefore 237 = 3 \times 79$$

$$\therefore 1500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5$$

$$\frac{237}{1500} = \frac{\cancel{3} \times 79}{2^2 \times \cancel{3} \times 5^3}$$

$$= \frac{79}{2^2 \times 5^3}$$

$$= \frac{79 \times 2}{2^3 \times 5^3 \times 2} \quad \left(\text{Multiplying both numerator and denominator by '2'} \right)$$

$$= \frac{158}{2^3 \times 5^3}$$

$$= \frac{158}{(10)^3}$$

$$= 0.158$$

$$\therefore \frac{237}{1500} = 0.158$$

Solution (6):

Given rational number $\frac{257}{5000}$

Prime factorization of denominator 5000

$$\therefore 5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$$

$$= 2^3 \times 5^4$$

Thus, denominator of rational number is in the form $2^m \times 5^n$,

where $m=3$ and $n=4$.

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$$\therefore \frac{257}{5000} = \frac{257}{2^3 \times 5^4}$$

$$= \frac{257 \times 2}{2^3 \times 5^4 \times 2}$$

$$= \frac{514}{2^4 \times 5^4}$$

(By multiplying numerator and denominator by 2)

$$= \frac{514}{(2 \times 5)^4}$$

$$= \frac{514}{10^4}$$

$$= 0.0514$$

$\therefore$ Decimal expansion of $\frac{257}{5000}$ is 0.0514 .

Solution (7) :-

Decimal expansion of $\frac{1}{7}$:

	0.142857
7	10
	7
	30
	28
	20
	14
	60
	56
	40
	35
	50
	49
	1

← Remainder '1' is repeated.

$\therefore$ Decimal Expansion of $\frac{1}{7}$ is non-terminating repeating

$$\frac{1}{7} = 0.\overline{142857}$$

$\frac{2}{7}$ can be written as $2 \times \frac{1}{7}$

$$\begin{aligned}\text{Decimal Expansion of } \frac{2}{7} &= 2 \times \frac{1}{7} \\ &= 2 \times 0.\overline{142857} \\ &= 0.\overline{285714}\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{3}{7} &= 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} \\ &= 0.\overline{428571}\end{aligned}$$

$$\begin{aligned}\frac{4}{7} &= 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} \\ &= 0.\overline{571428}\end{aligned}$$

$$\begin{aligned}\frac{5}{7} &= 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} \\ &= 0.\overline{714285}\end{aligned}$$

$$\begin{aligned}\frac{6}{7} &= 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} \\ &= 0.\overline{857142}\end{aligned}$$

Solution (8) :-

(i) Let $x = 0.\overline{3} = 0.3333\ldots \rightarrow \textcircled{1}$

As there is one repeating digit after the decimal point so multiplying both sides of Eq (1) by 10.

$$10x = 3.333\ldots \rightarrow \textcircled{2}$$

Subtracting (1) from (2), we get.

$$10x - x = 3.333\ldots - 0.333\ldots$$

$$9x = 3$$

$$x = \frac{3}{9}$$

$$x = \frac{1}{3}$$

$\therefore x = 0.\bar{3} = \frac{1}{3}$, which is in $\frac{p}{q}$ form.

(ii) Let $x = 5.\bar{2} = 5.222\ldots \rightarrow \textcircled{1}$

As there is one repeating digit after the decimal point, so multiplying both sides of Eq $\textcircled{1}$ by 10.

$$10x = 52.222\ldots \rightarrow \textcircled{2}$$

Subtracting $\textcircled{1}$ from $\textcircled{2}$, we get

$$\begin{array}{r} 10x = 52.222\ldots \\ - x = 5.222\ldots \\ \hline 9x = 47.000 \end{array}$$

$$x = \frac{47}{9}$$

$\therefore x = 5.\bar{2} = \frac{47}{9}$, which is in $\frac{p}{q}$ form.

(iii) Let $x = 0.404040\ldots \rightarrow \textcircled{1}$

As there is two repeating digit after the decimal point, so multiplying both sides of Eq $\textcircled{1}$ by 100

$$100x = 40.404040 \rightarrow \textcircled{2}$$

Subtracting $\textcircled{1}$ from $\textcircled{2}$, we get.

$$\begin{array}{r} 100x = 40.404040\ldots \\ - x = 0.404040\ldots \\ \hline 99x = 40.000 \end{array}$$

$$99x = 40$$

$$x = \frac{40}{99}$$

$\therefore x = 0.404040 \dots = \frac{40}{99}$, which is in $\frac{p}{q}$ form.

(iv) Let $x = 0.4\bar{7} = 0.4777 \dots$

$$x = 0.4777 \dots \rightarrow (1)$$

There is one non-repeating digit after the decimal point, multiplying both sides of (1) by 10.

$$10x = 4.777 \dots \rightarrow (2)$$

As there is one repeating digit after the decimal point, multiplying both sides of (2) by 10.

$$100x = 47.777 \dots \rightarrow (3)$$

Subtracting (2) from (3), we get.

$$100x = 47.777 \dots$$

$$10x = 4.777 \dots$$

(-)

$$90x = 43.000$$

$$90x = 43$$

$$x = \frac{43}{90}$$

$\therefore x = 0.4777 \dots = \frac{43}{90}$, which is in $\frac{p}{q}$ form.

(v) $0.1\overline{34}$

Let $x = 0.1343434\ldots \rightarrow \textcircled{1}$

There is one non-repeating digit after the decimal point, multiplying both sides of $\textcircled{1}$ by 10.

$10x = 1.343434\ldots \rightarrow \textcircled{2}$

As there are two repeating digits after the decimal point, so multiplying both sides of $\textcircled{2}$ by 100.

$1000x = 134.343434\ldots \rightarrow \textcircled{3}$

Subtracting $\textcircled{2}$ from $\textcircled{3}$, we get

$$\begin{array}{r} 1000x = 134.343434\ldots \\ 10x = 1.343434\ldots \\ \hline 990x = 133.000000 \end{array}$$

$$\begin{aligned} 990x &= 133 \\ x &= \frac{133}{990} \end{aligned}$$

$\therefore x = \frac{133}{990}$, which is in $\frac{p}{q}$ form.

(vi) Let $x = 0.\overline{001}$

$x = 0.001001\ldots \rightarrow \textcircled{1}$

As there are three repeating digits after the decimal point, so multiplying both sides ^① by 1000.

$1000x = 1.001001\ldots \rightarrow \textcircled{2}$

Subtracting $\textcircled{1}$ from $\textcircled{2}$, we get,

$$1000x = 1.001001 \dots$$

$$x = 0.001001 \dots$$

$$\begin{array}{r} (-) \\ \hline 999x = 1 \end{array}$$

$$x = \frac{1}{999}$$

$\therefore x = 0.\overline{001} = \frac{1}{999}$, which is in $\frac{p}{q}$ form.

Solution (9)

(i) $\sqrt{23}$

Square root of 23 by long division method.

	4.79583
4	23.0000000000
	16
87	700
	609
949	9100
	8541
9585	55900
	47925
95908	7,97500
	7,67,264
959163	30,23600
	28,77,489
	146111

$\therefore \sqrt{23} = 4.79583$, which has non-terminating and non-repeating decimal expansion.

So, it is an irrational number.

(ii) $\sqrt{225}$

Prime factorization of 225.

$$225 = 3 \times 3 \times 5 \times 5$$

$$225 = (3 \times 5)^2$$

$$\sqrt{225} = \sqrt{(3 \times 5)^2} = ((3 \times 5)^2)^{\frac{1}{2}}$$

$$\therefore \sqrt{225} = 3 \times 5 = 15.$$

$\sqrt{225} = 15$; which is a rational number.

(iii) 0.3796

Decimal Expansion of 0.3796 is terminating.

So, $0.3796 = \frac{3796}{10000}$ which is in $\frac{p}{q}$ form.

$\therefore 0.3796$ is a rational number.

(iv) $x = 7.478478 \dots \rightarrow \textcircled{1}$

As there are three repeating digits after the decimal point, so multiplying both sides of $\textcircled{1}$ by 1000.

$$1000x = 7478.478478 \dots \rightarrow \textcircled{2}$$

Subtracting $\textcircled{1}$ from $\textcircled{2}$, we get,

$$1000x = 7478.478478 \dots$$

$$x = 7.478478 \dots$$

(-)

$$999x = 7471.0$$

$$x = \frac{7471}{999}$$

$$\begin{array}{r|l} 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore x = 7.478478\ldots = \frac{7471}{999}$, which
is in $\frac{p}{q}$ form.

So, $7.478478\ldots$ is a rational number.

(v) $1.101001000100001\ldots$

From the above decimal expansion, we observed that after decimal point, number of zeros between two consecutive ones are increasing. So, it is a non-terminating and non-repeating decimal expansion.

$\therefore 1.101001000100001\ldots$ is an irrational number.

(vi) $345.\overline{0456}$

Let $x = 345.0456456\ldots \rightarrow (1)$

Multiplying by 10 on both sides of Eq. (1)

$10x = 3450.456456\ldots \rightarrow (2)$

As there are three repeating digits after the decimal point, so multiplying both sides of (2) by 1000.

$10000x = 3450456.456456\ldots \rightarrow (3)$

$$\begin{array}{rcl} (3) - (2) \Rightarrow & 10000x & = 3450456.456456\ldots \\ & 10x & = 345.456456\ldots \\ \hline (-) & & \\ & 9990x & = 3450111.0 \end{array}$$

$$\therefore 9990x = 3450111.$$

$$x = \frac{3450111}{9990},$$

which is in the form $\frac{p}{q}$

So, $345.04\overline{56}$ is a rational number.

Solution (10) :-

(i) Decimal Expansion of $\frac{1}{3}$ and $\frac{1}{2}$.

$$\therefore \frac{1}{3} = 0.333\ldots \\ = 0.\overline{3}$$

3	0.33
3	10
	9
3	10
	9

← remainder is repeating.

$$\therefore \frac{1}{2} = 0.5$$

2	0.5
2	10
	10
	0

There are infinite rational numbers between

$$\frac{1}{3} (= 0.\overline{3}) \text{ and } \frac{1}{2} (= 0.5).$$

One among them is $0.4040040004\ldots$

(ii) $-\frac{2}{5}$ and $\frac{1}{2}$.

Decimal expansion of $-\frac{2}{5}$ and $\frac{1}{2}$.

$\therefore -\frac{2}{5} = -0.4$

$$\begin{array}{r|l} & 0.4 \\ 5 & 20 \\ & 20 \\ \hline & 0 \end{array}$$

$\therefore \frac{1}{2} = 0.5$

$$\begin{array}{r|l} & 0.5 \\ 2 & 10 \\ & 10 \\ \hline & 0 \end{array}$$

There are many irrational numbers between $-\frac{2}{5}$ and $\frac{1}{2}$. One among them is 0.1010010001....

(iii) 0 and 0.1

There are infinite irrational numbers between 0 and 0.1. One among them is 0.06006000600006.....

Solution (11) :-

There are infinite irrational numbers between 2 and 3. Two among them are

2.0101001000100001....

2.919119111911119.....

Solution (12) :-

Decimal Expansion of $\frac{4}{9}$ and $\frac{7}{11}$

$$\frac{4}{9} = 0.44 \dots \dots$$

$$= 0.\overline{4}$$

9	0.44	
	40	
	36	
	40	
	36	
	4	← Remainder is repeating

$$\frac{7}{11} = 0.6363 \dots \dots$$

$$= 0.\overline{63}$$

11	0.63	
	70	
	66	
	40	
	33	
	7	← Remainder is repeating.

there are infinite rational number between

$$\frac{4}{9} (=0.\overline{4}) \text{ and } \frac{7}{11} (=0.\overline{63})$$

Two among them are $0.404004000400004 \dots \dots$,
 $0.515115111511115 \dots \dots$

Solution (13) :

$$\text{Value of } \sqrt{2} = 1.414 \dots \dots$$

$$\text{Value of } \sqrt{3} = 1.732 \dots \dots$$

there are many rational numbers between $\sqrt{2}$ and $\sqrt{3}$. One among them 1.6.

Finding Value of $\sqrt{2}$ and $\sqrt{3}$ by long division method.

$$\begin{array}{r}
 1.414 \\
 \hline
 1 \quad 2.00 \ 00 \ 00 \\
 \underline{1} \\
 24 \quad 100 \\
 \underline{96} \\
 281 \quad 400 \\
 \underline{281} \\
 2824 \quad 11900 \\
 \underline{11296} \\
 604
 \end{array}$$

$\therefore \sqrt{2} = 1.414 \dots$

$$\begin{array}{r}
 1.732 \\
 \hline
 1 \quad 3.00 \ 00 \ 00 \\
 \underline{1} \\
 27 \quad 200 \\
 \underline{189} \\
 343 \quad 1100 \\
 \underline{1029} \\
 3462 \quad 7100 \\
 \underline{6924} \\
 176
 \end{array}$$

$\therefore \sqrt{3} = 1.732 \dots$

Solution (14) :

$$2\sqrt{3} = \sqrt{2^2 \times 3} = \sqrt{4 \times 3} = \sqrt{12}$$

$$\therefore 2\sqrt{3} = \sqrt{12}$$

We have, $12 < 12.25 < 12.96 < 15$

$$\Rightarrow \sqrt{12} < \sqrt{12.25} < \sqrt{12.96} < \sqrt{15}$$

$$\sqrt{12} < \sqrt{(3.5)^2} < \sqrt{(3.6)^2} < \sqrt{15}$$

$$\sqrt{12} < 3.5 < 3.6 < \sqrt{15}$$

$\therefore$ 3.5 and 3.6 are two rational numbers between $\sqrt{12}$ and $\sqrt{15}$.

Solution (15) :

We have, $5 < 6 < 7$

$$\Rightarrow \sqrt{5} < \sqrt{6} < \sqrt{7}$$

$\therefore \sqrt{6}$ is an irrational number between $\sqrt{5}$ and $\sqrt{7}$.

Solution (16) :

We have, $3 < 5 < 6 < 7$

$$\Rightarrow \sqrt{3} < \sqrt{5} < \sqrt{6} < \sqrt{7}$$

$\therefore \sqrt{5}$ and $\sqrt{6}$ are two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$.

EXERCISE - 1.4

SOLUTION - 1

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Sol: $\sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 4\sqrt{5}$
 $= \sqrt{9}\sqrt{5} - 3\sqrt{4}\sqrt{5} + 4\sqrt{5}$
 $= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5}$
 $= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$
 $= (3 - 6 + 4)\sqrt{5}$
 $= (7 - 6)\sqrt{5}$
 $= \sqrt{5}$

(ii) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

Sol: $3\sqrt{3} + 2\sqrt{9 \times 3} + \frac{7}{\sqrt{3}}$
 $= 3\sqrt{3} + 2 \times \sqrt{9}\sqrt{3} + \frac{7}{\sqrt{3}}$
 $= 3\sqrt{3} + 2 \times 3\sqrt{3} + \frac{7}{\sqrt{3}}$
 $= 3\sqrt{3} + 6\sqrt{3} + \frac{7}{\sqrt{3}}$
 $= (3 + 6)\sqrt{3} + \frac{7}{\sqrt{3}}$
 $= 9\sqrt{3} + \frac{7}{\sqrt{3}}$

Multiplying and Dividing by " $\sqrt{3}$ "

$$= 9\sqrt{3} + \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 9\sqrt{3} + \frac{7\sqrt{3}}{3}$$

By cross multiplying

$$= \frac{3 \times 9\sqrt{3} + 7\sqrt{3}}{3}$$

$$= \frac{27\sqrt{3} + 7\sqrt{3}}{3}$$

$$= \frac{(27+7)\sqrt{3}}{3}$$

$$= \frac{34\sqrt{3}}{3}$$

(iii) $6\sqrt{5} \times 2\sqrt{5}$

Sol: $6 \times 2 \times \sqrt{5} \cdot \sqrt{5}$

$$= 12 \times (\sqrt{5})^2$$

$$= 12 \times 5$$

$$= 60$$

(iv) $8\sqrt{15} \div 2\sqrt{3}$

Sol: $\frac{8\sqrt{15}}{2\sqrt{3}} = \frac{8 \cdot \sqrt{5} \sqrt{3}}{2\sqrt{3}}$

$$= 4\sqrt{5}$$

$$(V) \quad \frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$$

$$\begin{aligned} \text{Sol:} \quad & \frac{\sqrt{6 \times 4}}{8} + \frac{\sqrt{9 \times 6}}{9} \\ &= \frac{\sqrt{6} \cdot \sqrt{4}}{8} + \frac{\sqrt{9} \cdot \sqrt{6}}{9} \\ &= \frac{2\sqrt{6}}{8} + \frac{3\sqrt{6}}{9} \\ &= \frac{1 \cdot \sqrt{6}}{4} + \frac{1 \cdot \sqrt{6}}{3} \\ &= \sqrt{6} \left[\frac{1}{4} + \frac{1}{3} \right] \\ &= \sqrt{6} \left[\frac{3+4}{12} \right] \\ &= \frac{7\sqrt{6}}{\sqrt{12}} \end{aligned}$$

$\therefore$ LCM of 4 and 3
is 12

$$(VI) \quad \frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$$

$$\begin{aligned} \text{Sol:} \quad & \frac{3}{\sqrt{2 \times 4}} + \frac{1}{\sqrt{2}} \\ &= \frac{3}{\sqrt{2} \cdot \sqrt{4}} + \frac{1}{\sqrt{2}} \\ &= \frac{3}{2\sqrt{2}} + \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \left(\frac{3}{2} + 1 \right) \\ &= \frac{1}{\sqrt{2}} \left(\frac{3+2}{2} \right) \end{aligned}$$

$\therefore$ LCM of 2 and 1
is 2

$$= \frac{1}{\sqrt{2}} \left(\frac{5}{2} \right)$$

$$= \frac{5}{2\sqrt{2}}$$

Multiply and Divide by " $\sqrt{2}$ "

$$= \frac{5}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{5\sqrt{2}}{2 \cdot \sqrt{2} \times \sqrt{2}}$$

$$= \frac{5\sqrt{2}}{2 \cdot (\sqrt{2})^2} = \frac{5\sqrt{2}}{2 \cdot 2}$$

$$= \frac{5\sqrt{2}}{4}$$

SOLUTION - 2

(i) $(5 + \sqrt{7})(2 + \sqrt{5})$

Sol: $5 \times 2 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{7} \cdot \sqrt{5}$

$$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{7 \times 5}$$

$$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$$

(ii) $(5 + \sqrt{5})(5 - \sqrt{5})$

Sol: $(5)^2 - (\sqrt{5})^2$

$$= 25 - 5$$

$$= 20$$

(iii) $(\sqrt{5} + \sqrt{2})^2$

Sol: $(\sqrt{5})^2 + (\sqrt{2})^2 + 2 \cdot \sqrt{5} \cdot \sqrt{2}$

$$= 5 + 2 + 2\sqrt{5 \times 2}$$

$$= 5 + 2 + 2\sqrt{10}$$

$$= 7 + 2\sqrt{10}$$

(iv) $(\sqrt{3} - \sqrt{7})^2$

Sol: $(\sqrt{3})^2 + (\sqrt{7})^2 - 2 \cdot \sqrt{3} \cdot \sqrt{7}$

$$= 3 + 7 - 2\sqrt{3 \times 7}$$

$$= 10 - 2\sqrt{21}$$

(v) $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$

Sol: $\sqrt{2} \cdot \sqrt{5} + \sqrt{2} \cdot \sqrt{7} + \sqrt{3} \cdot \sqrt{5} + \sqrt{3} \cdot \sqrt{7}$

$$= \sqrt{2 \times 5} + \sqrt{2 \times 7} + \sqrt{3 \times 5} + \sqrt{3 \times 7}$$

$$= \sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$$

(vi) $(4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$

Sol: $4\sqrt{3} - 4\sqrt{7} + \sqrt{5} \cdot \sqrt{3} - \sqrt{5} \cdot \sqrt{7}$

$$= 4\sqrt{3} - 4\sqrt{7} + \sqrt{5 \times 3} - \sqrt{5 \times 7}$$

$$= 4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$$

SOLUTION -3

~~~~~

(i)  $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

Sol:  $\sqrt{4 \times 2} + \sqrt{25 \times 2} + \sqrt{36 \times 2} + \sqrt{49 \times 2}$

$$= \sqrt{4} \cdot \sqrt{2} + \sqrt{25} \cdot \sqrt{2} + \sqrt{36} \cdot \sqrt{2} + \sqrt{49} \cdot \sqrt{2}$$

$$= 2\sqrt{2} + 5\sqrt{2} + 6\sqrt{2} + 7\sqrt{2}$$

$$= (2+5+6+7)\sqrt{2}$$

$$= 20 \times \sqrt{2}$$

$$= 20 \times 1.414$$

$$= 28.28$$

(ii)  $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$

Sol:  $3\sqrt{16 \times 2} - 2\sqrt{25 \times 2} + 4\sqrt{64 \times 2} - 20\sqrt{9 \times 2}$

$$= 3 \times 4\sqrt{2} - 2 \times 5\sqrt{2} + 4 \times 8\sqrt{2} - 20 \times 3\sqrt{2}$$

$$= 12\sqrt{2} - 10\sqrt{2} + 32\sqrt{2} - 60\sqrt{2}$$

$$= (12 - 10 + 32 - 60)\sqrt{2}$$

$$= -26 \times \sqrt{2}$$

$$= -26 \times 1.414$$

$$= -36.764$$

SOLUTION - 4

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(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$

Sol: $\sqrt{9 \times 3} + \sqrt{25 \times 3} + \sqrt{36 \times 3} - \sqrt{81 \times 3}$

$$= 3\sqrt{3} + 5\sqrt{3} + 6\sqrt{3} - 9\sqrt{3}$$

$$= (3+5+6-9)\sqrt{3}$$

$$= (14-9)\sqrt{3}$$

$$= 5 \times \sqrt{3}$$

$$= 5 \times 1.732$$

$$= 8.66$$

(ii) $3\sqrt{32} - 5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$

Sol: $5\sqrt{4 \times 3} - 3\sqrt{16 \times 3} + 6\sqrt{25 \times 3} + 7\sqrt{36 \times 3}$

$$= 5 \times 2\sqrt{3} - 3 \times 4\sqrt{3} + 6 \times 5\sqrt{3} + 7 \times 6\sqrt{3}$$

$$= 10\sqrt{3} - 12\sqrt{3} + 30\sqrt{3} + 42\sqrt{3}$$

$$= (10 - 12 + 30 + 42)\sqrt{3}$$

$$= (82 - 12)\sqrt{3}$$

$$= 70 \times \sqrt{3}$$

$$= 70 \times 1.732$$

$$= 121.24$$

SOLUTION - 5

(i), $\sqrt{\frac{4}{9}}$, $-\frac{3}{70}$, $\sqrt{\frac{7}{25}}$, $\sqrt{\frac{16}{5}}$

Sol: $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$

$= \frac{2}{3}$. It is in the form of $\frac{p}{q}$

and p, q are Integers

Therefore $\sqrt{\frac{4}{9}}$ is a rational number.

$\Rightarrow -\frac{3}{70}$ is a rational number.

$\Rightarrow \sqrt{\frac{7}{25}} = \frac{\sqrt{7}}{\sqrt{25}}$

$= \frac{\sqrt{7}}{5}$. Since $\sqrt{7}$ is not an Integer

Therefore, $\sqrt{\frac{7}{25}}$ is an Irrational number.

$\Rightarrow \sqrt{\frac{16}{5}} = \frac{\sqrt{16}}{\sqrt{5}}$

$= \frac{4}{\sqrt{5}}$. Since $\sqrt{5}$ is not an Integer

Therefore, $\sqrt{\frac{16}{5}}$ is an Irrational number.

(ii) $-\sqrt{\frac{2}{49}}$, $\frac{3}{200}$, $\sqrt{\frac{25}{3}}$, $-\sqrt{\frac{49}{16}}$

Sol: $-\sqrt{\frac{2}{49}} = -\frac{\sqrt{2}}{\sqrt{49}}$

$= -\frac{\sqrt{2}}{7}$. Since $\sqrt{2}$ is not an Integer

Therefore, $-\sqrt{\frac{2}{49}}$ is an irrational number

$\Rightarrow \frac{3}{200}$. It is in the form of $\frac{p}{q}$
and p, q are Integers . So, it
is a rational number

$\Rightarrow \sqrt{\frac{25}{3}} = \frac{\sqrt{25}}{\sqrt{3}}$

$= \frac{5}{\sqrt{3}}$ since $\sqrt{3}$ is not an Integer

Therefore, $\sqrt{\frac{25}{3}}$ is an irrational number

$\Rightarrow -\sqrt{\frac{49}{16}} = -\frac{\sqrt{49}}{\sqrt{16}}$

$= -\frac{7}{4}$

Therefore, $-\sqrt{\frac{49}{16}}$ is a rational number

SOLUTION - 6

(i) $-3\sqrt{2}$

Sol: Since $\sqrt{2}$ is an irrational number

Therefore, $-3\sqrt{2}$ will change into non-terminating non-recurring decimal

(ii) $\sqrt{\frac{256}{81}}$

Sol:- $\frac{\sqrt{256}}{\sqrt{81}} = \frac{16}{9}$

$= 1.777777$

Therefore, $\sqrt{\frac{256}{81}}$ will not change into non-terminating non-recurring decimal

(iii) $\sqrt{27 \times 16}$

Sol:- $\sqrt{27} \times \sqrt{16} = \sqrt{9 \times 3} \times 4$
 $= 4 \times 3\sqrt{3}$
 $= 12\sqrt{3}$

$\therefore \sqrt{3}$ is an irrational number

Therefore, $\sqrt{27 \times 16}$ will change into non-terminating non-recurring decimal

(iv) $\sqrt{\frac{5}{36}}$

Sol:- $\frac{\sqrt{5}}{\sqrt{36}} = \frac{\sqrt{5}}{6}$

$\therefore \sqrt{5}$ is an irrational number

Therefore, $\sqrt{\frac{5}{36}}$ will change into non-terminating non-recurring decimal

SOLUTION-7

(i) $3 - \sqrt{\frac{7}{25}}$

Sol: $3 - \frac{\sqrt{7}}{\sqrt{25}}$

$$= 3 - \frac{\sqrt{7}}{5}$$

$$= \frac{15 - \sqrt{7}}{5}$$

$\therefore \sqrt{7}$ is an irrational number

Therefore, $3 - \sqrt{\frac{7}{25}}$ is also an irrational number

(ii) $-\frac{2}{3} + \sqrt[3]{2}$

Sol: Since $\sqrt[3]{2}$ is an irrational number

Therefore, $-\frac{2}{3} + \sqrt[3]{2}$ is also an irrational number

(NOTE: Sum of rational and irrational number is irrational)

(iii) $\frac{3}{\sqrt{3}}$

Sol: $\frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = \sqrt{3}$

Since $\sqrt{3}$ is an irrational number

Therefore, $\frac{3}{\sqrt{3}}$ is also an irrational number.

iv) $-\frac{2}{7} \sqrt[3]{5}$

Sol: Since $\sqrt[3]{5}$ is an irrational number

Therefore, $-\frac{2}{7} \sqrt[3]{5}$ is also an irrational number.

(Note: Product of rational and irrational number is irrational)

v) $(2-\sqrt{3})(2+\sqrt{3})$

Sol: $2 \times 2 + 2\sqrt{3} - 2\sqrt{3} - (\sqrt{3})^2$

$$= 4 - 3$$

$$= 1$$

Therefore, $(2-\sqrt{3})(2+\sqrt{3})$ is a rational number

vi) $(3+\sqrt{5})^2$

Sol: $(3)^2 + (\sqrt{5})^2 + 2 \times 3 \times \sqrt{5}$

$$= 9 + 5 + 6\sqrt{5}$$

$$= 14 + 6\sqrt{5}$$

Since $\sqrt{5}$ is an irrational number

$(3+\sqrt{5})^2$ is also an irrational number

$$(VII), \left(\frac{2}{5}\sqrt{7}\right)^2$$

$$\text{Sol: } \left(\frac{2}{5}\right)^2 \cdot (\sqrt{7})^2$$

$$= \frac{4}{25} \times 7$$

$$= \frac{28}{25}$$

Therefore, $\left(\frac{2}{5}\sqrt{7}\right)^2$ is a rational number

$$(VIII), (3-\sqrt{6})^2$$

$$\text{Sol: } (3)^2 + (\sqrt{6})^2 - 2 \times 3 \times \sqrt{6}$$

$$= 9 + 6 - 6\sqrt{6}$$

$$= 15 - 6\sqrt{6}$$

Since $\sqrt{6}$ is an irrational number

$(3-\sqrt{6})^2$ is also an irrational number

SOLUTION - 8 :
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$$(i), \sqrt[3]{2}$$

Sol: Suppose that  $\sqrt[3]{2} = \frac{p}{q}$ , where  $p, q$  are integers,  $q > 0$ ,  $p$  and  $q$  have no common factors (except 1)

$$2 = \left[\frac{p}{q}\right]^3$$

$$p^3 = 2q^3 \rightarrow \textcircled{1}$$

As 2 divides  $2q^3 \Rightarrow 2$  divides  $p^3$

$\Rightarrow 2$  divides  $p$

Let  $p = 2K$ , where  $K$  is an integer

Substituting this value of ' $p$ ' in (1), we get

$$(2K)^3 = 2q^3$$

$$8K^3 = 2q^3$$

$$4K^3 = q^3$$

As 2 divides  $4K^3 \Rightarrow 2$  divides  $q^3$

$\Rightarrow 2$  divides  $q$

Thus  $p$  and  $q$  have a common factor "2".

This contradicts that  $p$  and  $q$  have no common factor (except 1).

Therefore,  $\sqrt[3]{2}$  is an irrational number.

(ii)  $\sqrt[3]{3}$

Sol: Suppose that  $\sqrt[3]{3} = \frac{p}{q}$ , where  $p, q$  are integers

$q > 0$ ,  $p$  and  $q$  have no common factors

(except 1).

$$3 = \left(\frac{p}{q}\right)^3$$

$$p^3 = 3q^3 \rightarrow (1)$$

As 3 divides  $3q^3 \Rightarrow 3$  divides  $p^3$

$\Rightarrow 3$  divides  $p$

Let  $P = 3K$ , where  $K$  is an Integer

Substituting this value of 'p' in ①, we get

$$(3K)^3 = 3q^3$$

$$27K^3 = 3q^3$$

$$9K^3 = q^3$$

As 3 divides  $9K^3 \Rightarrow 3$  divides  $q^3$

$\Rightarrow 3$  divides  $q$

Thus  $p$  and  $q$  have a common factor "3"  
This contradicts that  $p$  and  $q$  have no common factor (except 1).

Therefore,  $\sqrt[3]{3}$  is an irrational number.

(iii)  $\sqrt[4]{5}$

Sol: Suppose that  $\sqrt[4]{5} = \frac{p}{q}$ , where  $p, q$  are Integers  
 $q > 0$ ,  $p$  and  $q$  have no common factors  
(except 1)

$$5 = \left(\frac{p}{q}\right)^4$$

$$p^4 = 5q^4 \rightarrow \text{①}$$

As 5 divides  $5q^4 \Rightarrow 5$  divides  $p^4$

$\Rightarrow 5$  divides  $p$

Let  $p = 5K$ , where  $K$  is an Integer



Substituting this value of 'p' in ①, we get

$$(5K)^4 = 5q^4$$

$$625K^4 = 5q^4$$

$$125K^4 = q^4$$

As 5 divides  $125K^4 \Rightarrow 5$  divides  $q^4$

$\Rightarrow 5$  divides  $q$

Thus  $p$  and  $q$  have a common factor "5"

This contradicts that  $p$  and  $q$  have no common factors (except 1)

Therefore,  $\sqrt[4]{5}$  is an irrational number.

SOLUTION-9

(i)  $2\sqrt{3}, \frac{3}{\sqrt{2}}, -\sqrt{7}, \sqrt{15}$

Sol:  $\sqrt{4 \times 3} = \sqrt{12}$

$$\frac{3}{\sqrt{2}} = \sqrt{\frac{9}{2}}$$

$$= \sqrt{4.5}$$

$$\therefore \sqrt{12}, \sqrt{4.5}, -\sqrt{7}, \sqrt{15}$$

$$-\sqrt{7} < \sqrt{4.5} < \sqrt{12} < \sqrt{15}$$

The greatest real number is  $\sqrt{15}$

The smallest real number is  $-\sqrt{7}$

(ii)  $-3\sqrt{2}$ ,  $\frac{9}{\sqrt{5}}$ ,  $-4$ ,  $\frac{4}{3}\sqrt{5}$ ,  $\frac{3}{2}\sqrt{3}$

Sol:  $-3\sqrt{2} = -\sqrt{9 \times 2}$

$$= -\sqrt{18}$$

$$\frac{9}{\sqrt{5}} = \sqrt{\frac{81}{5}}$$

$$= \sqrt{16.2}$$

$$-4 = -\sqrt{16}$$

$$\frac{4}{3}\sqrt{5} = \sqrt{\frac{16 \times 5}{9}}$$

$$= \sqrt{\frac{80}{9}}$$

$$= \sqrt{8.89}$$

$$\frac{3}{2}\sqrt{3} = \sqrt{\frac{9 \times 3}{4}}$$

$$= \sqrt{\frac{27}{4}}$$

$$= \sqrt{6.75}$$

$$\therefore -\sqrt{18}, \sqrt{16.2}, \sqrt{8.89}, -\sqrt{16}, \sqrt{6.75}$$

$$-3\sqrt{2} < -4 < \frac{3}{2}\sqrt{3} < \frac{4}{3}\sqrt{5} < \frac{9}{\sqrt{5}}$$

The greatest real number is  $\frac{9}{\sqrt{5}}$

The smallest real number is  $-3\sqrt{2}$

### SOLUTION - 10

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(i) $3\sqrt{2}$, $2\sqrt{3}$, $\sqrt{15}$, 4

Sol: Write all the numbers as square roots under one radical

$$3\sqrt{2} = \sqrt{9} \times \sqrt{2} = \sqrt{18}$$

$$2\sqrt{3} = \sqrt{4} \times \sqrt{3} = \sqrt{12}$$

$$\sqrt{15} = \sqrt{15}$$

$$4 = \sqrt{16}$$

Since $12 < 15 < 16 < 18$

$$\Rightarrow \sqrt{12} < \sqrt{15} < \sqrt{16} < \sqrt{18}$$

$$\Rightarrow 2\sqrt{3} < \sqrt{15} < 4 < 3\sqrt{2}$$

Hence, the given numbers in ascending orders are

$$2\sqrt{3}, \sqrt{15}, 4, 3\sqrt{2}$$

(ii) $3\sqrt{2}$, $2\sqrt{8}$, 4, $\sqrt{50}$, $4\sqrt{3}$

Sol: Write all the numbers as square roots under one radical

$$3\sqrt{2} = \sqrt{9} \times \sqrt{2} = \sqrt{18}$$

$$2\sqrt{8} = \sqrt{4} \times \sqrt{8} = \sqrt{32}$$

$$4 = \sqrt{16}$$

$$\sqrt{50} = \sqrt{50}$$

$$4\sqrt{3} = \sqrt{16} \times \sqrt{3} = \sqrt{48}$$

Since $16 < 18 < 32 < 48 < 50$

$$\Rightarrow \sqrt{16} < \sqrt{18} < \sqrt{32} < \sqrt{48} < \sqrt{50}$$

$$\Rightarrow 4 < 3\sqrt{2} < 2\sqrt{8} < 4\sqrt{3} < \sqrt{50}$$

Hence, the given numbers in ascending orders are

$$4, 3\sqrt{2}, 2\sqrt{8}, 4\sqrt{3}, \sqrt{50}$$

SOLUTION - 11

(i) $\frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 4\sqrt{3}, 3\sqrt{\frac{6}{5}}$

Sol: Write all the numbers as square roots
under one radical

$$\frac{9}{\sqrt{2}} = \sqrt{\frac{81}{2}} = \sqrt{40.5}$$

$$\frac{3}{2}\sqrt{5} = \sqrt{\frac{9}{4} \times 5} = \sqrt{\frac{45}{4}} = \sqrt{11.25}$$

$$4\sqrt{3} = \sqrt{16 \times 3} = \sqrt{48}$$

$$3\sqrt{\frac{6}{5}} = \sqrt{9 \times \frac{6}{5}} = \sqrt{\frac{54}{5}} = \sqrt{10.8}$$

Since $48 > 40.5 > 11.25 > 10.8$

$$\Rightarrow \sqrt{48} > \sqrt{40.5} > \sqrt{11.25} > \sqrt{10.8}$$

$$\Rightarrow 4\sqrt{3} > \frac{9}{\sqrt{2}} > \frac{3}{2}\sqrt{5} > 3\sqrt{\frac{6}{5}}$$

Hence, the given numbers in descending orders

$$\text{are } 4\sqrt{3}, \frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 3\sqrt{\frac{6}{5}}$$

lin $\frac{5}{\sqrt{3}}$, $\frac{7}{3}\sqrt{2}$, $-\sqrt{3}$, $3\sqrt{5}$, $2\sqrt{7}$

Sol: Write all the numbers as square roots under one radical

$$\frac{5}{\sqrt{3}} = \sqrt{\frac{25}{3}} = \sqrt{8.33}$$

$$\frac{7}{3}\sqrt{2} = \sqrt{\frac{49}{9}} \times \sqrt{2} = \sqrt{\frac{98}{9}} = \sqrt{10.89}$$

$$-\sqrt{3} = -\sqrt{3}$$

$$3\sqrt{5} = \sqrt{9} \times \sqrt{5} = \sqrt{45}$$

$$2\sqrt{7} = \sqrt{4} \times \sqrt{7} = \sqrt{28}$$

Since $45 > 28 > 10.89 > 8.33 > -3$

$$\Rightarrow \sqrt{45} > \sqrt{28} > \sqrt{10.89} > \sqrt{8.33} > -\sqrt{3}$$

$$\Rightarrow 3\sqrt{5} > 2\sqrt{7} > \frac{7}{3}\sqrt{2} > \frac{5}{\sqrt{3}} > -\sqrt{3}$$

Hence, the given numbers in descending orders are $3\sqrt{5}$, $2\sqrt{7}$, $\frac{7}{3}\sqrt{2}$, $\frac{5}{\sqrt{3}}$, $-\sqrt{3}$

SOLUTION - 12

(i) $\sqrt[3]{2}$, $\sqrt{3}$, $\sqrt[6]{5}$

Sol: L.C.M. of 2, 3 and 6 is 6

$$\sqrt[3]{2} = 2^{\frac{1}{3}} = (2^{\frac{2}{2}})^{\frac{1}{6}} = (4)^{\frac{1}{6}}$$

$$\sqrt{3} = 3^{\frac{1}{2}} = (3^3)^{\frac{1}{6}} = (27)^{\frac{1}{6}}$$

$$\sqrt[6]{5} = 5^{\frac{1}{6}} = (5^1)^{\frac{1}{6}} = (5)^{\frac{1}{6}}$$

Since $4 < 5 < 27$

$$\Rightarrow (4)^{\frac{1}{6}} < (5)^{\frac{1}{6}} < (27)^{\frac{1}{6}}$$

$$\Rightarrow \sqrt[3]{2} < \sqrt[6]{5} < \sqrt{3}$$

Hence, the given numbers in ascending order are $\sqrt[3]{2}$, $\sqrt[6]{5}$, $\sqrt{3}$

EXERCISE - 1.5

SOLUTION - 1

(i) $\frac{3}{4\sqrt{5}}$

Sol: $\frac{3}{4\sqrt{5}} \times \frac{4\sqrt{5}}{4\sqrt{5}} = \frac{12\sqrt{5}}{16 \cdot (\sqrt{5})^2}$

$$= \frac{12\sqrt{5}}{16 \times 5}$$
$$= \frac{3\sqrt{5}}{20}$$

(ii) $\frac{5\sqrt{7}}{\sqrt{3}}$

Sol: $\frac{5\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{7 \times 3}}{(\sqrt{3})^2}$

$$= \frac{5\sqrt{21}}{3}$$

(iii) $\frac{3}{4-\sqrt{7}}$

Sol: $\frac{3}{4-\sqrt{7}} \times \frac{4+\sqrt{7}}{4+\sqrt{7}} = \frac{3(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})}$

$$= \frac{12 + 3\sqrt{7}}{16 - (\sqrt{7})^2}$$
$$= \frac{12 + 3\sqrt{7}}{16 - 7}$$
$$= \frac{12 + 3\sqrt{7}}{9}$$

$$= \frac{13(4 + 3\sqrt{7})}{9}$$

$$= \frac{4 + \sqrt{7}}{3}$$

(iv) $\frac{17}{3\sqrt{2} + 1}$

Sol: $\frac{17}{3\sqrt{2} + 1} \times \frac{3\sqrt{2} - 1}{3\sqrt{2} - 1} = \frac{17(3\sqrt{2} - 1)}{(3\sqrt{2} + 1)(3\sqrt{2} - 1)}$

$$= \frac{51\sqrt{2} - 17}{(3\sqrt{2})^2 - (1)^2}$$

$$= \frac{17(3\sqrt{2} - 1)}{9 \times 2 - 1}$$

$$= \frac{17(3\sqrt{2} - 1)}{18 - 1}$$

$$= \frac{17(3\sqrt{2} - 1)}{17}$$

$$= 3\sqrt{2} - 1$$

(v) $\frac{16}{\sqrt{41} - 5}$

Sol: $\frac{16}{\sqrt{41} - 5} \times \frac{\sqrt{41} + 5}{\sqrt{41} + 5} = \frac{16(\sqrt{41} + 5)}{(\sqrt{41})^2 - (5)^2}$

$$= \frac{16(\sqrt{41} + 5)}{41 - 25}$$

$$= \frac{16(\sqrt{41} + 5)}{16}$$

$$= \sqrt{41} + 5$$

$$(VI) \frac{1}{\sqrt{7}-\sqrt{6}}$$

$$\begin{aligned} \text{Sol: } \frac{1}{\sqrt{7}-\sqrt{6}} &\times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} \\ &= \frac{\sqrt{7}+\sqrt{6}}{7-6} \\ &= \sqrt{7}+\sqrt{6} \end{aligned}$$

$$(VII) \frac{1}{\sqrt{5}+\sqrt{2}}$$

$$\begin{aligned} \text{Sol: } \frac{1}{\sqrt{5}+\sqrt{2}} &\times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} \\ &= \frac{\sqrt{5}-\sqrt{2}}{5-2} \\ &= \frac{\sqrt{5}-\sqrt{2}}{3} \end{aligned}$$

$$(VIII) \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$\begin{aligned} \text{Sol: } \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} &\times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2-(\sqrt{3})^2} \\ &= \frac{(\sqrt{2})^2+(\sqrt{3})^2+2\cdot\sqrt{2}\cdot\sqrt{3}}{2-3} \\ &= \frac{2+3+2\sqrt{6}}{-1} \\ &= -(5+2\sqrt{6}) \\ &= -5-2\sqrt{6} \end{aligned}$$

SOLUTION-2

(i) $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$

Sol:- $\frac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}} = \frac{(7+3\sqrt{5})^2}{(7)^2 - (3\sqrt{5})^2}$

$$= \frac{(7)^2 + (3\sqrt{5})^2 + 2 \times 7 \times 3\sqrt{5}}{49 - 9 \times 5}$$
$$= \frac{49 + 45 + 42 \times 3\sqrt{5}}{49 - 45}$$
$$= \frac{94 + 42\sqrt{5}}{4}$$
$$= \frac{2(47 + 21\sqrt{5})}{4}$$
$$= \frac{47 + 21\sqrt{5}}{2}$$

(ii) $\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

Sol:- $\frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{(3-2\sqrt{2})^2}{(3)^2 - (2\sqrt{2})^2}$

$$= \frac{(3)^2 + (2\sqrt{2})^2 - 2 \times 3 \times 2\sqrt{2}}{9 - 4 \times 2}$$
$$= \frac{9 + 8 - 12\sqrt{2}}{9 - 8}$$
$$= \frac{17 - 12\sqrt{2}}{1}$$
$$= 17 - 12\sqrt{2}$$

$$(iii) \frac{5-3\sqrt{14}}{7+2\sqrt{14}}$$

$$\text{Sol: } \frac{5-3\sqrt{14}}{7+2\sqrt{14}} \times \frac{7-2\sqrt{14}}{7-2\sqrt{14}}$$

$$= \frac{5 \times 7 - 5 \times 2\sqrt{14} - 7 \times 3\sqrt{14} + 2 \times 3 \times \sqrt{14} \times \sqrt{14}}{(7)^2 - (2\sqrt{14})^2}$$

$$= \frac{35 - 10\sqrt{14} - 21\sqrt{14} + 6 \times 14}{49 - 4 \times 14}$$

$$= \frac{35 - 31\sqrt{14} + 84}{49 - 56}$$

$$= \frac{119 - 31\sqrt{14}}{-7}$$

$$= \frac{31\sqrt{14} - 119}{7}$$

SOLUTION - 3

$$(i) \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\text{Sol: } \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}$$

$$= \frac{7\sqrt{30} - 7 \times 3}{(\sqrt{10})^2 - (\sqrt{3})^2}$$

$$= \frac{7\sqrt{30} - 21}{10 - 3}$$

$$= \frac{7(\sqrt{30} - 3)}{7} = \sqrt{30} - 3$$

$$\begin{aligned}
 \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} &= \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} \\
 &= \frac{2\sqrt{30} - 2 \times 5}{(\sqrt{6})^2 - (\sqrt{5})^2} \\
 &= \frac{2\sqrt{30} - 10}{6 - 5} \\
 &= 2\sqrt{30} - 10
 \end{aligned}$$

$$\begin{aligned}
 \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} &= \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} \\
 &= \frac{3\sqrt{30} - 9 \times 2}{(\sqrt{15})^2 - (3\sqrt{2})^2} \\
 &= \frac{3\sqrt{30} - 18}{15 - 18} \\
 &= \frac{3(\sqrt{30} - 6)}{-2} \\
 &= -\sqrt{30} + 6
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \\
 &= \sqrt{30} - 3 - (2\sqrt{30} - 10) - (-\sqrt{30} + 6) \\
 &= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6 \\
 &= \cancel{\sqrt{30}} - 3 - \cancel{2\sqrt{30}} + 10 - 6 + \cancel{\sqrt{30}} \\
 &= 1
 \end{aligned}$$

SOLUTION -4

$$(i) \frac{3-\sqrt{5}}{3+2\sqrt{5}} = -\frac{19}{11} + a\sqrt{5}$$

$$\begin{aligned}\text{Sol: } \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}} &= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3)^2 - (2\sqrt{5})^2} \\ &= \frac{9 - 6\sqrt{5} - 3\sqrt{5} + 2(5)}{9 - 4(5)} \\ &= \frac{19 - 9\sqrt{5}}{9 - 20} \\ &= \frac{19 - 9\sqrt{5}}{-11} \\ &= -\frac{19}{11} + \frac{9}{11}\sqrt{5}\end{aligned}$$

$$\therefore -\frac{19}{11} + a\sqrt{5} = -\frac{19}{11} + \frac{9}{11}\sqrt{5}$$

$$\Rightarrow a = \frac{9}{11}$$

$$(ii) \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$$

$$\begin{aligned}\text{Sol: } \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} &= \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2 - (2\sqrt{3})^2} \\ &= \frac{3(2) + 2\sqrt{6} + 3\sqrt{6} + 2(3)}{9(2) - 4(3)} \\ &= \frac{6 + 5\sqrt{6} + 6}{18 - 12}\end{aligned}$$

$$= \frac{12 + 5\sqrt{6}}{6}$$

$$= 2 + \frac{5}{6}\sqrt{6}$$

$$= 2 - \left(-\frac{5}{6}\right)\sqrt{6}$$

$$\therefore a - b\sqrt{6} = 2 - \left(-\frac{5}{6}\right)\sqrt{6}$$

$$\Rightarrow a = 2 \quad ; \quad b = -\frac{5}{6}$$

$$(iii) \quad \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}b\sqrt{5}$$

$$\begin{aligned} \text{Sol: } \frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} &= \frac{(7+\sqrt{5})^2}{(7)^2 - (\sqrt{5})^2} \\ &= \frac{49 + (\sqrt{5})^2 + 2 \cdot 7 \cdot \sqrt{5}}{49 - 5} \\ &= \frac{49 + 5 + 14\sqrt{5}}{44} \end{aligned}$$

$$\begin{aligned} \frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}} &= \frac{(7-\sqrt{5})^2}{(7)^2 - (\sqrt{5})^2} \\ &= \frac{49 + (\sqrt{5})^2 - 2 \times 7 \times \sqrt{5}}{49 - 5} \\ &= \frac{49 + 5 - 14\sqrt{5}}{44} \end{aligned}$$

$$\begin{aligned} \therefore \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} &= \frac{54 + 14\sqrt{5}}{44} - \frac{54 - 14\sqrt{5}}{44} \\ &= \frac{54 + 14\sqrt{5} - 54 + 14\sqrt{5}}{44} \end{aligned}$$

$$= \frac{7 \cdot 28\sqrt{5}}{11 \cdot 44}$$

$$= \frac{7}{11} \times \sqrt{5}$$

$$\therefore a + \frac{7}{11}b\sqrt{5} = \frac{7}{11}\sqrt{5}$$

$$\Rightarrow a=0 ; b=1$$

SOLUTION -5:
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$$(i) \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = p+q\sqrt{5}$$

$$\text{Sol: } \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3)^2 - (\sqrt{5})^2}$$

$$= \frac{21 - 7\sqrt{5} + 9\sqrt{5} - 3(5)}{9-5}$$

$$= \frac{21 + 2\sqrt{5} - 15}{4}$$

$$= \frac{6+2\sqrt{5}}{4}$$

$$\frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(7-3\sqrt{5})(3+\sqrt{5})}{(3)^2 - (\sqrt{5})^2}$$

$$= \frac{21 + 7\sqrt{5} - 9\sqrt{5} - 3(5)}{9-5}$$

$$= \frac{21 - 2\sqrt{5} - 15}{4}$$

$$= \frac{6-2\sqrt{5}}{4}$$

$$\begin{aligned}
 \therefore \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} &= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4} \\
 &= \frac{6+2\sqrt{5} - 6 + 2\sqrt{5}}{4} \\
 &= \frac{4\sqrt{5}}{4} \\
 &= \sqrt{5}
 \end{aligned}$$

$$\therefore p+q\sqrt{5} = \sqrt{5}$$

$$\Rightarrow p=0 ; q=1$$

SOLUTION -6 :

$$(i) \frac{\sqrt{2}}{2+\sqrt{2}}$$

$$\begin{aligned}
 \text{Sol: } \frac{\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} &= \frac{\sqrt{2}(2-\sqrt{2})}{(2)^2 - (\sqrt{2})^2} \\
 &= \frac{2\sqrt{2} - 2}{4 - 2} \\
 &= \frac{2(\sqrt{2}-1)}{2} \\
 &= \sqrt{2}-1 \\
 &= 1.414-1 \\
 &= 0.414
 \end{aligned}$$

$$(ii) \frac{1}{\sqrt{3}+\sqrt{2}}$$

$$\begin{aligned} \text{Sol: } \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} &= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{\sqrt{3}-\sqrt{2}}{3-2} \\ &= \sqrt{3}-\sqrt{2} \\ &= 1.732 - 1.414 \\ &= 0.318 \end{aligned}$$

SOLUTION - 7 :

$$(i) a = 2+\sqrt{3}$$

$$\begin{aligned} \text{Sol: } \frac{1}{a} &= \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \\ &= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2} \\ &= \frac{2-\sqrt{3}}{4-3} \\ &= 2-\sqrt{3} \end{aligned}$$

$$\begin{aligned} \therefore a - \frac{1}{a} &= 2+\sqrt{3} - (2-\sqrt{3}) \\ &= 2+\sqrt{3} - 2 + \sqrt{3} \\ &= 2\sqrt{3} \end{aligned}$$

### SOLUTION -8

(i)  $x = 1 - \sqrt{2}$

Sol: Given  $x = 1 - \sqrt{2}$

$$\therefore \frac{1}{x} = \frac{1}{1 - \sqrt{2}} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$$

$$= \frac{1 + \sqrt{2}}{(1)^2 - (\sqrt{2})^2}$$

$$= \frac{1 + \sqrt{2}}{1 - 2}$$

$$= -(1 + \sqrt{2})$$

$$\therefore \left(x - \frac{1}{x}\right)^4 = (1 - \sqrt{2} - (-1 - \sqrt{2}))^4$$

$$= (1 - \sqrt{2} + 1 + \sqrt{2})^4$$

$$= 2^4$$

$$= 16$$

### SOLUTION -9

(i)  $x = 5 - 2\sqrt{6}$

Sol: Given  $x = 5 - 2\sqrt{6}$

$$\therefore \frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$$

$$= \frac{5 + 2\sqrt{6}}{(5)^2 - (2\sqrt{6})^2}$$

$$= \frac{5 + 2\sqrt{6}}{25 - 24}$$

$$= 5 + 2\sqrt{6}$$

$$\therefore x + \frac{1}{x} = (5 - 2\sqrt{6}) + (5 + 2\sqrt{6})$$

$$= 10$$

We know that  $(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$

$$\Rightarrow x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$$

$$= (10)^2 - 2$$

$$= 100 - 2$$

$$= 98$$

SOLUTION - 10

$$(i) p = \frac{2-\sqrt{5}}{2+\sqrt{5}} ; q = \frac{2+\sqrt{5}}{2-\sqrt{5}}$$

Sol:  $p+q = \frac{2-\sqrt{5}}{2+\sqrt{5}} + \frac{2+\sqrt{5}}{2-\sqrt{5}}$

$$= \frac{(2-\sqrt{5})^2 + (2+\sqrt{5})^2}{(2)^2 - (\sqrt{5})^2}$$

$$= \frac{(4+5-4\sqrt{5}) + (4+5+4\sqrt{5})}{4-5}$$

$$= \frac{18}{-1}$$

$$\therefore p+q = -18$$



$$\begin{aligned}
 \text{(ii)} \quad p - q &= \frac{2-\sqrt{5}}{2+\sqrt{5}} - \frac{2+\sqrt{5}}{2-\sqrt{5}} \\
 &= \frac{(2-\sqrt{5})^2 - (2+\sqrt{5})^2}{(2)^2 - (\sqrt{5})^2} \\
 &= \frac{(4+5-4\sqrt{5}) - (4+5+4\sqrt{5})}{4-5} \\
 &= \frac{9-4\sqrt{5} - 9-4\sqrt{5}}{-1} \\
 &= \frac{-8\sqrt{5}}{-1} \\
 &= 8\sqrt{5}
 \end{aligned}$$

$$\text{(iii)} \quad p^2 + q^2$$

Sol: We know that

$$(p+q)^2 = p^2 + q^2 + 2pq$$

$$\therefore pq = \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2+\sqrt{5}}{2-\sqrt{5}} = 1$$

$$\therefore p+q = -18$$

$$\begin{aligned}
 \Rightarrow p^2 + q^2 &= (p+q)^2 - 2pq \\
 &= (-18)^2 - 2 \times 1 \\
 &= 324 - 2 \\
 &= 322
 \end{aligned}$$

(iv)  $p^2 - q^2$

Sol:-  $\therefore p^2 - q^2 = (p+q)(p-q)$   
 $= (-18)(8\sqrt{5})$   
 $= -144\sqrt{5}$

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